Effects of Three-Dimensional Geometry and Radial Electric Field on ITG Turbulence and Zonal Flows

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Introduction
Tokamak

\[ B = B_0 (1 - \varepsilon_t \cos \theta) \]

Helical System

\[ B = B_0 \left[ 1 - \varepsilon_t \cos \theta - \varepsilon_h \cos \left( L \theta - M \zeta \right) \right] \]
Helical geometry influences ITG mode and zonal flow.

Eigenfunction of linear ITG mode electrostatic potential

Zonal-flow response (GAM, residual ZF)

Watanabe et al. NF2007

Sugama & Watanabe PoP2006
Gyrokinetic Equations (for ITG Turbulence)

\[ k_\perp \rho_i \approx 1, \quad k_\perp \rho_e << 1 \]

Ion gyrokinetic equation for \( \delta f(x, v_\parallel, \mu, t) \)

\[
\left[ \frac{\partial}{\partial t} + v_\parallel \hat{b} \cdot \nabla + v_d \cdot \nabla - \mu \left( \hat{b} \cdot \nabla \Omega \right) \frac{\partial}{\partial v_\parallel} \right] \delta f + \frac{c}{B_0} \{ \psi, \delta f \} = \left( v_* - v_d - v_\parallel \hat{b} \right) \cdot \frac{e \nabla \psi}{T_i} F_M + C(\delta f)
\]

Diamagnetic drift

\[ v_* = - \frac{cT_i}{eL_n B_0} \left[ 1 + \eta_i \left( \frac{mv^2}{2T_i} - \frac{3}{2} \right) \right] \hat{y}, \quad \mu = \frac{v_\perp^2}{2\Omega} \]

Gyrocenter drift

\[ v_d \cdot \nabla \]

Mirror force

\[-\mu (\hat{b} \cdot \nabla \Omega) \frac{\partial}{\partial v_\parallel} \]

Effects of magnetic geometry

Quasineutrality condition & Adiabatic electron assumption

\[
\int J_0(k_\perp v_\perp / \Omega) \delta f \, d^3 \nu - \left[ 1 - \Gamma_0(k_\perp^2) \right] \frac{e \phi}{T_i} = \frac{e}{T_e} \left( \phi - \langle \phi \rangle \right), \quad k_\perp^2 = \left( k_x + s_z k_y \right)^2 + k_y^2
\]

Ion polarization
Linear ITG Mode Analysis for High-$T_i$ LHD plasmas
Fluctuation in High-\(T_i\) discharge in LHD

K. Tanaka et al., to be appeared in Plasma Fusion Res.

\(t = 1.833s\)

\(t = 2.233s\) (High \(T_i\))

- Fluctuation peak exists at \(\rho = 0.8-1.0\) in space, \(k_{p\rho_1} \sim 0.26\) in wavenumber.

- Fluctuation peak exists at \(\rho = 0.5-0.8\) in space, \(k_{p\rho_1} \sim 0.45\) in wavenumber.
Results from Linear ITG Mode Analyses by GKV-X
(See Poster by M. Nunami)

Radial profiles of $\gamma_{\text{max}}$

- Growth rates are peaked at
  $\rho \sim 0.65$ (t=2.233s),
  $\rho \sim 0.85$ (t=1.833s).

Growth rates of ITG modes

- There exists ITG unstable region.
- Maximum growth rates exists at
  $k_0 \rho_i \sim 0.35$ (t=2.233s),
  $k_0 \rho_i \sim 0.20$ (t=1.833s),
in poloidal wavenumber space.
Zonal Flows and ITG Turbulence in Helical Systems
For low collisionality, better confinement is observed in the inward-shifted magnetic configurations, where lower neoclassical ripple transport but more unfavorable magnetic curvature driving pressure-gradient instabilities are anticipated.

Anomalous transport is also improved in the inward shifted configuration.

Scenario:
Neoclassical optimization contributes to reduction of anomalous transport by enhancing the zonal-flow level.
Collisionless Time Evolution of Zonal Flows in Helical Systems
[Sugama & Watanabe, PRL (2005), Phys.Plasmas (2006)]

Response of the zonal-flow potential to a given initial potential
\[ \langle \phi_k (t) \rangle = K(t) \langle \phi_k (0) \rangle \]

**Response function**  
\[ K(t) = K_{GAM}(t)[1 - K_L(0)] + K_L(t) \]

\[ K(t = 0) = 1 \quad K(t) \to K_L(t), \quad K_{GAM}(t) \to 0 \text{ as } t \to +\infty \]

**GAM response function**  
\[ K_{GAM}(t) = \cos(\omega_G) \exp(-\gamma t) \]

**Long-time response function**
\[ K_L(t) = \frac{1 - (2/\pi)^{1/2} \langle (2\varepsilon_H)^{1/2} \{1 - g_{il}(t,\theta)\} \rangle}{1 + G + \langle E(t) \rangle (n_0 \langle k^2_{\perp} \rho_{ti}^2 \rangle)} \]

\( E(t) \) represents effects of shielding of potential due to helical-ripple-trapped particles.
\[ E(t) = \frac{2}{\pi} n_0 \left[ \frac{\langle (2\varepsilon_H)^{1/2} \{1 - g_{il}(t,\theta)\} \rangle - \frac{3}{2} \langle k^2_{\perp} \rho_{ti}^2 \rangle \langle (2\varepsilon_H)^{1/2} \{1 - g_{il}(t,\theta)\} \rangle}{T_e \langle (2\varepsilon_H)^{1/2} \{1 - g_{el}(t,\theta)\} \rangle} \right] + \frac{T}{T_e} \frac{\langle (2\varepsilon_H)^{1/2} \{1 - g_{el}(t,\theta)\} \rangle}{\langle (2\varepsilon_H)^{1/2} \{1 - g_{el}(t,\theta)\} \rangle} \]

\[ B = B_0 \left[ 1 - \varepsilon_i \cos \theta - \varepsilon_h \cos (L\theta - M\zeta) \right] \]

\( \varepsilon_h = 0.1 \quad (L=2, \quad M=10) \)
Smaller $\chi_i$ and larger zonal flows are found in the saturated turbulent state for the inward-shifted configuration than for the standard one!

Larger residual zonal flow is found for the inward-shifted case.

Watanabe, Sugama & Ferrando, PRL(2008)
Sugama, Watanabe & Ferrando, PFR(2009)

The GKV turbulence simulations were carried out by the Earth Simulator (JAMSTEC).

Results from GKV simulation (flux tube, $E_r = 0$)

Turbulent thermal diffusivity and squared zonal-flow potential

Larger residual zonal flow is found for the inward-shifted case.
Effects of Equilibrium Electric Field $E_r$ on Zonal Flows in Helical Systems

In helical systems $E_r$ is given from ambipolar condition of radial particle fluxes. $E_r$ reduces neoclassical ripple transport.

How does $E_r$ influence zonal flows and anomalous transport?
Effects of $E_r$ on gyrokinetic equation and zonal flows

Gyrokinetic equation for $k_\perp = k_r \nabla r$

\[
\left[ \frac{\partial}{\partial t} + v_\parallel \hat{b} \cdot \nabla + i k_r \cdot v_d - \mu (\hat{b} \cdot \nabla \Omega) \frac{\partial}{\partial v_\parallel} + \omega_E \frac{\partial}{\partial \alpha} \right] \delta f = -i k_r \cdot v_d \frac{e \langle \phi(x + \rho) \rangle}{T_i} F_M
\]

angular velocity due to ExB drift $\omega_E = -\frac{c E_r}{r_0 B_0}$  
field line label $\alpha = \theta - \zeta / q$

Gyrophase average of zonal-flow potential

In helical systems, $\alpha$-dependence appears in $k_r \cdot v_d$ and $\mu (\hat{b} \cdot \nabla \Omega)$.

Therefore, even if the zonal-flow potential $\phi$ is independent of $\alpha$, $\delta f$ comes to depend on $\alpha$.

Thus, $\omega_E$ influences $\delta f$ and accordingly $\phi$ through quasineutrality condition.
Classification of particle orbits in the presence of $E_r$

$\Delta E \sim r_0 \frac{V_{dr}}{V_{E\times B}}$

radial displacement of helically-trapped particle

trapping parameter

$\kappa^2 = \frac{1 - \lambda B_0 [1 - \varepsilon_T(\theta) - \varepsilon_H(\theta)]}{2\lambda B_0 \varepsilon_H(\theta)}$

$\lambda = \frac{1}{2} \frac{mv^2}{\mu}$

Cary et al., PF (1988)
Wakatani (1998)
Solution of gyrokinetic equation to describe long-time evolution of zonal flows [Sugama & Watanabe, PoP(2009)]

Perturbed particle distribution function

\[ \delta f_k(t) = -\frac{e}{T} \phi_k(t) F_M \left[ 1 - e^{-i k_r \rho_r} e^{-i k_r \Delta_r} \left( \int e^{i k_r \rho_r} \Delta_r J_0(r) \right) \right] \]

+ \[ e^{-i k_r \rho_r} e^{-i k_r \Delta_r} \left( \int \delta f_k^{(g)}(0) + F_M \int_0^t S_k(t) dt \right) \]

Average along the orbit

\[ \langle \cdots \rangle_{\text{orbit}} = \oint \frac{dl}{(dl/dt)} \frac{dl}{(dl/dt)} \]

For particles which show transitions

\[ \oint \frac{dl}{(dl/dt)} = (1 - P_t) \int \left[ \frac{dl}{\left| v_{\|} \right|} \right] + \int \left[ \frac{dl}{\left| v_{\|} \right|} \right] + P_t \int \frac{d\theta}{\omega_E} \]

Polarization (classical & neoclassical)

Initial condition & Turbulence source

Average along the orbit

\[ \langle \cdots \rangle_{\text{orbit}} = \oint \frac{dl}{(dl/dt)} \frac{dl}{(dl/dt)} \]

For particles which show transitions

\[ \oint \frac{dl}{(dl/dt)} = (1 - P_t) \int \left[ \frac{dl}{\left| v_{\|} \right|} \right] + \int \left[ \frac{dl}{\left| v_{\|} \right|} \right] + P_t \int \frac{d\theta}{\omega_E} \]

Perturbed particle distribution function

\[ \rho_r \cdots \text{gyro motion} \]

\[ \Delta_r \cdots \text{drift motion} \]

ExB drift

transition points

helically trapped

toroidally trapped

\[ P_t \approx \frac{4 \sqrt{2}}{\pi} \left( \frac{c E_r}{v B_0} \right) \left( \frac{R_0 q}{r_0} \right) \left( \varepsilon_H \right)^{-1/2} \left( \frac{\partial \varepsilon_H / \partial \theta}{\partial (\varepsilon_H - \varepsilon_T) / \partial \theta} \right) \]
Long-time zonal-flow response to the initial condition and turbulence source

For radial wavenumbers $k_r \rho_{ti} < 1$ (ITG turbulence) and $k_r \Delta E < 1$, the zonal-flow potential is derived from the quasineutrality condition as \cite{Sugama & Watanabe, PoP(2009)}

$$
e \frac{\langle \phi_k(t) \rangle}{T_i} = \frac{1}{n_0^0} \int d^3v \left[ 1 + i k_r \left( \Delta_{ir} - \langle \Delta_{ir} \rangle_{\text{orbit}} \right) \right] \left[ \delta f^{(g)}_i(0) + F_M \int_0^t S_{ik}(t) dt \right] (k_r \rho_{ti})^2 \left[ 1 + G_p + G_t + \left( M_p^2 (G_{ht} + G_h)(1 + T_e/T_i) \right) \right]$$

Geometrical factors $G$'s represents shielding effects of neoclassical polarization due to particles motions in different orbits.

- $G \propto \text{(population)}$
- $G_p$ : passing
- $G_t$ : toroidally-trapped
- $G_{ht}$ : helically-trapped (unclosed orbit)
- $G_h$ : toroidally-trapped (closed orbit)

Zonal-flow generation can be enhanced when $G_{ht}$ and $G_h$ decreases with \textbf{neoclassical optimization} (which reduces radial drift velocity $V_{dr}$) and when \textbf{poloidal Mach number} $M_p = \left| \frac{cE_r/B_0}{r \nu_{ti}/R_q} \right|$ increases with increasing $E_r$ and using heavier ions.
Response to the initial condition

Assume the initial distribution to have Maxwellian dependence
\[ \delta f_{ik}^{(g)}(0) = [\delta n_k^{(g)}(0)/n_0] F_M \]
\[ \delta n_k^{(g)}(0)/n_0 = (k r_{hi})^2 e \phi(0)/T_i \]

Then, we obtain
\[ \langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + M_p^{-2} (G_{ht} + G_h)(1 + T_e/T_i)} \]

(no turbulence source)

For the single-helicity configuration
\[ B = B_0[1 - \varepsilon_i \cos \theta - \varepsilon_h \cos(L \theta - M \xi)] \quad (\varepsilon_h \text{ : independent of } \theta) \]

No transitions occur. \[ G_{ht} = 0, \quad G_h = (15 \pi / 4) q^2 (2 \varepsilon_h)^{1/2} \]

\[ \langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + (15 \pi / 4) M_p^{-2} q^2 (2 \varepsilon_h)^{1/2} (1 + T_e/T_i)} \]

This corresponds to the case considered by previous works.

Mynick & Boozer, PoP(2007)
Sugama, Watanabe & Ferrando, PFR(2008)
The residual zonal-flow potential as a function of $k_r \rho_{ti}$ for $M_p = 0$ and $M_p = 0.3$

Different $k_r \rho_{ti}$ dependences for $M_p = 0$ and $M_p = 0.3$ are theoretically predicted and confirmed by simulation.

Theoretical results are derived by assuming $k_r \rho_{ti} \ll 1$.

[To be published in CPP]
Momentum Balance and Radial Electric Field in Quasisymmetric Systems with Stellarator Symmetry
Basic Boltzmann Kinetic Equation for description of Collisional and Turbulent Transport

Equilibrium magnetic field
\[ B = \psi' \nabla s \times \nabla \theta + \chi' \nabla \zeta \times \nabla s = B_s \nabla s + B_\theta \nabla \theta + B_\zeta \nabla \zeta \]

Boltzmann kinetic equation
\[
\left[ \frac{\partial}{\partial t} + v \cdot \nabla + \frac{e_a}{m_a} \left\{ \left( E + \hat{E} \right) + \frac{1}{c} v \times \left( B + \hat{B} \right) \right\} \cdot \frac{\partial}{\partial v} \right] (f_a + \hat{f}_a) = C_a(f_a + \hat{f}_a)
\]

Ensemble-averaged kinetic equation
\[
\frac{d}{dt} f_a \equiv \left[ \frac{\partial}{\partial t} + v \cdot \nabla + \frac{e_a}{m_a} \left( E + \frac{1}{c} v \times B \right) \cdot \frac{\partial}{\partial v} \right] f_a = \langle C'_a \rangle_{\text{ens}} + D_a
\]
\[
D_a = -\frac{e_a}{m_a} \left\langle \left( \hat{E} + \frac{1}{c} v \times \hat{B} \right) \cdot \frac{\partial \hat{f}_a}{\partial v} \right\rangle_{\text{ens}}
\]
The gyrophase ($\xi$) -average part and the oscillating part of an arbitrary function $F$ is defined by

\[
\bar{F} \equiv (2\pi)^{-1} \int d\xi F \quad \text{and} \quad \tilde{F} \equiv F - \bar{F}
\]

respectively.

**Particle flux**

\[
\Gamma_a \equiv \langle \Gamma_a \cdot \nabla s \rangle \equiv \left\langle \int d^3v \bar{f}_a \mathbf{v} \cdot \nabla s \right\rangle
\]

**Heat flux**

\[
\frac{q_a}{T_a} \equiv \frac{\langle q_a \cdot \nabla s \rangle}{T_a} \equiv \left\langle \int d^3v \bar{f}_a \left( \frac{m_a v^2}{2T_a} - \frac{5}{2} \right) \mathbf{v} \cdot \nabla s \right\rangle
\]

The ensemble-averaged kinetic equation is divided as

\[
\bar{\mathcal{L}}(\bar{f}_a + \tilde{f}_a) = \langle \mathcal{C}_a \rangle_{\text{ens}} + \bar{D}_a, \quad \Omega_a \frac{\partial \bar{f}_a}{\partial \xi} = \bar{\mathcal{L}} f_a - \langle \mathcal{C}_a \rangle_{\text{ens}} - \bar{D}_a
\]

Second order part of $\tilde{f}_a$ in $\delta \sim \rho / L$

\[
\tilde{f}_{a2} = \tilde{f}_{aN} + \tilde{f}_{aH} + \tilde{f}_{aC} + \tilde{f}_{aA} \equiv \frac{1}{\Omega_a} \int \xi \, d\xi \left[ \bar{\mathcal{L}} \bar{f}_{a1} + \bar{\mathcal{L}} \tilde{f}_{a1} - C_a^{\bar{\mathcal{L}}} (\bar{f}_{a1}) - \bar{D}_a \right]
\]

\[
\Gamma_a = \Gamma_{a_{\text{ncl}}} + \Gamma_{a_{\text{cl}}} + \Gamma_{a_{\text{anom}}}, \quad q_a = q_{a_{\text{ncl}}} + q_{a_{\text{cl}}} + q_{a_{\text{anom}}}
\]
Momentum Balance

\[ \frac{\partial}{\partial t} \left( n_a m_a u_a \right) = -\nabla \cdot P_a + n_a e_a \left( E + \frac{u_a}{c} \times B \right) + F_{a1} + K_{a1} \]

density \: \: n_a \equiv \int d^3v \: f_a \quad \text{particle flux} \quad n_a u_a \equiv \int d^3v \: f_a v

pressure tensor \: \: \: P_a \equiv \int d^3v \: f_a m_a v v

friction force \: \: \: F_{a1} \equiv \int d^3v \: C_a(f_a) m_a v

turbulent electromagnetic force \: \: \: K_{a1} \equiv \int d^3v \: D_a v

\[ \sum_a K_{a1} = \nabla \cdot \left\langle \frac{1}{4\pi} \left( \hat{E} \hat{E} + \hat{B} \hat{B} \right) - \frac{1}{8\pi} \left( \hat{E}^2 + \hat{B}^2 \right) I \right\rangle_{\text{ens}} - \frac{1}{4\pi c} \frac{\partial}{\partial t} \left\langle \hat{E} \times \hat{B} \right\rangle_{\text{ens}} \]
\[ = \nabla \cdot T_{EM} - \frac{\partial}{\partial t} \left( \frac{S_{EM}}{c^2} \right), \]
Momentum Balance in the direction tangential to the flux surface

\[
\frac{\partial}{\partial t} \sum_a n_a m_a \left\{ c_1 \left( u_{a\theta} + \frac{(S_{EM})_{\theta}}{c^2} \right) + c_2 \left( u_{a\zeta} + \frac{(S_{EM})_{\zeta}}{c^2} \right) \right\}
\]

\[
= -\frac{1}{V'} \frac{\partial}{\partial s} \left[ V' \left( \nabla s \cdot \left( \sum_a P_a - T_{EM} \right) \cdot \left( c_1 \frac{\partial x}{\partial \theta} + c_2 \frac{\partial x}{\partial \zeta} \right) \right) \right] + \frac{1}{c} \left( -c_1 \psi' + c_2 \chi' \right) \sum_a e_a \langle n_a u^s_a \rangle
\]

\((s, \theta, \zeta) : \text{Hamada coordinates}\)

The surface-averaged radial current

\[
\sum_a e_a \Gamma_a \equiv \sum_a e_a \langle n_a u^s_a \rangle = -\frac{1}{4\pi} \frac{\partial}{\partial t} \langle E^s \rangle
\]
Quasisymmetry


\[ c_1 \frac{\partial B}{\partial \theta} + c_2 \frac{\partial B}{\partial \zeta} = 0 \]

quasi-axi-symmetry \quad (c_1, c_2) = (0, 1)

quasi-poloidal-symmetry \quad (c_1, c_2) = (1, 0)

The \( O(\delta) \) viscosity component in the quasisymmetry direction vanishes:

\[
\left\langle \left( c_1 \frac{\partial x}{\partial \theta} + c_2 \frac{\partial x}{\partial \zeta} \right) \cdot \left[ \nabla \cdot \left\{ P_{||a} b b + P_{\perp a} (I - b b) \right\} \right] \right\rangle
= - \left\langle \left( P_{||a} - P_{\perp a} \right) \left( c_1 \frac{\partial x}{\partial \theta} + c_2 \frac{\partial x}{\partial \zeta} \right) \cdot \nabla \ln B \right\rangle = 0
\]

The ambipolarity \( \sum_a e_a \langle n_a u_a^s \rangle = 0 \)

is satisfied automatically up to \( O(\delta) \).
Stellarator Symmetry

Magnetic field strength

\[ B(s, -\theta, -\zeta) = B(s, \theta, \zeta) \]

Magnetic field components

\[ B^\theta(s, -\theta, -\zeta) = B^\theta(s, \theta, \zeta), \quad B^\zeta(s, -\theta, -\zeta) = B^\zeta(s, \theta, \zeta) \]
\[ B_\theta(s, -\theta, -\zeta) = B_\theta(s, \theta, \zeta), \quad B_\zeta(s, -\theta, -\zeta) = B_\zeta(s, \theta, \zeta) \]
\[ B_s(s, -\theta, -\zeta) = -B_s(s, \theta, \zeta), \]

Metric tensor components

\[ g_{ss}(s, -\theta, -\zeta) = g_{ss}(s, \theta, \zeta), \quad g_{\theta\theta}(s, -\theta, -\zeta) = g_{\theta\theta}(s, \theta, \zeta), \]
\[ g_{\theta\zeta}(s, -\theta, -\zeta) = g_{\theta\zeta}(s, \theta, \zeta), \quad g_{\zeta\zeta}(s, -\theta, -\zeta) = g_{\zeta\zeta}(s, \theta, \zeta), \]
\[ g_{s\theta}(s, -\theta, -\zeta) = -g_{s\theta}(s, \theta, \zeta), \quad g_{s\zeta}(s, -\theta, -\zeta) = -g_{s\zeta}(s, \theta, \zeta), \]
\[ g(s, -\theta, -\zeta) = g(s, \theta, \zeta), \]
Parity Transformation associated with Stellarator Symmetry

Expansion in \( \eta \sim \delta \sim \rho / L \)

\[
f_a(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t, \eta) = f_{aM}(s, v, \eta^2 t) + \eta f_{a1}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t) + \eta^2 f_{a2}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t) + \cdots,
\]

\[
\Phi(s, \theta, \zeta, t, \eta) = \eta \Phi_1(s, \eta^2 t) + \eta^2 \Phi_2(s, \theta, \zeta, \eta^2 t)
\]

Parity operator \( \mathcal{P} \) is defined by

\[
(\mathcal{P} Q)(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t, \eta) \equiv Q(s, -\theta, -\zeta, v^s, -v^\theta, -v^\zeta, t, -\eta)
\]

In the presence of stellarator symmetry, Boltzmann and Maxwell equations are invariant under parity transformation

\[
f_a + \hat{f}_a \quad \longrightarrow \quad \mathcal{P}(f_a + \hat{f}_a)
\]

\[
E_s + \hat{E}_s, \quad E_\theta + \hat{E}_\theta, \quad E_\zeta + \hat{E}_\zeta \quad \longrightarrow \quad -\mathcal{P}(E_s + \hat{E}_s), \quad \mathcal{P}(E_\theta + \hat{E}_\theta), \quad \mathcal{P}(E_\zeta + \hat{E}_\zeta)
\]

\[
B_s + \hat{B}_s, \quad B_\theta + \hat{B}_\theta, \quad B_\zeta + \hat{B}_\zeta \quad \longrightarrow \quad -\mathcal{P}(B_s + \hat{B}_s), \quad \mathcal{P}(B_\theta + \hat{B}_\theta), \quad \mathcal{P}(B_\zeta + \hat{B}_\zeta)
\]
Parity of solutions

\( \mathcal{P} f_a = f_a, \quad -\mathcal{P} \Phi = \Phi \)

\[ f_j(s, -\theta, -\zeta, v^s, -v^\theta, -v^\zeta, \eta^2 t) = (-1)^j f_j(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t) \]

\[ \Phi_j(s, -\theta, -\zeta, \eta^2 t) = (-1)^{j-1} \Phi_j(s, \theta, \zeta, \eta^2 t) \]

When \( j \) is even, the \( O(\delta^j) \) part of radial transport fluxes of poloidal and toroidal momentum vanish.

\[ \left\langle \left( P_a^{(j)} \right)_\theta^s \right\rangle = \left\langle \left( P_a^{(j)} \right)_\zeta^s \right\rangle = \left\langle \left( T_{EM}^{(j)} \right)_\theta^s \right\rangle = \left\langle \left( T_{EM}^{(j)} \right)_\zeta^s \right\rangle = 0 \quad \text{(for even } j) \]
Momentum Balance in Quasisymmetric Systems with Stellarator Symmetry

In quasisymmetric systems with stellarator symmetry, the momentum transport fluxes vanish up to $O(\delta^2)$, and the ambipolarity is automatically satisfied up to $O(\delta^2)$.

The momentum balance equation determining $E_s$ is of $O(\delta^3)$:

$$
\frac{\partial}{\partial t} \left[ \frac{(c_2 \chi' - c_1 \psi')}{4\pi c} \left\{ \langle |\nabla s|^2 \rangle + \frac{4\pi c^2}{(c_2 \chi' - c_1 \psi')^2} \left\langle \left| \frac{1}{\partial \theta} + \frac{c_2}{\partial \zeta} \right|^2 \right\rangle \right\} E_s 
+ \sum_{a} \frac{m_a}{(c_2 \chi' - c_1 \psi')} \left\{ -\frac{c}{e_a} \frac{\partial p_a}{\partial s} \left\langle \left| \frac{1}{\partial \theta} + \frac{c_2}{\partial \zeta} \right|^2 \right\rangle + \frac{n_a V' c_1^2}{4\pi^2} \langle B_\theta + c_2 B_\zeta \rangle \langle c_2 u_a^\theta - c_1 u_a^\xi \rangle \right\} \right] 
= -\frac{1}{V'} \frac{\partial}{\partial S} \left[ V' \langle \nabla s \cdot \left( \sum_{a} P_a^{(3)} - T_{EM}^{(3)} \right) \cdot \left( \frac{1}{\partial \theta} + \frac{c_2}{\partial \zeta} \right) \rangle \right]
$$
Summary

- Fluctuations observed in a high $T_i$ LHD plasma are considered as ITG modes predicted from linear calculation by GKV-X.

- Zonal-flow response theory and simulation show that zonal flow generation and turbulence regulation are enhanced when the radial displacements of helical-ripple-trapped particles are reduced either by neoclassical optimization of the helical geometry lowering the radial drift velocity or by strengthening the radial electric field $E_r$ to boost the poloidal rotation.

- The $E_r$ effects appear through the poloidal Mach number $M_p$. For the same magnitude of $E_r$, higher zonal-flow response is obtained by using ions with heavier mass (favorable deviation from gyro-Bohm scaling).

- The momentum balance equation determining $E_r$ in quasisymmetric helical system with stellarator symmetry is shown to be of $O(\delta^3)$ by using a novel parity operator.