Entropy transfer processes in kinetic plasma turbulence and zonal flows

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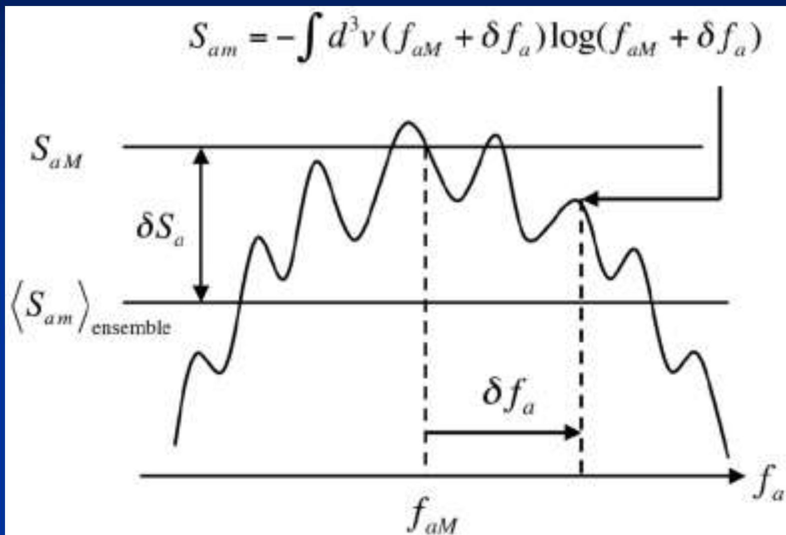
Generation of Fine Structures

- Shear operators $$U(Z_i,\ldots)\frac{\partial f(Z_i, Z_j\ldots)}{\partial Z_j}$$

  in the phase space generate fine structures of δf, and transfer the perturbations from macro- to micro-scales (where Z_i, and Z_j are a pair of the independent coordinates)
  - Linear parallel advection, ballistic modes $\exp(ikvt)$
  - Nonlinear wave-particle interactions, trapping, etc.
  - Parallel advection in ExB turbulence (W-S)
  - ExB with FLR effect (Schekochihin, Tatsuno, Plunk)
  - Magnetic drift and mirror force terms

Entropy Variable δS



$$
\begin{array}{c} \delta S = S_M - S_m \\ f = F_M + \delta f \end{array}, \quad
\begin{cases}
\delta S = \left\langle \int d^3v \, \delta f^2 / 2F_M \right\rangle \\
S_M = \left\langle \int d^3v F_M \ln F_M \right\rangle \\
S_m = \left\langle \int d^3v \, f \ln f \right\rangle
\end{cases}
$$

- Entropy variable δS is a good measure for fluctuations of f in the phase space.

Outline

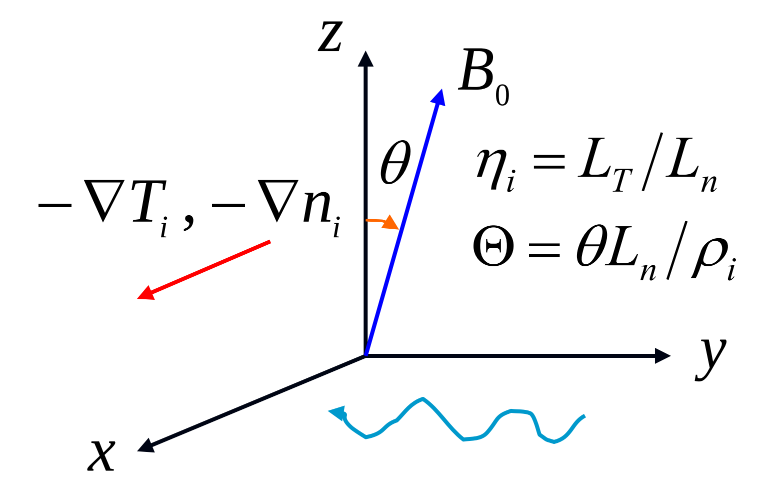
- Entropy production, transfer, and dissipation processes in the slab ITG turbulence
  - Simulation and theoretical models
  - Relation to the kinetic-fluid closure models
- Zonal flow dynamics and entropy balance in toroidal systems
  - Zonal flow response in toroidal systems
  - Zonal flow generation and entropy transfer
- Summary

Kinetic simulation of entropy production, transfer and dissipation processes in slab ITG turbulence

Slab ITG Turbulence Model

- Consider a reduced kinetic equation given by integrating the GK equation for v_{perp} in a 2-D slab with uniform B_0

$$\partial_t \tilde{f}_{\mathbf{k}}(v_{\|}) + ik_y\Theta v_{\|}\tilde{f}_{\mathbf{k}}(v_{\|}) + \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''}\left(k'_y k''_x - k'_x k''_y\right)\Psi_{\mathbf{k}'}\tilde{f}_{\mathbf{k}''}(v_{\|}) = -ik_y\Psi_{\mathbf{k}}F_M(v_{\|})\left[1+\frac{\eta_i}{2}\left(v_{\|}^2-1-k^2\right)+\Theta v_{\|}\right]+C\left(\tilde{f}_{\mathbf{k}}\right)$$

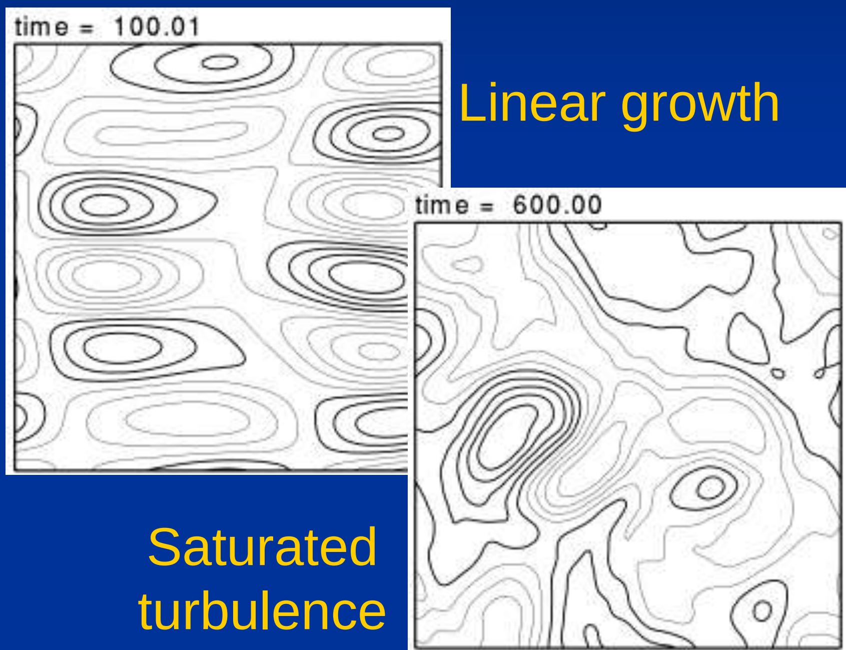


- Constant gradients for T & n (Instability drive)
- No zonal flow case
- Adiabatic electrons

$$\Psi_{\mathbf{k}} = e^{-k^2/2}\Phi_{\mathbf{k}} = \frac{e^{-k^2}}{2-\Gamma_0(k^2)-\delta(k_y)}\int dv_{\|}\tilde{f}_{\mathbf{k}}(v_{\|})$$

Simulation of the collisionless slab ITG turbulence

Linear growth



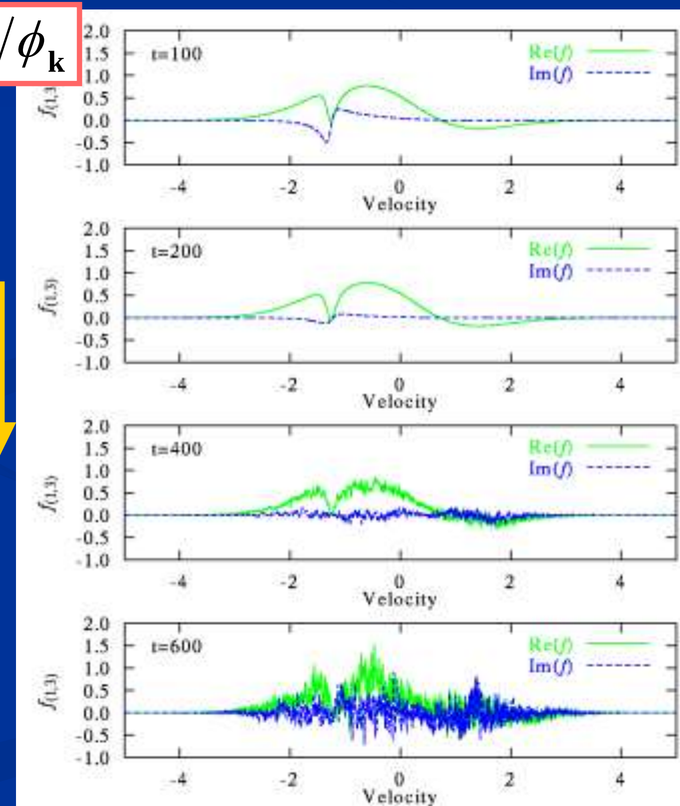
Saturated turbulence

Electrostatic potential (Stream Function)

Velocity-space structure of δf

$f_{\mathbf{k}} / \phi_{\mathbf{k}}$

Time



Watanabe & Sugama, PoP (2002)

Entropy balance derived from GK

$$\boxed{\frac{d}{dt}\left\{\delta S + W\right\} = \eta_i Q_i + D}$$

$$\begin{cases} \delta S = \sum_{\mathbf{k}} \int dv_{\|} \left|\tilde{f}_{\mathbf{k}}\right|^2 \big/ 2F_M \\ Q_i = \sum_{\mathbf{k}} \int dv_{\|} \left(-ik_y e^{-k^2/2}\Phi_{\mathbf{k}}\right) v_{\|}^2 \tilde{f}_{-\mathbf{k}} / 2 \\ W = \sum_{\mathbf{k}} \left(2 - \Gamma_0(k^2) - \delta(k_y)\right) \left|\Phi_{\mathbf{k}}\right|^2 \big/ 2 \\ D = \sum_{\mathbf{k}} \int dv_{\|} \left(\tilde{f}_{-\mathbf{k}} / F_M\right) C\left[\tilde{f}_{\mathbf{k}}\right] \end{cases}$$

(entropy variable)

(turbulent energy flux)

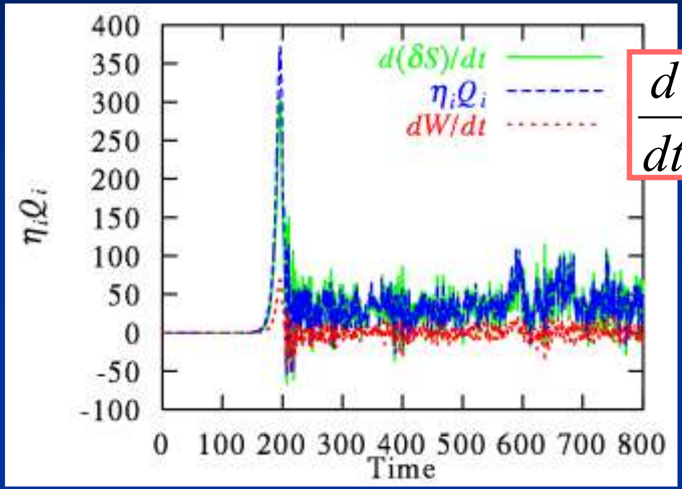
(potential energy)

(collisional dissipation < 0)

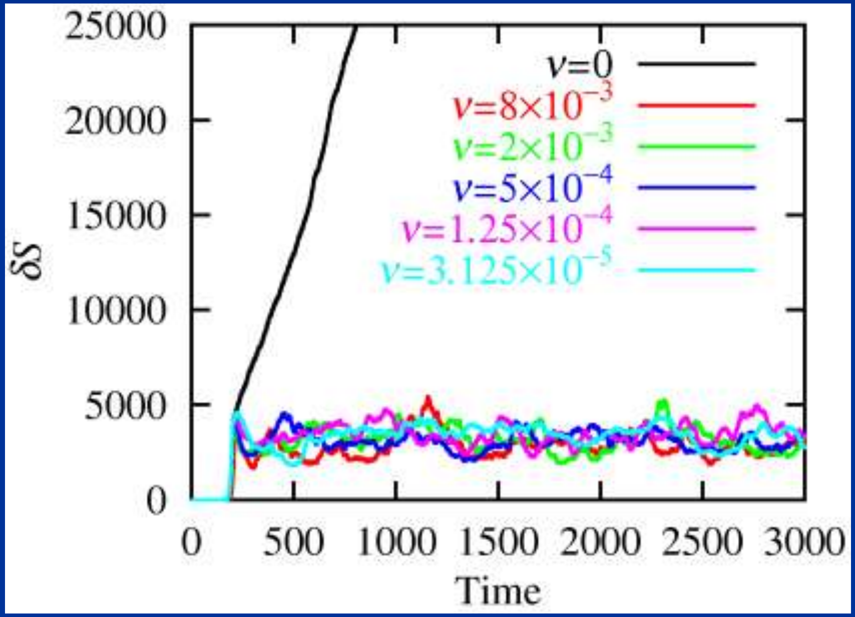
- Source terms with constant n and T_i gradients drive the drift wave turbulence producing the entropy variable.
- The entropy balance equation relates δS with the transport flux and collisionnal dissipation.

Entropy Balance in Simulations

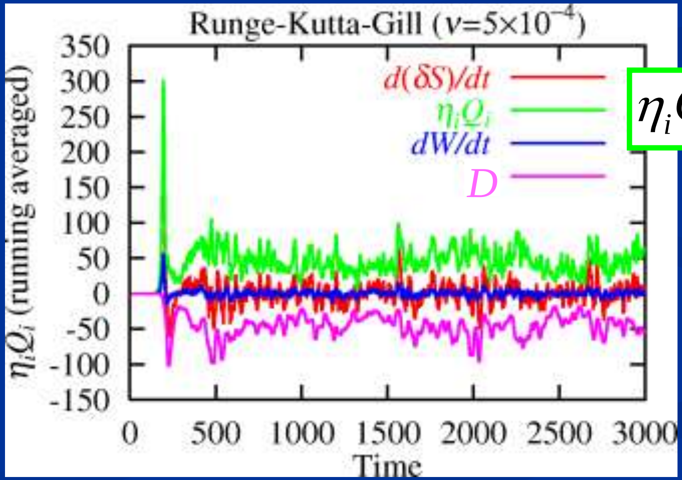
Collisionless (quasisteady)



$$\frac{d}{dt}\delta S \approx \eta_i Q_i$$



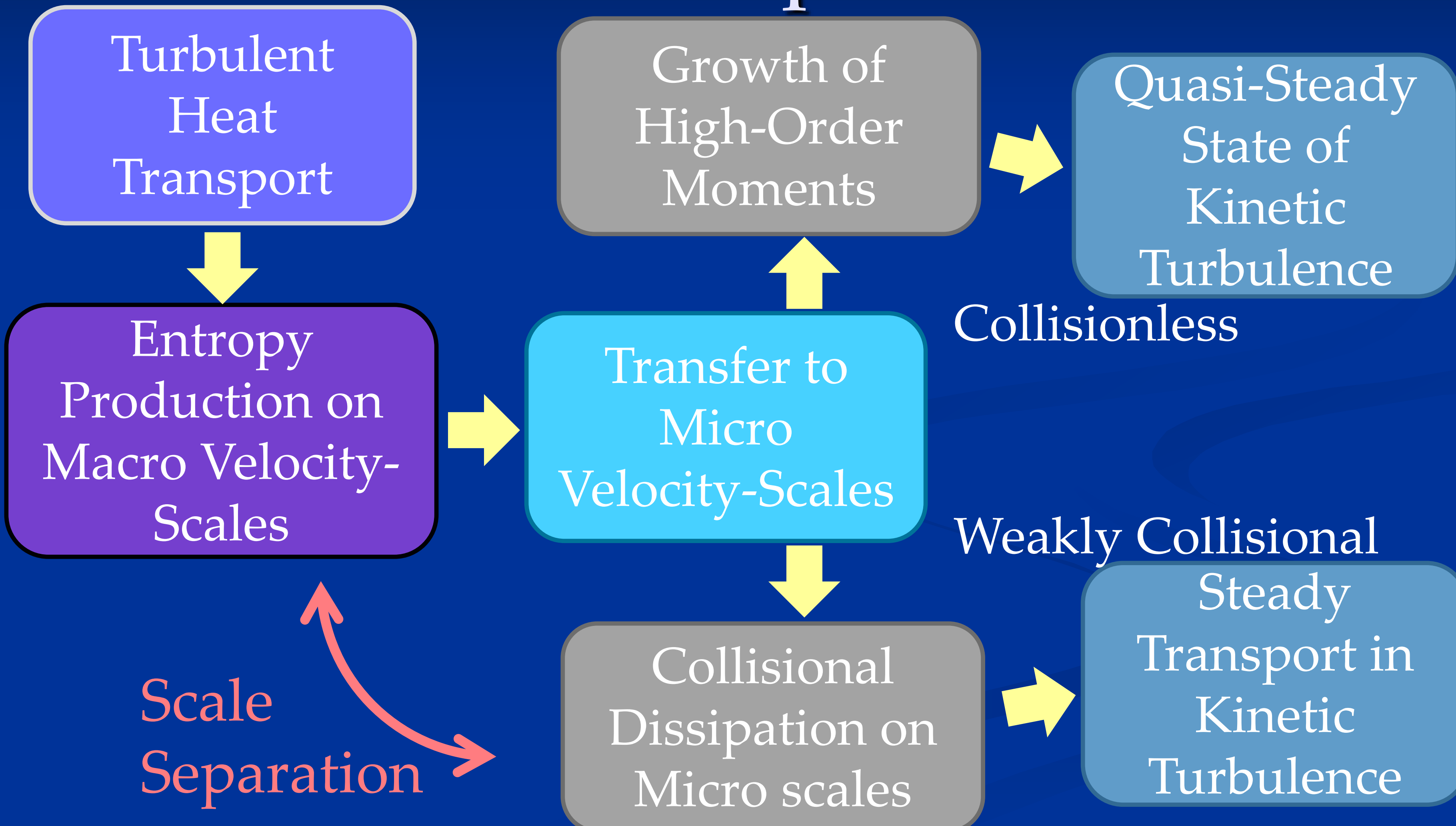
Weakly-Collisional (statistically steady)



$$\eta_i Q_i \approx -D$$

(Lenard-Berstein collision model)

Entropy Production, Transfer, and Dissipation



Spectral analysis of the distribution function

Hermite Spectral Eq. for δS

- Hermite Polynomial Expansion

$$\delta S_n \equiv \sum_{\mathbf{k}} \delta S_{\mathbf{k},n} \equiv \sum_{\mathbf{k}} \frac{1}{2} n! |\hat{f}_{\mathbf{k},n}|^2,$$

$$\tilde{f}_{\mathbf{k}}(v_\parallel) = \sum_{n=0}^{\infty} \hat{f}_{\mathbf{k},n} H_n(v_\parallel) F_M(v_\parallel).$$

$$H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n}(e^{-x^2/2})$$

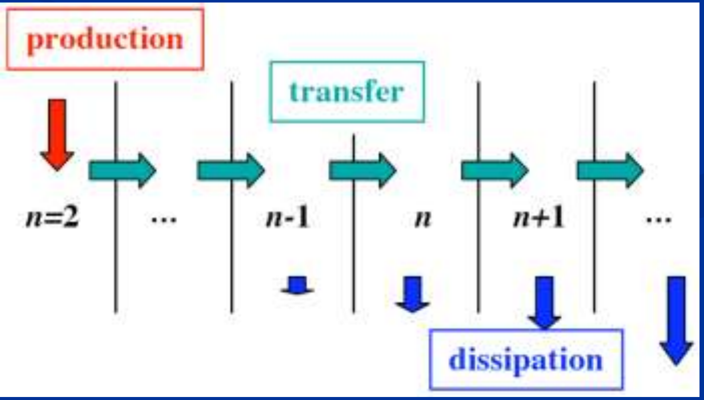
- Hermite Spectral eq. of the entropy variable density δS_n

$$\frac{d}{dt}\left[\delta S_n + \delta_{n,1} \frac{1}{2} \sum_{\mathbf{k}} |\phi_{\mathbf{k}}|^2 \{2 - \Gamma_0(b_{\mathbf{k}})\}\right]$$
$$= J_{n-1/2} - J_{n+1/2} + \delta_{n,2} \eta_i Q_i - 2\nu n\, \delta S_n,$$

- Transfer function for δS_n in n

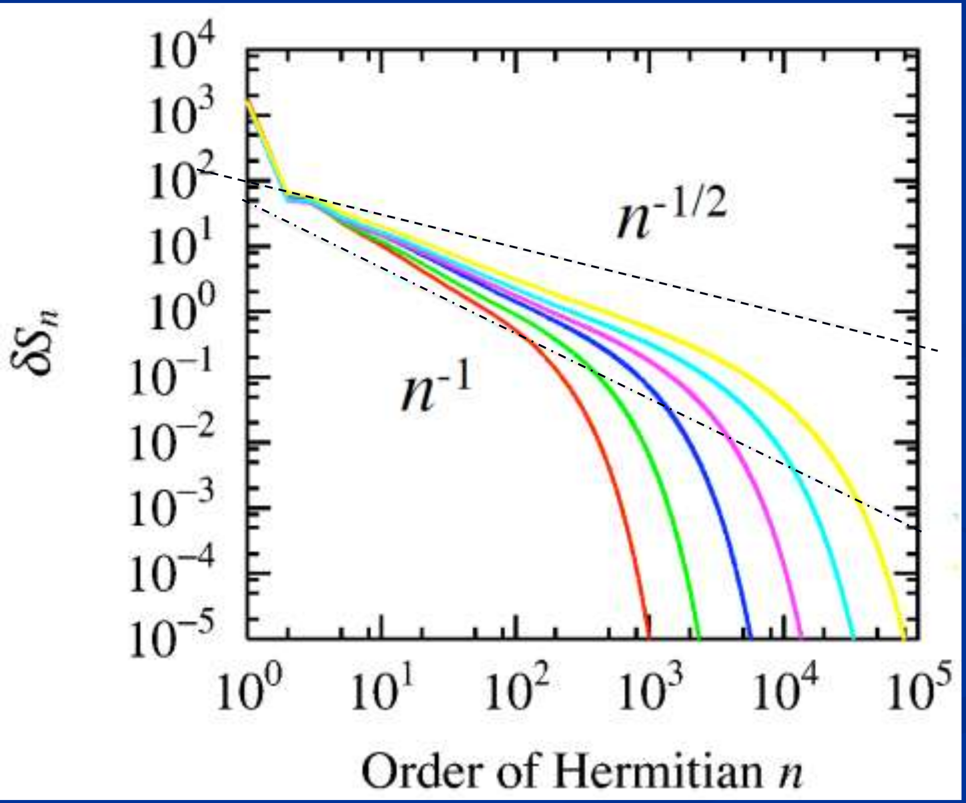
$$J_{n-1/2} \equiv \sum_{\mathbf{k}} \Theta k_y n! \,\mathrm{Im}(\hat{f}_{\mathbf{k},n-1} \hat{f}^*_{\mathbf{k},n}),$$

$$J_{n+1/2} \equiv \sum_{\mathbf{k}} \Theta k_y (n+1)! \,\mathrm{Im}(\hat{f}_{\mathbf{k},n} \hat{f}^*_{\mathbf{k},n+1}),$$



δS_n Spectrum

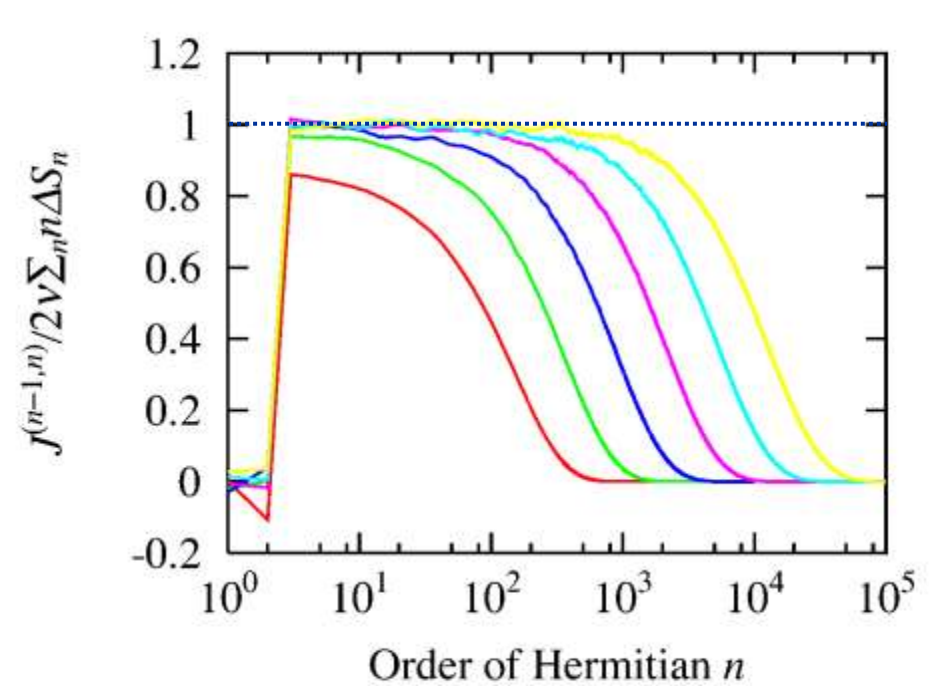
- Spectrum of the entropy variable density



ν is changed from 0.002 to 0.002/1024 by factor of 1/4.

"Inertial Sub-range" with Constant Transfer Function J_n

- Flux function of the entropy variable density in the n-space

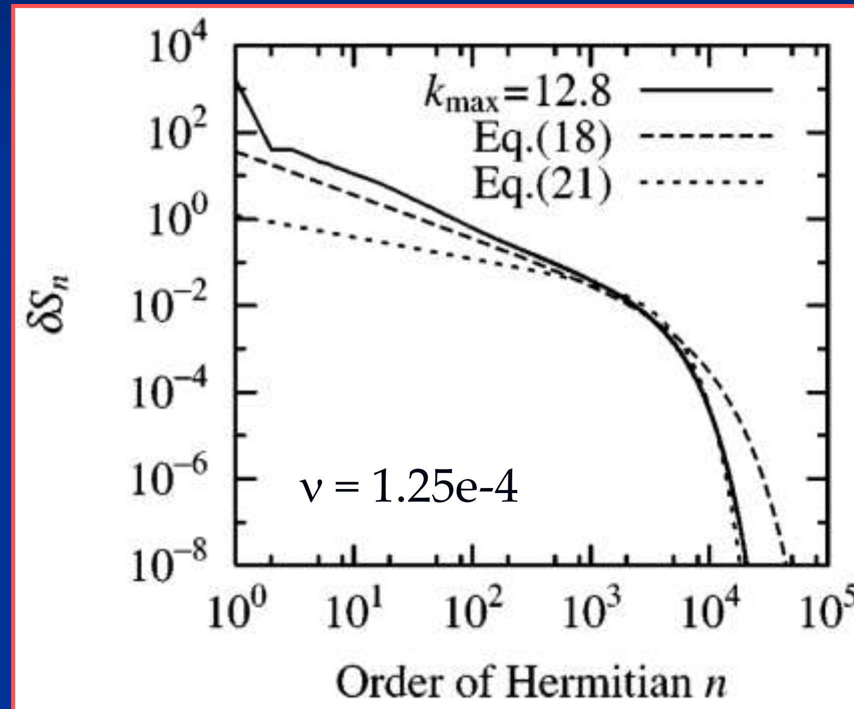


ν is changed from 0.002 to 0.002/1024 by factor of 1/4.

- J is constant in a dissipation-free range where δS_n is transferred, that is, the "inertial sub-range".

Limiting Form of Entropy Variable Spectrum

- A theoretical model for δS_n
- Mixing theory of a passive scalar in the homogeneous isotropic turbulence
  - Large Prandtl number
  - Short wave-length regime (<< Kolmogolov scale)
- Effective wavenumber of δf is increased by turbulence ($k \sim n^{0.5} \sim l$).
- Finite resolution in the real space is taken into account.

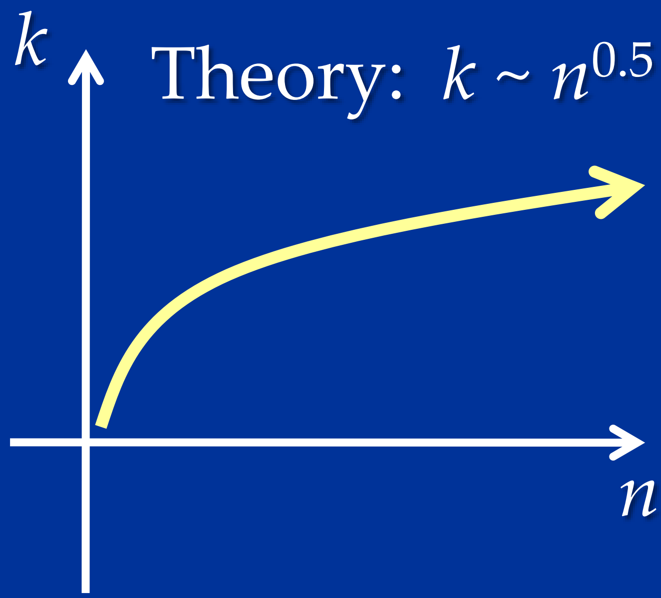


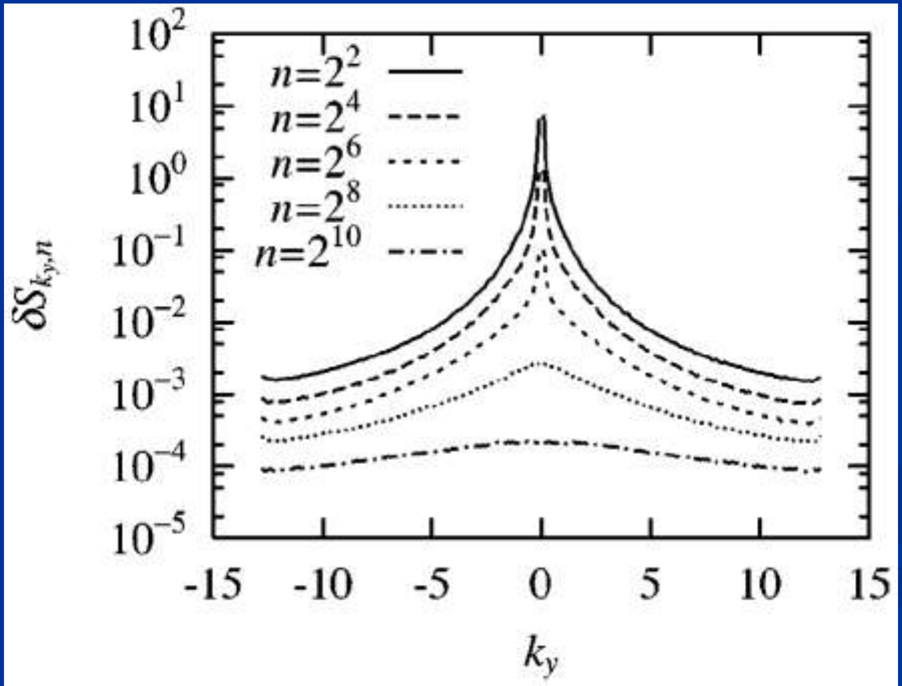
$$\delta S_n = \frac{\sigma}{2\gamma n}\exp\left(-\frac{\nu n}{\gamma}\right),$$

$$\delta S_n = \frac{\sigma}{2\gamma_M\sqrt{n}}\exp\left(-\frac{2}{3}\frac{\nu n^{3/2}}{\gamma_M}\right)$$

Profiles of $\delta S_{k,n}$

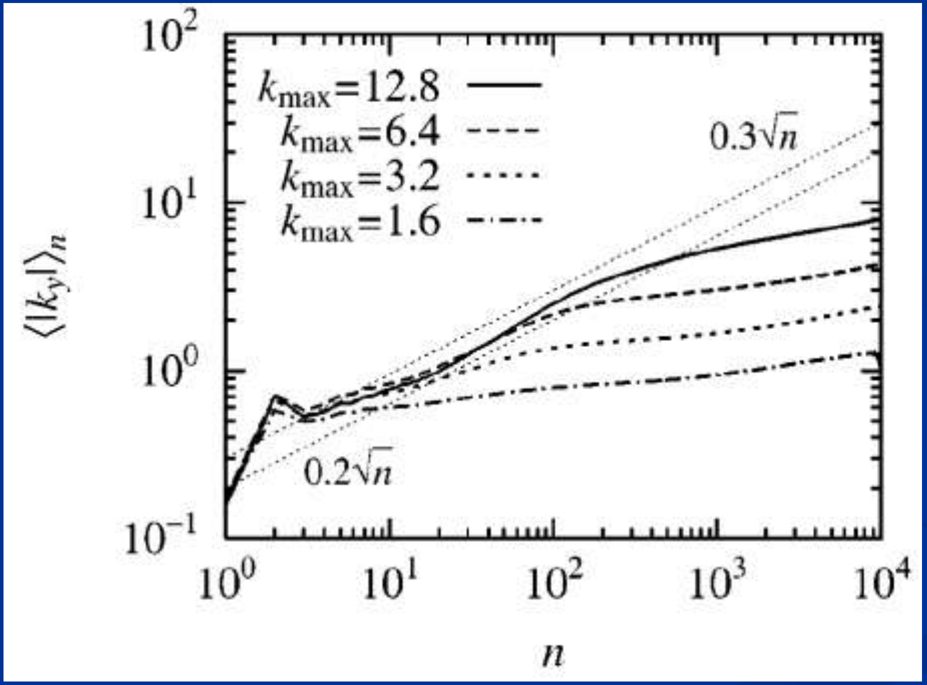
- $\delta S_{k,n}$ reaches to the maximum k (especially for high-n) due to the turbulent cascade (or stretching).





Growth of $\langle k_y \rangle$ in turbulence

- Effective wavenumber $\langle k_y \rangle$ is roughly proportional to $n^{1/2}$ (thus, ~ l) for large k_{max}.



$$\langle \cdot \rangle_n \equiv \left(\Sigma_{\mathbf{k}} |\hat{f}_{\mathbf{k},n}|^2 \cdot\right) / \left(\Sigma_{\mathbf{k}} |\hat{f}_{\mathbf{k},n}|^2\right)$$

Relation to kinetic-fluid closure models

Collisionless fluid equations

$$\partial_t n_{\mathbf{k}} + i k_{\|} n_0 u_{\mathbf{k}} - i\omega_{*i} n_0 \left(1 - \frac{b_{\mathbf{k}}}{2}\eta_i\right)\frac{e\Psi_{\mathbf{k}}}{T_i}$$

$$-\frac{c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} [\mathbf{b}\cdot(\mathbf{k}'\times\mathbf{k}'')]\Psi_{\mathbf{k}'} n_{\mathbf{k}''} = 0,$$

$$n_0 m_i \partial_t u_{\mathbf{k}} + i k_{\|}(T_i n_{\mathbf{k}} + n_0 T_{\mathbf{k}} + n_0 e\Psi_{\mathbf{k}})$$

$$-\frac{n_0 m_i c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} [\mathbf{b}\cdot(\mathbf{k}'\times\mathbf{k}'')]\Psi_{\mathbf{k}'} u_{\mathbf{k}''} = 0,$$

$$n_0 \partial_t T_{\mathbf{k}} + i k_{\|}(2 n_0 T_i u_{\mathbf{k}} + q_{\mathbf{k}}) - i \ldots$$

$$-\frac{n_0 c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} [\mathbf{b}\cdot(\mathbf{k}'\times \ldots$$

where $n_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_{\|} f_{\mathbf{k}}(v_{\|},t)$, $n_0 u_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_{\|} \times f_{\mathbf{k}}(v_{\|},t) v_{\|}$, $n_0 T_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_{\|} f_{\mathbf{k}}(v_{\|},t)(m_i v_{\|}^2 - T_i)$, and $q_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_{\|} f_{\mathbf{k}}(v_{\|},t)(m_i v_{\|}^3 - 3T_i v_{\|})$. Here all nonlinear terms result from the $\mathbf{E}\times\mathbf{B}$ drift.

- Fluid equations derived from the kinetic ones by taking velocity-space moments.
- The parallel heat flux q_k is taken into account.

$$\exp(-b_{\mathbf{k}}/2) n_{\mathbf{k}} - n_0 \frac{e\phi_{\mathbf{k}}}{T_i}[1 - \Gamma_0(b_{\mathbf{k}})] = \frac{e\phi_{\mathbf{k}}}{T_e} \quad (\text{for } k_{\|} \neq 0),$$

Fluid & Kinetic Entropy Balance

- From the fluid equations,

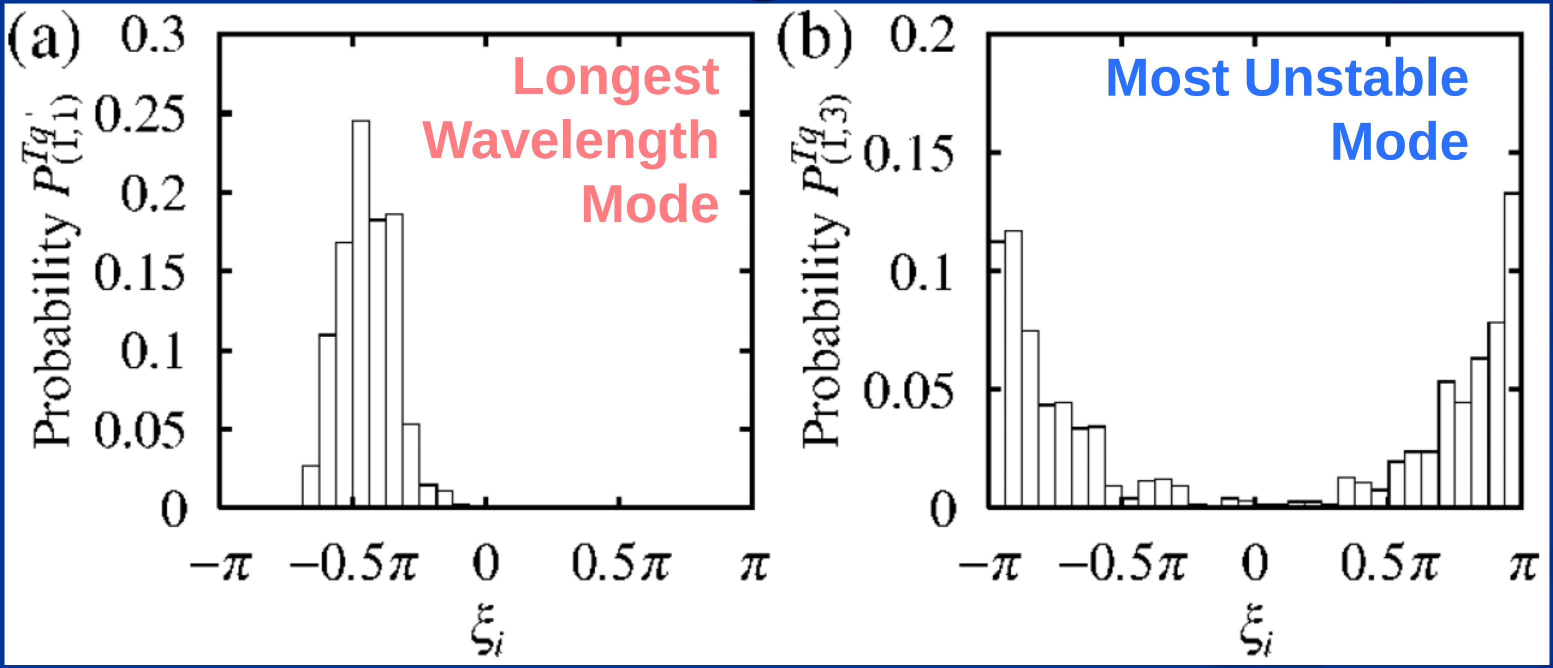
$$\frac{d}{dt}\sum_{\mathbf{k}} n_0\left(\frac{1}{2}\left|\frac{n_{\mathbf{k}}}{n_0}\right|^2+\frac{1}{2}\left|\frac{u_{\mathbf{k}}}{v_t}\right|^2+\frac{1}{4}\left|\frac{T_{\mathbf{k}}}{T_i}\right|^2+\frac{T_e}{2T_i}\left|\frac{e\phi_{\mathbf{k}}}{T_e}\right|^2\left[1+\frac{T_e}{T_i}\{1-\Gamma_0(b_{\mathbf{k}})\}\right]\right)$$
$$=\frac{\mathbf{q}_\perp}{T_i}\cdot(-\nabla\ \ln T_i)+\sum_{\mathbf{k}}\mathrm{Re}\left(\frac{T_{\mathbf{k}}}{2T_i^2}ik_\parallel q_{\mathbf{k}}^*\right),$$

- From the kinetic equation and its Hermite expansion,

$$\frac{d}{dt}\sum_{\mathbf{k}}\left(\int dv_\parallel\frac{|f_{\mathbf{k}}|^2}{2F_M}+\frac{n_0T_e}{2T_i}\left|\frac{e\phi_{\mathbf{k}}}{T_e}\right|^2\left[1+\frac{T_e}{T_i}\{1-\Gamma_0(b_{\mathbf{k}})\}\right]\right)$$
$$=\frac{\mathbf{q}_\perp}{T_i}\cdot(-\nabla\ \ln T_i).$$
$$=\frac{\mathbf{q}_\perp}{T_i}\cdot(-\nabla\ \ln T_i),$$

Phase angle ξ between $T_{\mathbf{k}}$ & $q_{\mathbf{k}}$

- Kinetic simulation results provide useful information for making a closure model



$$P^{Tq}_{\mathbf{k}}(\xi_i) \equiv \int_{t_1}^{t_2} dt \int_{\xi_i-\Delta\xi/2}^{\xi_i+\Delta\xi/2} d\xi_{\mathbf{k}} A_{\mathbf{k}}(\xi_{\mathbf{k}}) \Big/ \int_{t_1}^{t_2} dt \int_{-\pi}^{\pi} d\xi_{\mathbf{k}} A_{\mathbf{k}}(\xi_{\mathbf{k}}),$$

where $A_{\mathbf{k}}(\xi_{\mathbf{k}}) = |T_{\mathbf{k}}||q_{\mathbf{k}}|$, $\Delta\xi = \pi/16$, and $\xi_i = -\pi + (i+\frac{1}{2})\Delta\xi$

Zonal flow response in toroidal systems from a point of view of the entropy transfer

GK eqs. for toroidal flux tube

- GK ordering + Flute Reduction + Periodic (*x*,*y*)

$$\left[\frac{\partial}{\partial t}+v_{\|}\hat{\mathbf{b}}\cdot\nabla+\mathbf{v}_d\cdot\nabla-\mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)\frac{\partial}{\partial v_{\|}}\right]\delta f+\frac{c}{B_0}\{\psi,\delta f\}=\left(\mathbf{v}_*-\mathbf{v}_d-v_{\|}\hat{\mathbf{b}}\right)\cdot\frac{e\nabla\psi}{T_i}F_M+C(\delta f)$$

- Co-centric & Circular Flux Surface with Constant Shear and Gradients (for tokamak)

$$\mathbf{v}_d\cdot\nabla=-\frac{v_{\|}^2+\Omega_0\mu}{\Omega_0 R_0}\left[\left(\cos\theta+\hat{s}\theta\sin\theta\right)\frac{\partial}{\partial y}+\sin\theta\frac{\partial}{\partial x}\right],$$

$$\mathbf{v}_*=-\frac{cT_i}{eL_nB_0}\left[1+\eta_i\left(\frac{mv^2}{2T_i}-\frac{3}{2}\right)\right]\hat{\mathbf{y}},\mu=\frac{v_\perp^2}{2\Omega}$$

- Quasi-Neutrality + Adiabatic Electron

$$\int J_0(k_\perp v_\perp/\Omega)\delta f\,\mathrm{d}^3v-\left[1-\Gamma_0\left(k_\perp^2\right)\right]\frac{e\phi}{T_i}=\frac{e}{T_e}\left(\phi-\langle\phi\rangle_{\mathrm{FS}}\right),k_\perp^2=\left(k_x+\hat{s}\theta k_y\right)^2+k_y^2$$

Collisionless Damping of Zonal Flow in Tokamak

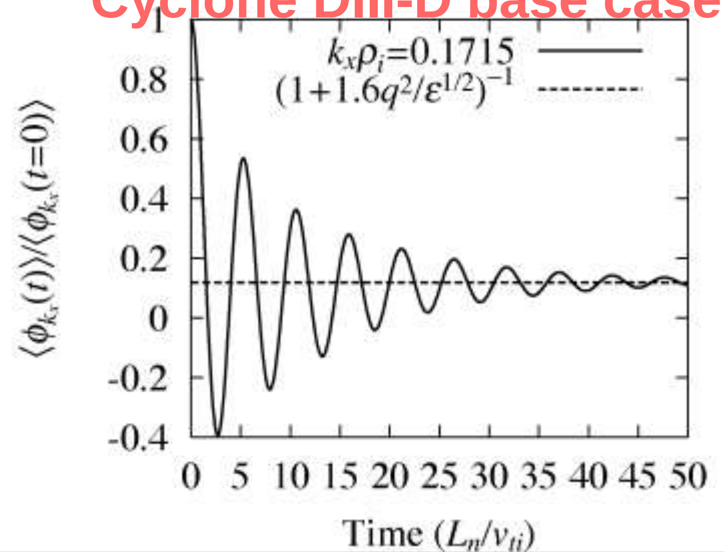
- Consider the linearized GK equation for the zonal flow component of n=0.
- Initial value problem for n=0 mode with $\delta f(t=0)=F_M$
- The residual zonal flow is important to regulating turbulent transport

(Rosenbluth & Hinton, 1998)

Residual Zonal Flow (response kernel)

$$K = \frac{\langle \phi_{k_x}(t=\infty) \rangle}{\langle \phi_{k_x}(t=0) \rangle} \approx \frac{1}{1+1.6q^2/\varepsilon^{1/2}}$$

Cyclone DIII-D base case



Conservation Law for the Zonal Flow (n=0) Components

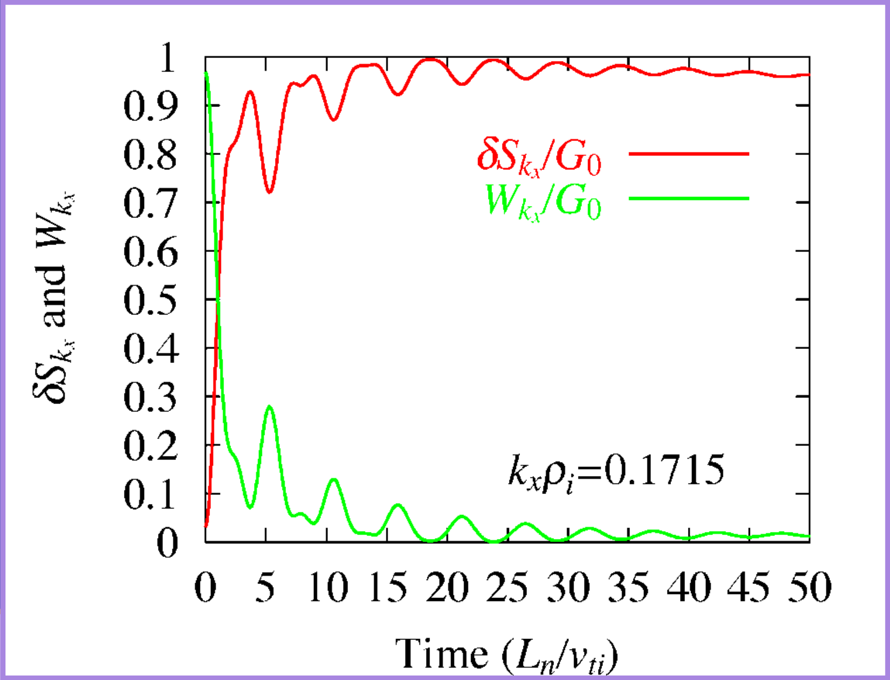
- From the gyrokinetic equation for k_y=0,

$$\boxed{\frac{dG}{dt} \equiv \frac{d}{dt}\left(\delta S_{k_x} + W_{k_x}\right) = 0}$$

… Subset of the entropy balance equation

$$\delta S = \left\langle \int d^3v \left|\tilde{f}_{k_x}\right|^2 \Big/ 2F_M \right\rangle$$

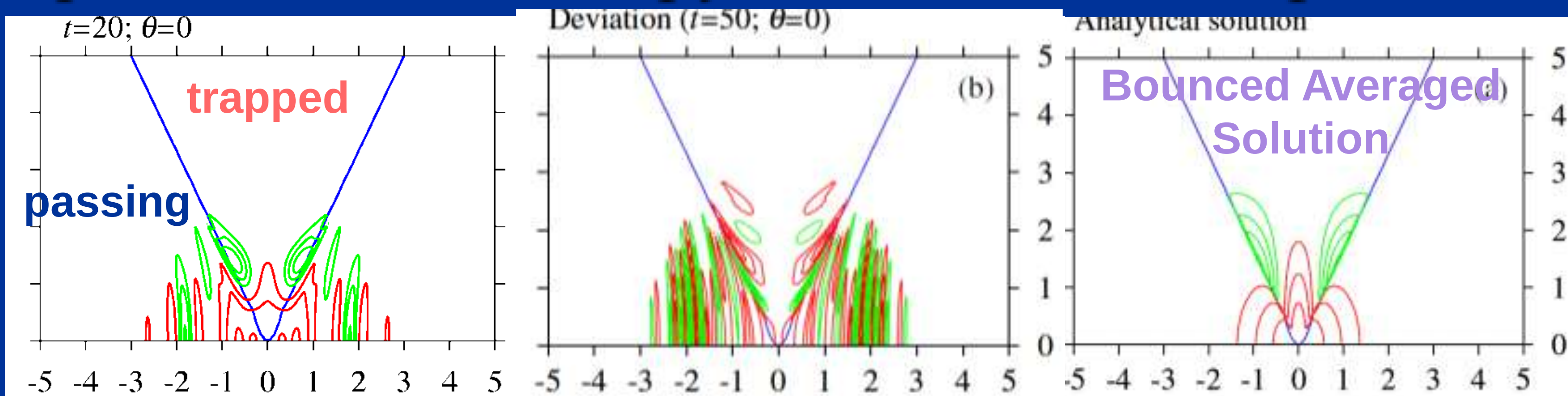
$$W = \frac{1}{2}\left\langle \left(1 - \Gamma_0(k^2) + \frac{T_i}{T_e}\right) \left|\Phi_{k_x}\right|^2 \right\rangle - \frac{T_i}{2T_e}\left|\left\langle \Phi_{k_x} \right\rangle\right|^2$$



Entropy variable δS increases during the zonal flow damping.

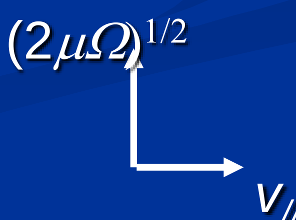
Velocity-Space Structures of f during the ZF Damping

- Decrease of W_{kx} with the invariant G means increase of δS_{kx} as well as generation of fine-scale structures of f due to phase mixing by passing particles <=> Entropy transfer in the v-space



- Coherent structures for trapped particles

  <=> Neoclassical Polarization



Collisionless Response of Zonal Flows in Helical System

- Radial drift motion of helical-ripple-trapped particles causes additional polarization effect of the zonal flow potential due to the phase mixing, and influences its response function.

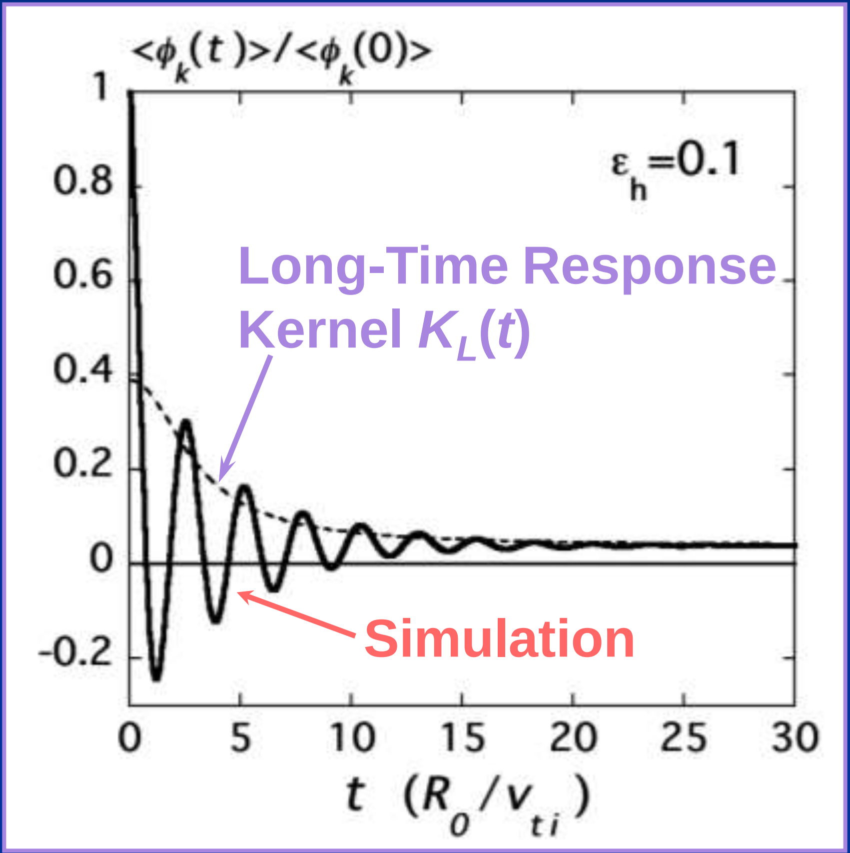
Long-time Response Function for the initial Maxwellian

$$\boxed{\mathcal{K}_L(t) \equiv \frac{1-(2/\pi)\langle(2\epsilon_H)^{1/2}\{1-g_{i1}(t,\theta)\}\rangle}{1+G+\mathcal{E}(t)/\left(n_0\langle k_\perp^2 a_i^2\rangle\right)}}$$

$$\mathcal{E}(t) = \left\langle \int_{\kappa^2<1} d^3 v F_{i0} J_0^2 (1 - e^{-ik_r \bar{v}_{dri} t}) \right\rangle + \frac{T_i}{T_e} \left\langle \int_{\kappa^2<1} d^3 v F_{e0} (1 - e^{-ik_r \bar{v}_{dre} t}) \right\rangle$$

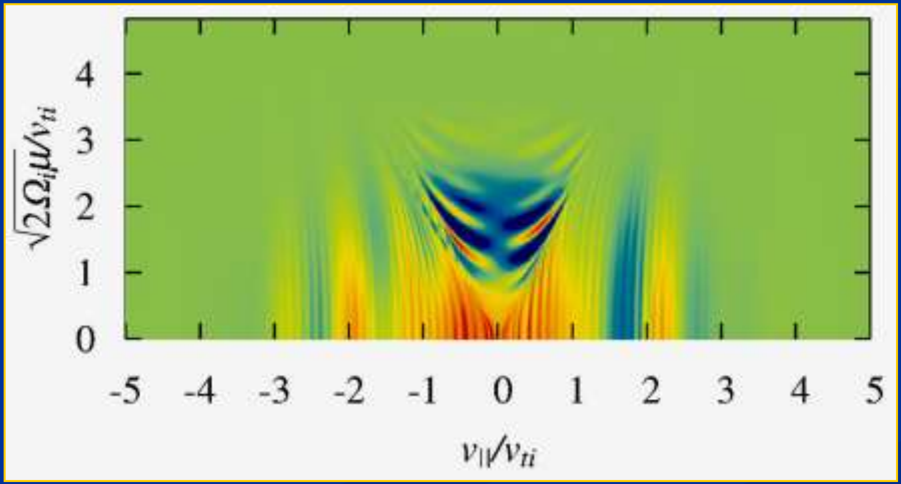
Simulation of Zonal Flow Damping in Helical Systems

(L = 2, M = 10)



(q = 1.5, ε_t = 0.1, $k_r\, a_i$ =0.131)

- Radial drift of helical-ripple-trapped particles is found as well as the ballistic-like motion of passing particles.



Velocity distribution function for θ=8π/13 at t = 6.23 R_0/v_{ti}.

Entropy balance in toroidal ITG/ETG turbulence

Entropy Balance Eq, Again

$$\frac{\partial}{\partial t}\sum_{\mathbf{k}_\perp}\left[\sum_a T_a\left\langle\!\!\left\langle\int d^3v\frac{|\delta f_{a\mathbf{k}_\perp}|^2}{2f_{aM}}\right\rangle\!\!\right\rangle+\frac{1}{8\pi}\langle\langle|\mathbf{E}_{\mathbf{k}_\perp}|^2+|\mathbf{B}_{\mathbf{k}_\perp}|^2\rangle\rangle\right]$$

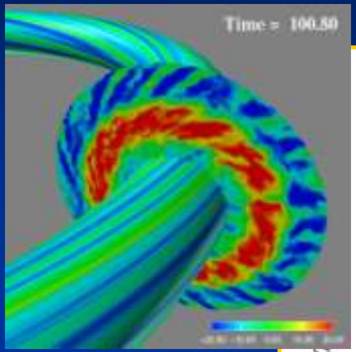
$$=\sum_a T_a(J^A_{a1}X^A_{a1}+J^A_{a2}X^A_{a2})$$

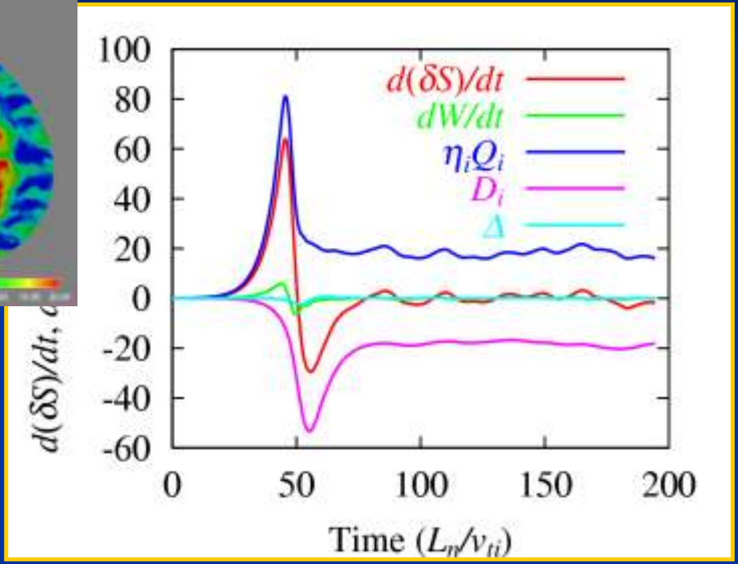
$$+\sum_{\mathbf{k}_\perp}\sum_{a,b}T_a\left\langle\!\!\left\langle\int d^3v\frac{\delta f^*_{a\mathbf{k}_\perp}}{f_{aM}}C^L_{ab}(\delta f_{a\mathbf{k}_\perp},\delta f_{b\mathbf{k}_\perp})\right\rangle\!\!\right\rangle.$$

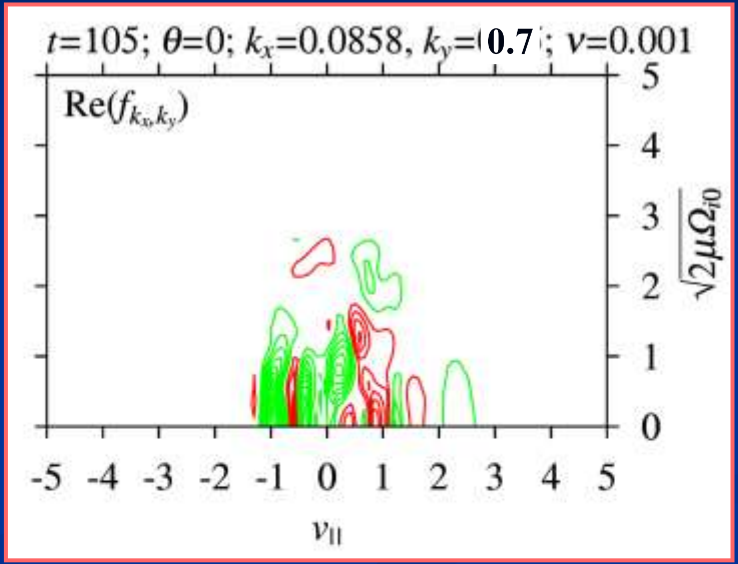
$$[J^A_{a1},J^A_{a2}]\equiv\left[\Gamma^A_a,\frac{q^A_a}{T_a}\right]\equiv\mathrm{Re}\left\langle\!\!\left\langle\int d^3v\left[1,\left(x_a^2-\frac{5}{2}\right)\right]\times\sum_{\mathbf{k}_\perp}h^*_{a\mathbf{k}_\perp}\left(-i\frac{c}{B}\psi_{a\mathbf{k}_\perp}\mathbf{k}_\perp\times\mathbf{b}\right)\cdot\nabla s\right\rangle\!\!\right\rangle$$

$$[X^A_{a1},X^A_{a2}]\equiv\left[-\frac{\partial\ln p_a}{\partial s}-\frac{e_a}{T_a}\frac{\partial\Phi}{\partial s},-\frac{\partial\ln T_a}{\partial s}\right]$$

Entropy Balance in Tokamak ITG Turbulence







- The steady and constant transport flux is obtained with satisfying the entropy balance, which enables one to accurately evaluate χ_i.
- The perturbed distribution function shows fine velocity-space structures far from the Maxwellian.

Entropy Transfer to ZFs

- Consider

$$\sum_{\mathbf{k}_\perp} = \sum_{\mathbf{k}_\perp(\mathrm{Z})} + \sum_{\mathbf{k}_\perp(\mathrm{NZ})}$$

- Self-adjointness of the collision operator guarantees the collisional dissipation $D_i < 0$ for each k.
- Thus, the entropy transfer function $T(\mathrm{NZ}\text{->}\mathrm{Z}) > 0$ in the statistically steady turbulence

(Sugama, Watanabe & Nunami, 2009)

$$\frac{\partial}{\partial t}\sum_{\mathbf{k}_\perp(\mathrm{NZ})}\left[\sum_a T_a\left\langle\!\left\langle\int d^3v\frac{|\delta f_{a\mathbf{k}_\perp}|^2}{2f_{aM}}\right\rangle\!\right\rangle + \frac{1}{8\pi}\langle\langle|\mathbf{E}_{\mathbf{k}_\perp}|^2 + |\mathbf{B}_{\mathbf{k}_\perp}|^2\rangle\rangle\right]$$
$$= \sum_a T_a(J_{a1}^A X_{a1}^A + J_{a2}^A X_{a2}^A) - \mathcal{T}(\mathrm{NZ}\to\mathrm{Z})$$
$$+ \sum_{\mathbf{k}_\perp(\mathrm{NZ})}\sum_{a,b} T_a\left\langle\!\left\langle\int d^3v\frac{\delta f^*_{a\mathbf{k}_\perp}}{f_{aM}}C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp})\right\rangle\!\right\rangle, \quad (67)$$

and

$$\frac{\partial}{\partial t}\sum_{\mathbf{k}_\perp(\mathrm{Z})}\left[\sum_a T_a\left\langle\!\left\langle\int d^3v\frac{|\delta f_{a\mathbf{k}_\perp}|^2}{2f_{aM}}\right\rangle\!\right\rangle + \frac{1}{8\pi}\langle\langle|\mathbf{E}_{\mathbf{k}_\perp}|^2 + |\mathbf{B}_{\mathbf{k}_\perp}|^2\rangle\rangle\right]$$
$$= \mathcal{T}(\mathrm{NZ}\to\mathrm{Z}) + \sum_{\mathbf{k}_\perp(\mathrm{Z})}\sum_{a,b} T_a$$
$$\times\left\langle\!\left\langle\int d^3v\frac{\delta f^*_{a\mathbf{k}_\perp}}{f_{aM}}C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp})\right\rangle\!\right\rangle, \quad (68)$$

Entropy Transfer Function

- Entropy transfer from non-zonal to zonal modes

$$\mathcal{T}(\mathrm{NZ}\rightarrow\mathrm{Z}) \equiv \sum_a T_a \left\langle\left\langle \frac{c}{B} \sum_{\mathbf{k}_\perp(\mathrm{Z})} \sum_{\mathbf{k}'_\perp(\mathrm{NZ})} \sum_{\mathbf{k}''_\perp(\mathrm{NZ})} \delta_{\mathbf{k}'_\perp+\mathbf{k}''_\perp,\mathbf{k}} \left[\mathbf{b}\cdot(\mathbf{k}'_\perp\times\mathbf{k}''_\perp)\right] \int d^3v \frac{1}{f_{aM}} \mathrm{Re}[\psi_{a\mathbf{k}'_\perp} h_{a\mathbf{k}''_\perp} h^*_{a\mathbf{k}_\perp}] \right\rangle\right\rangle$$

- In the fluid limit with cold ions,

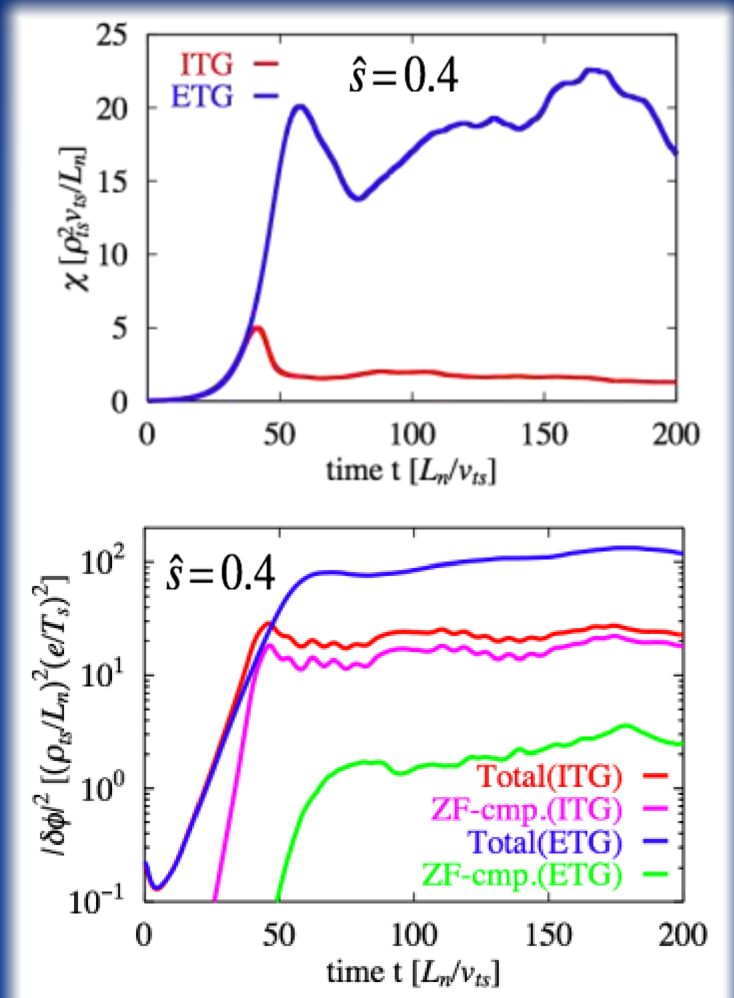
$$\mathcal{T}(\mathrm{NZ}\rightarrow\mathrm{Z}) \simeq \left\langle\left\langle \frac{n_0 m_i c^3}{2B^3} \sum_{\mathbf{k}_\perp(\mathrm{Z})} \sum_{\mathbf{k}'_\perp(\mathrm{NZ})} \sum_{\mathbf{k}''_\perp(\mathrm{NZ})} \delta_{\mathbf{k}'_\perp+\mathbf{k}''_\perp,\mathbf{k}} [\mathbf{b}\cdot(\mathbf{k}'_\perp\times\mathbf{k}''_\perp)][(k''_\perp)^2-(k'_\perp)^2]\mathrm{Re}[\phi_{\mathbf{k}'_\perp}\phi_{\mathbf{k}''_\perp}\phi^*_{\mathbf{k}_\perp}] \right\rangle\right\rangle$$

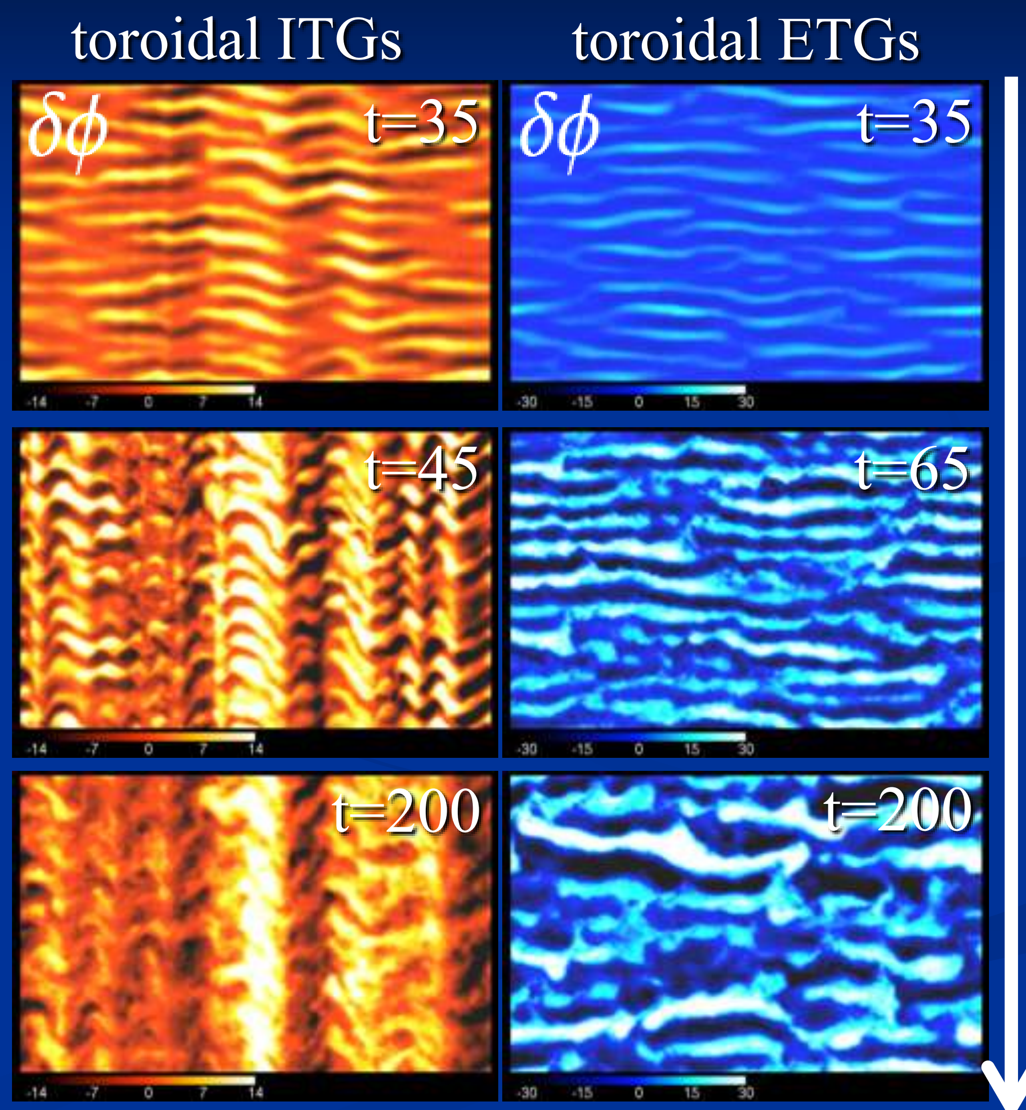
$$= \left\langle\left\langle \sum_{\mathbf{k}_\perp(\mathrm{Z})} \sum_{\mathbf{k}'_\perp(\mathrm{NZ})} \sum_{\mathbf{k}''_\perp(\mathrm{NZ})} \delta_{\mathbf{k}'_\perp+\mathbf{k}''_\perp,\mathbf{k}}\ \mathrm{Re}[\mathbf{v}_{E\mathbf{k}'_\perp}\mathbf{v}_{E\mathbf{k}''_\perp} : (i\mathbf{k}_\perp \mathbf{v}_{E\mathbf{k}_\perp})^*] \right\rangle\right\rangle$$

represents product of the Reynolds stress due to the non-zonal ExB drift velocity and the zonal ExB flow shear.

ITG and ETG turbulence simulations

- Time evolutions of heat diffusivity and potential fluctuation





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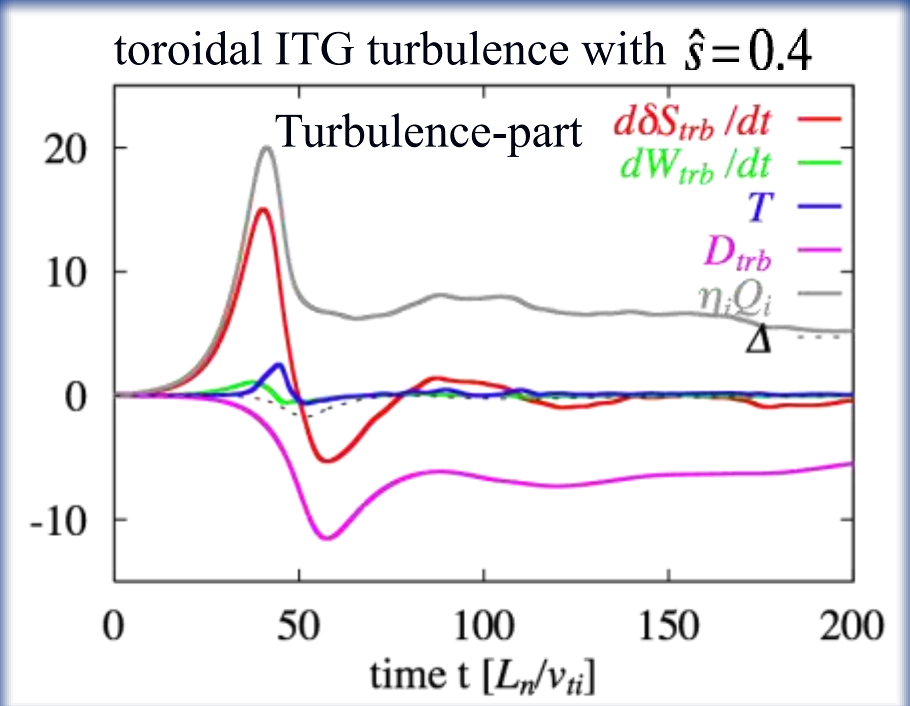
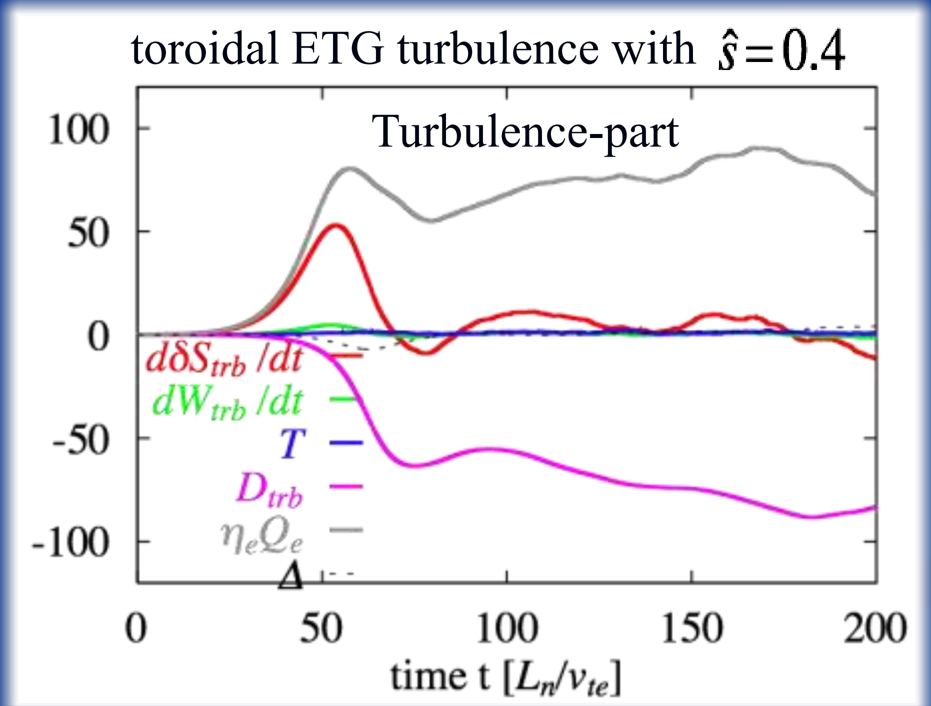
Entropy balance relation: Turbulence part

- Comparison of entropy balance in toroidal ITG and ETG turbulence

$$\frac{d}{dt}(\delta S_{\mathrm{trb}} + W_{\mathrm{trb}}) = \eta_s Q_s + D_{\mathrm{trb}} - T_{(\mathrm{trb}\to\mathrm{zf})}, \quad \boxed{\overline{\eta_s Q_s} - \overline{T_{(\mathrm{trb}\to\mathrm{zf})}} = -\overline{D_{\mathrm{trb}}}}$$





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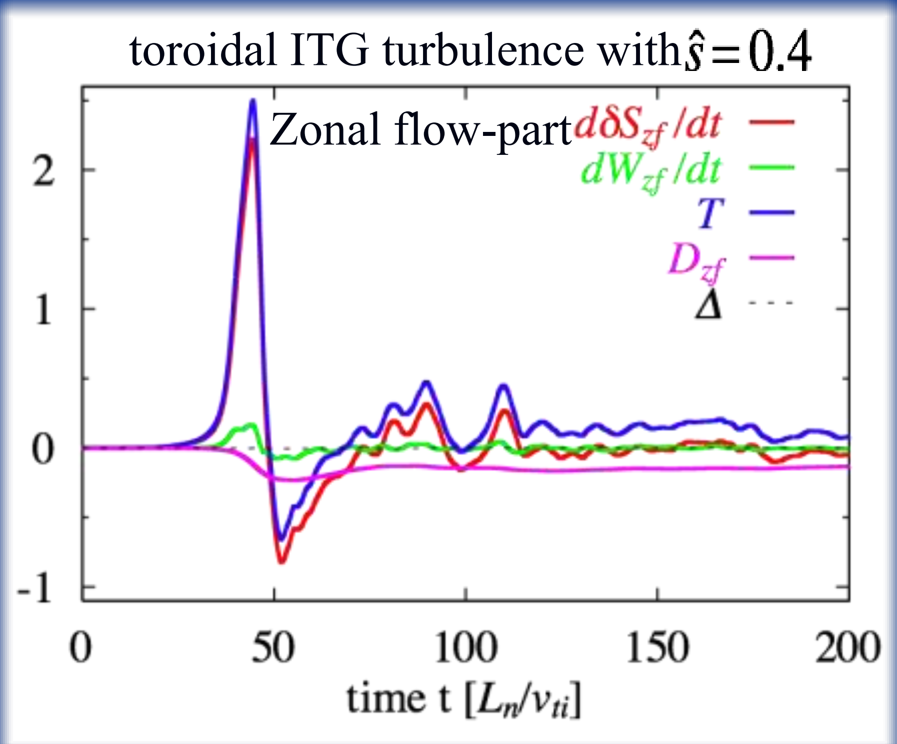
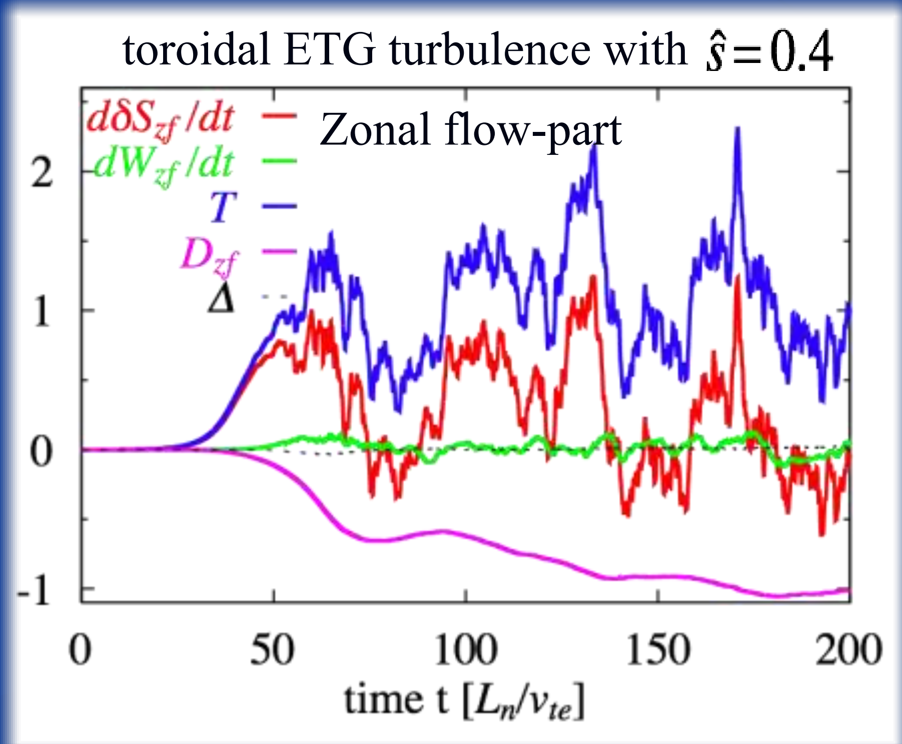
Entropy balance relation: Zonal flow part

- Comparison of entropy balance in toroidal ITG and ETG turbulence.

$$\frac{d}{dt}(\delta S_{\rm zf} + W_{\rm zf}) = T_{(\rm trb\to zf)} + D_{\rm zf}, \quad \boxed{\overline{T_{(\rm trb\to zf)}} = -\overline{D_{\rm zf}} > 0}$$





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Summary

- We discussed entropy production, transfer, and dissipation processes in plasma turbulence and zonal flows.
  - Entropy balance eq. describes transfer of the entropy variable in the phase space, and provides us a good measure for the steady and quasi-steady states of plasma turbulent transport and zonal flow dynamics.
  - Entropy transfer from macro to micro scales through the "inertial sub-range" is related to generation of fine-scale structures of f by shear operators.
  - ZF damping and generation are described as entropy transfer processes in the phase space (l- and k-spaces, respectively).