GROKINETICS IN LABORATORY AND ASTROPHYSICAL PLASMAS
Workshop "Kinetic-scale turbulence in laboratory and space plasmas: emprical constraints, fundamental concepts and unsolved problems"
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Effects of Three-Dimensional Geometry and Radial Electric Field on ITG Turbulence and Zonal Flows

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OUTLINE

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- **Linear ITG Mode Analysis for High-T_{i} LHD plasmas**
- **Zonal Flows and ITG Turbulence in Helical Systems**
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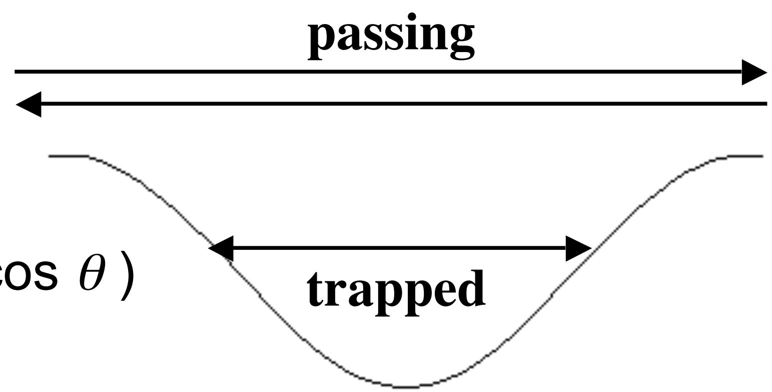
Introduction

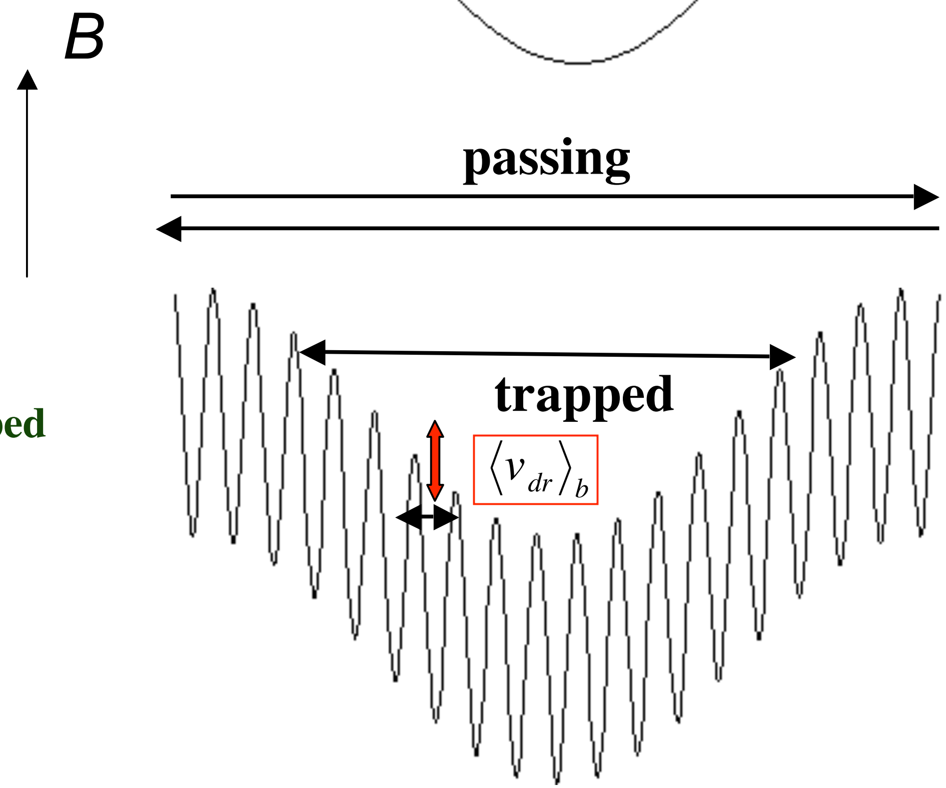
Classification of particle orbits

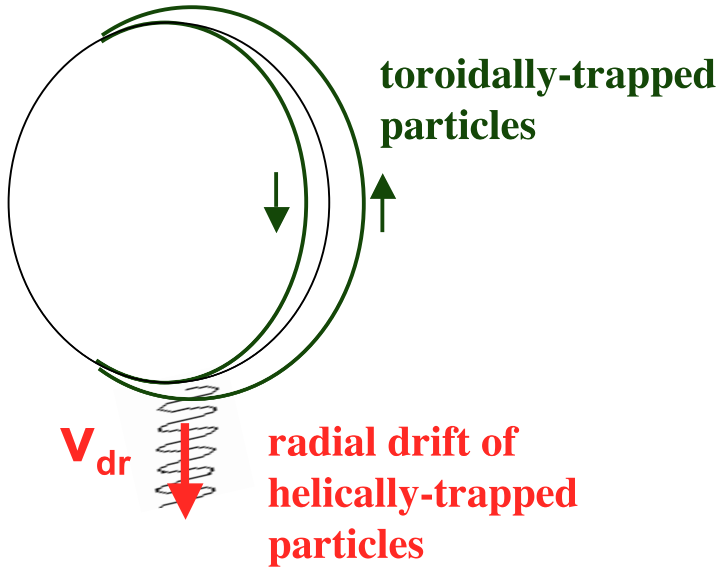
Tokamak

$B = B_0\ (1 - \varepsilon_t \cos\theta)$



Helical System

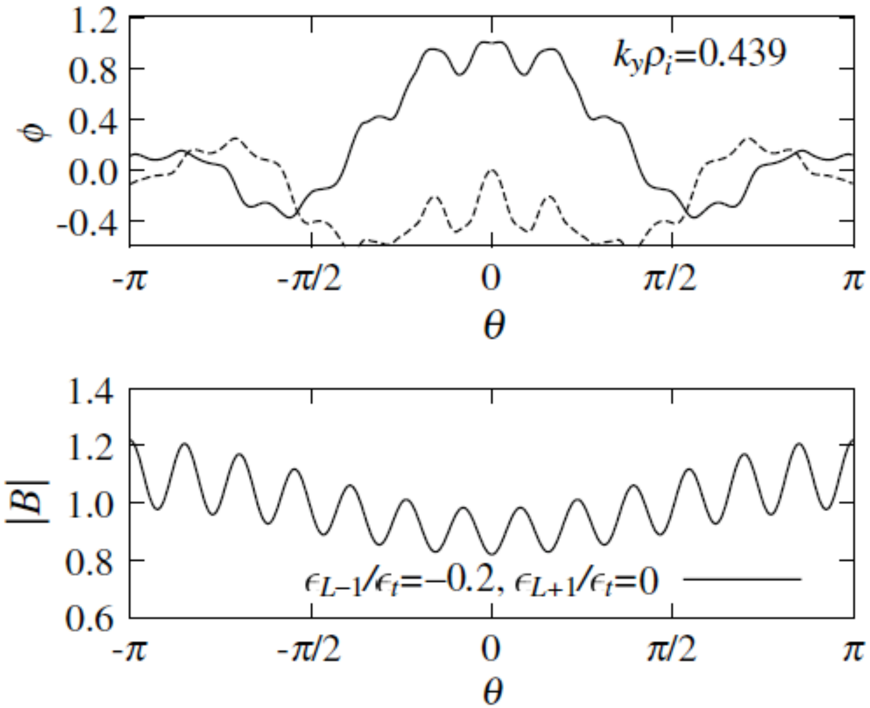




$B = B_0\ [\ 1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\zeta)\ ]$

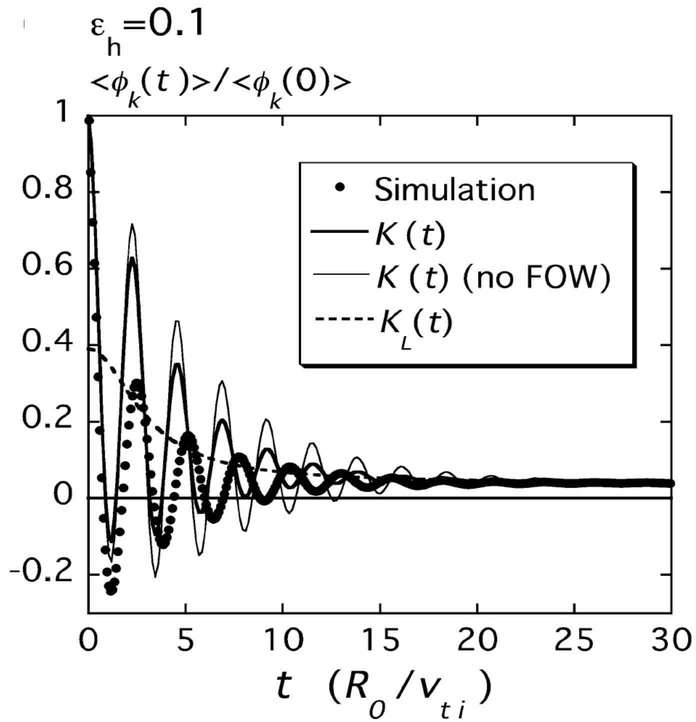
Helical geometry influences ITG mode and zonal flow.

Eigenfunction of linear ITG mode electrostatic potential



Watanabe et al. NF2007

Zonal-flow response (GAM, residual ZF)



Sugama & Watanabe PoP2006

Gyrokinetic Equations (for ITG Turbulence)

$k_\perp \rho_i \approx 1, \quad k_\perp \rho_e << 1$

Ion gyrokinetic equation for $\delta f(x, v_{\|}, \mu, t)$

$$\boxed{\left[\frac{\partial}{\partial t} + v_\| \hat{\mathbf{b}} \cdot \nabla + \mathbf{v}_d \cdot \nabla - \mu\left(\hat{\mathbf{b}} \cdot \nabla \Omega\right)\frac{\partial}{\partial v_\|}\right]\delta f + \frac{c}{B_0}\{\psi, \delta f\} = \left(\mathbf{v}_* - \mathbf{v}_d - v_\| \hat{\mathbf{b}}\right) \cdot \frac{e \nabla \psi}{T_i} F_M + C(\delta f)}$$

Diamagnetic drift $\quad \mathbf{v}_* = -\frac{cT_i}{eL_n B_0}\left[1 + \eta_i\left(\frac{mv^2}{2T_i} - \frac{3}{2}\right)\right]\hat{\mathbf{y}}, \quad \mu = \frac{v_\perp^2}{2\Omega}$

Gyrocenter drift $\quad \mathbf{v}_d \cdot \nabla$

Mirror force $\quad -\mu(\mathbf{b} \cdot \nabla \Omega)\, \partial / \partial v_\|$

Effects of magnetic geometry

Quasineutrality condition & Adiabatic electron assumption

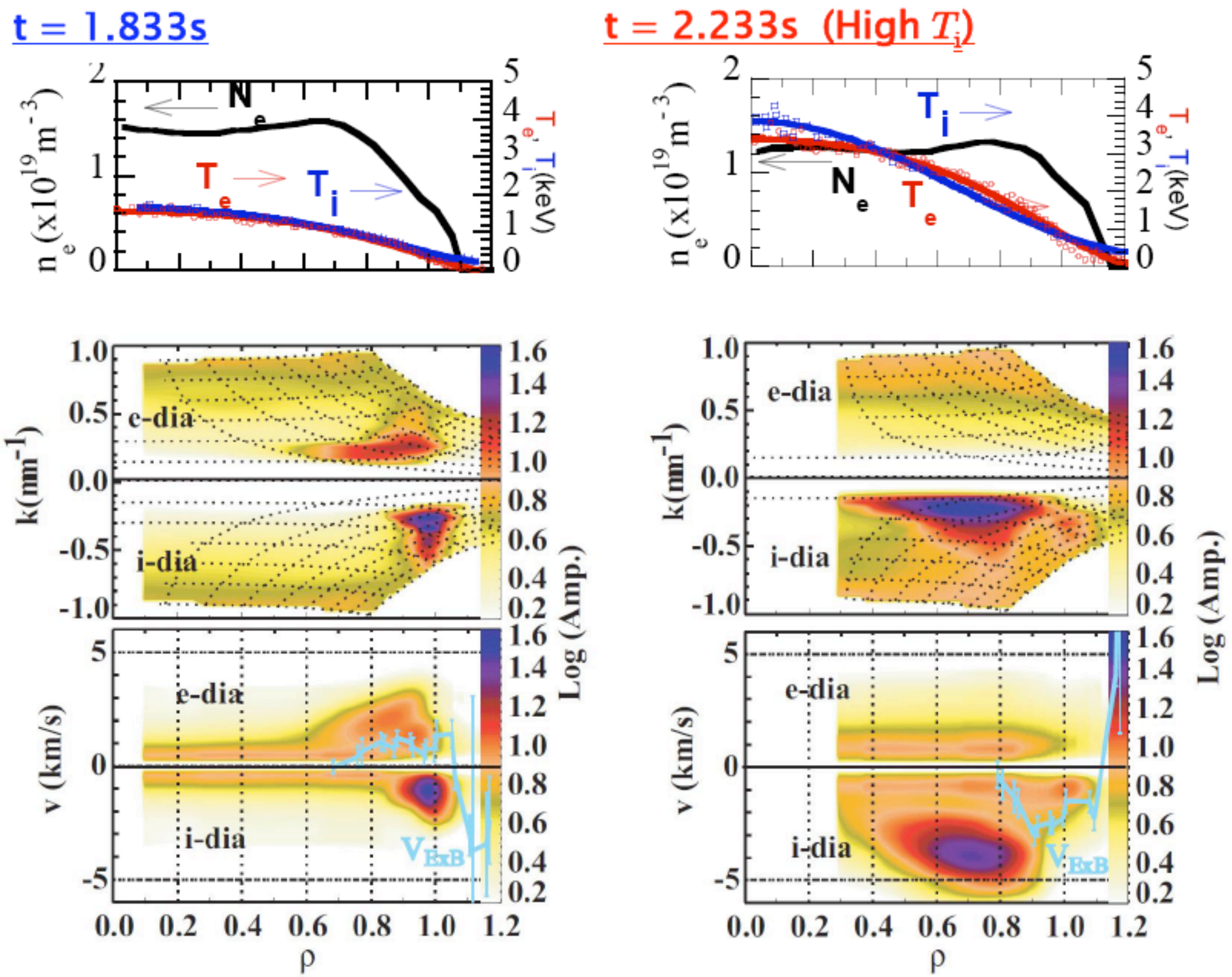
$$\boxed{\int J_0(k_\perp v_\perp / \Omega) \delta f \, \mathrm{d}^3 v - \left[1 - \Gamma_0\left(k_\perp^2\right)\right]\frac{e\phi}{T_i} = \frac{e}{T_e}\left(\phi - \langle \phi \rangle\right), \quad k_\perp^2 = \left(k_x + \hat{s} z k_y\right)^2 + k_y^2}$$

Ion polarization

Linear ITG Mode Analysis for High-T_i LHD plasmas

Fluctuation in High-T_i discharge in LHD

K. Tanaka et al., to be appeared in Plasma Fusion Res.

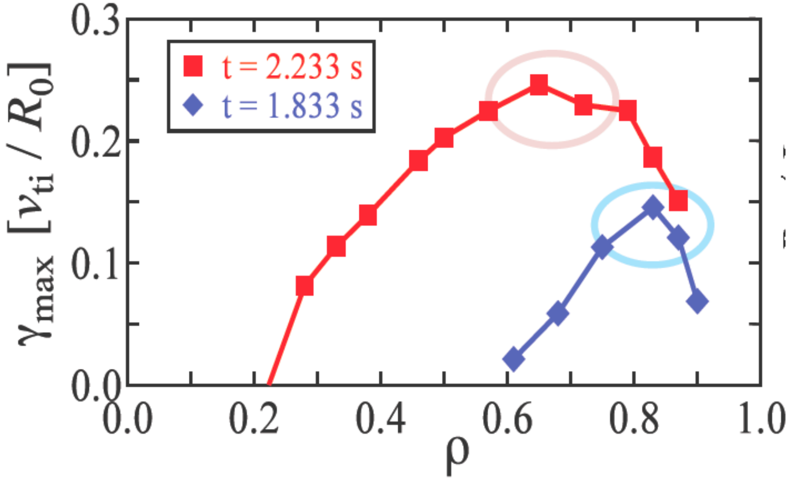


- Fluctuation peak exists at ρ=0.8–1.0 in space, $k_p \rho_i$~0.26 in wavenumber.
- Fluctuation peak exists at ρ=0.5–0.8 in space, $k_p \rho_i$~0.45 in wavenumber.

Results from Linear ITG Mode Analyses by GKV-X

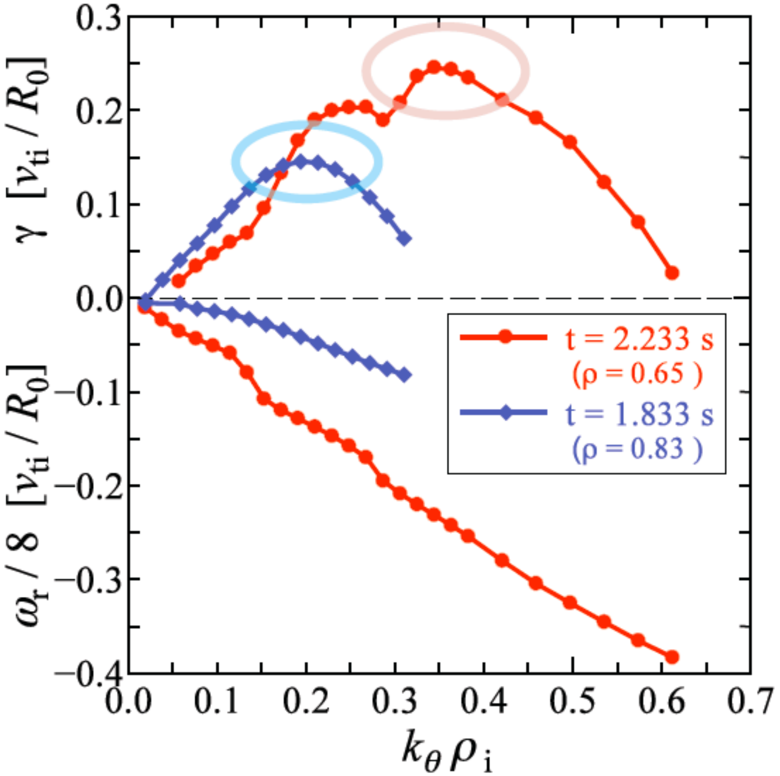
(See Poster by M. Nunami)

Radial profiles of γ_{max}



▪Growth rates are peaked at
$\rho \sim 0.65$ (t=2.233s),
$\rho \sim 0.85$ (t=1.833s).

Growth rates of ITG modes



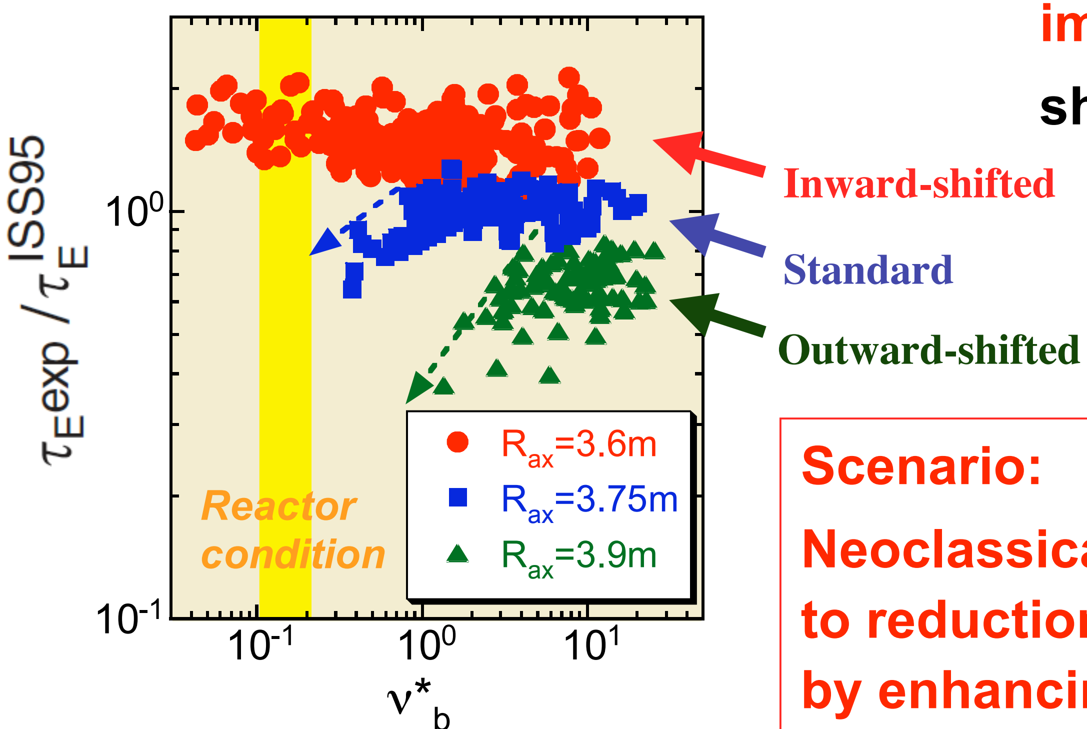
▪There exists ITG unstable region.

▪Maximum growth rates exists at
$k_\theta \rho_i \sim 0.35$ (t=2.233s),
$k_\theta \rho_i \sim 0.20$ (t=1.833s),
in poloidal wavenumber space.

Zonal Flows and ITG Turbulence in Helical Systems

Results from LHD experiments

For low collisionality, better confinement is observed in the inward-shifted magnetic configurations, where lower neoclassical ripple transport but more unfavorable magnetic curvature driving pressure-gradient instabilities are anticipated.



H. Yamada *et al.* (PPCF2001)

Anomalous transport is also improved in the inward shifted configuration.

Scenario:

Neoclassical optimization contributes to reduction of anomalous transport by enhancing the zonal-flow level.

Collisionless Time Evolution of Zonal Flows in Helical Systems

[Sugama & Watanabe, PRL (2005), Phys.Plasmas (2006)]

Response of the zonal-flow potential to a given initial potential $\boxed{k\,\rho_i < 1}$

$$\langle \phi_k(t) \rangle = K(t)\, \langle \phi_k(0) \rangle$$

<u>Response function</u> = GAM component + Residual component

$$K(t) = K_{GAM}(t)[1 - K_L(0)] + K_L(t)$$

$K(t=0) = 1 \qquad K(t) \to K_L(t),\ K_{GAM}(t) \to 0 \text{ as } t \to +\infty$

$B = B_0\,[\,1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\zeta)\,]$

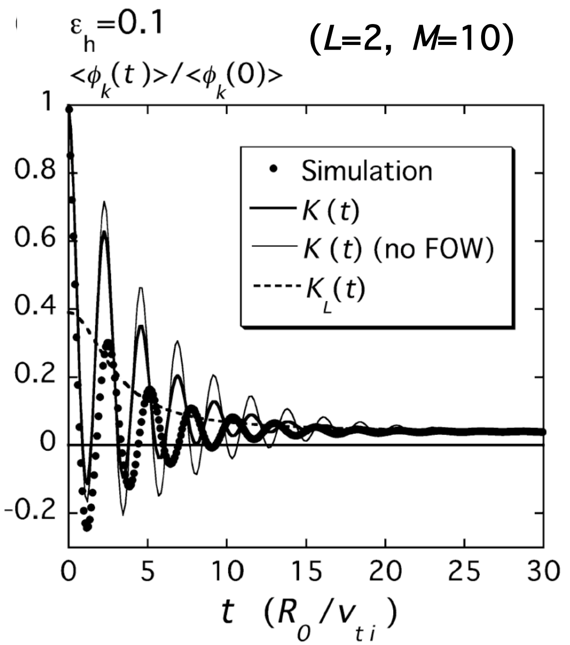
GAM response function $K_{GAM}(t) = \cos(\omega_G)\exp(-|\gamma|t)$

Long-time response function

$$\boxed{K_L(t) = \frac{1 - (2/\pi)^{1/2}\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle}{1 + G + E(t) \Big/ \left(n_0 \left\langle k_\perp^2 \rho_{ti}^2 \right\rangle \right)}}$$

E(t) **represents effects of shielding of potential due to helical-ripple-trapped particles.**

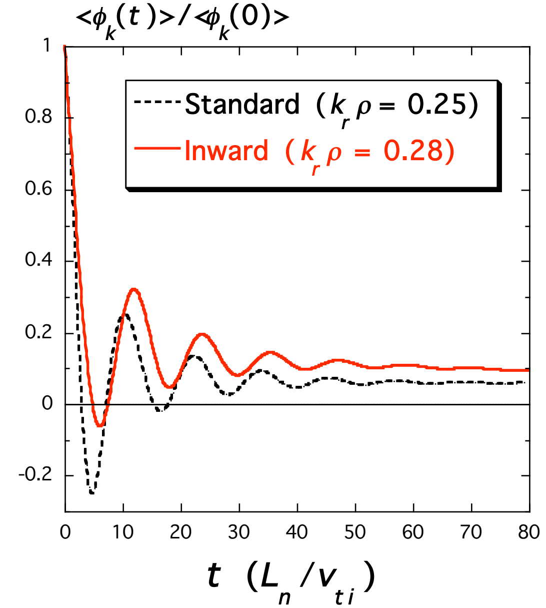
$$E(t) = \frac{2}{\pi} n_0 \left[\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle - \frac{3}{2}\left\langle k_\perp^2 \rho_{ti}^2 \right\rangle \left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle + \frac{T_i}{T_e}\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{e1}(t,\theta)\}\right\rangle \right]$$



Results from GKV simulation (flux tube, $E_r = 0$)

Smaller χ_i and larger zonal flows are found in the saturated turbulent state for the inward-shifted configuration than for the standard one !

Linear time evolution of zonal-flow potential



Larger residual zonal flow is found for the inward-shifted case.

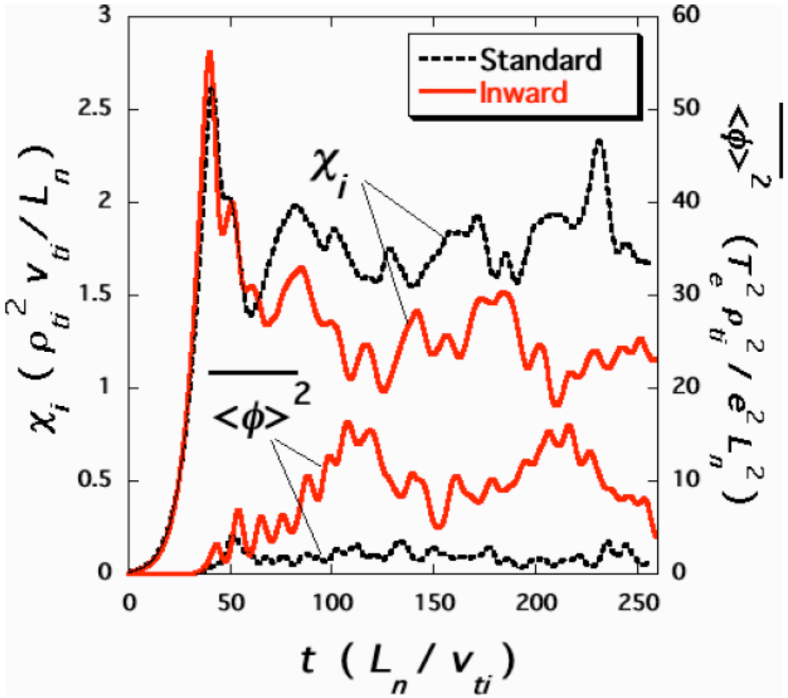
Watanabe, Sugama & Ferrando, PRL(2008)
Sugama, Watanabe & Ferrando, PFR(2009)

EARTH SIMULATOR

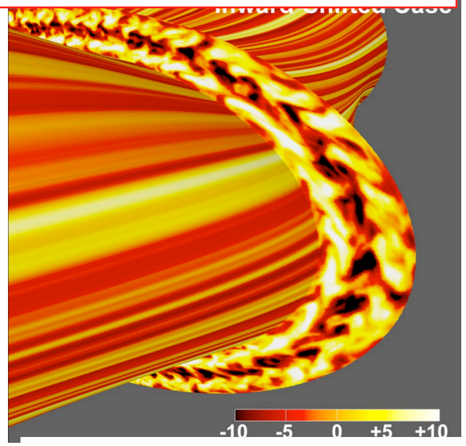
The GKV turbulence simulations were carried out by the Earth Simulator (JAMSTEC).

ITG turbulence

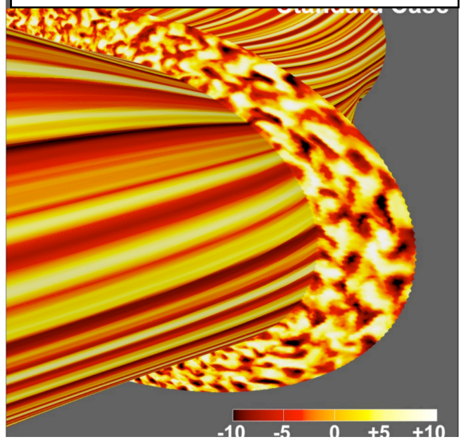
Turbulent thermal diffusivity and squared zonal-flow potential



inward-shifted configuration



standard configuration



Potential contours obtained from six copies of flux tube

Effects of Equilibrium Electric Field E_r on Zonal Flows in Helical Systems

In helical systems

E_r is given from ambipolar condition of radial particle fluxes.

E_r reduces neoclassical ripple transport.

How does E_r influence zonal flows and anomalous transport?

Effects of E_r on gyrokinetic equation and zonal flows

gyrophase average of zonal-flow potential

Gyrokinetic equation for $\mathbf{k}_\perp = k_r \nabla r$

$$\left[\frac{\partial}{\partial t} + v_\parallel \hat{\mathbf{b}}\cdot\nabla + i\mathbf{k}_r\cdot\mathbf{v}_d - \mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)\frac{\partial}{\partial v_\parallel} + \omega_E \frac{\partial}{\partial \alpha}\right]\delta f = -i\mathbf{k}_r\cdot\mathbf{v}_d \frac{e\langle\phi(\mathbf{x}+\rho)\rangle}{T_i} F_M$$

angular velocity due to ExB drift $\omega_E = -\dfrac{c\,E_r}{r_0 B_0}$

field line label $\alpha = \theta - \zeta / q$

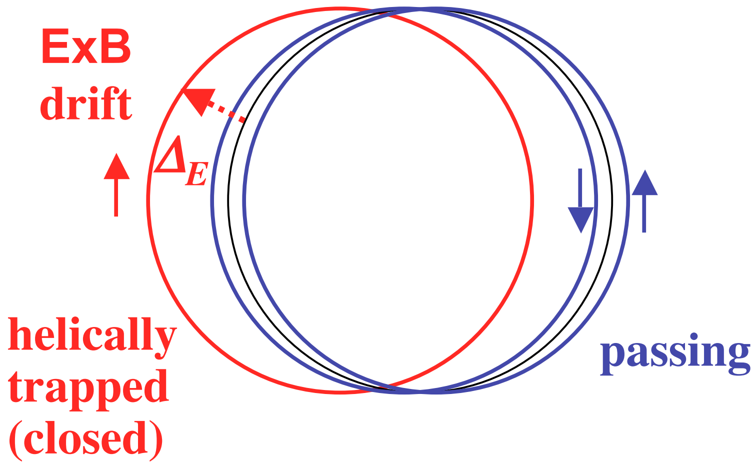
In helical systems, α -dependence appears in $\mathbf{k}_r \cdot \mathbf{v}_d$ and $\mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)$.

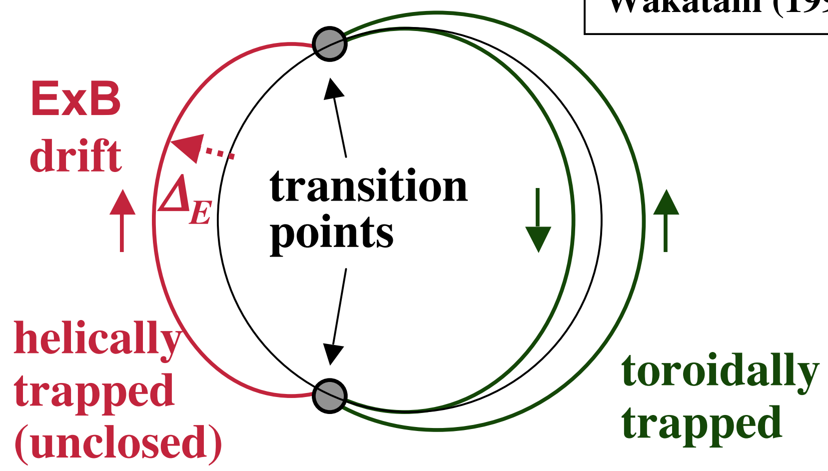
Therefore, even if the zonal-flow potential ϕ is independent of α , δf comes to depend on α.

Thus, ω_E influences δf and accordingly ϕ through quasineutrality condition.

Classification of particle orbits in the presence of E_r

Cary *et al.*, PF (1988)
Wakatani (1998)





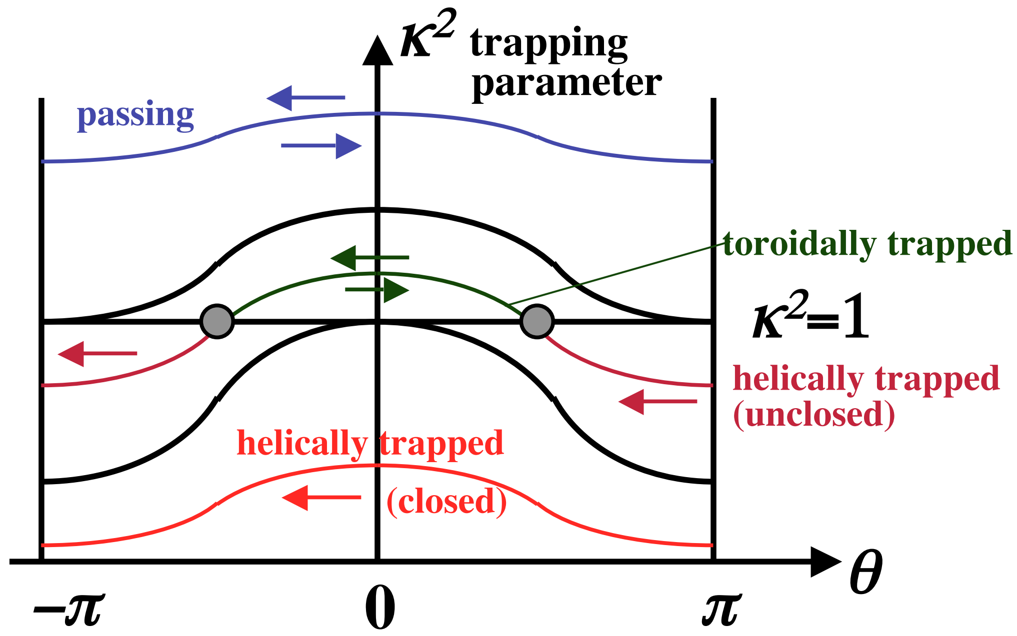
$$\boxed{\Delta_E \sim r_0 \frac{v_{dr}}{v_{E\times B}}}$$

radial displacement of
helically-trapped particle

trapping parameter

$$\kappa^2 = \frac{1-\lambda B_0[1-\varepsilon_T(\theta)-\varepsilon_H(\theta)]}{2\lambda B_0 \varepsilon_H(\theta)}$$

$$\lambda = \frac{1}{2}mv^2 \Big/ \mu$$



Solution of gyrokinetic equation to describe long-time evolution of zonal flows [Sugama & Watanabe, PoP(2009)]

Perturbed particle distribution function

$\rho_r \cdots$ **gyro motion**

$\Delta_r \cdots$ **drift motion**

$$\delta f_k(t) = -\frac{e}{T}\phi_k(t) F_M \left[1 - e^{-ik_r \rho_r} e^{-ik_r \Delta_r} \left\langle e^{ik_r \Delta_r} J_0(k_r \rho_r) \right\rangle_{\text{orbit}} \right]$$

→ **Polarization (classical & neoclassical)**

$$+ e^{-ik_r \rho_r} e^{-ik_r \Delta_r} \left\langle e^{ik_r \Delta_r} \left[\delta f_k^{(g)}(0) + F_M \int_0^t S_k(t)\, dt \right] \right\rangle_{\text{orbit}}$$

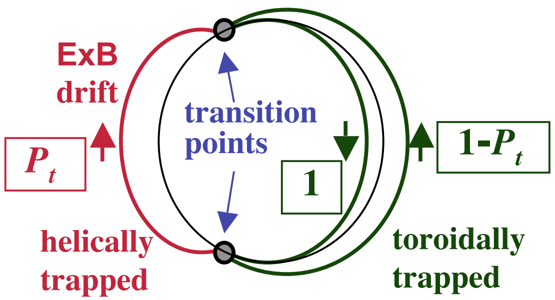
→ **Initial condition & Turbulence source**

Average along the orbit

$$\langle \cdots \rangle_{\text{orbit}} = \oint \cdots \frac{dl}{(dl/dt)} \bigg/ \oint \frac{dl}{(dl/dt)}$$

$$\boxed{P_t \approx \frac{4\sqrt{2}}{\pi} \left(\frac{c\, E_r}{v\, B_0} \right) \left(\frac{R_0 q}{r_0} \right) \left[(\varepsilon_H)^{-1/2} \frac{\partial \varepsilon_H / \partial \theta}{\partial(\varepsilon_H - \varepsilon_T)/\partial \theta} \right]_{P-}}$$

For particles which show transitions



$1\text{-}P_t$

P_t transition probability

$$\oint \frac{dl}{(dl/dt)} = (1 - P_t) \int_{\substack{\kappa^2 > 1 \\ v_\parallel > 0}} \frac{dl}{|v_\parallel|} + \int_{\substack{\kappa^2 > 1 \\ v_\parallel < 0}} \frac{dl}{|v_\parallel|} + P_t \int_{\kappa^2 < 1} \frac{d\theta}{\omega_E}$$

toroidally trapped **helically trapped**

Long-time zonal-flow response to the initial condition and turbulence source

For radial wavenumbers $k_r\rho_{ti} < 1$ (ITG turbulence) and $k_r\Delta_E < 1$, the zonal-flow potential is derived from the quasineutrality condition as [Sugama & Watanabe, PoP(2009)]

$$\boxed{\frac{e}{T_i}\langle \phi_k(t)\rangle = \frac{\left\langle n_0^{-1}\int d^3v\left[1 + i k_r\left(\Delta_{ir} - \langle \Delta_{ir}\rangle_{\text{orbit}}\right)\right]\left[\delta f_{ik}^{(g)}(0) + F_M\int_0^t S_{ik}(t)\,dt\right]\right\rangle}{(k_r\rho_{ti})^2\left[1 + G_p + G_t + M_p^{-2}(G_{ht} + G_h)(1 + T_e/T_i)\right]}}$$

Geometrical factors G's represents shielding effects of neoclassical polarization due to particles motions in different orbits.

$G \propto$ (population) $\times (\Delta_r/\rho)^2$

G_p : passing

G_t : toroidally-trapped

G_{ht} : helicallly-trapped (unclosed orbit)

G_h : toroidally-trapped (closed orbit)

Zonal-flow generation can be enhanced when

G_{ht} and G_h decreases with neoclassical optimization (which reduces radial drift velocity v_{dr})

and when poloidal Mach number $\boxed{M_p \equiv \left|(cE_r/B_0)/(r v_{ti}/Rq)\right|}$ increases

with increasing E_r and using heavier ions.

Response to the initial condition

Assume the initial distribution to have Maxwellian dependence $\delta f_{ik}^{(g)}(0) = [\delta n_k^{(g)}(0)/n_0] F_M$

Then, we obtain $\delta n_k^{(g)}(0)/n_0 = (k_r \rho_{ti})^2 e\phi(0)/T_i$

$$\boxed{\langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + M_p^{-2}(G_{ht} + G_h)(1 + T_e/T_i)}}$$

(no turbulence source)

For the single-helicity configuration

$$B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\varsigma)] \quad (\varepsilon_h : \text{independent of } \theta)$$

No transitions occur. $G_{ht} = 0, \quad G_h = (15\pi/4) q^2 (2\varepsilon_h)^{1/2}$

$$\boxed{\langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + (15\pi/4) M_p^{-2} q^2 (2\varepsilon_h)^{1/2} (1 + T_e/T_i)}}$$

This corresponds to the case considered by previous works.

Mynick & Boozer, PoP(2007)

Sugama, Watanabe & Ferrando, PFR(2008)

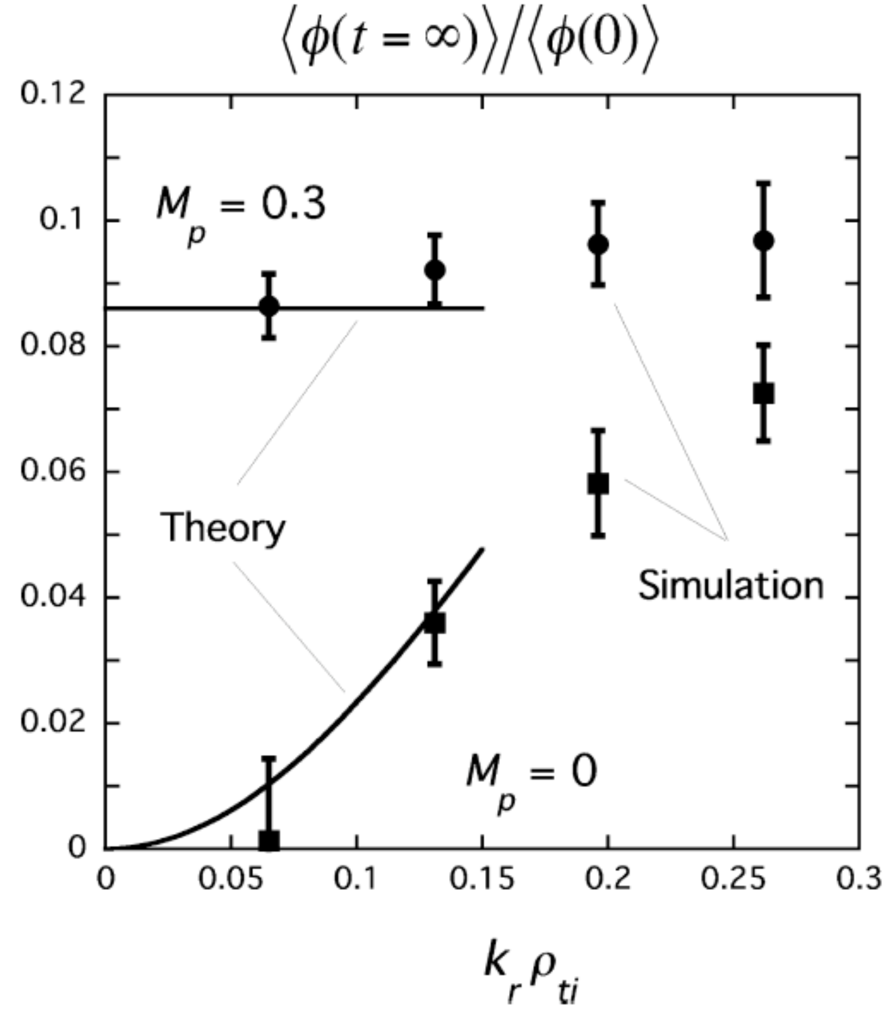
The residual zonal-flow potential as a function of $k_r \rho_{ti}$ for $M_p = 0$ and $M_p = 0.3$

Different $k_r \rho_{ti}$ dependences for $M_p = 0$ and $M_p = 0.3$ are theoretically predicted and confirmed by simulation.

Theoretical results are derived by assuming $k_r \rho_{ti} << 1$.

[To be published in CPP]

Inward-shited configuration



Momentum Balance and Radial Electric Field in Quasisymmetric Systems with Stellarator Symmetry

Basic Boltzmann Kinetic Equation for description of Collisional and Turbulent Transport

Equilibrium magnetic field

$$\mathbf{B} = \psi' \nabla s \times \nabla \theta + \chi' \nabla \zeta \times \nabla s = B_s \nabla s + B_\theta \nabla \theta + B_\zeta \nabla \zeta$$

Boltzmann kinetic equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_a}{m_a} \left\{ \left(\mathbf{E} + \hat{\mathbf{E}} \right) + \frac{1}{c} \mathbf{v} \times \left(\mathbf{B} + \hat{\mathbf{B}} \right) \right\} \cdot \frac{\partial}{\partial \mathbf{v}} \right] (f_a + \hat{f}_a) = C_a (f_a + \hat{f}_a)$$

Ensemble-averaged kinetic equation

$$\frac{d}{dt} f_a \equiv \left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_a}{m_a} \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial}{\partial \mathbf{v}} \right] f_a = \langle C_a \rangle_{\mathrm{ens}} + \mathcal{D}_a$$

$$\mathcal{D}_a = -\frac{e_a}{m_a} \left\langle \left(\hat{\mathbf{E}} + \frac{1}{c} \mathbf{v} \times \hat{\mathbf{B}} \right) \cdot \frac{\partial \hat{f}_a}{\partial \mathbf{v}} \right\rangle_{\mathrm{ens}}$$

Classical, Neoclassical, and Anomalous Transport of Particles and Heat [Sugama et al. PoP1996]

The gyrophase (ξ) -average part and the oscillating part of an aribtrary function F is defined by $\overline{F} \equiv (2\pi)^{-1} \oint d\xi \; F$ **and** $\widetilde{F} \equiv F - \overline{F}$ **respectively.**

Particle flux $$\Gamma_a \equiv \langle \mathbf{\Gamma}_a \cdot \nabla s \rangle \equiv \left\langle \int d^3v \; \widetilde{f}_a \mathbf{v} \cdot \nabla s \right\rangle$$

Heat flux $$\frac{q_a}{T_a} \equiv \frac{\langle \mathbf{q}_a \cdot \nabla s \rangle}{T_a} \equiv \left\langle \int d^3v \; \widetilde{f}_a \left(\frac{m_a v^2}{2T_a} - \frac{5}{2} \right) \mathbf{v} \cdot \nabla s \right\rangle$$

The ensemble-averaged kinetic equation is divided as

$$\overline{\mathcal{L}(\overline{f}_a + \widetilde{f}_a)} = \left\langle \overline{C}_a \right\rangle_{\text{ens}} + \overline{\mathcal{D}}_a, \qquad \Omega_a \frac{\partial \widetilde{f}_a}{\partial \xi} = \widetilde{\mathcal{L} f_a} - \left\langle \widetilde{C}_a \right\rangle_{\text{ens}} - \widetilde{\mathcal{D}}_a$$

$$\mathcal{L} \equiv d/dt + \Omega_a \partial / \partial \xi$$

Second order part of $\widetilde{f}_a$ **in** $\delta \sim \rho / L$

$$\widetilde{f}_{a2} = \widetilde{f}_a^N + \widetilde{f}_a^H + \widetilde{f}_a^C + \widetilde{f}_a^A \equiv \frac{1}{\Omega_a} \int^\xi d\xi \left[\widetilde{\mathcal{L} \overline{f}_{a1}} + \widetilde{\mathcal{L} \widetilde{f}_{a1}} - C_a^L(\widetilde{f}_{a1}) - \widetilde{D}_a \right]$$

$$\Gamma_a = \Gamma_a^{\text{ncl}} + \Gamma_a^{\text{cl}} + \Gamma_a^{\text{anom}} \qquad q_a = q_a^{\text{ncl}} + q_a^{\text{cl}} + q_a^{\text{anom}}$$

Momentum Balance

$$\frac{\partial}{\partial t}(n_a m_a \mathbf{u}_a) = -\nabla \cdot \mathbf{P}_a + n_a e_a \left(\mathbf{E} + \frac{\mathbf{u}_a}{c} \times \mathbf{B}\right) + \mathbf{F}_{a1} + \mathbf{K}_{a1}$$

density $n_a \equiv \int d^3v \; f_a$ **particle flux** $n_a \mathbf{u}_a \equiv \int d^3v \; f_a \mathbf{v}$

pressure tensor $\mathbf{P}_a \equiv \int d^3v \; f_a m_a \mathbf{v}\mathbf{v}$

friction force $\mathbf{F}_{a1} \equiv \int d^3v \; C_a(f_a) m_a \mathbf{v}$

turbulent electromagnetic force $\mathbf{K}_{a1} \equiv \int d^3v \; \mathcal{D}_a \mathbf{v}$

$$\begin{aligned}\sum_a \mathbf{K}_{a1} &= \nabla \cdot \left\langle \frac{1}{4\pi}\left(\hat{\mathbf{E}}\hat{\mathbf{E}} + \hat{\mathbf{B}}\hat{\mathbf{B}}\right) - \frac{1}{8\pi}\left(\hat{E}^2 + \hat{B}^2\right)\mathbf{I}\right\rangle_{\text{ens}} - \frac{1}{4\pi c}\frac{\partial}{\partial t}\left\langle \hat{\mathbf{E}} \times \hat{\mathbf{B}}\right\rangle_{\text{ens}} \\ &= \nabla \cdot \mathbf{T}_{EM} - \frac{\partial}{\partial t}\left(\frac{\mathbf{S}_{EM}}{c^2}\right),\end{aligned}$$

Momentum Balance in the direction tangential to the flux surface

(c_1, c_2 : constants)

$$\frac{\partial}{\partial t}\sum_a \left\langle n_a m_a \left\{ c_1\left(u_{a\theta} + \frac{(S_{EM})_\theta}{c^2}\right) + c_2\left(u_{a\zeta} + \frac{(S_{EM})_\zeta}{c^2}\right)\right\}\right\rangle$$

$$= -\frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle \nabla s \cdot \left(\sum_a \mathbf{P}_a - \mathbf{T}_{EM}\right) \cdot \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta}\right)\right\rangle\right] + \frac{1}{c}\left(-c_1\psi' + c_2\chi'\right)\sum_a e_a \left\langle n_a u_a^s \right\rangle$$

(s, θ, ζ) : Hamada coordinates

The surface-averaged radial current

$$\sum_a e_a \Gamma_a \equiv \sum_a e_a \left\langle n_a u_a^s \right\rangle = -\frac{1}{4\pi}\frac{\partial}{\partial t}\left\langle E^s \right\rangle$$

Quasisymmetry

[Boozer(1983), Nuhrenberg(1988), Helander&Simakov (2008)]

$$c_1 \frac{\partial B}{\partial \theta} + c_2 \frac{\partial B}{\partial \zeta} = 0$$

quasi-axi-symmetry $(c_1,\ c_2) = (0,\ 1)$

quasi-poloidal-symmetry $(c_1,\ c_2) = (1,\ 0)$

The $O(\delta)$ viscosity component in the quasisymmetry direction vanishes :

$$\left\langle \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \left[\nabla \cdot \left\{ P_{\|a} \mathbf{bb} + P_{\perp a} \left(\mathbf{I} - \mathbf{bb} \right) \right\} \right] \right\rangle$$

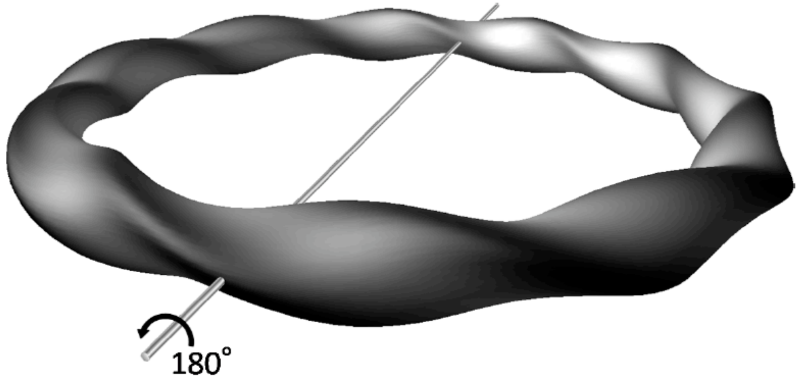
$$= -\left\langle \left(P_{\|a} - P_{\perp a} \right) \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \nabla \ln B \right\rangle = 0$$

The ambipolarity $\sum_a e_a \langle n_a u_a^s \rangle = 0$

is satisfied automatically up to $O(\delta)$.

Stellarator Symmetry

Magnetic field strength



$$B(s, -\theta, -\zeta) = B(s, \theta, \zeta)$$

Magnetic field components

$$B^\theta(s, -\theta, -\zeta) = B^\theta(s, \theta, \zeta), \qquad B^\zeta(s, -\theta, -\zeta) = B^\zeta(s, \theta, \zeta)$$
$$B_\theta(s, -\theta, -\zeta) = B_\theta(s, \theta, \zeta), \qquad B_\zeta(s, -\theta, -\zeta) = B_\zeta(s, \theta, \zeta)$$
$$B_s(s, -\theta, -\zeta) = -B_s(s, \theta, \zeta),$$

Metric tensor components

$$g_{ss}(s, -\theta, -\zeta) = g_{ss}(s, \theta, \zeta), \qquad g_{\theta\theta}(s, -\theta, -\zeta) = g_{\theta\theta}(s, \theta, \zeta),$$
$$g_{\theta\zeta}(s, -\theta, -\zeta) = g_{\theta\zeta}(s, \theta, \zeta), \qquad g_{\zeta\zeta}(s, -\theta, -\zeta) = g_{\zeta\zeta}(s, \theta, \zeta),$$
$$g_{s\theta}(s, -\theta, -\zeta) = -g_{s\theta}(s, \theta, \zeta), \qquad g_{s\zeta}(s, -\theta, -\zeta) = -g_{s\zeta}(s, \theta, \zeta),$$
$$g(s, -\theta, -\zeta) = g(s, \theta, \zeta),$$

Parity Transformation associated with Stellarator Symmetry

Expansion in $\eta \sim \delta \sim \rho / L$ (Put $e_a \to \eta^{-1} e_a$ in Boltzmann and Maxwell eqs.)

$$f_a(s,\theta,\zeta,v^s,v^\theta,v^\zeta,t,\eta) = f_{aM}(s,v,\eta^2 t) + \eta f_{a1}(s,\theta,\zeta,v^s,v^\theta,v^\zeta,\eta^2 t) + \eta^2 f_{a2}(s,\theta,\zeta,v^s,v^\theta,v^\zeta,\eta^2 t) + \cdots,$$

$$\Phi(s,\theta,\zeta,t,\eta) = \eta \Phi_1(s,\eta^2 t) + \eta^2 \Phi_2(s,\theta,\zeta,\eta^2 t)$$

Parity operator $\mathcal{P}$ **is defined by**

$$(\mathcal{P}Q)(s,\theta,\zeta,v^s,v^\theta,v^\zeta,t,\eta) \equiv Q(s,-\theta,-\zeta,v^s,-v^\theta,-v^\zeta,t,-\eta)$$

In the presence of stellarator symmetry, Boltzmann and Maxwell equations are invariant under parity transformation

$$f_a + \hat{f}_a \longrightarrow \mathcal{P}(f_a + \hat{f}_a)$$

$$E_s + \hat{E}_s,\ E_\theta + \hat{E}_\theta,\ E_\zeta + \hat{E}_\zeta \longrightarrow -\mathcal{P}(E_s + \hat{E}_s),\ \mathcal{P}(E_\theta + \hat{E}_\theta),\ \mathcal{P}(E_\zeta + \hat{E}_\zeta)$$

$$B_s + \hat{B}_s,\ B_\theta + \hat{B}_\theta,\ B_\zeta + \hat{B}_\zeta \longrightarrow -\mathcal{P}(B_s + \hat{B}_s),\ \mathcal{P}(B_\theta + \hat{B}_\theta),\ \mathcal{P}(B_\zeta + \hat{B}_\zeta)$$

Momentum Transport Fluxes in Stellarator Symmetric Systems

Parity of solutions

$$\mathcal{P} f_a = f_a, \qquad -\mathcal{P}\Phi = \Phi$$

$$f_j(s, -\theta, -\zeta, v^s, -v^\theta, -v^\zeta, \eta^2 t) = (-1)^j f_j(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t)$$

$$\Phi_j(s, -\theta, -\zeta, \eta^2 t) = (-1)^{j-1} \Phi_j(s, \theta, \zeta, \eta^2 t)$$

When j is even, the $O(\delta^j)$ part of radial transport fluxes of poloidal and toroidal momentum vanish.

$$\left\langle \left(P_a^{(j)}\right)^s_\theta \right\rangle = \left\langle \left(P_a^{(j)}\right)^s_\zeta \right\rangle = \langle (T_{EM}^{(j)})^s_\theta \rangle = \langle (T_{EM}^{(j)})^s_\zeta \rangle = 0 \quad (\text{for even } j)$$

Momentum Balance in Quasisymmetric Systems with Stellarator Symmetry

In quasisymmetric systems with stellarator symmetry, the momentum transport fluxes vanish up to $O(\delta^2)$, and the ambipolarity is automatically satisfied up to $O(\delta^2)$.

The momentum balance equation determing E_s is of $O(\delta^3)$:

$$\frac{\partial}{\partial t}\left[\frac{(c_2\chi' - c_1\psi')}{4\pi c}\left\{\langle|\nabla s|^2\rangle + \frac{4\pi c^2 \sum_a n_a m_a}{(c_2\chi' - c_1\psi')^2}\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle\right\} E_s\right.$$
$$\left. + \sum_a \frac{m_a}{(c_2\chi' - c_1\psi')}\left\{-\frac{c}{e_a}\frac{\partial p_a}{\partial s}\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle + \frac{n_a V'}{4\pi^2}\langle c_1 B_\theta + c_2 B_\zeta\rangle\left\langle c_2 u_a^\theta - c_1 u_a^\zeta\right\rangle\right\}\right]$$
$$= -\frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle \nabla s \cdot \left(\sum_a \mathbf{P}_a^{(3)} - \mathbf{T}_{EM}^{(3)}\right)\cdot\left(c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\varsigma}\right)\right\rangle\right]$$

Summary

- **Fluctuations observed in a high T_i LHD plasma are considered as ITG modes predicted from linear calculation by GKV-X.**

- **Zonal-flow response theory and simulation show that zonal flow generation and turbulence regulation are enhanced when the radial displacements of helical-ripple-trapped particles are reduced either by neoclassical optimization of the helical geometry lowering the radial drift velocity or by strengthening the radial electric field E_r to boost the poloidal rotation.**

- **The E_r effects appear through the poloidal Mach number M_p.**
  For the same magnitude of E_r, higher zonal-flow response is obtained by using ions with heavier mass (favorable deviation from gyro-Bohm scaling).

- **The momentum balance equation determining E_r in quasisymmetric helical system with stellarator symmetry is shown to be of $O(\delta^3)$ by using a novel parity operator.**