GROKINETICS IN LABORATORY AND ASTROPHYSICAL PLASMAS
Workshop "Kinetic-scale turbulence in laboratory and space plasmas: emprical constraints, fundamental concepts and unsolved problems"
Issac Newton Institute for Mathematical Sciences, Cambridge, July 23, 2010

Effects of Three-Dimensional Geometry and Radial Electric Field on ITG Turbulence and Zonal Flows

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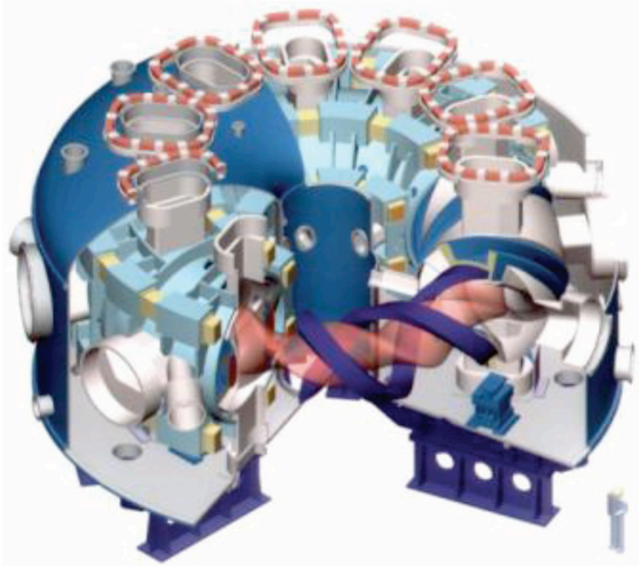
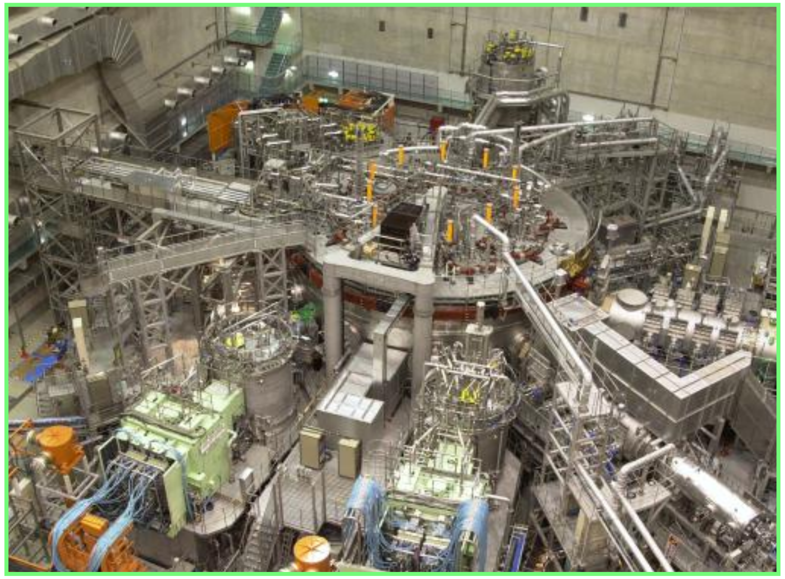
Introduction

Large Helical Device (LHD)





Heliotron configuration

No net plasma current required

➡ Suitable for steady-state operation

R = 3.9 m

a = 0.6 m

V = 30 m^3

B = 3 ~ 4 T

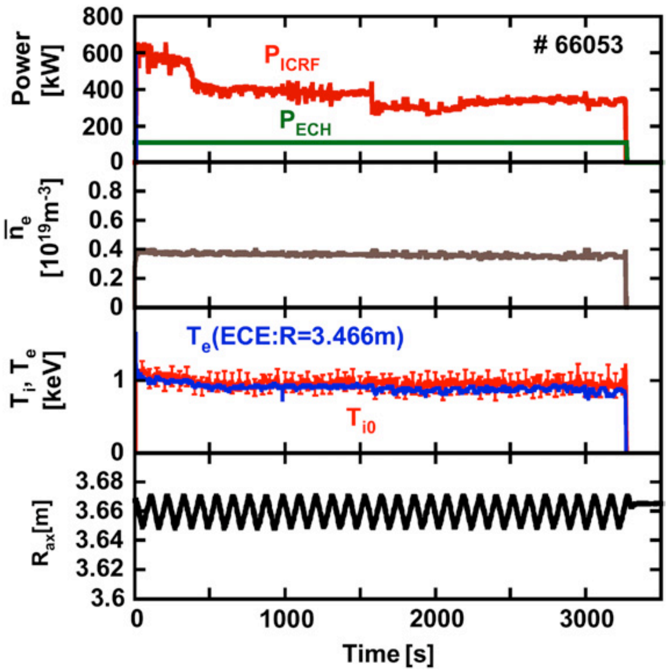
Max. parameters

n = 1.1 x 10^{21} m^{-3}

T_e = 15 keV

T_H = 5.2 keV

<β> = 5.1 %

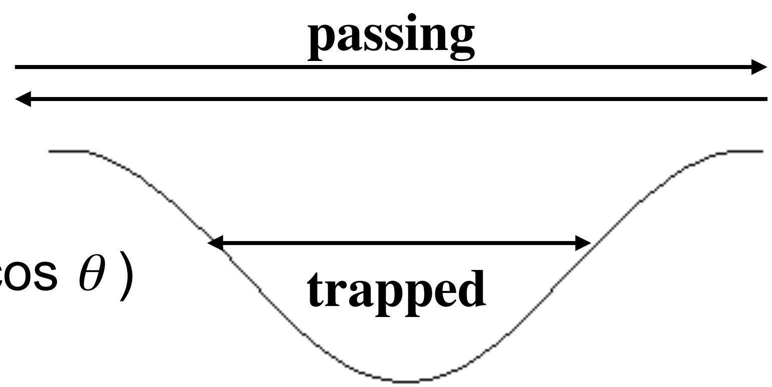


1-hour discharge

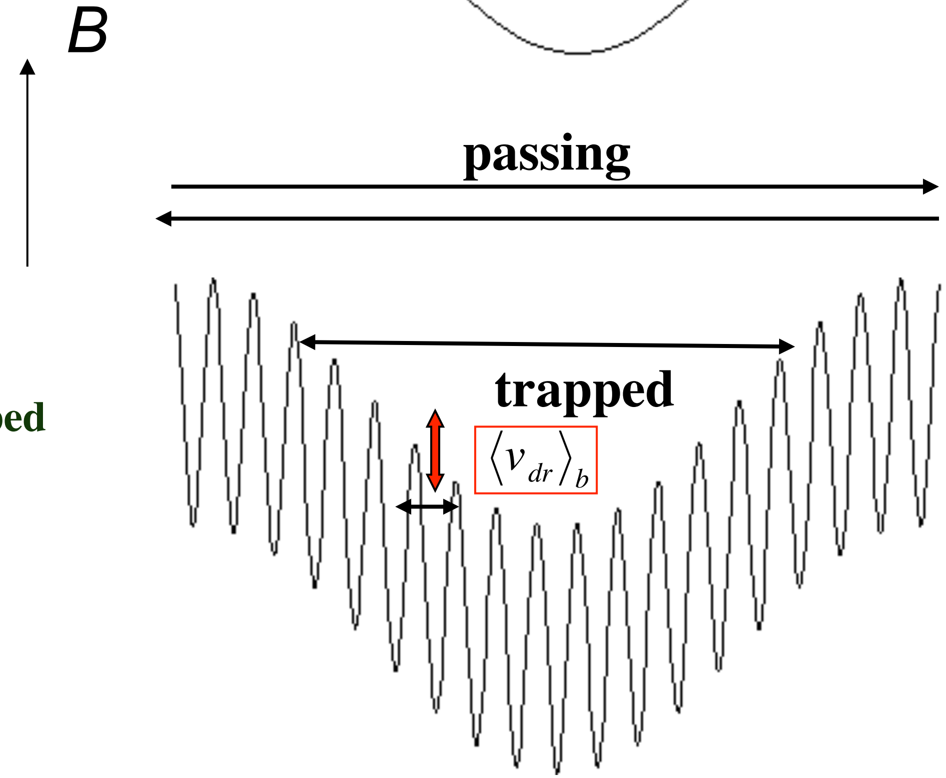
Classification of particle orbits

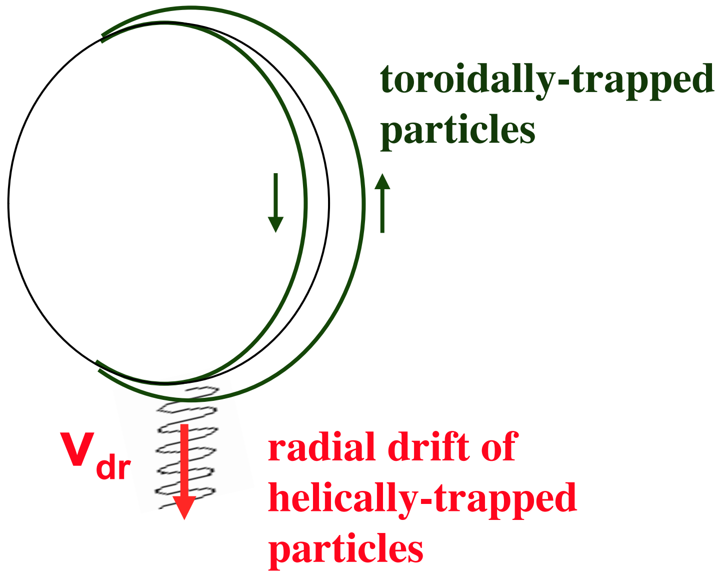
Tokamak

$B = B_0\,(1 - \varepsilon_t \cos\theta)$



Helical System

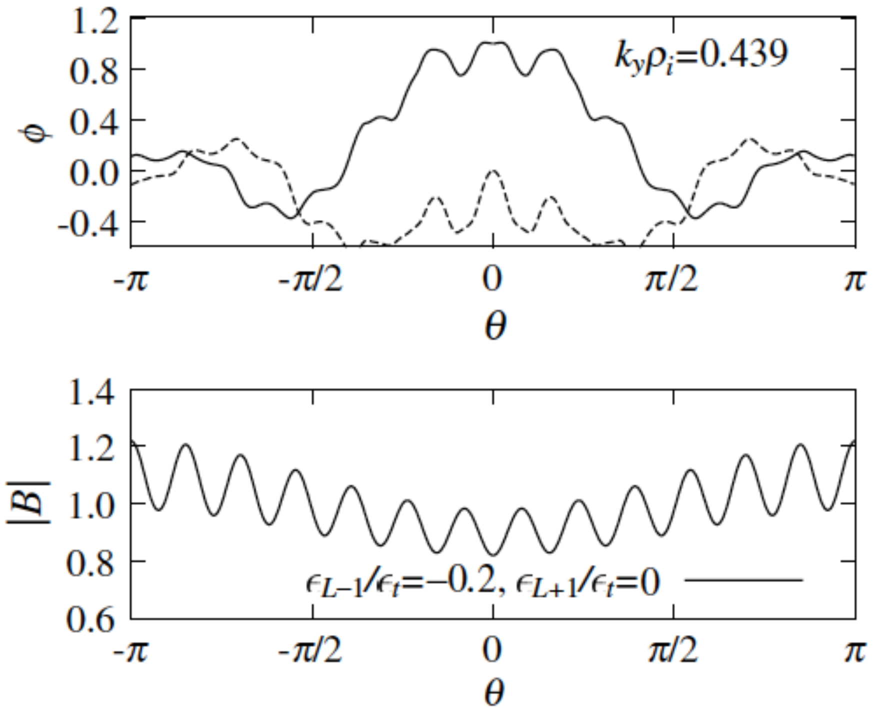




$B = B_0\,[\,1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\zeta)\,]$

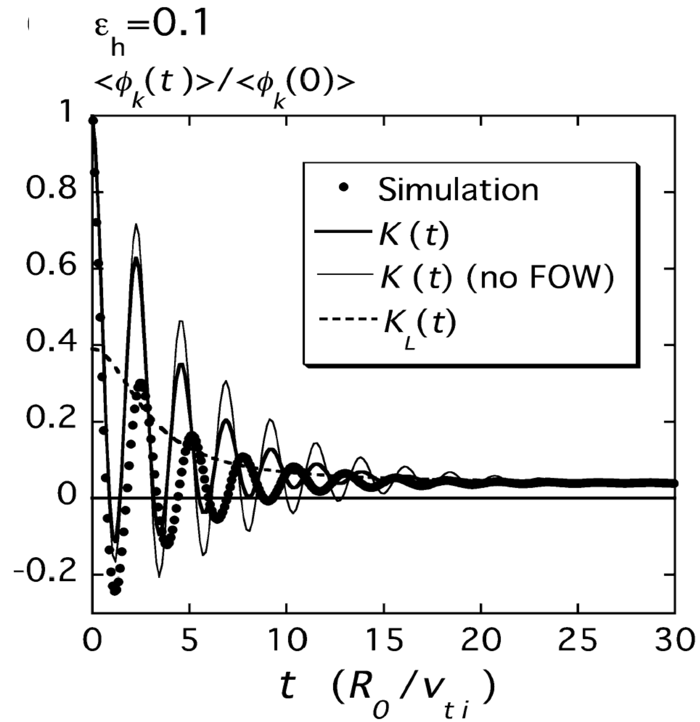
Helical geometry influences ITG mode and zonal flow.

Eigenfunction of linear ITG mode electrostatic potential



Watanabe et al. NF2007

Zonal-flow response (GAM, residual ZF)



Sugama & Watanabe PoP2006

Gyrokinetic Equations (for ITG Turbulence)

$k_\perp \rho_i \approx 1, \quad k_\perp \rho_e << 1$

Ion gyrokinetic equation for $\delta f(x, v_{||}, \mu, t)$

$$\boxed{\left[\frac{\partial}{\partial t} + v_\| \hat{\mathbf{b}} \cdot \nabla + \mathbf{v}_d \cdot \nabla - \mu\left(\hat{\mathbf{b}} \cdot \nabla\Omega\right)\frac{\partial}{\partial v_\|}\right]\delta f + \frac{c}{B_0}\{\psi, \delta f\} = \left(\mathbf{v}_* - \mathbf{v}_d - v_\| \hat{\mathbf{b}}\right) \cdot \frac{e\nabla\psi}{T_i} F_M + C(\delta f)}$$

Diamagnetic drift $\quad \mathbf{v}_* = -\frac{cT_i}{eL_n B_0}\left[1 + \eta_i\left(\frac{mv^2}{2T_i} - \frac{3}{2}\right)\right]\hat{\mathbf{y}}, \quad \mu = \frac{v_\perp^2}{2\Omega}$

Gyrocenter drift $\quad \mathbf{v}_d \cdot \nabla$

Mirror force $\quad -\mu\left(\mathbf{b} \cdot \nabla\Omega\right) \partial / \partial v_\|$

Effects of magnetic geometry

Quasineutrality condition & Adiabatic electron assumption

$$\boxed{\int J_0\left(k_\perp v_\perp / \Omega\right)\delta f \, \mathrm{d}^3 v - \left[1 - \Gamma_0\left(k_\perp^2\right)\right]\frac{e\phi}{T_i} = \frac{e}{T_e}\left(\phi - \langle\phi\rangle\right), \quad k_\perp^2 = \left(k_x + \hat{s} z k_y\right)^2 + k_y^2}$$

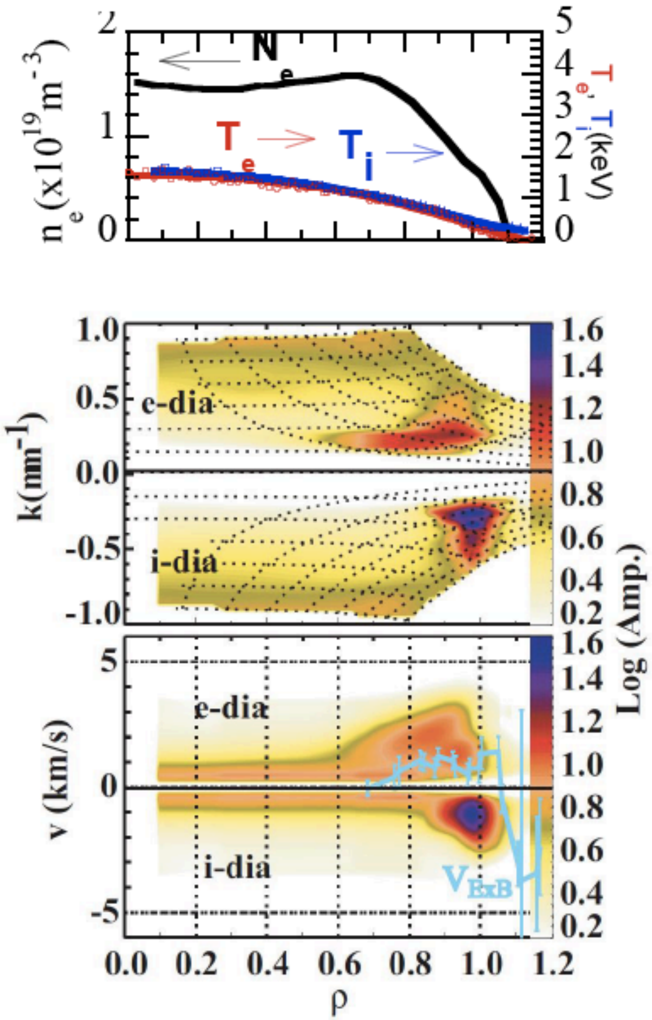
Ion polarization

Linear ITG Mode Analysis for High-T_i LHD plasmas

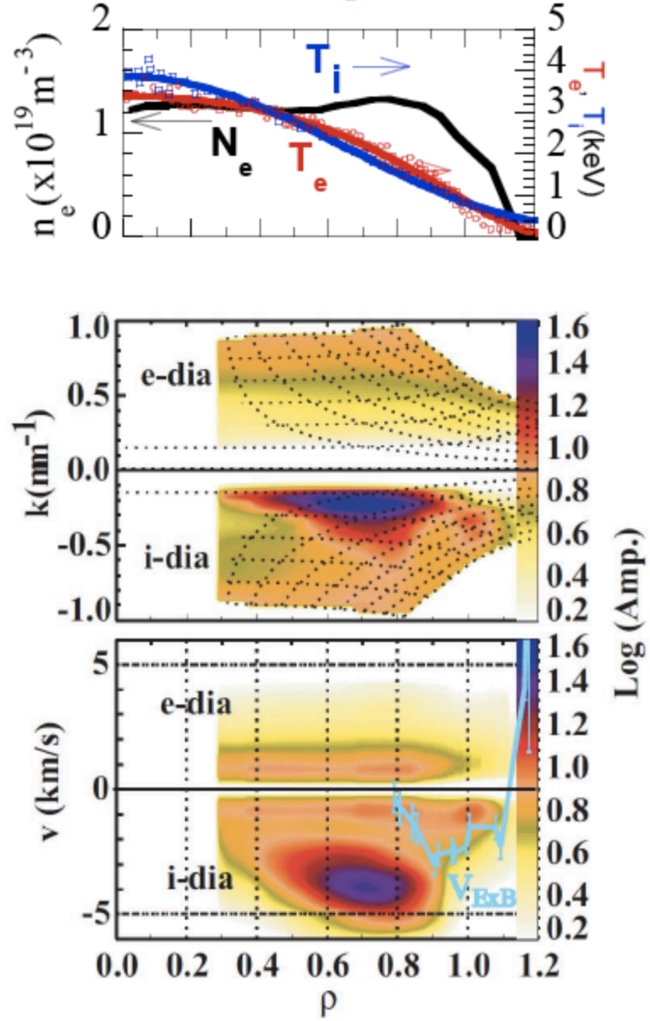
Fluctuation in High-T_i discharge in LHD

K. Tanaka et al., to be appeared in Plasma Fusion Res.

t = 1.833s



t = 2.233s (High T_i)

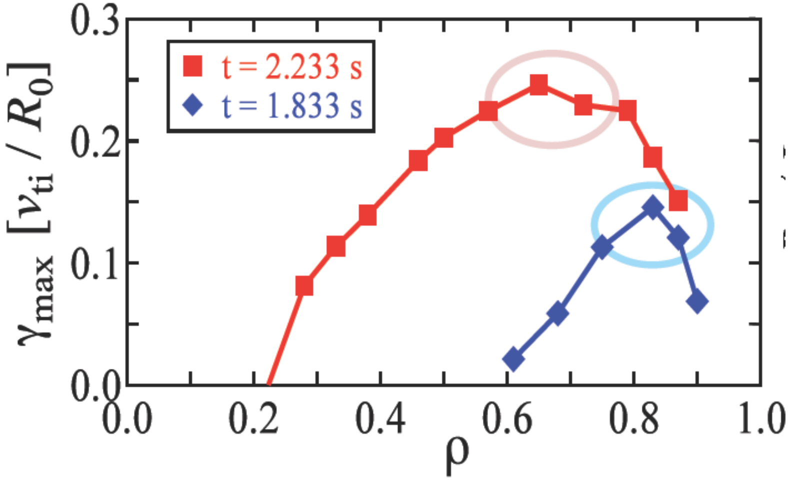


- Fluctuation peak exists at ρ=0.8-1.0 in space, $k_p\rho_i$~0.26 in wavenumber.
- Fluctuation peak exists at ρ=0.5-0.8 in space, $k_p\rho_i$~0.45 in wavenumber.

Results from Linear ITG Mode Analyses by GKV-X

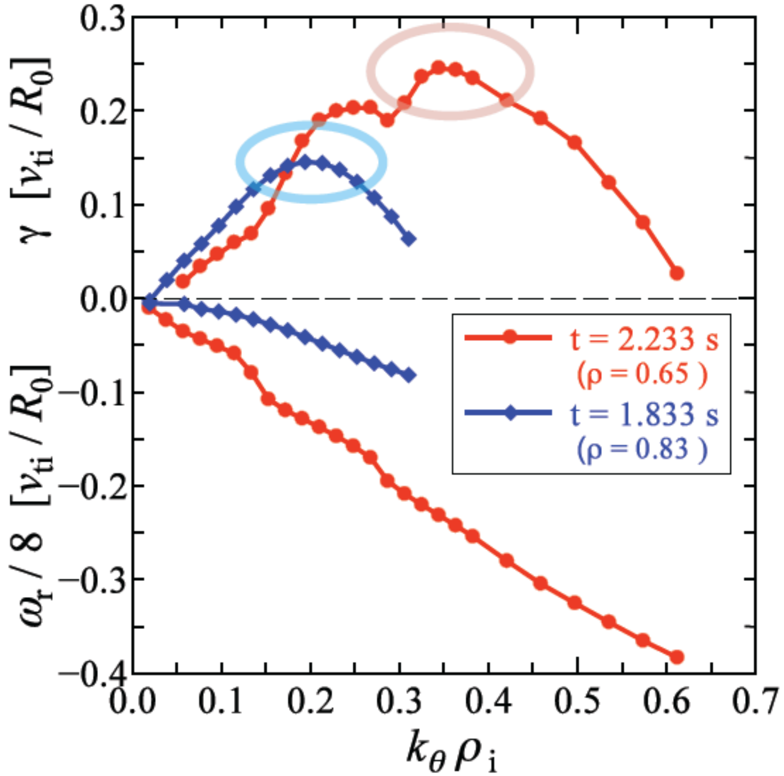
(See Poster by M. Nunami)

Radial profiles of γ_{max}



▪Growth rates are peaked at
ρ～0.65 (t=2.233s),
ρ～0.85 (t=1.833s).

Growth rates of ITG modes



▪There exists ITG unstable region.

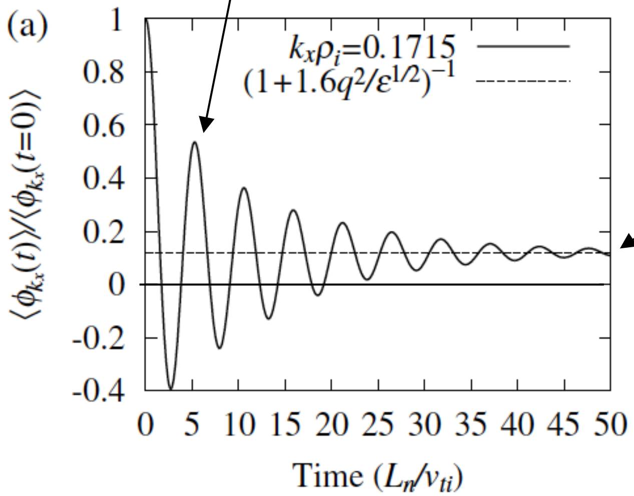
▪Maximum growth rates exists at
$k_\theta \rho_i$～0.35 (t=2.233s),
$k_\theta \rho_i$～0.20 (t=1.833s),
in poloidal wavenumber space.

Zonal Flows and ITG Turbulence

Gyrokinetic Simulation of EXB Zonal Flow Damping in Tokamaks

Watanabe & Sugama,
Nucl.Fusion 46, 24(2006)

GAM

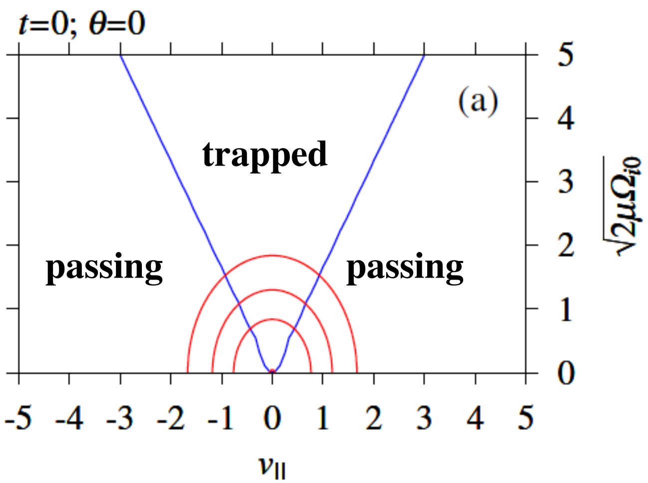


Undamped residual flow
[Rosenbluth & Hinton, PRL(1998)]

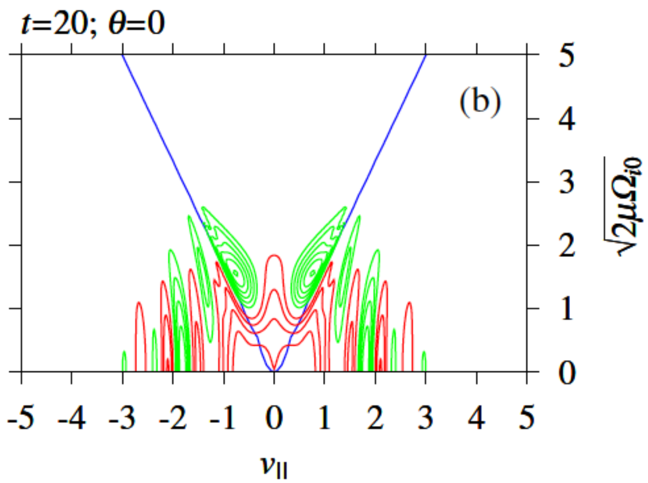
$$\phi_{k_r0}(\infty) = \phi_{k_r0}(0)/(1+1.6q^2/\epsilon^{1/2})$$

After GAM oscillations are damped in the collisionless process (Landau damping), the zonal-flow potential approaches the theoretical value predicted by the Rosenbluth-Hinton theory.

initial (t=0) distribution



t increases

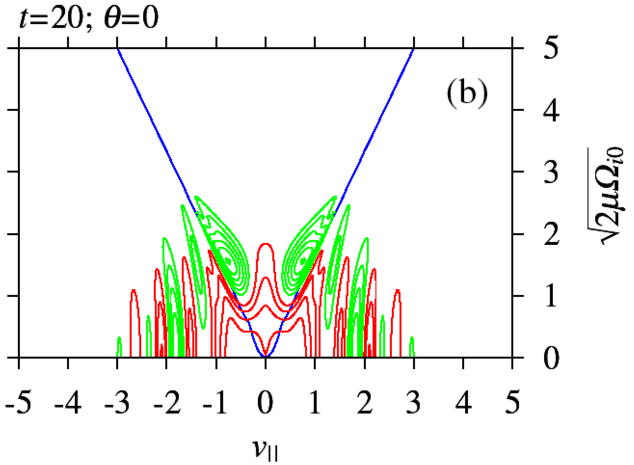


Results from
Gyrokinetic Vlasov (GKV) code

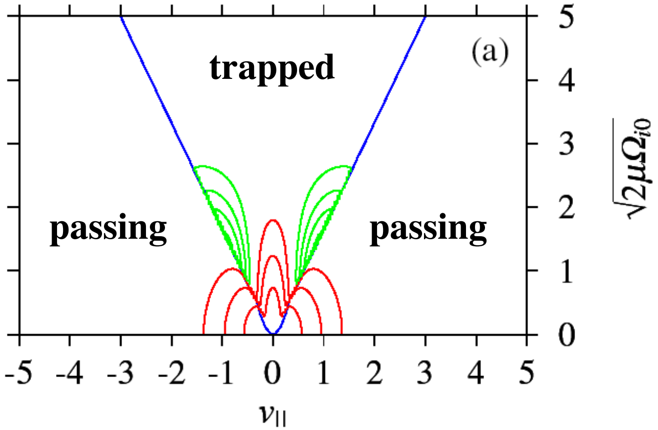
Real part of the ion gyrocenter distribution function $\delta f(v_{||},\mu)$

Structures of the perturbed gyrocenter distribution for zonal-flow components (tokamak case)

Simulation results



Analytical solution **(rapid oscillations dropped)**



The gyrocenter distribution for residual zonal flow part can be described by the analytical solution.

$$f_{k_x,0}(t) = F_{\mathrm{M}} \frac{e\langle \phi_{k_x,0}(0) \rangle}{T_{\mathrm{i}}} [k_x^2 \rho_{\mathrm{i}}^2 + \{\mathrm{i}k_x(\overline{\rho_b} - \rho_b) + k_x^2(\rho_b \overline{\rho_b} - \tfrac{1}{2}\overline{\rho_b^2} - \tfrac{1}{2}\rho_b^2)/(1 + 1.6q^2/\epsilon^{1/2})]$$

$\Downarrow$

Useful information to derive a kinetic-fluid closure model

Closure Model for Zonal Flow Dynamics in Tokamaks (I)

Sugama, Watanabe & Horton, PoP(2007)

Parallel heat fluxes

$$\left[q_{\|},\, q_{\perp}\right] \equiv \int d^3v\, \delta f \left[\left(mv_{\|}^2 - 3T\right) v_{\|},\ \left(\frac{1}{2}mv_{\perp}^2 - T\right) v_{\|}\right]$$

Fourth-order moments

$$\left[\delta r_{\|\|},\, \delta r_{\|\perp},\, \delta r_{\perp\perp}\right] \equiv \int d^3v\, \delta f \left[mv_{\|}^4,\ \frac{1}{2}mv_{\|}^2 v_{\perp}^4,\ \frac{1}{4}mv_{\perp}^4\right]$$

$$q = q_{\|}^{(l)} + q_{\|}^{(s)}$$

(l) long-time behavior (residual zonal flow) + (s) short-time behavior (GAM damping)

using the analytical solution δf

$$q_{\|\mathbf{k}_\perp}^{(l)} = -2q_{\perp\mathbf{k}_\perp}^{(l)} = 2p_0 U_{\mathbf{k}_\perp}[B - (\beta_2/\beta_1)B^2]$$

$$U_{\mathbf{k}_\perp} \equiv \beta_1\left(\beta_1 - \langle B^{-2}\rangle\right)^{-1}\left[\langle u_{\|\mathbf{k}_\perp}/B\rangle - \langle B^{-2}\rangle\langle Bu_{\|\mathbf{k}_\perp}(t=0)\rangle - (\beta_1 n_0)^{-1}\langle B^{-2}\rangle\left\langle \int d^3v\, F_0 R_{\mathbf{k}_\perp}(t)\overline{(v_{\|}/B)}\right\rangle\right].$$

$$\beta_1 = \frac{15}{4}\int_0^{B_M} d\lambda/\langle B/(1-\lambda B)^{1/2}\rangle$$

$$\beta_2 = \frac{3}{2}\int_0^{B_M} \lambda d\lambda/\langle B/(1-\lambda B)^{1/2}\rangle$$

$$R_{\mathbf{k}_\perp}(t) = \int_0^t dt' S_{\mathbf{k}_\perp}(t')$$

different model from Beer & Hammett (1998)

Closure Model for Zonal Flow Dynamics in Tokamaks (II)

$$q = q_{\|}^{(l)} + q_{\|}^{(s)}$$

(l) long-time behavior (residual zonal flow) + (s) short-time behavior (GAM damping)

Hammett-Perkins type model

$$q_{\|}^{(s)} = -2\sqrt{\frac{2}{\pi}}\, i\, n_0 v_t \sum_m \frac{m}{|m|}\delta T_{\| m}\, e^{im\theta}$$

$$q_{\perp}^{(s)} = -\sqrt{\frac{2}{\pi}}\, i\, n_0 v_t \sum_m \frac{m}{|m|}\delta T_{\perp m}\, e^{im\theta}$$

$$n_0 \delta T_{\|} = \delta p_{\|} - T\delta n$$

$$n_0 \delta T_{\perp} = \delta p_{\perp} - T\delta n$$

Fourth-order variables

$$\left(\delta r_{\|\|\|}, \delta r_{\|\perp}, \delta r_{\perp\perp}\right) = (3,1,2)\times T v_t^2\, \delta n^{(g)}$$

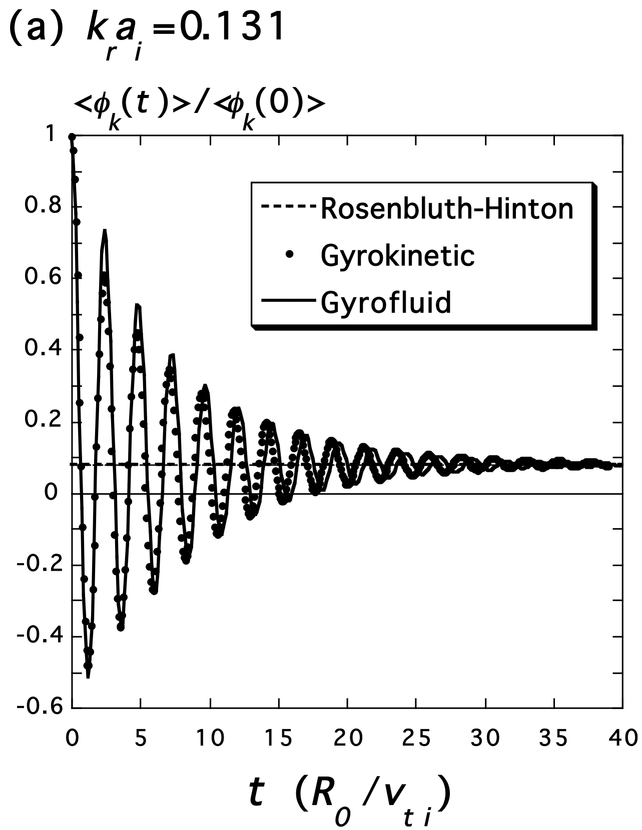
where the Maxwellian part of the perturbed distribution is taken into account.

ITG-Mode-Driven Zonal Flow in Tokamaks

Gyrofluid equations for ions combined with the quasineutrality condition

$$e^{-b_i/2}\left(\frac{\delta n_{i\mathbf{k}_\perp}^{(\mathrm{g})}}{n_0}-\frac{b_i}{2}\frac{\delta T_{i\perp\mathbf{k}_\perp}}{T_i}\right)-\frac{e\phi_{\mathbf{k}_\perp}}{T_i}\left[1-\Gamma_0(b_i)\right] \;=\; \frac{e}{T_e}\left(\phi_{\mathbf{k}_\perp}-\langle\phi_{\mathbf{k}_\perp}\rangle\right)$$

Gyrofluid simulation shows a GAM damping process toward the same residual zonal-flow level as given by gyrokinetic simulation and the Rosenbluth-Hinton theory.



Rosenbluth-Hinton formula

$$K_{\mathrm{R-H}} = 1/(1+1.6q^2/\varepsilon_t^{1/2})$$

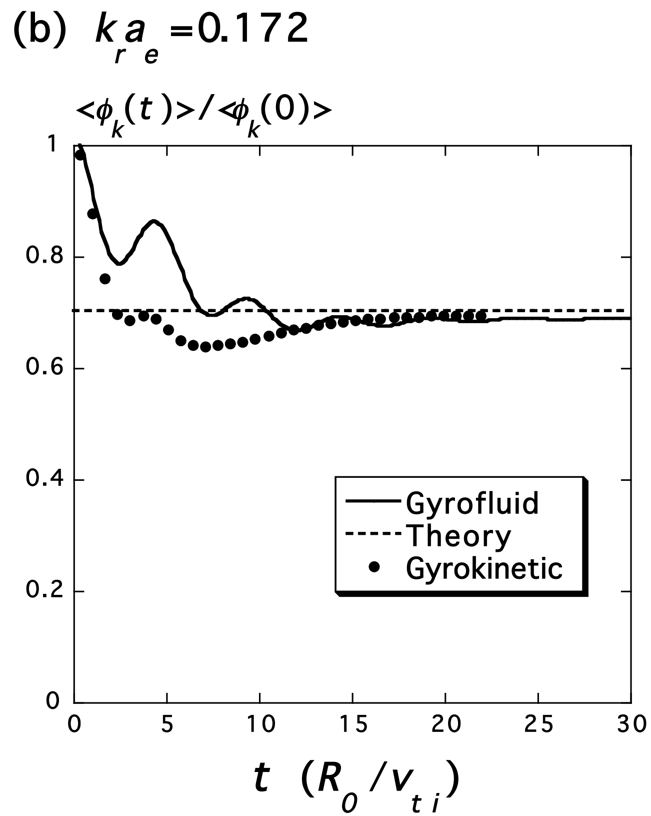
ETG-Mode-Driven Zonal Flow

Gyrofluid equations for electrons combined with the Poisson equation

$$e^{-b_e/2}\left(\frac{\delta n^{(\mathrm{g})}_{e\mathbf{k}_\perp}}{n_0} - \frac{b_e}{2}\frac{\delta T_{e\perp\mathbf{k}_\perp}}{T_e}\right) + \frac{e\phi_{\mathbf{k}_\perp}}{T_e}\left[1-\Gamma_0(b_e)+k_\perp\lambda^2_{De}\right] = -\frac{e\phi_{\mathbf{k}_\perp}}{T_i}$$

Gyrofluid simulation shows the same residual zonal-flow level as given by gyrokinetic simulation and the analytical theory.

$$\boxed{\phi_{\mathbf{k}_\perp}(t) = \frac{T_e/T_i + k_\perp^2(a_e^2+\lambda_{De}^2)}{T_e/T_i + k_\perp^2 a_e^2[1+1.6(1+T_e/T_i)q^2/\varepsilon^{1/2}] + k_\perp^2\lambda_{De}^2}\phi_{\mathbf{k}_\perp}(0)}$$



Collisionless Time Evolution of Zonal Flows in Helical Systems

[Sugama & Watanabe, PRL (2005), Phys.Plasmas (2006)]

Response of the zonal-flow potential to a given initial potential $\boxed{k\,\rho_i < 1}$

$$\langle \phi_k(t) \rangle = K(t)\, \langle \phi_k(0) \rangle$$

Response function = GAM component + Residual component

$$K(t) = K_{GAM}(t)[1 - K_L(0)] + K_L(t)$$

$$K(t=0) = 1 \qquad K(t) \to K_L(t),\; K_{GAM}(t) \to 0 \text{ as } t \to +\infty$$

$B = B_0\,[\,1 - \varepsilon_t \cos\,\theta - \varepsilon_h \cos\,(L\theta - M\zeta)\,]$

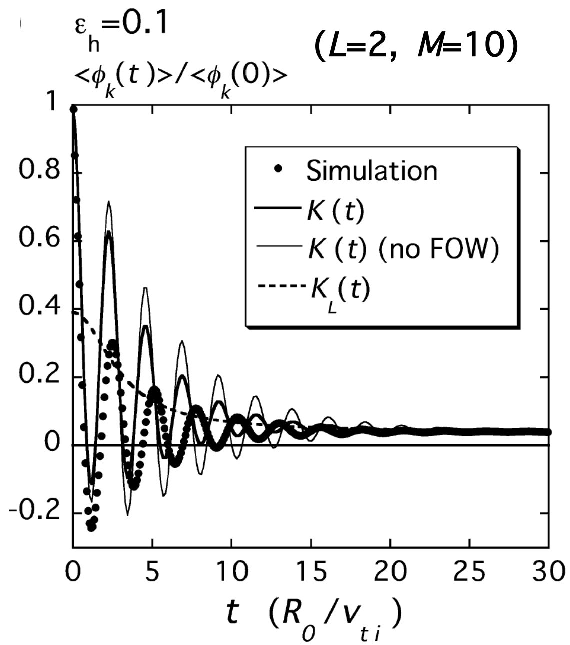
GAM response function $K_{GAM}(t) = \cos(\omega_G)\exp(-|\gamma|t)$

Long-time response function

$$\boxed{K_L(t) = \frac{1 - (2/\pi)^{1/2}\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta\}\right\rangle}{1 + G + E(t)\Big/\left(n_0\left\langle k_\perp^2 \rho_{ti}^2\right\rangle\right)}}$$

***E(t)* represents effects of shielding of potential due to helical-ripple-trapped particles.**

$$E(t) = \frac{2}{\pi} n_0 \left[\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle - \frac{3}{2}\left\langle k_\perp^2 \rho_{ti}^2 \right\rangle \left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle + \frac{T_i}{T_e}\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{e1}(t,\theta)\}\right\rangle \right]$$



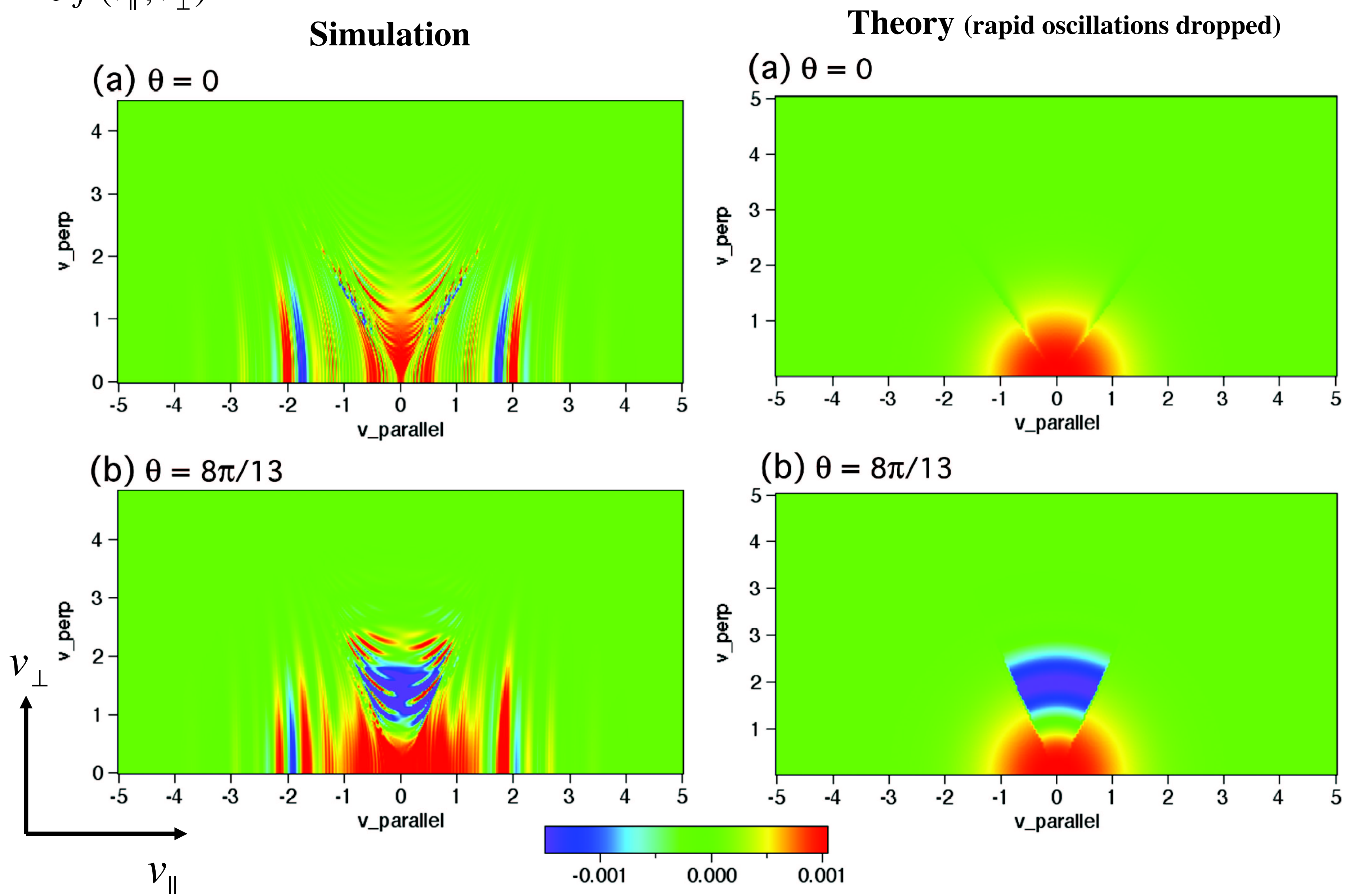
Perturbed gyrocener distribution

$\delta f\,(v_{\parallel}, v_{\perp})$

Helical plasma

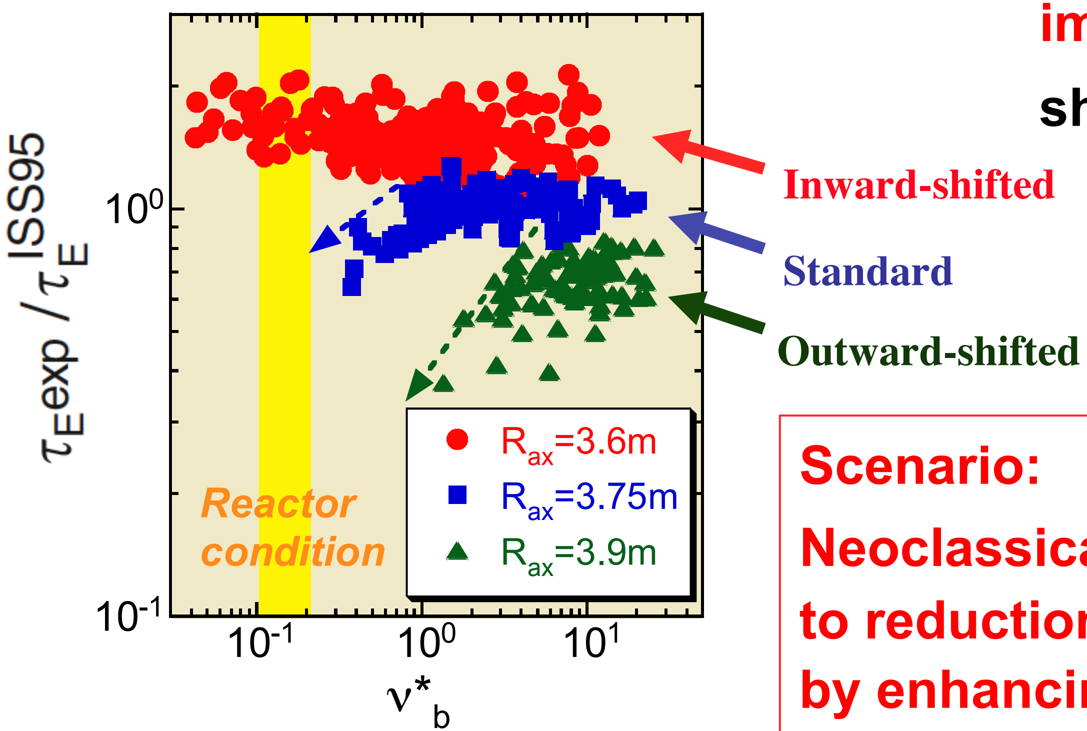
$q = 1.5, \ \varepsilon_h = 0.1, \ L = 2, \ M = 10$

$t = 12.5 \ (R_0/v_{ti})$



Results from LHD experiments

For low collisionality, better confinement is observed in the inward-shifted magnetic configurations, where lower neoclassical ripple transport but more unfavorable magnetic curvature driving pressure-gradient instabilities are anticipated.



H. Yamada *et al.* (PPCF2001)

Anomalous transport is also improved in the inward shifted configuration.

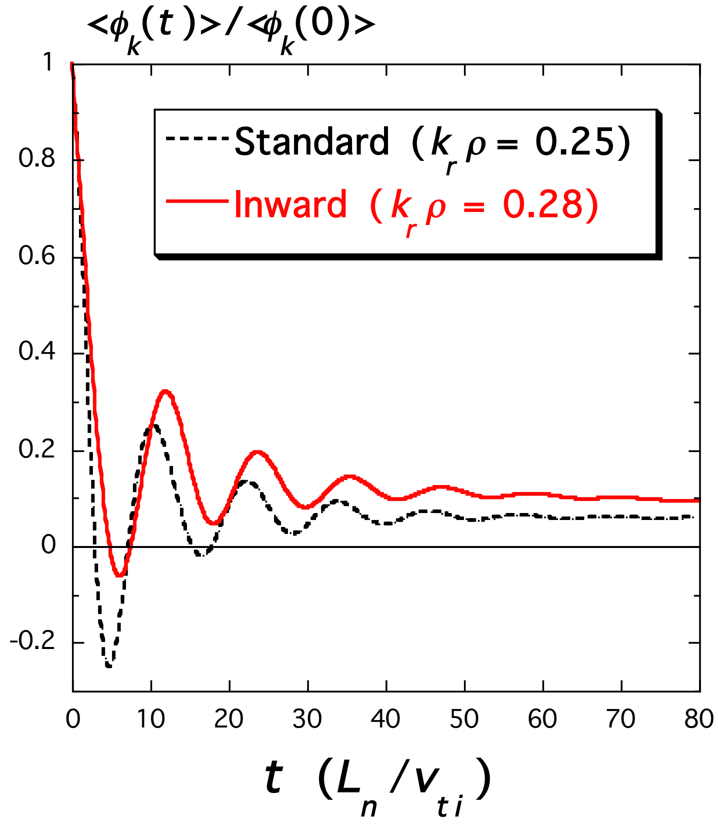
Scenario:

Neoclassical optimization contributes to reduction of anomalous transport by enhancing the zonal-flow level.

Standard and Inward-shifted configurations

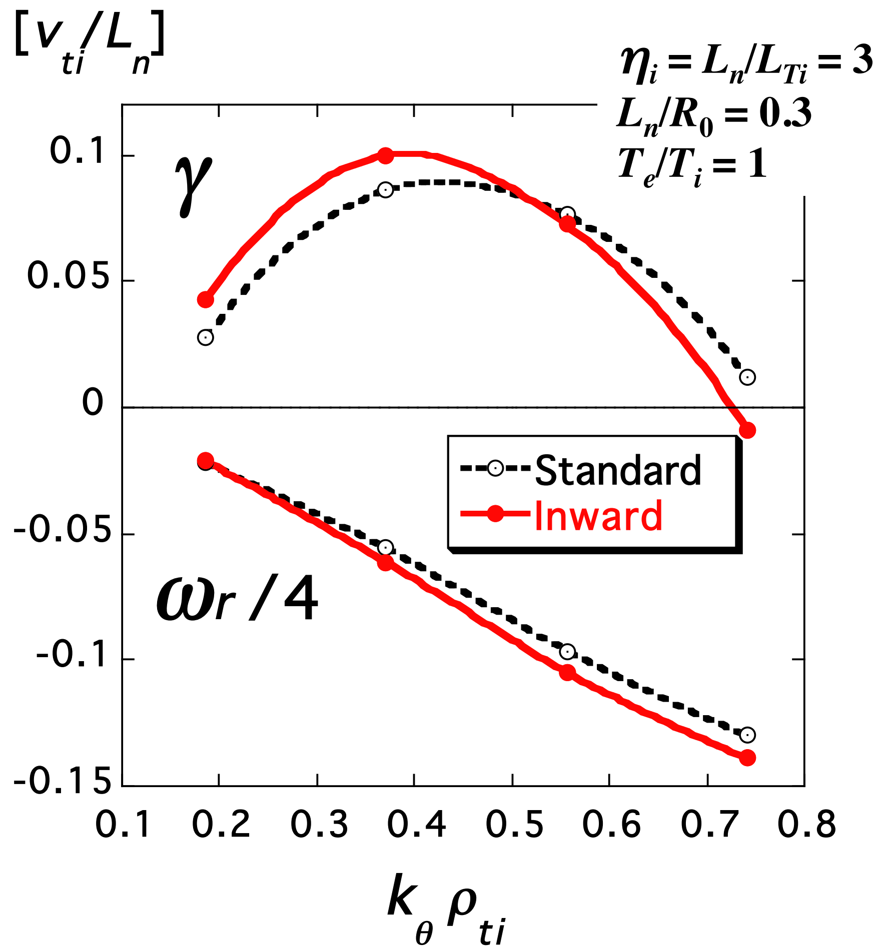
For the inward-shifted case, more unfavorable curvature but lower q and higher magnetic shear s.

Larger residual zonal flow is found for the inward-shifted case.



Response of zonal-flow potential to a given initial potential

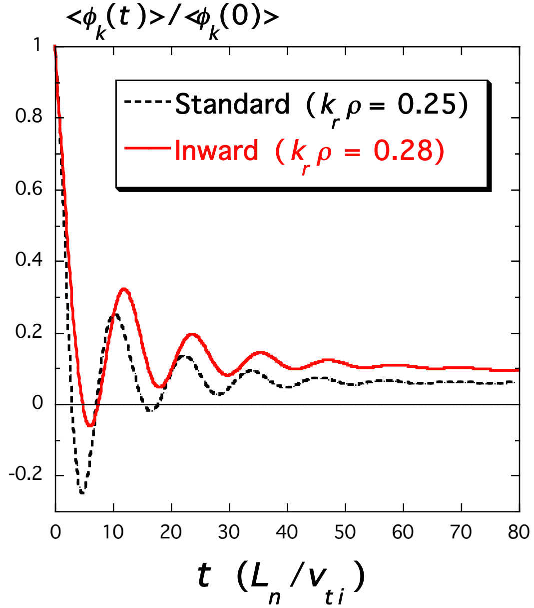
The maximum ITG growth rate is slightly larger for the inward-shifted case.



Results from GKV simulation (flux tube, $E_r = 0$)

Smaller χ_i and larger zonal flows are found in the saturated turbulent state for the inward-shifted configuration than for the standard one !

Linear time evolution of zonal-flow potential

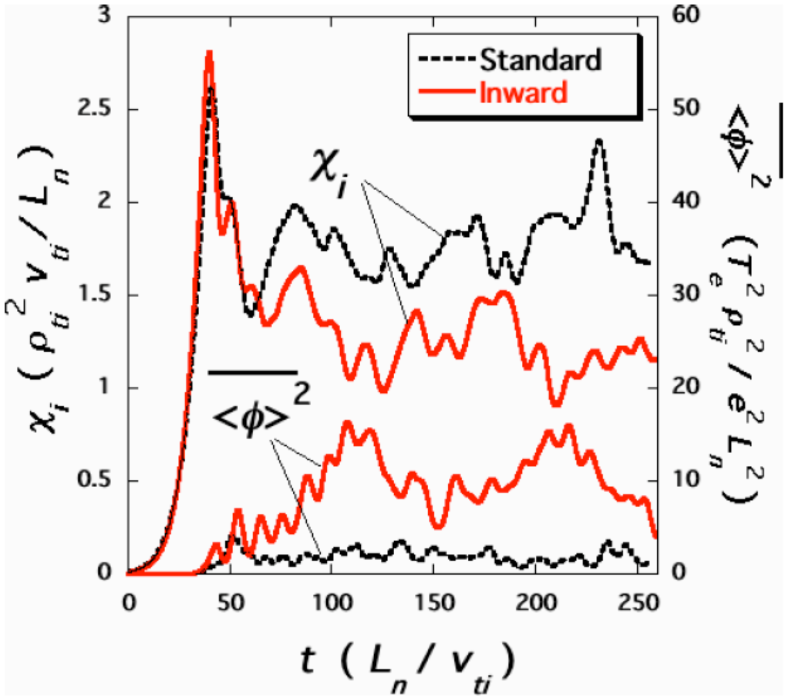


Larger residual zonal flow is found for the inward-shifted case.

Watanabe, Sugama & Ferrando, PRL(2008)
Sugama, Watanabe & Ferrando, PFR(2009)

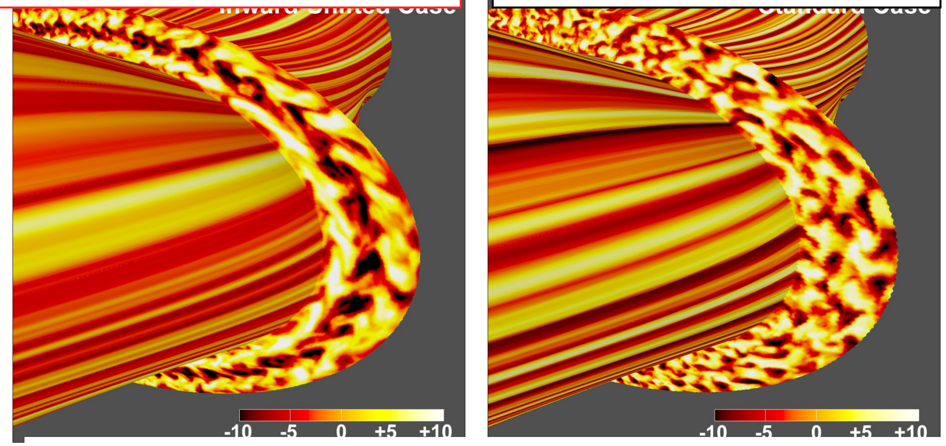
ITG turbulence

Turbulent thermal diffusivity and squared zonal-flow potential



inward-shifted configuration

standard configuration



Potential contours obtained from six copies of flux tube

The GKV turbulence simulations were carried out by the Earth Simulator (JAMSTEC).

Effects of Equilibrium Electric Field E_r on Zonal Flows in Helical Systems

In helical systems

E_r is given from ambipolar condition of radial particle fluxes.

E_r reduces neoclassical ripple transport.

How does E_r influence zonal flows and anomalous transport?

Effects of E_r on gyrokinetic equation and zonal flows

gyrophase average of
zonal-flow potential

Gyrokinetic equation for $\mathbf{k}_\perp = k_r \nabla r$

$$\left[\frac{\partial}{\partial t} + v_\parallel \hat{\mathbf{b}}\cdot\nabla + i\mathbf{k}_r\cdot\mathbf{v}_d - \mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)\frac{\partial}{\partial v_\parallel} + \omega_E\frac{\partial}{\partial\alpha}\right]\delta f = -i\mathbf{k}_r\cdot\mathbf{v}_d\,\frac{e\left\langle\phi(\mathbf{x}+\rho)\right\rangle}{T_i}F_M$$

angular velocity
due to ExB drift

$$\omega_E = -\frac{c\,E_r}{r_0 B_0}$$

field line label

$$\alpha = \theta - \zeta/q$$

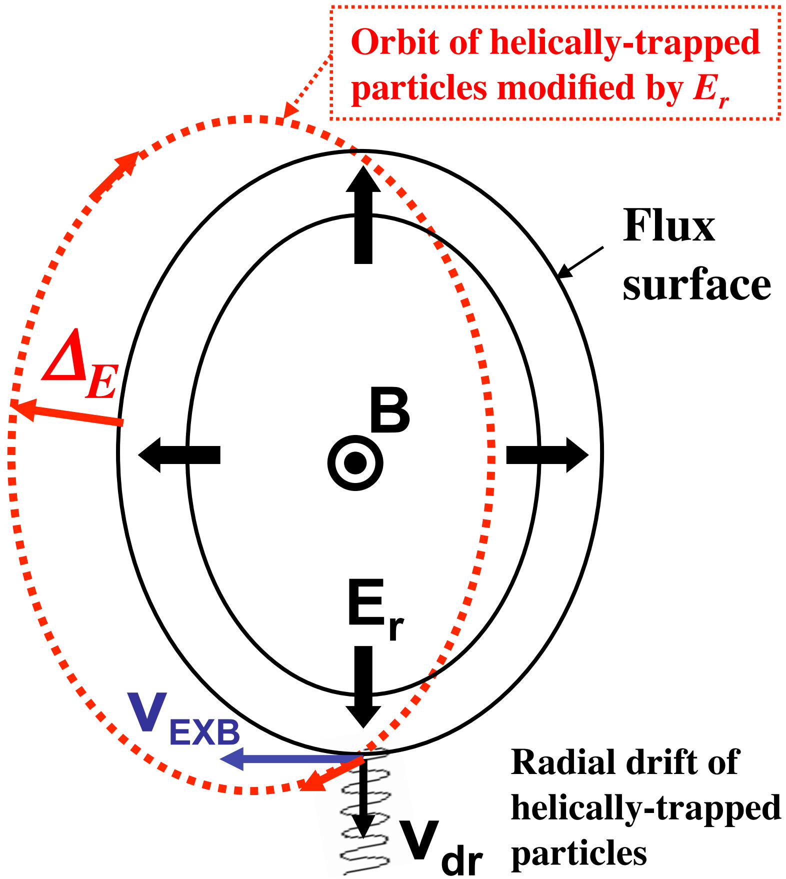
In helical systems, α -dependence appears in $\mathbf{k}_r\cdot\mathbf{v}_d$ **and** $\mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)$ **.**

Therefore, even if the zonal-flow potential ϕ is independent of α ,
δf comes to depend on α .

Thus, ω_E influences δf and accordingly ϕ through quasineutrality condition.

Effects of Equilibrium E_r on Zonal-Flow Response

Equilibrium E_r field generates a ExB component to the velocity.

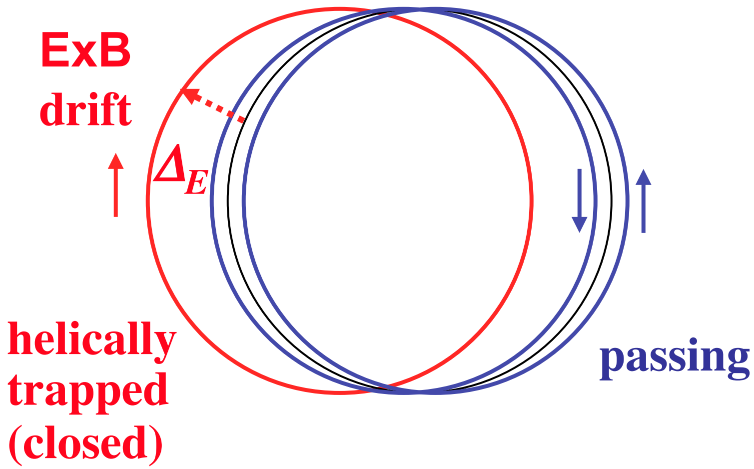


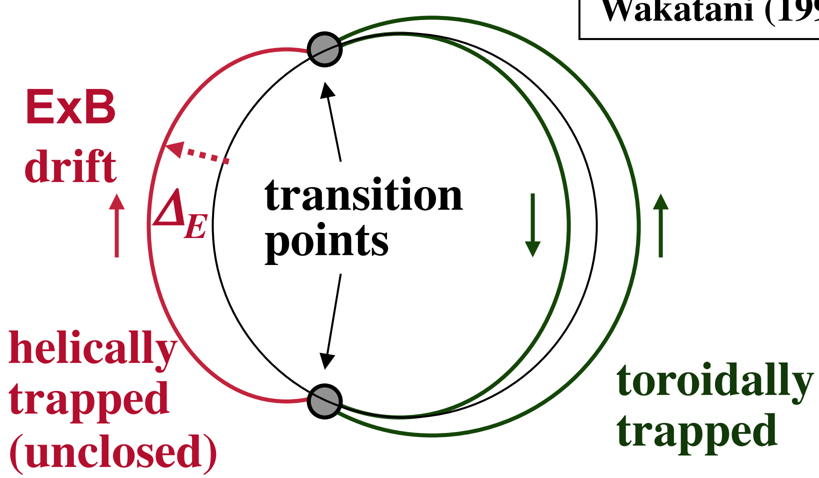
Poloidal ExB rotation of helically-trapped particles with reduced radial displacements Δ_E will decrease the shielding of zonal-flow potential and increase its response.

Mynick & Boozer, PoP(2007)
Action-Angle Formulation

Classification of particle orbits in the presence of E_r

Cary *et al.*, PF (1988)
Wakatani (1998)





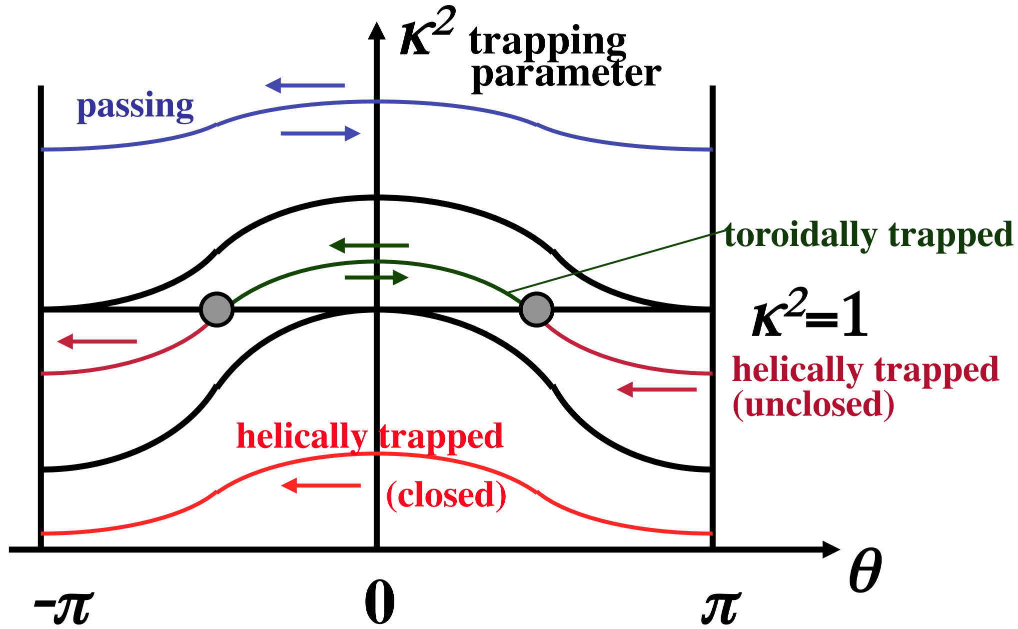
$$\boxed{\Delta_E \sim r_0 \frac{v_{dr}}{v_{E\times B}}}$$

radial displacement of
helically-trapped particle

trapping parameter

$$\kappa^2 = \frac{1-\lambda B_0[1-\varepsilon_T(\theta)-\varepsilon_H(\theta)]}{2\lambda B_0 \varepsilon_H(\theta)}$$

$$\lambda = \frac{1}{2}mv^2 \Big/ \mu$$



Solution of gyrokinetic equation to describe long-time evolution of zonal flows [Sugama & Watanabe, PoP(2009)]

Perturbed particle distribution function

$\rho_r \cdots$ **gyro motion**

$\Delta_r \cdots$ **drift motion**

$$\delta f_k(t) = -\frac{e}{T}\phi_k(t) F_M \left[1 - e^{-ik_r\rho_r} e^{-ik_r\Delta_r} \left\langle e^{ik_r\Delta_r} J_0(k_r\rho_r) \right\rangle_{\text{orbit}} \right] \longrightarrow$$

Polarization (classical & neoclassical)

$$+ e^{-ik_r\rho_r} e^{-ik_r\Delta_r} \left\langle e^{ik_r\Delta_r} \left[\delta f_k^{(g)}(0) + F_M \int_0^t S_k(t)\,dt \right] \right\rangle_{\text{orbit}} \longrightarrow$$

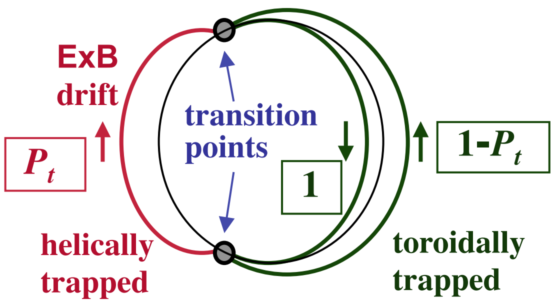
Initial condition & Turbulence source

Average along the orbit

$$\langle \cdots \rangle_{\text{orbit}} = \oint \cdots \frac{dl}{(dl/dt)} \bigg/ \oint \frac{dl}{(dl/dt)}$$

$$\boxed{P_t \approx \frac{4\sqrt{2}}{\pi}\left(\frac{c\,E_r}{v\,B_0}\right)\left(\frac{R_0 q}{r_0}\right)\left[(\varepsilon_H)^{-1/2}\frac{\partial\varepsilon_H/\partial\theta}{\partial(\varepsilon_H-\varepsilon_T)/\partial\theta}\right]_{P-}}$$

For particles which show transitions



1-P_t **P_t transition probability**

$$\oint \frac{dl}{(dl/dt)} = (1-P_t)\int_{\substack{\kappa^2>1 \\ v_\parallel>0}} \frac{dl}{|v_\parallel|} + \int_{\substack{\kappa^2>1 \\ v_\parallel<0}} \frac{dl}{|v_\parallel|} + P_t \int_{\kappa^2<1} \frac{d\theta}{\omega_E}$$

toroidally trapped **helically trapped**

Long-time zonal-flow response to the initial condition and turbulence source

For radial wavenumbers $k_r\rho_{ti} < 1$ (ITG turbulence) and $k_r\Delta_E < 1$, the zonal-flow potential is derived from the quasineutrality condition as [Sugama & Watanabe, PoP(2009)]

$$\boxed{\frac{e}{T_i}\langle\phi_k(t)\rangle = \frac{\left\langle n_0^{-1}\int d^3v\left[1+ik_r\left(\Delta_{ir}-\langle\Delta_{ir}\rangle_{\text{orbit}}\right)\right]\left[\delta f_{ik}^{(g)}(0)+F_M\int_0^t S_{ik}(t)\,dt\right]\right\rangle}{(k_r\rho_{ti})^2\left[1+G_p+G_t+M_p^{-2}(G_{ht}+G_h)(1+T_e/T_i)\right]}}$$

Geometrical factors G's represents shielding effects of neoclassical polarization due to particles motions in different orbits.

$G \propto$ (population) $\times(\Delta_r/\rho)^2$

G_p : passing

G_t : toroidally-trapped

G_{ht} : helicallly-trapped (unclosed orbit)

G_h : toroidally-trapped (closed orbit)

Zonal-flow generation can be enhanced when

G_{ht} and G_h decreases with neoclassical optimization (which reduces radial drift velocity v_{dr})

and when poloidal Mach number $\boxed{M_p \equiv \left|(cE_r/B_0)/(rv_{ti}/Rq)\right|}$ **increases**

with increasing E_r and using heavier ions.

Response to the initial condition

Assume the initial distribution to have Maxwellian dependence $\delta f_{ik}^{(g)}(0) = [\delta n_k^{(g)}(0)/n_0] F_M$

Then, we obtain $\delta n_k^{(g)}(0)/n_0 = (k_r \rho_{ti})^2 e\phi(0)/T_i$

$$\boxed{\langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + M_p^{-2}(G_{ht} + G_h)(1 + T_e/T_i)}}$$

(no turbulence source)

For the single-helicity configuration

$$B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\varsigma)] \quad (\varepsilon_h : \text{independent of } \theta)$$

No transitions occur. $G_{ht} = 0, \quad G_h = (15\pi/4) q^2 (2\varepsilon_h)^{1/2}$

$$\boxed{\langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + (15\pi/4) M_p^{-2} q^2 (2\varepsilon_h)^{1/2} (1 + T_e/T_i)}}$$

This corresponds to the case considered by previous works.

Mynick & Boozer, PoP(2007)
Sugama, Watanabe & Ferrando, PFR(2008)

Extention of GKV code to poloidally global model

GKV code is extended from the flux tube to the poloidally global model for studying effects of E_r on zonal flows in helical systems [Watanabe, IAEA FEC 2008].

$\alpha \equiv \theta - \zeta / q$ **: field-line label** ζ **: toroidal angle**

- **Linear simulations for time evolution of zonal flows are done using**
  129 Fourier modes in the α direction ,
  1, 536 grid points in the ζ direction, and
  (512, 48) grid points in the ($v_{\parallel}$, μ) space
  for a fixed radial wavenumber k_r .

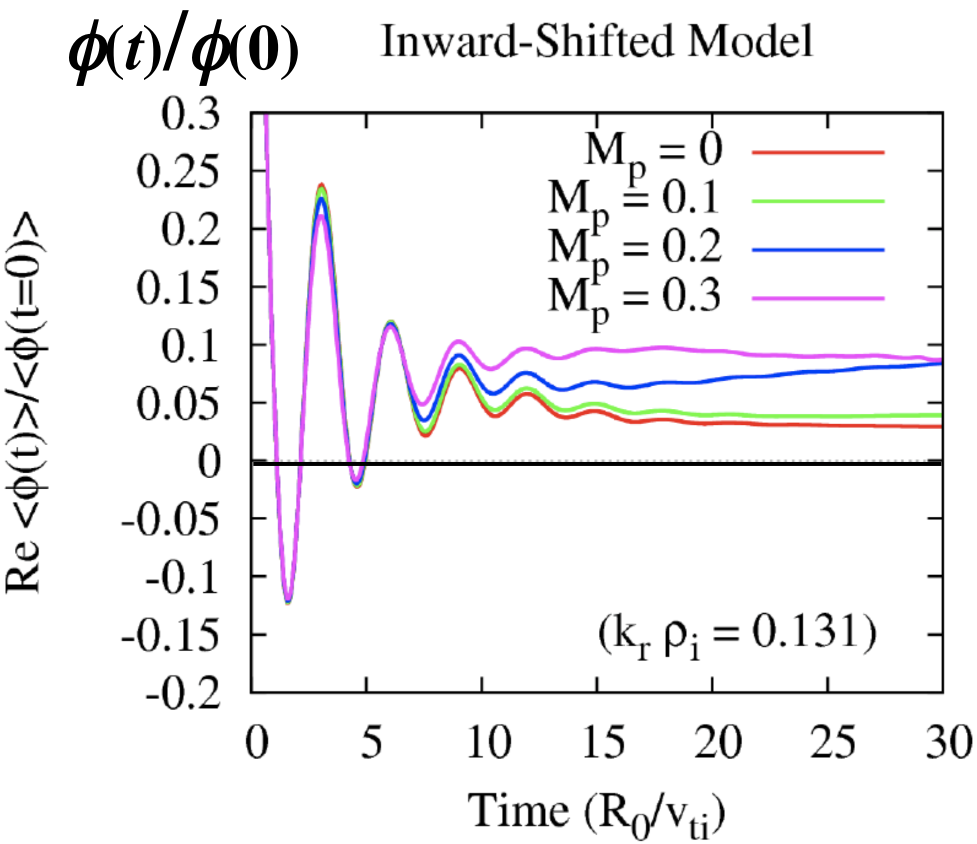
- **Standard configuration model (single helicity) :**

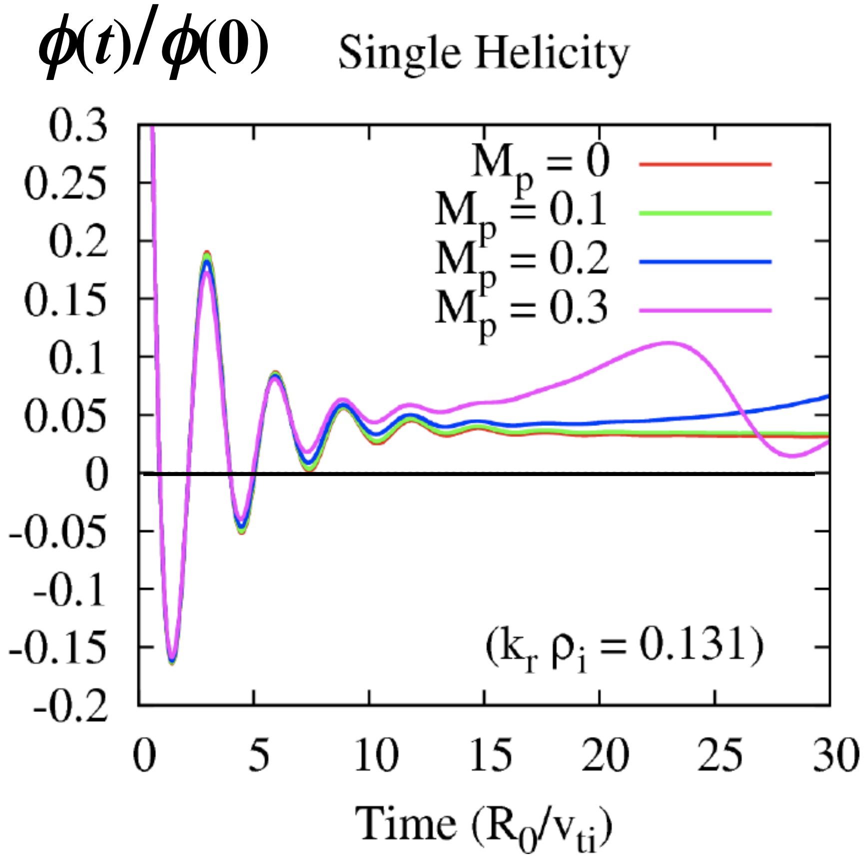
$$B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\varsigma)], \quad \varepsilon_t = 0.1, \quad \varepsilon_h = 0.1, \quad q = 1.5, \quad L = 2, \quad M = 10$$

- **Inward-shifted configuration model :**
  Sideband helicity components $\left(\varepsilon_{L+1} = -0.02, \quad \varepsilon_{L-1} = -0.08 \right)$
  are included.

Collisionless time evolution of zonal flows in helical configurations with E_r

It is clearly shown for the inward-shifted model configuration that the residual zonal-flow potential amplitude (observed after Landau damping of GAM) is enhaced by increasing E_r .





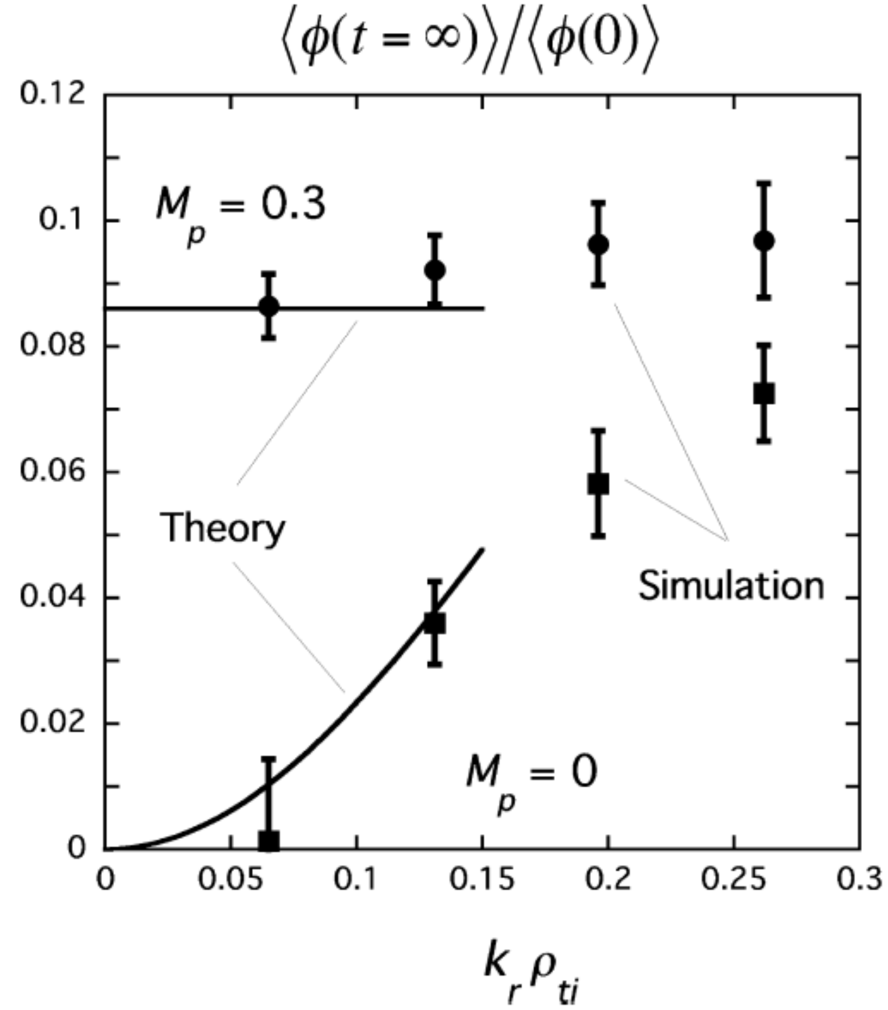
The residual zonal-flow potential as a function of $k_r \rho_{ti}$ for $M_p = 0$ and $M_p = 0.3$

Inward-shited configuration

Different $k_r \rho_{ti}$ dependences for $M_p = 0$ and $M_p = 0.3$ are theoretically predicted and confirmed by simulation.

Theoretical results are derived by assuming $k_r \rho_{ti} << 1$.

[To be published in CPP]



Dependence of the residual zonal-flow potential on the poloidal Mach Number (M_p) for $k_r \rho_{ti} = 0.065$

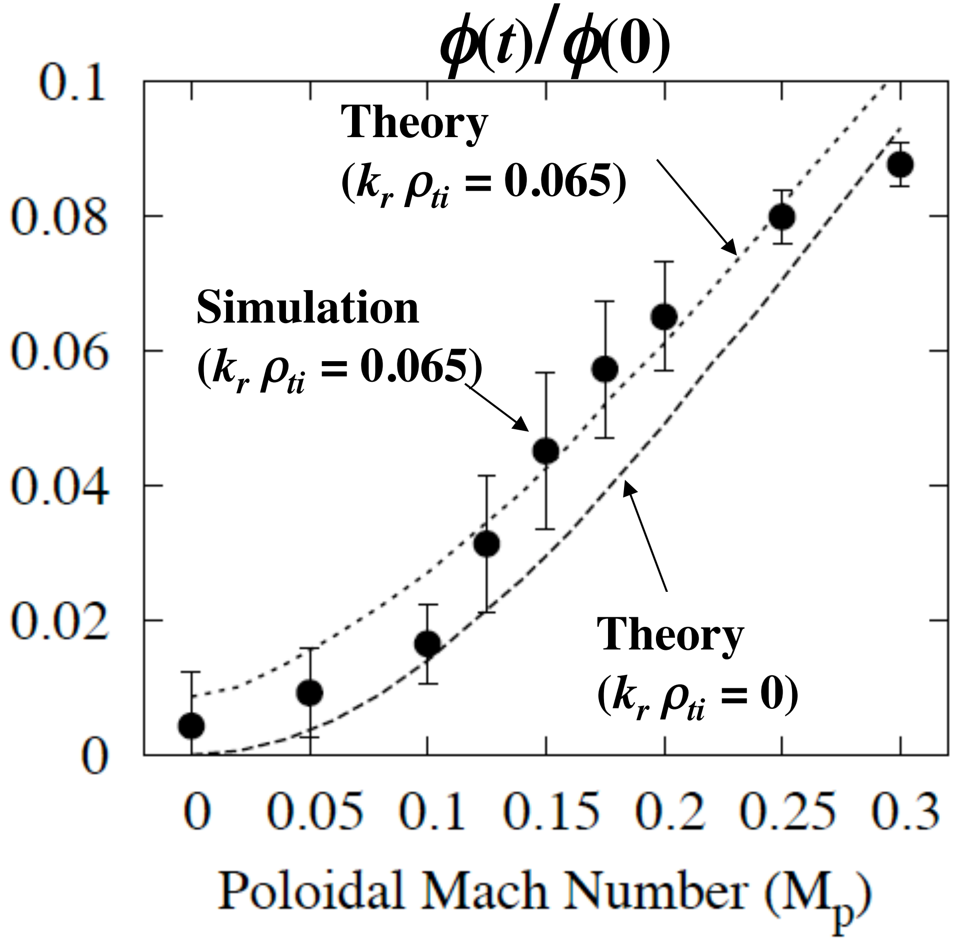
Residual zonal-flow potential increases with increasing M_p .

Qualitative agreement between theory and simulation is verified.

More details are found in poster by T.-H. Watanabe

[submitted to PPCF]

Inward-shited model configuration



Momentum Balance and Radial Electric Field in Quasisymmetric Systems with Stellarator Symmetry

Basic Boltzmann Kinetic Equation for description of Collisional and Turbulent Transport

Equilibrium magnetic field

$$\mathbf{B} = \psi' \nabla s \times \nabla \theta + \chi' \nabla \zeta \times \nabla s = B_s \nabla s + B_\theta \nabla \theta + B_\zeta \nabla \zeta$$

Boltzmann kinetic equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_a}{m_a} \left\{ \left(\mathbf{E} + \hat{\mathbf{E}} \right) + \frac{1}{c} \mathbf{v} \times \left(\mathbf{B} + \hat{\mathbf{B}} \right) \right\} \cdot \frac{\partial}{\partial \mathbf{v}} \right] (f_a + \hat{f}_a) = C_a(f_a + \hat{f}_a)$$

Ensemble-averaged kinetic equation

$$\frac{d}{dt} f_a \equiv \left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_a}{m_a} \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial}{\partial \mathbf{v}} \right] f_a = \langle C_a \rangle_{\mathrm{ens}} + \mathcal{D}_a$$

$$\mathcal{D}_a = -\frac{e_a}{m_a} \left\langle \left(\hat{\mathbf{E}} + \frac{1}{c} \mathbf{v} \times \hat{\mathbf{B}} \right) \cdot \frac{\partial \hat{f}_a}{\partial \mathbf{v}} \right\rangle_{\mathrm{ens}}$$

Classical, Neoclassical, and Anomalous Transport of Particles and Heat [Sugama et al. PoP1996]

The gyrophase (ξ) -average part and the oscillating part of an aribtrary function F is defined by $\overline{F} \equiv (2\pi)^{-1} \oint d\xi \; F$ **and** $\widetilde{F} \equiv F - \overline{F}$ **respectively.**

Particle flux $$\Gamma_a \equiv \langle \mathbf{\Gamma}_a \cdot \nabla s \rangle \equiv \left\langle \int d^3v \; \widetilde{f}_a \mathbf{v} \cdot \nabla s \right\rangle$$

Heat flux $$\frac{q_a}{T_a} \equiv \frac{\langle \mathbf{q}_a \cdot \nabla s \rangle}{T_a} \equiv \left\langle \int d^3v \; \widetilde{f}_a \left(\frac{m_a v^2}{2T_a} - \frac{5}{2} \right) \mathbf{v} \cdot \nabla s \right\rangle$$

The ensemble-averaged kinetic equation is divided as

$$\overline{\mathcal{L}(\overline{f}_a + \widetilde{f}_a)} = \left\langle \overline{C}_a \right\rangle_{\text{ens}} + \overline{\mathcal{D}}_a, \qquad \Omega_a \frac{\partial \widetilde{f}_a}{\partial \xi} = \widetilde{\mathcal{L} f_a} - \left\langle \widetilde{C}_a \right\rangle_{\text{ens}} - \widetilde{\mathcal{D}}_a$$

$$\mathcal{L} \equiv d/dt + \Omega_a \partial/\partial\xi$$

Second order part of $\widetilde{f}_a$ **in** $\delta \sim \rho / L$

$$\widetilde{f}_{a2} = \widetilde{f}_a^N + \widetilde{f}_a^H + \widetilde{f}_a^C + \widetilde{f}_a^A \equiv \frac{1}{\Omega_a} \int^{\xi} d\xi \left[\widetilde{\mathcal{L}\overline{f}_{a1}} + \widetilde{\mathcal{L}\widetilde{f}_{a1}} - C_a^L(\widetilde{f}_{a1}) - \widetilde{D}_a \right]$$

$$\Gamma_a = \Gamma_a^{\text{ncl}} + \Gamma_a^{\text{cl}} + \Gamma_a^{\text{anom}} \qquad q_a = q_a^{\text{ncl}} + q_a^{\text{cl}} + q_a^{\text{anom}}$$

Momentum Balance

$$\frac{\partial}{\partial t}(n_a m_a \mathbf{u}_a) = -\nabla \cdot \mathbf{P}_a + n_a e_a \left(\mathbf{E} + \frac{\mathbf{u}_a}{c} \times \mathbf{B}\right) + \mathbf{F}_{a1} + \mathbf{K}_{a1}$$

density $n_a \equiv \int d^3v \; f_a$ **particle flux** $n_a \mathbf{u}_a \equiv \int d^3v \; f_a \mathbf{v}$

pressure tensor $\mathbf{P}_a \equiv \int d^3v \; f_a m_a \mathbf{v}\mathbf{v}$

friction force $\mathbf{F}_{a1} \equiv \int d^3v \; C_a(f_a) m_a \mathbf{v}$

turbulent electromagnetic force $\mathbf{K}_{a1} \equiv \int d^3v \; \mathcal{D}_a \mathbf{v}$

$$\begin{aligned}\sum_a \mathbf{K}_{a1} &= \nabla \cdot \left\langle \frac{1}{4\pi}\left(\hat{\mathbf{E}}\hat{\mathbf{E}} + \hat{\mathbf{B}}\hat{\mathbf{B}}\right) - \frac{1}{8\pi}\left(\hat{E}^2 + \hat{B}^2\right)\mathbf{I} \right\rangle_{\mathrm{ens}} - \frac{1}{4\pi c}\frac{\partial}{\partial t}\left\langle \hat{\mathbf{E}} \times \hat{\mathbf{B}} \right\rangle_{\mathrm{ens}} \\ &= \nabla \cdot \mathbf{T}_{EM} - \frac{\partial}{\partial t}\left(\frac{\mathbf{S}_{EM}}{c^2}\right),\end{aligned}$$

Momentum Balance in the direction tangential to the flux surface

(c_1, c_2 : constants)

$$\frac{\partial}{\partial t}\sum_a \left\langle n_a m_a \left\{ c_1\left(u_{a\theta} + \frac{(S_{EM})_\theta}{c^2}\right) + c_2\left(u_{a\zeta} + \frac{(S_{EM})_\zeta}{c^2}\right)\right\}\right\rangle$$

$$= -\frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle \nabla s \cdot \left(\sum_a \mathbf{P}_a - \mathbf{T}_{EM}\right) \cdot \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta}\right)\right\rangle\right] + \frac{1}{c}\left(-c_1\psi' + c_2\chi'\right)\sum_a e_a \left\langle n_a u_a^s \right\rangle$$

(s, θ, ζ) : Hamada coordinates

The surface-averaged radial current

$$\sum_a e_a \Gamma_a \equiv \sum_a e_a \left\langle n_a u_a^s \right\rangle = -\frac{1}{4\pi}\frac{\partial}{\partial t}\left\langle E^s \right\rangle$$

Quasisymmetry

[Boozer(1983), Nuhrenberg(1988), Helander&Simakov (2008)]

$$c_1 \frac{\partial B}{\partial \theta} + c_2 \frac{\partial B}{\partial \zeta} = 0$$

quasi-axi-symmetry $(c_1,\ c_2) = (0, 1)$

quasi-poloidal-symmetry $(c_1,\ c_2) = (1, 0)$

The $O(\delta)$ viscosity component in the quasisymmetry direction vanishes :

$$\left\langle \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \left[\nabla \cdot \left\{ P_{\|a} \mathbf{bb} + P_{\perp a} \left(\mathbf{I} - \mathbf{bb} \right) \right\} \right] \right\rangle$$

$$= - \left\langle \left(P_{\|a} - P_{\perp a} \right) \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \nabla \ln B \right\rangle = 0$$

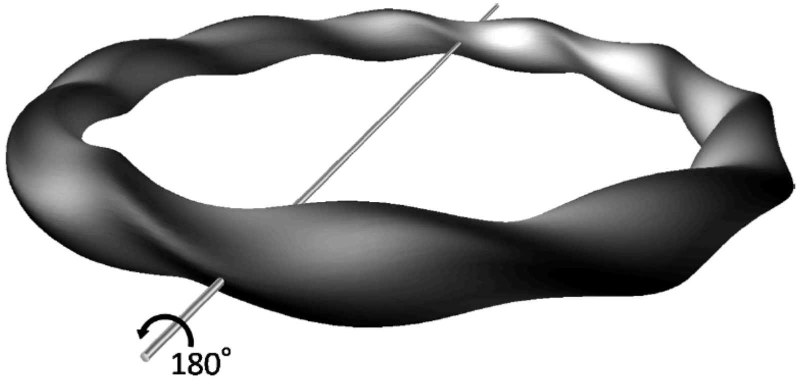
The ambipolarity $\sum_a e_a \langle n_a u_a^s \rangle = 0$

is satisfied automatically up to $O(\delta)$.

Stellarator Symmetry

Magnetic field strength

$$B(s,-\theta,-\zeta) = B(s,\theta,\zeta)$$



Magnetic field components

$$B^\theta(s,-\theta,-\zeta) = B^\theta(s,\theta,\zeta), \qquad B^\zeta(s,-\theta,-\zeta) = B^\zeta(s,\theta,\zeta)$$

$$B_\theta(s,-\theta,-\zeta) = B_\theta(s,\theta,\zeta), \qquad B_\zeta(s,-\theta,-\zeta) = B_\zeta(s,\theta,\zeta)$$

$$B_s(s,-\theta,-\zeta) = -B_s(s,\theta,\zeta),$$

Metric tensor components

$$g_{ss}(s,-\theta,-\zeta) = g_{ss}(s,\theta,\zeta), \qquad g_{\theta\theta}(s,-\theta,-\zeta) = g_{\theta\theta}(s,\theta,\zeta),$$

$$g_{\theta\zeta}(s,-\theta,-\zeta) = g_{\theta\zeta}(s,\theta,\zeta), \qquad g_{\zeta\zeta}(s,-\theta,-\zeta) = g_{\zeta\zeta}(s,\theta,\zeta),$$

$$g_{s\theta}(s,-\theta,-\zeta) = -g_{s\theta}(s,\theta,\zeta), \qquad g_{s\zeta}(s,-\theta,-\zeta) = -g_{s\zeta}(s,\theta,\zeta),$$

$$g(s,-\theta,-\zeta) = g(s,\theta,\zeta),$$

Parity Transformation associated with Stellarator Symmetry

Expansion in $\eta \sim \delta \sim \rho / L$ (Put $e_a \to \eta^{-1} e_a$ in Boltzmann and Maxwell eqs.)

$$f_a(s,\theta,\zeta,v^s,v^\theta,v^\zeta,t,\eta) = f_{aM}(s,v,\eta^2 t) + \eta f_{a1}(s,\theta,\zeta,v^s,v^\theta,v^\zeta,\eta^2 t) + \eta^2 f_{a2}(s,\theta,\zeta,v^s,v^\theta,v^\zeta,\eta^2 t) + \cdots,$$

$$\Phi(s,\theta,\zeta,t,\eta) = \eta\Phi_1(s,\eta^2 t) + \eta^2\Phi_2(s,\theta,\zeta,\eta^2 t)$$

Parity operator $\mathcal{P}$ **is defined by**

$$(\mathcal{P}Q)(s,\theta,\zeta,v^s,v^\theta,v^\zeta,t,\eta) \equiv Q(s,-\theta,-\zeta,v^s,-v^\theta,-v^\zeta,t,-\eta)$$

In the presence of stellarator symmetry, Boltzmann and Maxwell equations are invariant under parity transformation

$$f_a + \hat{f}_a \longrightarrow \mathcal{P}(f_a + \hat{f}_a)$$

$$E_s + \hat{E}_s,\ E_\theta + \hat{E}_\theta,\ E_\zeta + \hat{E}_\zeta \longrightarrow -\mathcal{P}(E_s + \hat{E}_s),\ \mathcal{P}(E_\theta + \hat{E}_\theta),\ \mathcal{P}(E_\zeta + \hat{E}_\zeta)$$

$$B_s + \hat{B}_s,\ B_\theta + \hat{B}_\theta,\ B_\zeta + \hat{B}_\zeta \longrightarrow -\mathcal{P}(B_s + \hat{B}_s),\ \mathcal{P}(B_\theta + \hat{B}_\theta),\ \mathcal{P}(B_\zeta + \hat{B}_\zeta)$$

Momentum Transport Fluxes in Stellarator Symmetric Systems

Parity of solutions

$$\mathcal{P} f_a = f_a, \qquad -\mathcal{P}\Phi = \Phi$$

$$f_j(s, -\theta, -\zeta, v^s, -v^\theta, -v^\zeta, \eta^2 t) = (-1)^j f_j(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t)$$

$$\Phi_j(s, -\theta, -\zeta, \eta^2 t) = (-1)^{j-1} \Phi_j(s, \theta, \zeta, \eta^2 t)$$

When j is even, the $O(\delta^j)$ part of radial transport fluxes of poloidal and toroidal momentum vanish.

$$\left\langle \left(P_a^{(j)}\right)_\theta^s \right\rangle = \left\langle \left(P_a^{(j)}\right)_\zeta^s \right\rangle = \langle (T_{EM}^{(j)})_\theta^s \rangle = \langle (T_{EM}^{(j)})_\zeta^s \rangle = 0 \quad (\text{for even } j)$$

Momentum Balance in Quasisymmetric Systems with Stellarator Symmetry

In quasisymmetric systems with stellarator symmetry, the momentum transport fluxes vanish up to $O(\delta^2)$, and the ambipolarity is automatically satisfied up to $O(\delta^2)$.

The momentum balance equation determing E_s is of $O(\delta^3)$:

$$\frac{\partial}{\partial t}\left[\frac{(c_2\chi' - c_1\psi')}{4\pi c}\left\{\langle|\nabla s|^2\rangle + \frac{4\pi c^2 \sum_a n_a m_a}{(c_2\chi' - c_1\psi')^2}\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle\right\} E_s\right.$$

$$\left. + \sum_a \frac{m_a}{(c_2\chi' - c_1\psi')}\left\{-\frac{c}{e_a}\frac{\partial p_a}{\partial s}\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle + \frac{n_a V'}{4\pi^2}\langle c_1 B_\theta + c_2 B_\zeta\rangle \left\langle c_2 u_a^\theta - c_1 u_a^\zeta\right\rangle\right\}\right]$$

$$= -\frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle \nabla s \cdot \left(\sum_a \mathbf{P}_a^{(3)} - \mathbf{T}_{EM}^{(3)}\right)\cdot\left(c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\varsigma}\right)\right\rangle\right]$$

Momentum Balance in Toroidally Rotating Tokamaks with Toroidal Velocity $V \sim v_{Ti}$ [Sugama& Horton, PoP1998]

Toroidal flow is proportional to the radial electric field

$$V_0 = RV^{\zeta} = -Rc\frac{\partial \Phi_0(\Psi)}{\partial \Psi}$$

The momentum balance equation determining E_s is of $O(\delta^2)$:

$$\frac{\partial}{\partial t}\left\langle \left(\sum_a m_a n_a\right)\left(1+\frac{v_{PA}^2}{c^2}\right) R^2 V^{\zeta}\right\rangle + \frac{1}{V'}\frac{\partial}{\partial \Psi}\left(V' \sum_a \Pi_a\right) = \sum_a \left\langle \int d^3v \; m_a v_{\zeta} (\mathcal{D}_a + \mathcal{I}_a)\right\rangle$$

Toroidal momentum flux is of $O(\delta^2)$.

$$\Pi_a = \Pi_a^{\mathrm{cl}} + \Pi_a^{\mathrm{ncl}} + \Pi_a^{H} + \Pi_a^{(E)} + \Pi_a^{\mathrm{anom}}$$

Neoclassical and Anomalous Toroidal Momentum Fluxes in Toroidally Rotating Tokamaks

Neoclassical toroidal momentum flux of $O(\delta^2)$.

$$\Pi_a^{\mathrm{ncl}} + \Pi_a^{H} = -m_a c I V^{\zeta} \langle n_a R^2 \rangle \frac{\langle B E_{\parallel}^{(A)} \rangle}{\langle B^2 \rangle} - \frac{m_a c}{2 e_a} \left\langle \int d^3 v \left[m_a \left(R^2 V^{\zeta} + \frac{I}{B} v_{\parallel}' \right) + \mu \frac{R^2 B_P^2}{B} \right] C_a^L(\bar{g}_a) \right\rangle$$

$$v_{\parallel}' \mathbf{b} \cdot \nabla \bar{g}_a - C_a^L(\bar{g}_a) = \frac{1}{T_a} f_{a0} (W_{a1} X_{a1} + W_{a2} X_{a2} + W_{aV} X_V + W_{aE} X_E)$$

$$X_V \equiv -\frac{\partial V^{\zeta}}{\partial \Psi} = c \frac{\partial^2 \Phi_0}{\partial \Psi^2} \qquad W_{aV} \equiv \frac{m_a c}{2 e_a} v_{\parallel}' \mathbf{b} \cdot \nabla \left[m_a \left(R^2 V^{\zeta} + \frac{I}{B} v_{\parallel}' \right)^2 + \mu \frac{R^2 B_P^2}{B} \right]$$

Anomalous toroidal momentum flux of $O(\delta^2)$.

$$\Pi_a^{A} = \left\langle \left\langle \int d^3 v \ \hat{h}_a(\mathbf{X}) \hat{w}_{aV}(\mathbf{X}) \right\rangle \right\rangle$$

$$\hat{w}_{aV}(\mathbf{X}) = \left\langle -\frac{c}{B} \nabla \left(\hat{\phi}(\mathbf{x}) - \frac{1}{c} (\mathbf{V}_0 + \mathbf{v}') \cdot \hat{\mathbf{A}}(\mathbf{x}) \right) \times \mathbf{b} \cdot \nabla \Psi m_a (\mathbf{V}_0 + \mathbf{v}') \cdot (R \hat{\boldsymbol{\zeta}}) \right\rangle_{\mathbf{X}}$$

$$\left[\frac{\partial}{\partial t} + \left(\mathbf{V}_0 + v_{\parallel}' \mathbf{b} + \mathbf{v}_{da} - \frac{c}{B} \nabla \hat{\psi}_a(\mathbf{X}) \times \mathbf{b} \right) \cdot \nabla \right] \hat{h}_a(\mathbf{X}) - \langle C_a^L [\hat{f}_a(\mathbf{X} + \boldsymbol{\rho}_a)] \rangle_{\mathbf{X}}$$

$$= \frac{c}{B} \nabla \hat{\psi}_a(\mathbf{X}) \times \mathbf{b} \cdot \left[\nabla + \left\{ \frac{e_a}{T_a} \frac{\partial \langle \Phi_1 \rangle}{\partial \Psi} + \frac{m_a}{T_a} \left(R^2 V^{\zeta} + \frac{I}{B} v_{\parallel}' \right) \frac{\partial V^{\zeta}}{\partial \Psi} \right\} \nabla \Psi \right] f_{a0} + \frac{e_a}{T_a} \left[\left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla \right) \hat{\psi}_a(\mathbf{X}) \right] f_{a0}$$

Quasi-axisymmetric System with Toroidal Velocity $V \sim v_{Ti}$

Toroidal flow $\quad \mathbf{V} = V^{\zeta}\dfrac{\partial \mathbf{x}}{\partial \zeta}, \qquad V^{\zeta} = -c\dfrac{\Phi_0'(s)}{\chi'(s)} = \mathcal{O}(v_{Ti})$

Equilibrium force balance

$$\left(\sum_a n_a m_a\right)\mathbf{V}\cdot\nabla\mathbf{V} = \frac{1}{c}\mathbf{J}\times\mathbf{B} - \nabla P$$

Toroidal component

$$\frac{1}{2}\left(\sum_a n_a m_a\right)(V^{\zeta})^2\frac{\partial g_{\zeta\zeta}}{\partial \zeta} = \frac{\chi'}{c}J^s = \frac{B^{\theta}}{c}\left(\frac{\partial B_{\zeta}}{\partial \theta} - \frac{\partial B_{\theta}}{\partial \zeta}\right)$$

Generally, $\quad \partial g_{\zeta\zeta}/\partial\zeta \neq 0 \qquad$ **Therefore,** $\quad J^s \neq 0$

Then, neither Boozer nor Hamada coordinates can be constructed. Thus, high toroidal velocity on the order of ion thermal velocity does not seem to be allowed by simple quasiaxisymmetry condition only.

Summary

- **Fluctuations observed in a high T_i LHD plasma are considered as ITG modes predicted from linear calculation by GKV-X.**

- **Zonal-flow response theory and simulation show that zonal flow generation and turbulence regulation are enhanced when the radial displacements of helical-ripple-trapped particles are reduced either by neoclassical optimization of the helical geometry lowering the radial drift velocity or by strengthening the radial electric field E_r to boost the poloidal rotation.**

- **The E_r effects appear through the poloidal Mach number M_p.**
  For the same magnitude of E_r, higher zonal-flow response is obtained by using ions with heavier mass (favorable deviation from gyro-Bohm scaling).

- **The momentum balance equation determining E_r in quasisymmetric helical system with stellarator symmetry is shown to be of $O(\delta^3)$**
  by using a novel parity operator.