Nonlinear, Kinetic Models of Drift-Tearing Modes in Magnetised Plasmas

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Acknowledgements:
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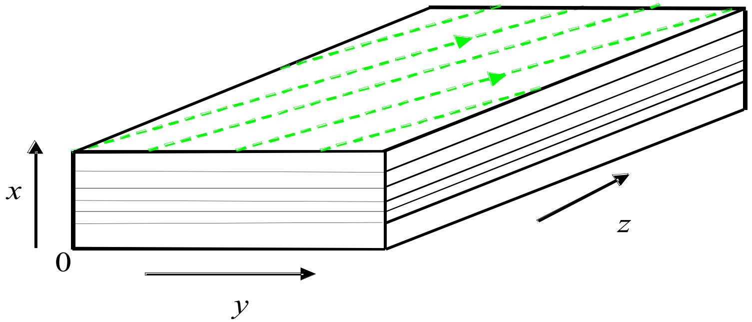
Outline

- Tearing modes in magnetised plasmas
  - The basic physics of drift-tearing modes
- Why are we interested?
- The challenges of modelling drift-tearing modes
- Summary

Magnetic geometry

- We adopt a "sheared slab" geometry as our reference state

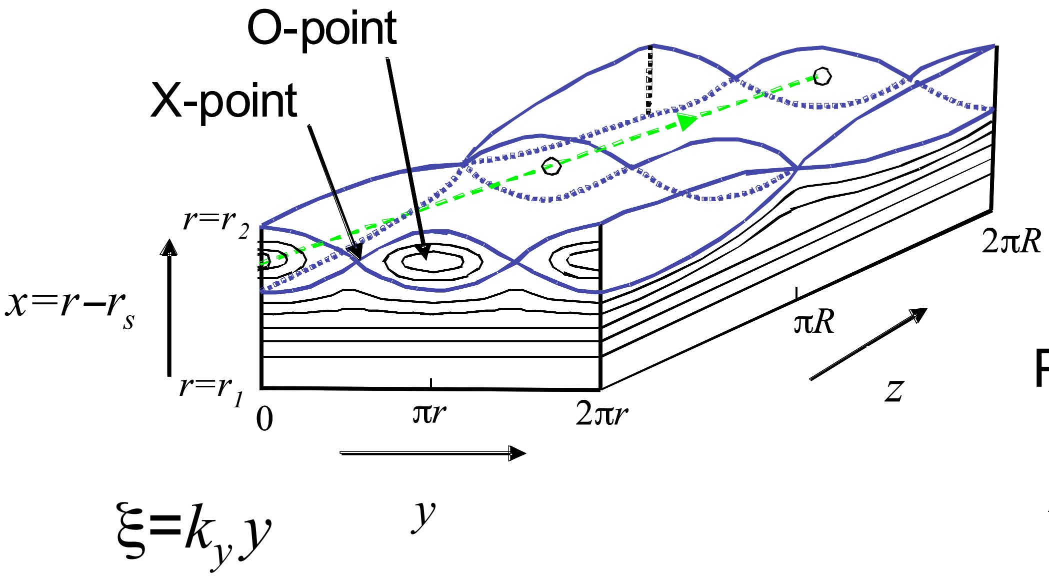


$$\boldsymbol{B} = B_0 \nabla z - \nabla \psi \times \nabla z$$

$$\psi = -\frac{B_0 x^2}{2L_s}$$

Magnetic geometry

- We adopt a “sheared slab” geometry and introduce a magnetic island



$$\boldsymbol{B} = B_0 \nabla z - \nabla \psi \times \nabla z$$

$$\psi = -\frac{B_0 x^2}{2L_s} + \tilde{\psi} \cos k_y y$$

Provides an island of half-width

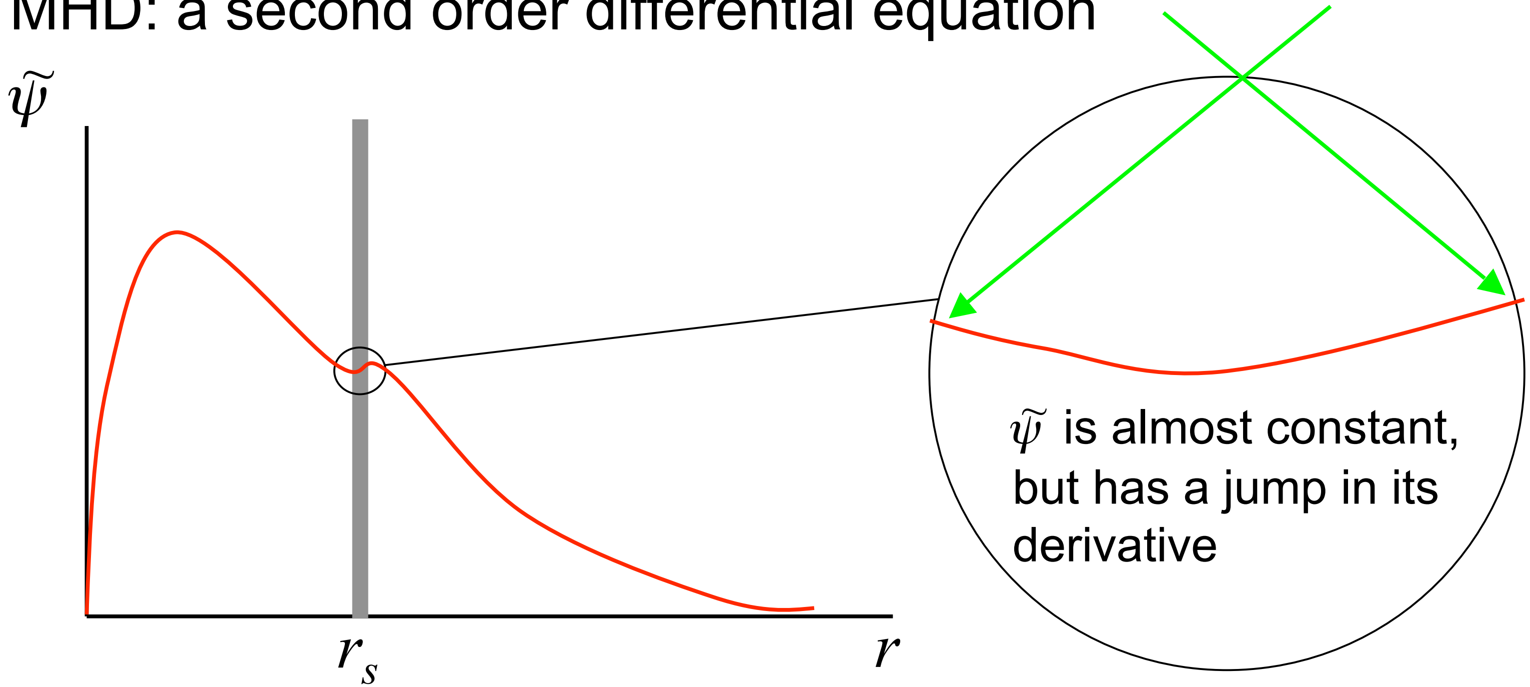
$$w = 2\left(\frac{L_s \tilde{\psi}}{B_0}\right)^{1/2}$$

- We consider the responses of the electrons and ions to this magnetic island, and the associated, self-consistent potential, φ

- The current perturbation evaluated from these responses determines whether drift effects amplify or suppress islands

The basic formalism: The concept of Δ'

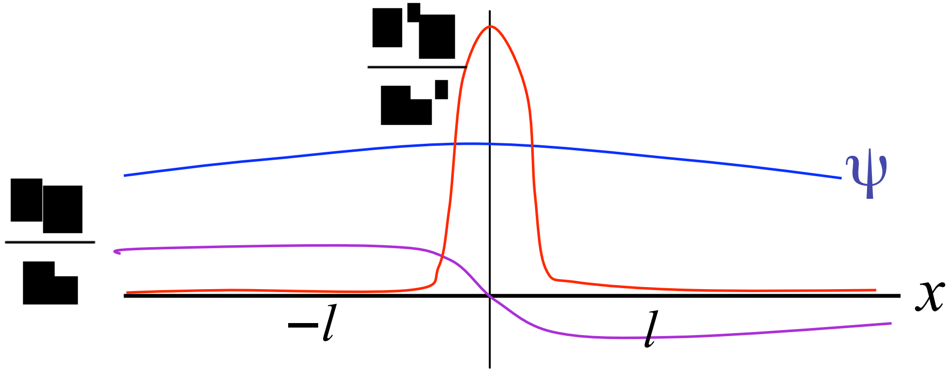
- Away from the rational surface, $\tilde{\psi}$ is determined by the equations of ideal MHD: a second order differential equation



  – Predicts that ψ has a discontinuous derivative at the rational surface $r=r_s$
  – This is conventionally parameterised by Δ′: $\Delta' = \frac{1}{\psi}\left[\left.\frac{d\psi}{dr}\right|_{r=r_s^+} - \left.\frac{d\psi}{dr}\right|_{r=r_s^-}\right]$
  – Δ' characterises global properties of the equilibrium (in particular the free energy in the current density gradient)

The basic formalism: The layer

- The discontinuous derivative arises because of currents, localised around the rational surface, where ideal MHD breaks down



- Ampere's law for long, thin islands: $\frac{d^2\tilde{\psi}}{dx^2}\cos\xi \approx \frac{\boldsymbol{B}\cdot(\nabla\times\delta\boldsymbol{B})}{B} = \mu_0 J_{\|}$

- Integrate this over a period in ξ and out to a large distance, l, from the rational surface (w<<l<<r_s):

$$\Delta'\tilde{\psi} = 2\mu_0 \int_{-\infty}^{\infty} dx \oint d\xi J_{\|} \cos\xi$$

- This is our basic tearing mode equation

Classes of tearing modes

- We have shown that the basic tearing mode equation is:

$$\Delta'\tilde{\psi} = 2\mu_0 \int_{-\infty}^{\infty} dx \oint d\xi J_{\|} \cos\xi$$

- The left hand side represents contributions from the global equilibrium profiles
  – Indeed Δ' represents the free energy available in the equilibrium current profile to drive the tearing mode

- The right hand side represents currents localised in a narrow layer in the vicinity of the rational surface

- Different classes of tearing mode can be thought of as arising from different models for the localised current density, $J_{\|}$

Basic Rutherford theory

- Consider the Ohmic current due to the induced electric field as the island evolves, and an electrostatic piece

$$\eta J_{\parallel} = E_{\parallel} = \frac{\partial \tilde{\psi}}{\partial t}\cos\xi - \nabla_{\parallel}\varphi$$

- In the absence of perpendicular drifts, perpendicular currents are zero, and so we have $\nabla\cdot\boldsymbol{J} = \nabla_{\parallel}J_{\parallel}=0$

- Thus, by averaging around flux surfaces, we eliminate φ to derive

$$J_{\parallel} = \frac{1}{\eta}\frac{\partial \tilde{\psi}}{\partial t}\langle\cos\xi\rangle \qquad \langle\nabla_{\parallel}\cdots\rangle = 0$$

- Recall our basic equation: $\Delta'\tilde{\psi} = 2\mu_0\int_{-\infty}^{\infty}dx\oint d\xi J_{\parallel}\cos\xi$

- Relating ψ to the island width, w, we then arrive at Rutherford's eqn:

$$\boxed{0.82\tau_r\frac{dw}{dt} = r_s^2\Delta'} \qquad \tau_r = \frac{\mu_0 r_s^2}{\eta} \qquad w = 2\left(\frac{L_s\tilde{\psi}}{B_0}\right)^{1/2}$$

Drift magnetic islands

$$0.82\frac{\tau_r}{r_s}\frac{dw}{dt} = r_s\Delta' - \frac{8L_s r_s}{B_0 w^2}\int_{-\infty}^{\infty} dx \oint d\xi J_{\parallel}\cos\xi$$

- A convenient picture is provided by considering the ExB drift
- For island width w comparable to the ion Larmor radius:
  - ions experience a gyro-averaged electric field
  - electrons experience the local electric field
  - the differing effective ExB drifts provides a perpendicular current: the ion polarisation current
- The divergence of this perpendicular current is not zero
- This drives a parallel current, which influences tearing mode stability
- Important for islands with a width ~few ρ_i

Motivation:
Why are we interested?

Why are we interested? 1. Transport

- Chains of small scale magnetic islands on adjacent rational surfaces can result in stochastic magnetic field regions, and enhance electron transport
  - Indeed, the self-consistent electrostatic potential associated with the island structures could also drive particle and ion thermal transport in principle

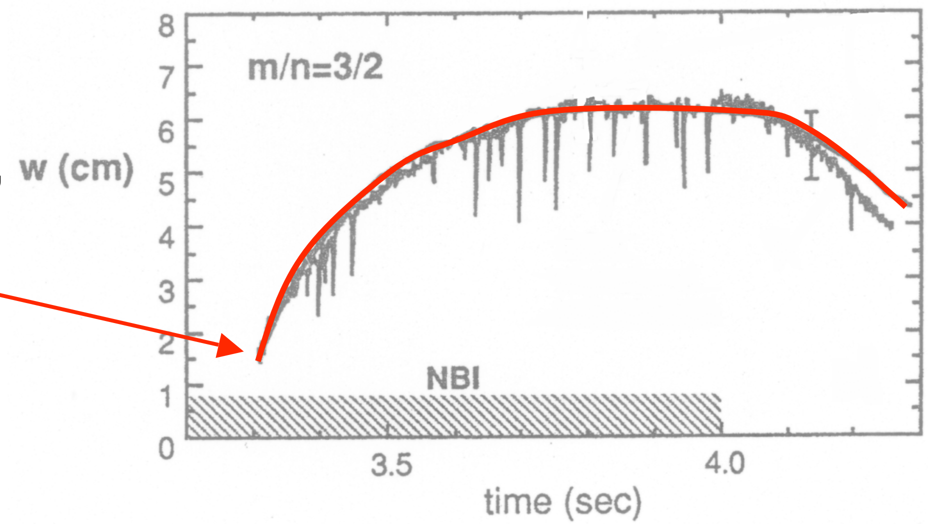
Why are we interested? 2. MHD

- In a tokamak, the perturbation in the bootstrap current can provide an additional drive (ie the neoclassical tearing mode, or NTM)

$$J_{bs} \sim -\frac{\varepsilon^{1/2}}{B_\theta}\frac{dp}{dr} \qquad 0.82\frac{\tau_r}{r_s}\frac{dw}{dt} = r_s\Delta' - a_{bs}\frac{L_q}{L_p}\frac{\beta_\theta}{w}$$

  – Arbitrarily small islands are amplified to large width: $w = a_{bs}\frac{L_q}{L_p}\frac{\beta_\theta}{r_s\Delta'}$
  – An effective (soft) limit on the pressure

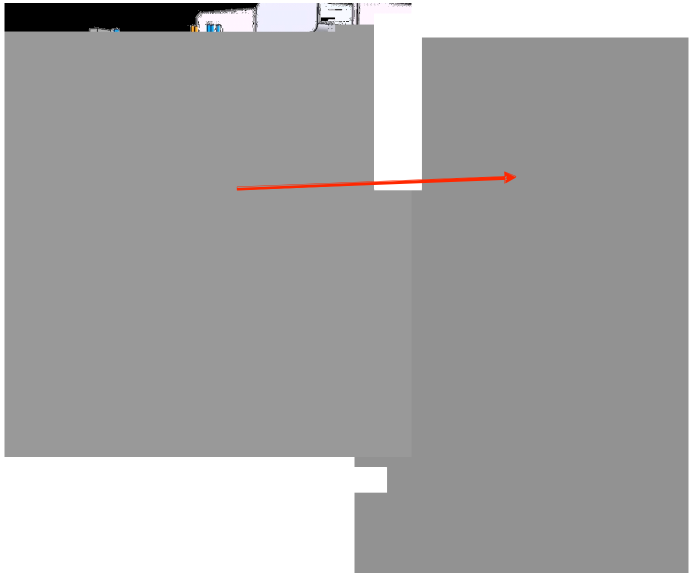
- Experimentally, there is observed to be a threshold for island growth, w~ ion orbit width
  – Drift effects are important to understand this threshold

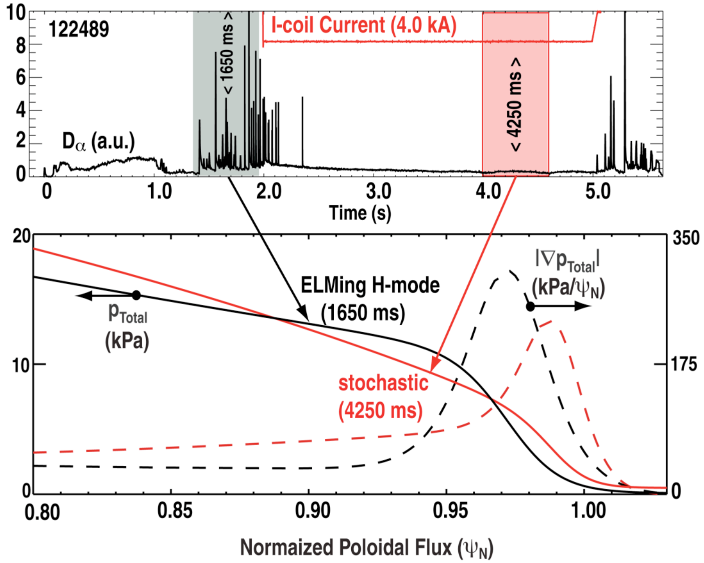


Z. Chang et al, PRL 1995

Why are we interested?
3. ELM suppression

- Periodic eruptions of plasma, called ELMs, are driven by the strong edge pressure gradient
  - If uncontrolled, they would cause excessive erosion in ITER
  - DIII-D has demonstrated ELM control using magnetic perturbations from coils to degrade the edge confinement and reduce the pressure gradient
- How does the plasma respond to magnetic perturbation from the coils
  - Do they create small islands, or does the plasma screen/heal them?





See, for example, T Evans Nature Phys (2006)

Calculating the Plasma Response:
The challenges of drift-tearing mode theory

The particle response

- We employ the non-linear gyro-kinetic equation for the particle responses to the imposed magnetic perturbation, $A_{\parallel} = -\tilde{\psi}\cos\xi$
  - Perturb about a Maxwellian reference state $F_M(x)$
  - Work in island rest frame, so total potential $\Phi=-E_x x + \varphi$
  - Nonlinear, non-adiabatic part of response

$$\omega_E \left.\frac{\partial g_j}{\partial \xi}\right|_x - k_{\parallel} v_{\parallel} \left.\frac{\partial g_j}{\partial \xi}\right|_{\psi} - \frac{1}{B_0^2}\left(\boldsymbol{B}_0 \times \nabla\langle\varphi\rangle_{\alpha}\right)\nabla g_j - C(g_j) = \frac{q_j}{T_j} F_{Mj}\left(\omega_E - \omega_{*j}^T\right)\left.\frac{\partial}{\partial \xi}\right|_x \left(\langle\varphi\rangle_{\alpha} - v_{\parallel} A_{\parallel}\right)$$

Gyroaverage

$$A_{\parallel} = -\tilde{\psi}\cos\xi$$

Magnetic perturbation causing the island

$$\omega_E = \frac{k_y E_x}{B_0}$$

ExB flow of plasma in reference state

$$\xi = k_y y$$

$$k_{\parallel} = -\frac{k_y x}{L_s}$$

The electrons

- The electrons are characterised by fast parallel velocities

$$\omega_E \frac{\partial g_e}{\partial \xi}\bigg|_x - k_\parallel v_\parallel \frac{\partial g_e}{\partial \xi}\bigg|_\psi - \frac{1}{B_0^2}\left(\boldsymbol{B}_0 \times \nabla\langle\varphi\rangle_\alpha\right)\nabla g_e - C(g_e) = \frac{q_e}{T_e} F_{Me}\left(\omega_E - \omega_{*e}^T\right)\frac{\partial}{\partial \xi}\bigg|_x \left(\langle\varphi\rangle_\alpha - v_\parallel A_\parallel\right)$$

  – The response is dominated by the parallel dynamics

- This can be integrated, and combined with the adiabatic piece to give the electron density:

$$n_e = \left(1 + \frac{e\varphi}{T_e}\right) n_0(x) + \frac{w}{L_n}\left(\frac{\omega_E - \omega_{*e}}{\omega_{*e}}\right)\left(\frac{x}{w} - h(\psi)\right) n_0$$

- Note that this can be written in the more familiar form:

$$n_e = \left(1 + \frac{e\Phi}{T_e}\right) n_0(0) - \frac{w}{L_n}\left(\frac{\omega_E - \omega_{*e}}{\omega_{*e}}\right) h(\psi) n_0 \qquad \Phi = -E_x x + \varphi$$

  – i.e. adiabatic, and constant on the perturbed field lines of the island
  – So $h(\psi)$ represents the density profile

    ⇒undetermined as we have no transport physics

The ions

- The ion response is dominated by the cross-field drifts:

$$\omega_E \left.\frac{\partial g_i}{\partial \xi}\right|_x - \cancel{k_\parallel v_\parallel \left.\frac{\partial g_i}{\partial \xi}\right|_\psi} - \frac{1}{B_0^2}\left(\boldsymbol{B}_0 \times \nabla\langle\varphi\rangle_\alpha\right)\nabla g_i - \cancel{C(g_i)} = \frac{q_i}{T_i} F_{Mi}\left(\omega_E - \omega_{*i}^T\right)\left.\frac{\partial}{\partial \xi}\right|_x \left(\langle\varphi\rangle_\alpha - \cancel{v_\parallel A_\parallel}\right)$$

- We can write the solution in terms of the linear response and an arbitrary function of the total potential: $\Phi = -E_x x + \varphi$

$$g_i\left(\langle\Phi\rangle_\alpha\right) = \frac{q_i}{T_i}\frac{\left(\omega_E - \omega_{*i}^T\right)}{\omega_E}\langle\varphi\rangle_\alpha F_{Mi} + K\left(\langle\Phi\rangle_\alpha\right)F_{Mi}$$

- Quasi-neutrality then provides an equation for φ ($\rho_i \ll w$)

$$\rho_s^2 \frac{\partial^2 \hat{\varphi}}{\partial x^2} + \left[\frac{T_i}{T_e}\frac{\omega_E - \omega_{*e}}{\omega_E - \omega_{*i}(1+\eta_i)}\right]\hat{\varphi} = \frac{w}{L_n}\frac{\omega(\omega - \omega_{*e})}{\omega_{*e}\left(\omega - \omega_{*i}(1+\eta_i)\right)}\left(\frac{x}{w} - h(\psi)\right) - \frac{\omega_E}{\omega_E - \omega_{*i}(1+\eta_i)}K(\Phi)$$

- The two profiles $h(\psi)$ and $K(\Phi)$ are determined by transport physics
  - we can define a transport model and determine $h(\psi)$ and $K(\Phi)$, or
  - we can adopt models for $h(\psi)$ and $K(\Phi)$, consistent with b.c.s

The transport profiles

- Self-consistent models for $K(\Phi)$ have been considered, but generally one simply adopts K=0

- Turn to $h(\psi)$:
  - perturbatively introduce a heuristic model for diffusion into the electron equation

$$k_{\parallel} v_{\parallel} \left. \frac{\partial g_e}{\partial \xi} \right|_{\psi} = \frac{q_e v_{\parallel}}{T_e} F_{Me} \left(\omega_E - \omega_{*e}^T \right) \left. \frac{\partial A_{\parallel}}{\partial \xi} \right|_{x}$$

The transport profiles

- Self-consistent models for $K(\varphi)$ have been considered, but generally one simply adopts K=0
- Turn to $h(\psi)$:
  - perturbatively introduce a heuristic model for diffusion into the electron equation

$$k_{\|} v_{\|} \left.\frac{\partial g_e}{\partial \xi}\right|_{\psi} - D\frac{\partial^2 g_e}{\partial x^2} = \frac{q_e v_{\|}}{T_e} F_{Me}\left(\omega_E - \omega_{*e}^T\right)\left.\frac{\partial A_{\|}}{\partial \xi}\right|_x$$

  - Averaging over the island flux surfaces provides: $D\left\langle \frac{\partial^2 g_e}{\partial x^2}\right\rangle = 0$
  - Provides $h(\psi)$ in terms of elliptic integrals: $h(\psi) = \frac{\Theta(\Omega-1)}{2\sqrt{2}}\int_1^{\Omega}\frac{d\Omega'}{P(\Omega')}$

$$\Omega = -\frac{\psi}{\tilde{\psi}} \qquad P(\Omega) = \frac{1}{2\pi}\oint\sqrt{\Omega+\cos\xi}\, d\xi$$

  - $h(\psi)$ is zero inside the island (Ω<1)
  - $dh/d\psi$ is discontinuous at the separatrix (Ω=1)

$\nabla\cdot\boldsymbol{J}$=0 provides our equation for $\boldsymbol{J}_{||}$

- The current perturbation is derived from the equation $\nabla\cdot\boldsymbol{J}$=0
  - This, in turn, is derived by integrating the electron and ion gyrokinetic equations over velocity space and summing (imposing quasi-neutrality)

- After some algebra, one finds: $\nabla_{||}J_{||} = \frac{n_0 e\tau}{B^2}\frac{(\omega_E - \omega_{*i})}{\omega_{*e}}\frac{w}{L_n}\frac{dh}{d\psi}(\boldsymbol{B}\times\nabla\psi)\cdot\nabla\phi_i$

Polarisation current

$\phi_i = \langle\varphi\rangle_\alpha - \varphi$ is zero if FLR effects are absent

- This can be integrated to provide the current density to be inserted in our island evolution equation:

$$J_{||} = \frac{4n_0 e}{B}\frac{(\omega_E - \omega_{*i})}{\omega_{*i}}\frac{L_s}{L_n}\frac{1}{w}\frac{dh}{d\psi}\left(\phi_i - \langle\phi_i\rangle\right)$$

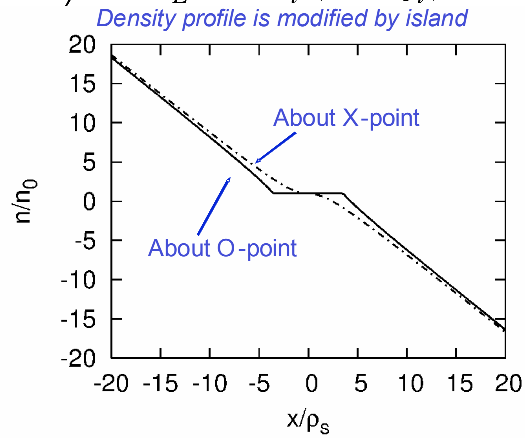
Flux surface average

The "full" system: and a twist

$$\rho_s^2 \frac{\partial^2 \hat{\varphi}}{\partial x^2} + \left[\frac{T_i}{T_e} \frac{\omega_E - \omega_{*e}}{\omega_E - \omega_{*i}(1+\eta_i)} \right] \hat{\varphi} = \frac{w}{L_n} \frac{\omega_E(\omega_E - \omega_{*e})}{\omega_{*e}(\omega_E - \omega_{*i}(1+\eta_i))} \left(\frac{x}{w} - h(\psi) \right) - \frac{\omega_E}{\omega_E - \omega_{*i}(1+\eta_i)} K(\hat{\varphi})$$

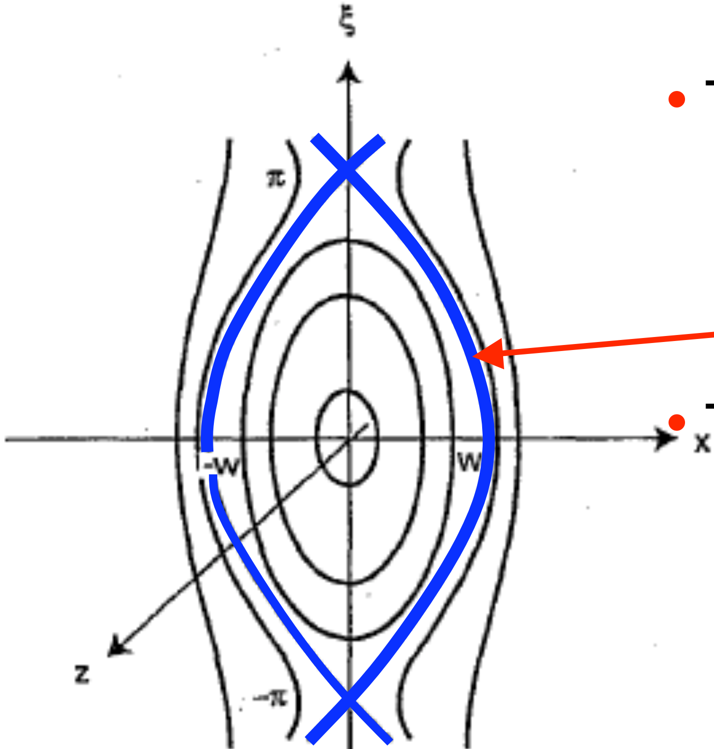
$$J_\parallel = \frac{4 n_0 e}{B} \frac{(\omega_E - \omega_{*i})}{\omega_{*i}} \frac{L_s}{L_n} \frac{1}{w} \frac{dh}{d\psi} \left(\phi_i - \overline{\phi}_i \right)$$

$$0.82 \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta' - \frac{8 L_s r_s}{B_0 w^2} \int_{-\infty}^{\infty} dx \oint d\xi J_\parallel \cos\xi$$



- Note that our quasi-neutrality condition requires $\rho_s \frac{\partial}{\partial x} << 1$
  - We then have $\hat{\varphi} = \frac{w}{L_n} \omega_E \left(\frac{x}{w} - h(\psi) \right)$
  - In addition $J_\parallel \sim \phi_i \sim \rho_s^2 \frac{\partial^2 \varphi}{\partial x^2}$
- As the derivative of $h(\psi)$ is discontinuous at the island separatrix, there is a δ-function contribution to the current density there

The separatrix layer



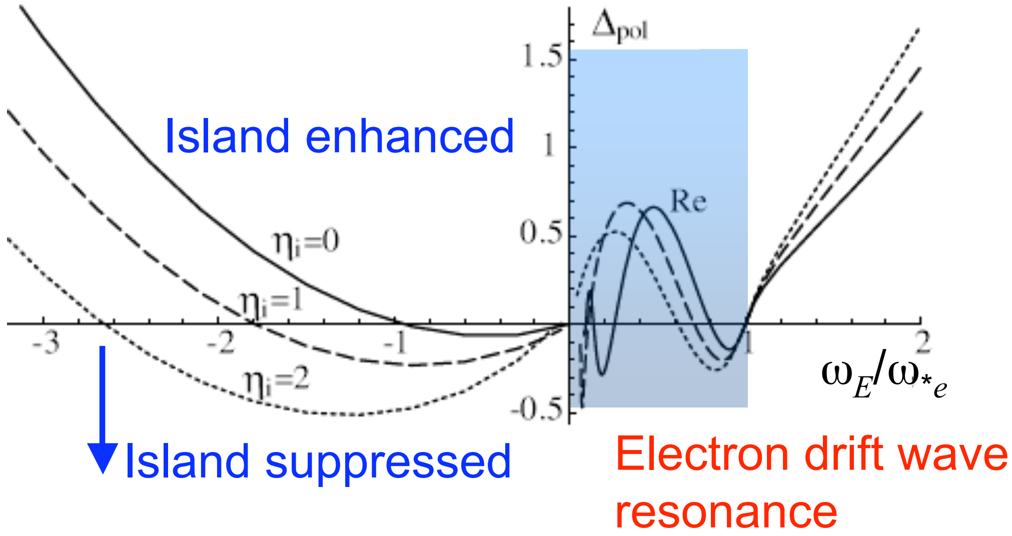
- There are two contributions to $J_{||}$
  - a $\sim(\rho_s/w)^2$ contribution from the region outside the separatrix layer
  - a very large current sheet at the separatrix
- These contributions
  - oppose each other
  - make a comparable (but opposite) contribution to the island evolution

$$0.82\frac{\tau_r}{r_s}\frac{dw}{dt} = r_s\Delta' - \frac{8L_s r_s}{B_0 w^2}\int_{-\infty}^{\infty}dx\oint d\xi J_{||}^{outer}\cos\xi - \frac{8L_s r_s}{B_0 w^2}\int_{-\infty}^{\infty}dx\oint d\xi J_{||}^{layer}\cos\xi$$

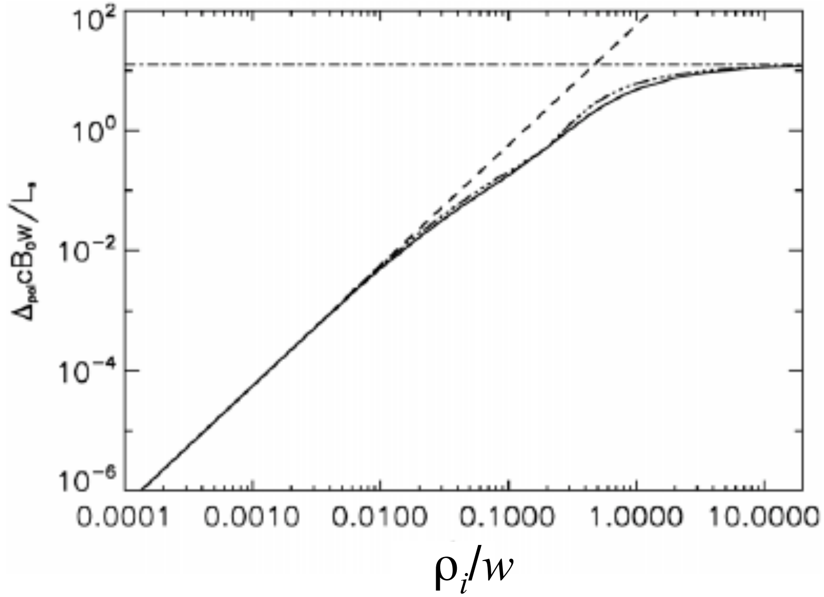
- A quantitative calculation of $J_{||}$ in each region is essential to describe the island evolution accurately
  - the layer is a major challenge

The separatrix layer: FLR effects

- Ion Larmor radius effects cannot (in general) be treated perturbatively in the layer
  - Full FLR must be retained to treat the layer region accurately



Waelbroeck, Connor, Wilson PRL (2001)



James, Wilson PPCF (2006)

- Note that whether islands are enhanced or suppressed by FLR effects depends on the propagation frequency, ω_E
  - This is treated as an input parameter here

The separatrix layer: Electron drift wave resonance

Consider our equation for the potential (with $K(\varphi)=0$)

$$-\left(1+\frac{T_e}{T_i}\right)\hat{\varphi}+\frac{T_e}{T_i}\frac{(\omega_E-\omega_{*i})}{\omega_E}\langle\hat{\varphi}\rangle_\alpha=\frac{w}{L_n}\frac{(\omega_E-\omega_{*e})}{\omega_{*e}}\left(\frac{x}{w}-h(\psi)\right)$$

– The left hand side is simply the standard linearised eigenmode equation for electrostatic modes (electron drift wave and ITG modes)

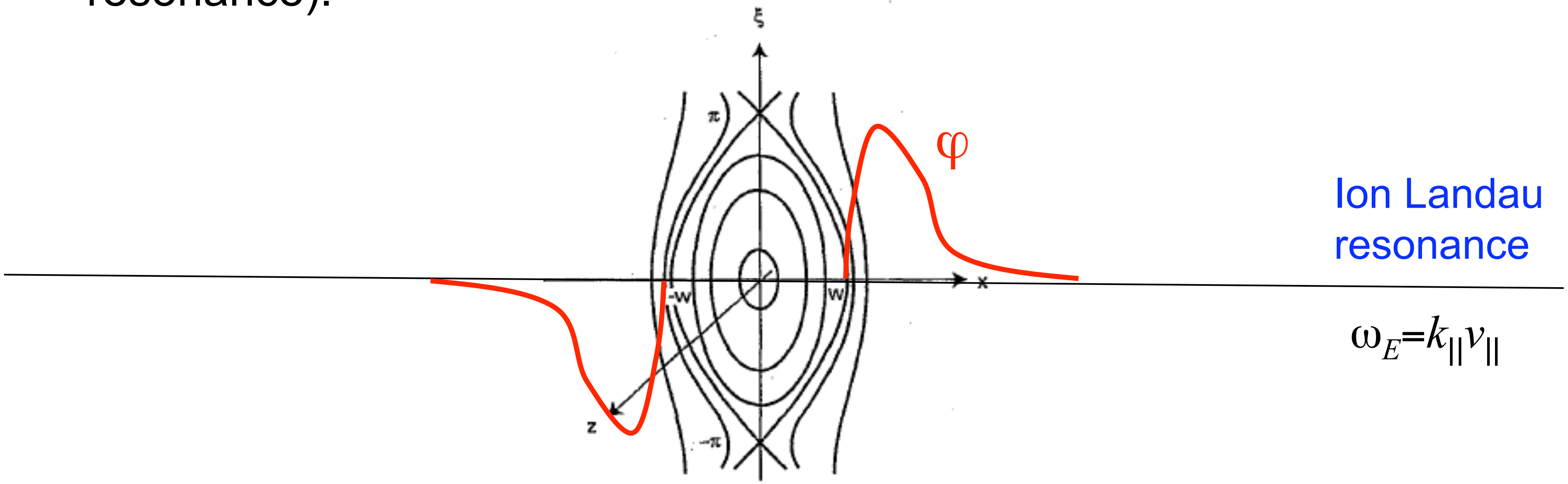
– The electron response to the magnetic perturbation of the island (right hand side) provides a drive

– The electron drift wave is driven at a frequency $0 < \omega(k_x) < \omega_{*e}$

– The separatrix layer has a range of k_x, one of which drives a resonance with the electron drift wave for $0 < \omega(k_x) < \omega_{*e}$

Electron drift wave resonance: The potential is not localised

- Recall that in the ion gyrokinetic equation, we ordered $\omega_E >> k_{||} v_{||}$
  - Thus the ion Landau resonance, which is far from the island, is ordered out of the problem when the potential is localised (ie no drift wave resonance).



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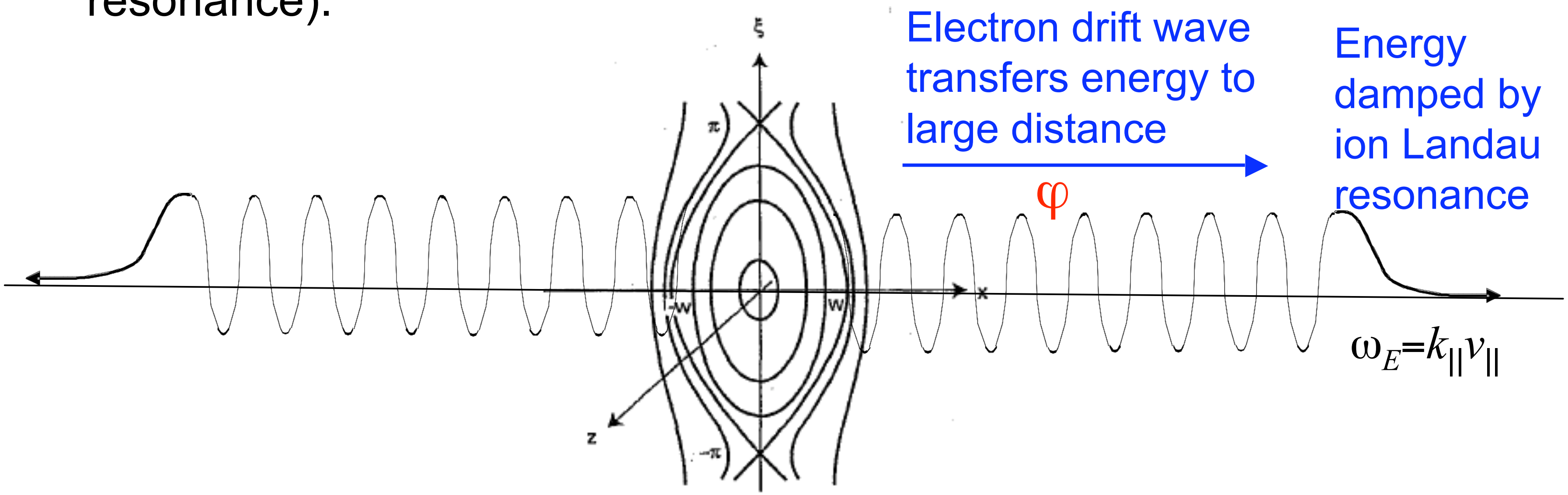


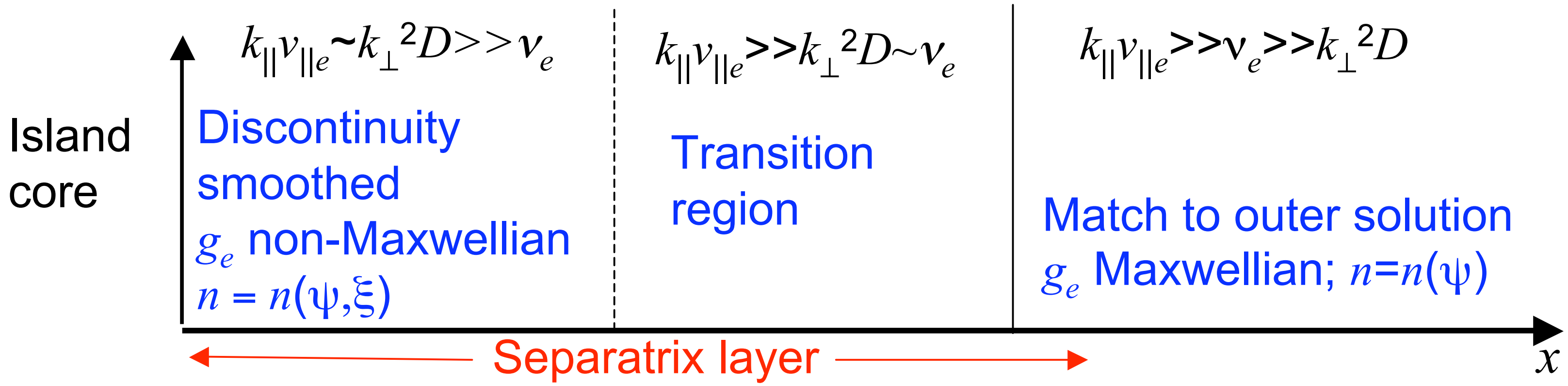
- The electron drift wave propagates out to large distance from island
  - This coupling to the electron drift wave allows the ion Landau resonance to be tapped (likely influencing ω_E through shear damping)

Resolving the separatrix layer: Cross-field transport

- Recall that the discontinuity in $h(\psi)$ is created by the rapid transport of electrons along the island field lines
  - we treated diffusion perturbatively (via a heuristic model)
  - in the layer this treatment is invalid: parallel streaming and cross-field diffusion balance

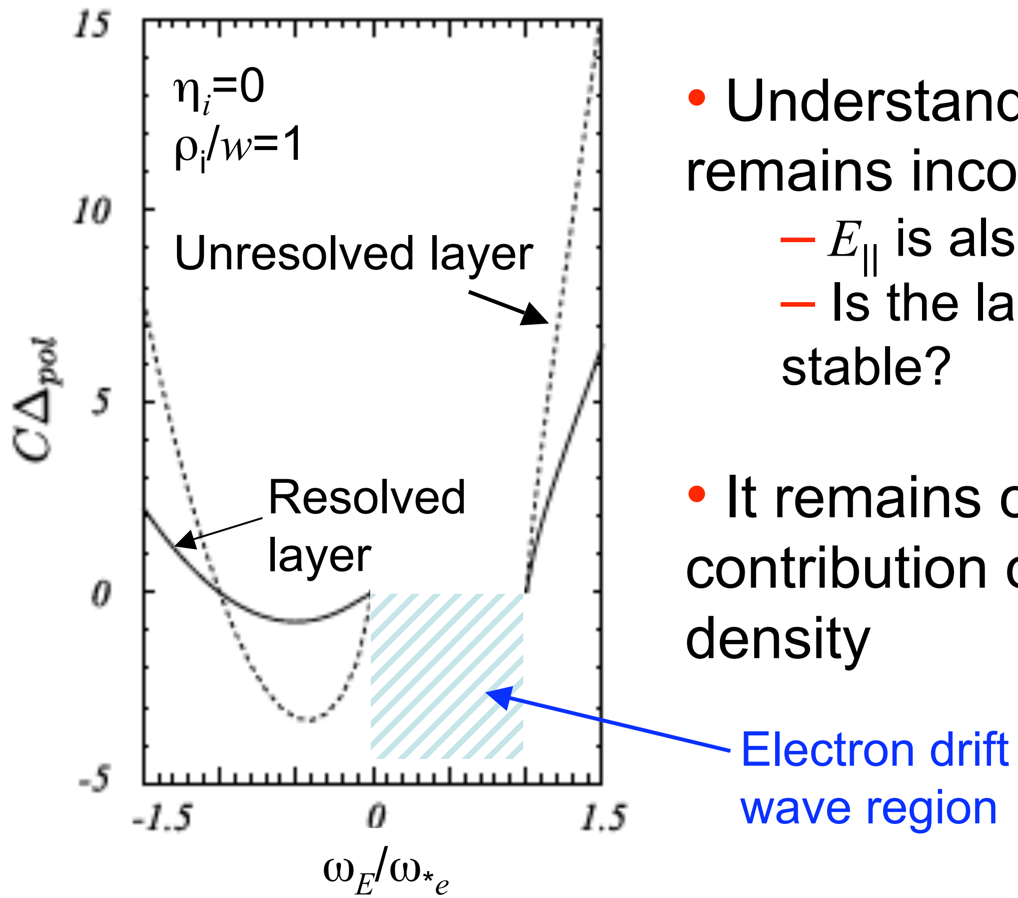
$$k_{\|}v_{\|}\left.\frac{\partial g_e}{\partial \xi}\right|_{\psi} - D\frac{\partial^2 g_e}{\partial x^2} - C(g_e) = \frac{q_e v_{\|}}{T_e}F_{Me}\left(\omega_E - \omega_{*e}^{T}\right)\left.\frac{\partial A_{\|}}{\partial \xi}\right|_{x}$$

  - This system has been solved (*Hazeltine, Catto and Helander (1998); James, Wilson and Connor (2010)*)



Resolving the separatrix layer: Influence on island evolution

- A proper treatment of the separatrix layer has a significant influence on its contribution to the island evolution



- Understanding of the layer region remains incomplete
  - $E_{||}$ is also likely large here
  - Is the large current density in this region stable?

- It remains crucial to determine the net contribution of the outer + layer current density

The physics of $K(\varphi)$ and ω_E

- We can think of $K(\varphi)$ as representing the flow profile in the vicinity of the island

- ω_E, on the other hand, characterises a global property of the flow

- Both must be determined within a self-consistent model, and require dissipation
  - collisional dissipation (viscosity)
  - non-ambipolar turbulent transport processes
  - Landau resonances

- We have seen how the ion Landau resonance could be tapped via the electron drift wave
  - what about the electron Landau resonance?

The electron Landau resonance

- Recall the approximations made for the electron response:

$$\cancel{\omega_E \frac{\partial g_e}{\partial \xi}\bigg|_x} - k_\parallel v_\parallel \frac{\partial g_e}{\partial \xi}\bigg|_\psi - \frac{1}{B_0^2}\cancel{\left(\boldsymbol{B}_0 \times \nabla \langle \varphi \rangle_\alpha\right)\cdot \nabla g_e} - \cancel{C(g_e)} = \frac{q_e}{T_e} F_{Me}\left(\omega_E - \omega_{*e}^T\right)\frac{\partial}{\partial \xi}\bigg|_x \left(\cancel{\langle \varphi \rangle_\alpha} - v_\parallel A_\parallel\right)$$

- Let us re-introduce the reference flow, ω_E, to provide the Landau resonance

$$\omega_E \frac{\partial g_e}{\partial \xi}\bigg|_x - k_\parallel v_\parallel \frac{\partial g_e}{\partial \xi}\bigg|_\psi = -k_\parallel v_\parallel \frac{\partial g_e}{\partial \xi}\bigg|_y = -\frac{q_e}{T_e} F_{Me}\left(\omega_E - \omega_{*e}^T\right) v_\parallel \frac{\partial A_\parallel}{\partial \xi}\bigg|_x$$

$$y = \left(\frac{L_s}{k_y w v_\parallel}\right)^2 \left(\omega_E - k_\parallel v_\parallel\right)^2 - \frac{1}{2}\cos\xi$$

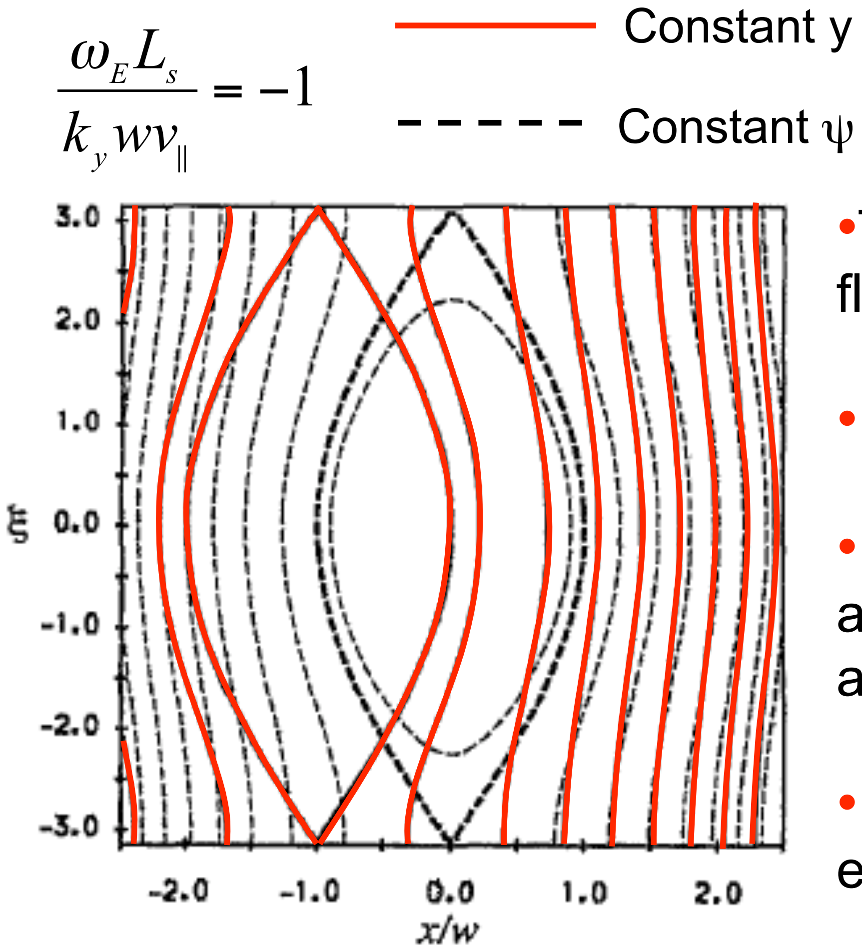
- Thus $\quad g_e = \dfrac{w}{L_n} F_{Me} \dfrac{\omega - \omega_{*e}^T}{\omega_{*e}} \left[\dfrac{L_s}{k_y w v_\parallel}\left(\omega_E - k_\parallel v_\parallel\right) - h(y)\right]$

- This means that the electron distribution function is constant on surfaces of constant y, rather than ψ

Electron Landau damping is enabled through collisions

- Compare magnetic flux (ψ) surfaces with drift (y) surfaces:

$$\frac{\omega_E L_s}{k_y w v_{\parallel}} = -1$$



- The electron distribution function is flattened across the Landau resonance
- This removes Landau damping
- Retaining a level of collisions maintains a gradient across the Landau resonance, and restores the Landau damping
- Collisions may play an important role, even when they are rare

Connor and Wilson, Phys Plas (1995)

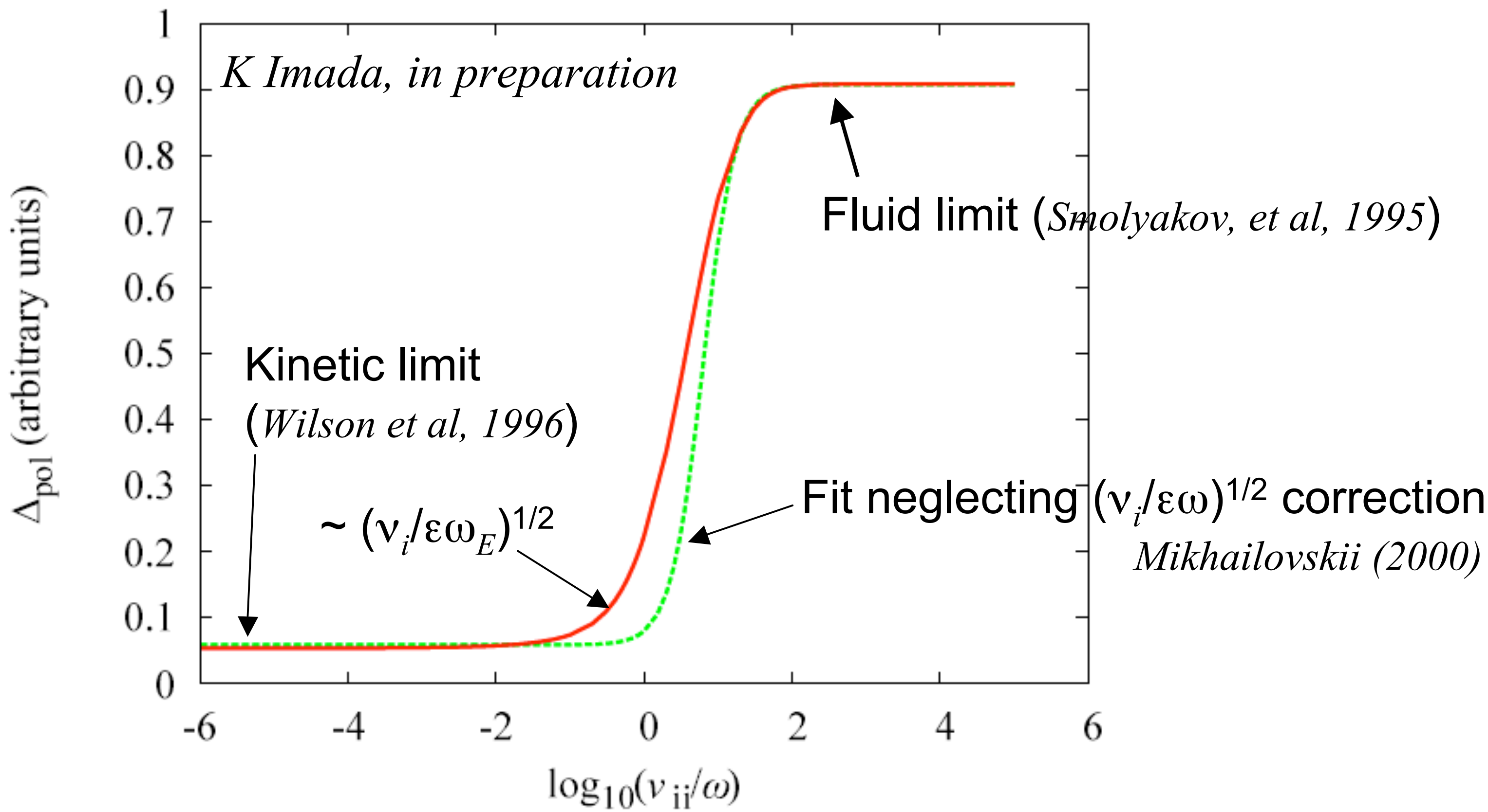
Toroidal effects

- The main influence of the toroidal geometry is the presence of trapped particles:
  - the bootstrap current perturbation, driving neoclassical tearing modes (which we shall not address)
  - the impact on the ion polarisation current

- A "neoclassical polarisation current" is generated due to the finite ion banana width
  - trapped ions experience orbit-averaged ExB drift
  - this provides a neoclassical polarisation current ~ $\varepsilon^{1/2}(\varepsilon^{1/2}\rho_{\theta i}/w)^2$
  - In a "collisional" regime $\nu_i/\varepsilon>\omega_E$ (but $\nu_*<<1$) the polarisation current is communicated to the passing particles, and amplified by a large factor ~$q^2/\varepsilon^{3/2}$

Neoclassical polarisation current

- The neoclassical polarisation current has a strong dependence on collision frequency



Summary

- Finite ion Larmor radius effects influence magnetic island evolution

- The theory is incomplete, complicated due to the existence of a narrow layer in the vicinity of the separatrix:
  - FLR effects crucial
  - Cross-field diffusion competes with parallel streaming
  - Parallel electric field important (not fully addressed)
  - Coupling to electron drift waves

- A complete theory must self-consistently determine flow profiles around the island and the propagation frequency, ω_E

- Toroidal geometry presents additional challenges for gyrokinetic codes
  - resolving a boundary layer between trapped and passing particles $\Delta v/v_{thi} \sim (\nu_i/\varepsilon\omega_E)^{1/2}$ is necessary (usually)

- Whether FLR is stabilising or destabilising remains unresolved
  - if it were destabilising, however, tokamak confinement would be terrible!