Alfvénic turbulence in tokamaks: 
from micro- to meso-scale fluctuations*

Fulvio Zonca\textsuperscript{1,2} in collaboration with Liu Chen\textsuperscript{2,3}

http://www.afs.enea.it/zonca

\textsuperscript{1}Associazione Euratom-ENEA sulla Fusione, C.R. Frascati, C.P. 65 - 00044 - Frascati, Italy.
\textsuperscript{2}Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou 310027, P.R.C.
\textsuperscript{3}Department of Physics and Astronomy, University of California, Irvine, CA 92697-2575

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Motivation

- The challenge of understanding fast particle collective behaviors in burning plasmas of fusion interest, is to develop a predictive capability for describing energetic particle confinement and its link to the dynamic evolution of thermal plasma profiles.

- Theory and simulation must play fundamental roles:
  
  - fusion plasmas are complex systems in which long time scale behaviors will be determined by cross-scale couplings of phenomena occurring on micro- and meso-spatiotemporal scales
  
  - existing experiments can look at these issues separately, since cross-scale couplings are not those of burning plasmas
  
  - mutual positive feedbacks are necessary between theory, simulations and experiments for V&V of present predictive capabilities
From micro- to meso-scales

- Drift wave plasma turbulence is well known and turbulent plasma transport is widely studied (ITG/TEM/ETG).

- Alfvénic turbulence has been addressed mostly for plasma edge conditions [Scott PPCF97], but much less studied in the tokamak core [Chen et al NF78, Tang et al NF80].

- The investigation of Alfvénic ITG activity [Zonca and Chen PPCF96] is more recent. It is of relevance to burning plasmas of fusion interest since it can be excited at acoustic frequencies over a broad range of scale-lengths, from thermal ion Larmor radius to the typical fast ion orbit width [Zonca and Chen POP99], with a smooth transition to MHD modes [Zonca and Chen PPCF06, NF07, NF09].

- There is a wide observation database of these phenomena, accumulated in the recent years after the first observations in DIII-D [Nazikian et al PRL06]
A “Sea of Core Localized Alfvén Eigenmodes” Observed in DIII-D Quiescent Double Barrier (QDB) plasmas

FIR scattering $l=3$, $l=2$, $l=1$

Simulation

- Bands of modes $m=n+l$, $l=1$, 2, ...
- Neutral beam injection opposite to plasma current: $V_{||} \approx 0.3 V_A$

R. Nazikian, et al. 06, PRL 96, 105006

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Fast particle transport by plasma turbulence

Significant interest on this topic was triggered by recent AUG [Günter et al NF07], JT-60U [Suzuki et al NF08] and DIII-D [Heidbrink et al PRL09] experimental results with NBI, showing evidence of anomalies in fast ion transport (clarified now: [Zhang et al POP10])

From Zhang et al POP 17 055902 (2010)
Diffusivity behaviors are consistent with theoretical predictions based on quasi-linear theory [Chen JGR99].

Intrinsic interest is mostly connected with explanation of present day experiments, with low characteristic values of $E/T$; e.g., evidence of ITG induced transport of NBI supra-thermal ions in DIII-D [Heidbrink et al PPCF09].

Results show that fast ion transport by micro/turbulence above the critical energy is negligible. Effects are expected on He ashes or medium energy supra/thermal tails: possible good news?

Fairly complete reconstruction of original theoretical works, e.g. [White and Mynick PFB89], theoretical issues, and experimental evidence in recent paper [Zhang et al POP10]. See also Heidbrink et al 2010 EPS invited talk [PPCF10].
Fast ion transports in burning plasmas

- Alfvén Eigenmodes (AE) modes are predicted to have small saturation levels and yield negligible transport unless stochastization threshold in phase space is reached [Berk and Breizman, PFB90; Sigmar et al PFB92].

- Strong energetic particle redistributions are predicted to occur above the Energetic Particle Modes (EPM) excitation threshold in 3D Hybrid MHD-Gyrokinetic simulations [Briguglio et al POP98].

- Nonlinear Dynamics of Burning Plasmas: energetic ion transport in burning plasmas has two components:
  - slow diffusive processes due to weakly unstable AEs and a residual component possibly due to plasma turbulence [Vlad et al PPCF05, Estrada-Mila et al POP06].
  - rapid transport processes with ballistic nature due to coherent non-linear interactions with EPM and/or low-frequency long-wavelength MHD: fast ion avalanches & experimental observation of Abrupt Large amplitude Events (ALE) on JT60-U [Shinohara et al PPCF04].
Phase space structures: fast ion resonant interactions with AE


- Transient losses $\approx \delta B_r/B$: resonant drift motion across the orbit-loss boundaries in phase space
- Diffusive losses $\approx (\delta B_r/B)^2$ above a stochastic threshold, due to stochastic diffusion in phase space across orbit-loss boundary
- Uncertainty in the stochastic threshold: $(\delta B_r/B) \lesssim 10^{-4}$ in the multiple mode case. Possibly reached via phase space explosion: “domino effect” [Berk et al POP96]

Lichtenberg & Lieberman
1983, Sp.-Ver. NY
ALE on JT-60U [Shinohara et al. NF01]

ALE = Abrupt Large amplitude Event

Courtesy of K. Shinohara and JT-60U
Fast ion transport: simulation and experiment

- Numerical simulations show fast ion radial redistributions, qualitatively similar to those by ALE on JT-60U.

Avalanches and NL EPM dynamics

Importance of toroidal geometry on wave-packet propagation and shape
Propagation of the unstable front

Gradient steepening and relaxation: spreading ... similar to turbulence
The form of nonlinear interactions in a torus

Channels for nonlinear interactions can be described via the three degrees of freedom of mode structures: the toroidal mode number $n$, the parallel mode structure reflecting the radial width of a single poloidal harmonic $m$, and radial mode envelope $A(r) = \exp i \int \theta_k d(nq)$.

Correspondingly, nonlinear interactions could occur via the following three channels: mode coupling between two $n$’s, distortion of the parallel mode structure, and modulation of the radial envelope.

- radial envelope modulations: ZF, GAM, zonal structures
- mode coupling: spectral transfer (cascade) via 3-wave interactions

Radial envelope modulation via generation of zonal flows dominates in ITG turbulence. Zonal structures (profile relaxation etc.) and/or GAM/ZF influence AE/EPM dynamics depending on the strength of mode drive.

ETG turbulence is regulated by nonlinear toroidal mode couplings. What effect on Alfvénic turbulence?
Three-wave interactions and spectral transfer

- This is still an open problem, largely unexplored:
  - **Alfvénic fluctuations** in toroidal plasmas of fusion interest cover all scales, from thermal ion Larmor radius up to characteristic energetic ion orbit widths (meso-scale AE/EPM) and macro-scales (MHD modes).
  - All scales involved: no unique approach to investigate spectral transfer processes (local vs non-local in $k$, anisotropy, density of states).
  - In realistic conditions, these dynamics are influenced by non-uniformities and toroidal geometries: wave-particle power exchange $\langle \mathbf{v} \cdot \delta \mathbf{E} \rangle = \mathbf{v}_d \cdot \langle \delta \mathbf{E} \rangle$; gyro-averaged forces $\langle \mathbf{v} \times \delta \mathbf{B} \rangle = -i\mathbf{v}_d \cdot \mathbf{k} \langle \delta \mathbf{A}_\parallel \rangle$

- Geometry and non-uniformity influence three-wave interactions (spectral transfer) via density of quasi-modes and scattering cross-section [Chen et al PPCF05] $\Rightarrow$ Filament-like structures ($|k_\parallel|qR \ll 1 \approx O(n^{-1/2})$) are naturally generated in toroidal systems.
Spectral transfer must be evaluated in competition with generation of ZF, GAM, zonal structures (e.g., phase-space holes and clumps if adiabatic processes dominate, which is not the general/most interesting case).

Most relevant open issue: determine hierarchy of relevant non-linear time scales in realistic conditions, where cross-scale couplings are reproducing those expected to occur in plasmas of fusion interest.
Zonal Flows and Zonal Structures

- Very disparate space-time scales of AE/EPM, MHD modes and plasma turbulence: complex self-organized behaviors of burning plasmas will be likely dominated by their nonlinear interplay via zonal flows and fields [Chen and Zonca ASICTP08]:
  - Effects of different ZF and ZS ($\omega, k_r$) spectra in cross/scale couplings
  - Peculiar roles of Alfvénic fluctuations in the acoustic frequency range: AITG, AE/EPM and MHD with similar frequencies

- Crucial role of toroidal geometry for Alfvénic fluctuations: fundamental importance of magnetic curvature couplings in both linear and nonlinear dynamics [Scott NJP05; Naulin et al POP05]

- Long time scale behaviors of zonal structures are important for the overall burning plasma performance: generators of NL (time varying) “equilibria”

- The corresponding stability determines the dynamics underlying the dissipation of zonal structures in collision-less plasmas and the nonlinear up-shift of thresholds for turbulent transport [Chen et al NF07].
Long time scale behaviors

- Depending on proximity to marginal stability, AE and EPM nonlinear evolutions can be predominantly affected by
  - spontaneous generation of zonal flows and fields [Chen et al NF01, Guzdar et al PRL01]
  - radial modulations in the fast ion profiles: for sufficiently strong fast ion drive, ZF effects are negligible $[(\beta_H/\beta_e)(R/L_{pH}) \gg \epsilon^{3/2}(T_H/q^2T_e)]$; Zonca et al, Varenna00

- AITG and strongly driven MHD modes behave similarly
Structures of the low-frequency SAW spectrum

- Low-frequency Shear Alfvén Wave (SAW) gap: \( \omega \sim \omega_{*i} \sim \omega_{ti}; \Lambda^2(\omega) = k_{||}^2 v_A^2 \)

  \( \Rightarrow \) (ideal MHD) accumulation point (at \( \omega = 0 \)) shifted by thermal ion kinetic effects [Zonca et al PPCF96]

  \( \Rightarrow \) new low-freq. gap! Kinetic Thermal Ion (KTI) gap [Chen NF07]
  - Diamagnetic drift: KBM [Biglari et al PRL91]
  - Thermal ion compressibility: BAE [Heidbrink et al PRL93]
  - \( \nabla T_i \) and wave-part. resonances: AITG [Zonca POP99]

  \( \Rightarrow \) unstable SAW accumulation point

  \( \Rightarrow \) “localization” \( \Rightarrow \) unstable discrete AITG mode

  \( \Rightarrow \) Excitation of BAE/EPM/AITG at all scales, form micro (thermal ions) to meso (fast ions)

- For physics analogy: BAE – GAM degeneracy [Zonca et al PPCF06; Chen et al NF07]. \( \Rightarrow \) Implications on cross-scale couplings mediated by zonal structures [Chen and Zonca ASICTP08; Varenna08].
Nonlinear Dynamics: local vs. global processes

- Alfvénic fluctuation spectrum in burning plasma is dense and consists of mode with characteristic frequencies and radial locations [Chen NF07]. However, nonlinear dynamics has been investigated so far mostly for one single mode.

- Mode saturation via wave-particle trapping [Berk et al PFB90, PLA97] has been successfully applied to explain pitchfork splitting of TAE spectral lines [Fasoli et al PRL98]: local distortion of the fast ion distribution function because of quasi-linear wave-particle interactions.

- Negligible transport by AE is expected because of these processes, unless stochastization threshold is reached in phase space possibly via “domino effect” [Berk et al POP96].
Such analyses generally assume proximity to marginal stability:

- One single low amplitude wave, such that linear mode structures can be assumed and drop out of the problem: radial uniform problem
- Finite background dissipation does not depend on the finite amplitude wave (no continuum damping)
- Wave dispersiveness is set by the background plasma (no beam mode/EPM)
- Frequency sweeping is absent or adiabatic (no mode-particle-pumping [White et al PF83])

These properties delineate an analogy between single Alfvén Eigenmode behaviors in burning plasmas with those of Langmuir waves (1D beam-plasma problem). However they show a strong contrast between dynamics of beam-mode in 1D systems and Energetic Particle Modes in toroidal devices [Zonca IFTS Lecture Notes Spring 2010].

Next: focus on the transition from weak to strong NL dynamic regime, where intrinsic EPM resonant nature plays a crucial role, with plasma non-uniformity and toroidal geometry. Control parameter is power density input: proximity to marginal stability does not hold.
Mode-particle pumping (fishbone)

  
  MHD ($\delta \phi, \delta A_\parallel$) with $\delta \phi = \delta \phi_0(r) \sin(n \varphi - m \theta - \omega t + \psi)$

$$
\begin{align*}
  r &\approx r_0 + \frac{v_{d0}}{\omega_b} \theta_b \cos(\omega_b t) + \langle \Delta r \rangle \\
  \langle \Delta r \rangle &\approx \frac{c m}{B r_0} \left[ \frac{\ell \omega_b + (m/q) \bar{\omega}_d}{\ell \omega_b + n \bar{\omega}_d} \right] \delta \phi_0 J_\ell(m \theta_b) \cos \psi \\
  \theta &\approx -\theta_b \sin(\omega_b t) + \langle \Delta \theta \rangle \\
  \psi &\approx \frac{\langle \Delta r \rangle}{r_0}(s - 1)n \bar{\omega}_d \\
  \omega &\approx n \bar{\omega}_d + \ell \omega_b \\
  n \bar{\omega}_d &\approx \omega_B^2 
\end{align*}
$$
Radial mode structures in toroidal geometry

- The notion of radial envelope: both linear and nonlinear interactions are affected by peculiar mode structures in toroidal geometry and by equilibrium non-uniformities
Transport, toroidal geometry, non-uniformities

- Energetic particle transport in toroidal systems has unique features that cannot be reproduced in simpler 1D geometries, except for special cases.

- Conservation of Hamiltonian in extended phase-space $\Rightarrow (\Delta r/L_p) \sim (\omega_{*p}/\omega)(\Delta E/E)$; drift-wave turbulence tends to maximize $\omega_{*}$ consistently with finite orbit averaging; for fast ions $|\omega_{*p}/\omega| \gg 1$

- Wave particle de-correlations are set by radial non-uniformities: magnetic shear and toroidal mode number for the circulating particles; magnetic shear (not only) but no dependence on the mode number explicitly for trapped particles precession resonance (origin of different behavior of transports in ITG vs TEM turbulences).
Either none or very limited/slow frequency chirping is allowed for Alfvénic modes (1D bump-on-tail paradigm): adiabatic hole-clump dynamics in phase-space near marginal stability [Berk et al PLA97, POP99; Lesur et al POP09] (strong drive case considered by [Vann et al PRL07])

Unique feature of EPM/fishbones with large/fast frequency chirping: resonant frequency locking removes wave particle decorrelation by radial non-uniformities $\Rightarrow$ resonance de-tuning in velocity space and fast convective transports as consequence of coherent wave and particle nonlinear dynamics (nonlinear dispersiveness compensates changing frequency)
Nonlinear initial value problem for EPM

Use the theoretical framework of the general fishbone-like dispersion relation: unified description for all Alfvénic modes in toroidal geometry [Chen and Zonca PPCF06, NF07]; focus on precession resonance for trapped particles with one monochromatic EPM with \( \omega(t) : |\dot{\omega}| \ll |\gamma(t)\omega(t)| \) [Zonca et al NF05; Zonca IFTS Lecture Notes Spring 2010]

\[
i \Lambda (\omega(t) + i\partial_t) A(r, t) = (\delta W_{f,MHD} + \delta W_{f,EP} + \delta W_{k,EPT}) A(r, t)
\]

\[
\delta W_{k,EPT} = \mathcal{I} \circ \int_{-\infty}^{\infty} \frac{k_\theta \tilde{\omega}_{dk} \hat{F}_0(\omega)/\partial r}{\omega_c \tilde{\omega}_{dk} - \omega(t) - \omega} e^{-i\omega t} d\omega
\]

\[
\delta W_{f,EP} = -\mathcal{I} \circ \frac{k_\theta}{\omega_c} \frac{\partial F_0}{\partial r} \tilde{J}_L^2
\]

\[
\mathcal{I} \equiv \frac{2\pi^2 e^2}{M c^2} \frac{q}{|s|} R_0 B_0 \sum_{v_\parallel/|v_\parallel| = \pm 1} \int_0^\infty \mathcal{E} d\mathcal{E} \int d\lambda \frac{\tilde{\omega}_{dk}}{k^2} \tau_b \tilde{J}^2
\]

\[
\tilde{J}^2 \equiv \int_{-\infty}^{\infty} J_0^2(\lambda_{Lk}) J_0^2(\lambda_{dk}) |\delta \phi_c(\theta, \theta_k)|^2 d\theta
\]

\[
\tilde{J}_L^2 \equiv \int_{-\infty}^{\infty} J_0^2(\lambda_{Lk}) |\delta \phi_c(\theta, \theta_k)|^2 d\theta
\]

This problem is already in the suitable nonlinear form via \( \hat{F}_0 \)
Within this approach, it is possible to systematically generate standard NL equations in the form (expand wave-packet propagation about envelope ray trajectories):

\[
\begin{align*}
\left\{ \frac{\omega^{-1}}{\omega} \partial_t - \gamma - \frac{\xi}{nq' \theta_k} \partial_r + i(\lambda + \xi) + i \frac{\lambda}{(nq' \theta_k)^2} \partial_r^2 \right\} A(r, t) &= \text{NL TERMS} \\
\theta_k \text{ solution of } D_R(r, \omega, \theta_k) &= 0 \text{ and} \\
\lambda &= \left( \frac{\theta_k^2}{2} \right) \frac{\partial^2 D_R/\partial \theta_k^2}{\omega \partial D_R/\partial \omega} ; \quad \xi = \frac{\theta_k (\partial D_R/\partial \theta_k) - \theta_k^2 (\partial^2 D_R/\partial \theta_k^2)}{\omega \partial D_R/\partial \omega} ; \quad \gamma = \frac{-D_I}{\partial D_R/\partial \omega} \\
\text{Equations admit the well-known local limit, which is readily recovered.}
\end{align*}
\]
The renormalized $\hat{F}_0$ expression

- The $F_0$ expression is obtained from the solution of the nonlinear GKE [Friedman and Chen PF82] with a source term $S$ (collisions are added trivially; note that only trapped particles are considered for simplicity, $\bar{v}_\parallel = 0$).

$$\frac{\partial F_0}{\partial t} = S - \frac{c}{B_0} \frac{i}{r} \left( \delta K_{-k} J_{0k} \delta \phi_k - \delta K_k J_{0-k} \delta \phi_{-k} \right)$$

- Decompose fluctuating particle responses into adiabatic and non-adiabatic

$$\delta F_k = \frac{e}{m} \delta \phi_k \frac{\partial}{\partial v^2/2} F_0 + \sum_{k_\perp} \exp \left( -i k_\perp \cdot \mathbf{v} \times \mathbf{b}/\omega_c \right) \overline{\delta H}_k$$

$$\overline{\delta H}_k = \delta K_k - \frac{e}{m} \frac{QF_0}{\omega_k} J_{0k} \delta \psi_k \quad \delta A_{||k} \equiv -i \left( \frac{c}{\omega} \right) \mathbf{b} \cdot \nabla \delta \psi_k$$

- Definition: $QF_0 = (\omega \partial \varepsilon + \omega_*^*) F_0$, $\omega_* F_0 = (mc/eB) (\mathbf{k} \times \mathbf{b}) \cdot \nabla F_0$. 
Solve the problem in the Laplace transform space;

\[ F_0(t) = \int_{-\infty}^{\infty} e^{-i\omega t} \hat{F}_0(\omega) d\omega, \quad \hat{F}_0(\omega) = (2\pi)^{-1} \int_0^{\infty} e^{i\omega t} F_0(t) dt. \]

\[
\delta \hat{K}_k(\omega) = \int_{-\infty}^{\infty} \frac{e}{m} \frac{\bar{\omega}_{dk}}{\bar{\omega}_{dk} - \omega} \frac{Q_{k,x}}{x} \hat{F}_0(\omega - x) J_{0k} J_{0dk}^2 \delta \phi_k(x) dx
\]

\[
\hat{F}_0(\omega) = \frac{i}{\omega} \hat{S}(\omega) + \frac{i}{2\pi \omega} F_0(0) - \frac{ck_\theta}{\omega B_0} \frac{\partial}{\partial r} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ J_{0k} \delta \phi_k(x) \delta K_{-k}(\omega - x) - J_{0-k} \delta \phi_{-k}(x) \delta K_k(\omega - x) \right] dx
\]

Assuming an implicit summation on \( k \)

\[
\hat{F}_0(\omega) = \frac{i}{\omega} \hat{S}(\omega) + \frac{i}{2\pi \omega} F_0(0) - \frac{ck_\theta}{\omega B_0} \frac{e}{m} \frac{\partial}{\partial r} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J_{0k}^2 J_{0dk}^2 \left[ \delta \phi_k(x) \frac{\bar{\omega}_{d-k}}{\bar{\omega}_{d-k} + x - \omega} \frac{Q_{-k,x'}}{x'} \right. \\
\left. \times \hat{F}_0(\omega - x - x') \delta \phi_{-k}(x') - \delta \phi_{-k}(x) \frac{\bar{\omega}_{dk}}{\bar{\omega}_{dk} + x - \omega} \frac{Q_{k,x'}}{x'} \hat{F}_0(\omega - x - x') \delta \phi_k(x') \right] dx dx'
\]
This solution is valid for strong distortions of the equilibrium distribution function and is the analogue of the Dyson equation \( G = G_0 + G_0 \Sigma G \), with \( G_0/G \) the bare/dressed propagators), as noted, e.g., by [Al’tshul’ and Karpman 65].

The description does not include power spectrum generation and spatial bunching and accurate treatment of phase mixing on scales much longer than the wave-particle trapping time. Not a limitation for EPM nonlinear dynamics ⇒ convective amplification of unstable front.
Monochromatic EPM: reasonable assumption, since EPM spectrum is peaked for optimizing resonance condition. \( \delta \phi(t) = \delta \ddot{\phi}(\tau) \exp(-i \omega(\tau)t), |\dot{\omega}(t)| \ll |\gamma(t)\omega(t)| \).

\[
\delta \hat{\phi}_k(\omega) = \frac{i \delta \ddot{\phi}_k(\tau)}{2\pi(\omega - \omega(\tau))}, \quad \delta \hat{\phi}_{-k}(\omega) = \frac{i \delta \ddot{\phi}_{-k}(\tau)}{2\pi(\omega + \omega^*(\tau))}
\]

\[
\hat{F}_0(\omega) = \frac{i}{\omega} \hat{S}(\omega) + \frac{i}{\omega} \text{St} \hat{F}_0(\omega) + \frac{i}{2\pi \omega} F_0(0) - \frac{ck_\theta}{\omega B_0} \frac{e}{m} \frac{\partial}{\partial r} J_{0k}^2 \cdot J_{0dk}^2 \left[ \frac{\bar{\omega}_{dk}}{\omega + \omega(\tau) - \bar{\omega}_{dk} \omega(\tau)} \right] \hat{F}_0(\omega - 2i\gamma(\tau)) |\delta \ddot{\phi}_k(\tau)|^2
\]

For evanescent drive and flat envelope, this problem is reduced to the wave-particle trapping and to [Berk et al PLA97, POP99] for \( \omega_B t \ll 1 \).

For increasing drive, EPM mode structures play a crucial role: particles are convected out efficiently since - with frequency locking - wave-particle resonances are decorrelated only in velocity space. \((nqs)^{-1} \lesssim (\gamma/\omega)(\omega_*/\omega) \lesssim (L_{pH}/nqr)^{1/2}\).
The EPM avalanche and convective EP transport

Assume isotropic slowing-down and EPM NL dynamics dominated by precession resonance, \((\beta_H/\beta_e)(R/L_{pH}) \gg \varepsilon^{3/2}(T_H/q^2T_e)\) [Zonca et al NF05].

\[
[D_R(\omega, \theta_k; s, \alpha) + iD_I(\omega, \theta_k; s, \alpha)] \partial_r A = \frac{3\pi \epsilon^{1/2}}{4\sqrt{2}} \alpha_H \overline{J}^2 \left[1 + \frac{\omega}{\omega_{dF}} \ln \left(\frac{\omega_{dF}}{\omega} - 1\right) \right]
\]

\[
+i\pi \frac{\omega}{\omega_{dF}} \left[ \partial_t A + \frac{\omega}{\omega_{dF}} \overline{J}^2 A^2 \frac{3\pi \epsilon^{1/2}}{4\sqrt{2}} k_\theta \rho_H \frac{T_H}{m_H} \partial_r^2 \partial_t^{-1} (\alpha_H \overline{J}^2 |A|^2) \right].
\]

This expression DECREASES DRIVE@ MAX |A|

INCREASES DRIVE NEARBY

Assume \(\alpha_H = \alpha_{H0} \exp(-x^2/L_p^2) \simeq \alpha_{H0}(1 - x^2/L_p^2)\), with \(x = (r - r_0)\). EPM structure has characteristic linear radial envelope width \(\Delta \approx (L_p/k_\theta)^{1/2}\) and is nonlinearly displaced to maximize the drive as

\[
(x_0/L_p) = \gamma_L^{-1} k_\theta \rho_H \left(T_H/M_H\right)^{1/2} \left(|A|/W_0\right), \quad W_0 = \text{NL EPM width}
\]
Importance of toroidal geometry on wave-packet propagation and shape
Crucial points and Summary

- Fluctuation induced fast particle transport in toroidal plasmas of fusion interest involves both micro- and meso-scales phenomena, as well as macro-scale MHD.

- Micro-turbulence induced transport of energetic particles is well described by quasi-linear theories, while meso-scale fluctuations exhibit both coherent and incoherent non-linear behaviors, corresponding to convective and diffusive transport events.

- Given a dense spectrum of Alfvénic fluctuations, spectral transfers (cascade) are strongly affected by non-uniformities and toroidal geometries. Meanwhile, three-wave couplings are in competition with zonal structure formations: determining an hierarchy of nonlinear time scales in realistic conditions remains an open problem.
There is a relationship of MHD and shear Alfvén waves in the kinetic thermal ion frequency gap with micro-turbulence, Zonal Flows and Geodesic Acoustic Modes, which has importance in determining long time scale dynamic behaviors in burning plasmas.

When drive is sufficiently strong, coherent nonlinear wave-particle interactions are the dominant processes, which determine energetic particle transport as avalanche phenomena, characterized by convective amplification of the unstable front and gradient steepening and relaxation, where plasma non-uniformities, mode structures and toroidal geometries are crucial elements.

Grazie — 謝謝您