Momentum balance in toroidal plasmas

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Outline

- General comments on structure and sources of rotation in tokamaks:
  - Turbulent (Reynolds) stress drive
  - Neoclassical effects
  - Gyroviscous stress, etc
- Momentum balance along the inhomogeneous magnetic fields
  - Coupling of particle, energy and momentum balance due to magnetic field curvature
  - Lagrange invariants and turbulent equipartition
  - Pinch and residual effects in density, energy and momentum transport

Motivation:Plasma rotation:

- Neoclassical rotation theory, e.g. poloidal flow damping, ...

- Zonal flows and Geodesic Acoustic modes; as low and finite frequency modes of poloidal rotation

- Spontaneous toroidal rotation

- Turbulent generation/anomalous transport of Zonal Flows/toroidal momentum

Momentum conservation

$$\sum_{i,e}\left(\frac{\partial}{\partial t}(nm\mathbf{V})+\nabla\cdot(nm\mathbf{V}\mathbf{V}+p\mathbf{I}+\mathbf{\Pi})\right)=\rho\mathbf{E}+\frac{1}{c}\mathbf{J}\times\mathbf{B},$$

Momentum exchange between plasma and electromagnetic field

$$n_\alpha\mathbf{V}_\alpha=\int\mathbf{v}f_\alpha d^3\mathbf{v}$$

$$p_\alpha=\frac{m}{3}\int v'^2 f_\alpha d^3\mathbf{v}$$

$$\mathbf{\Pi}_\alpha=m\int\left(\mathbf{v}'\mathbf{v}'-\frac{v'^2}{3}\mathbf{I}\right)f_\alpha d^3\mathbf{v}$$

$$\rho=e(n_i-n_e),\ \mathbf{J}=e(n_i\mathbf{V}_i-n_e\mathbf{V}_e)\,.$$

$\rho\ \rightarrow\ 0$ quasineutrality

$J\times B\rightarrow 0$ ambipolarity

unless RF drive, biasing, etc

$$\frac{\partial}{\partial t}\left(\frac{1}{4\pi c}\mathbf{E}\times\mathbf{B}\right)+\nabla\cdot\left(\frac{1}{8\pi}E^2\mathbf{I}-\frac{1}{4\pi}\mathbf{E}\mathbf{E}+\frac{1}{8\pi}B^2\mathbf{I}-\frac{1}{4\pi}\mathbf{B}\mathbf{B}\right)=-\rho\mathbf{E}-\frac{1}{c}\mathbf{J}\times\mathbf{B}.$$

Reduced equations

Exploit strong magnetic field, 1/B is an expansion parameter

$\omega / \omega_c << 1$ Low frequency

$k_\perp^{\,2} \rho_i^{\,2} << 1$ Small (but finite) Larmor radius

$$\frac{\partial}{\partial t}(n_\alpha m_\alpha \mathbf{V}_\alpha) + \nabla \cdot (n_\alpha m_\alpha \mathbf{V}_\alpha \mathbf{V}_\alpha + p_\alpha \mathbf{I} + \mathbf{\Pi}_\alpha) = e_\alpha n_\alpha \mathbf{E} + \frac{e}{c}\mathbf{V}_\alpha \times \mathbf{B} + \mathbf{R}_\alpha,$$

$$\mathbf{V} = V_{\|}\mathbf{b} + \mathbf{V}_\perp \qquad \mathbf{V}_\perp = \mathbf{V}_\perp^{(0)} + \ldots$$

$$\mathbf{V}_\perp^{(0)} = \frac{c}{B}\mathbf{e}_0 \times \nabla\phi + \frac{1}{mn\omega_c}\mathbf{e}_0 \times \nabla p$$

$$\mathbf{e}_0 = \mathbf{B}/B$$

Parallel degrees of freedom, require different expansion parameter, not always available; $\omega / k_{\|} v_T >> 1$

Further reduction

- Large aspect ratio, $V_\zeta \simeq V_{||}$
- Reynolds stress components:
  - $\mathbf{V}_\perp V_{||}$ - > parallel, transport of parallel momentum
  - $\mathbf{V}_\perp^{(0)}\mathbf{V}_\perp^{(0)}$ - > perpendicular components, e.g. zonal flow generation
- Viscosity contributions?
  - Parallel (neoclassical), gyroviscosity, perpendicular, ...?

Viscosity evolution equation (Grad)

$$
\begin{aligned}
&\frac{d}{dt}\mathbf{\Pi}+\mathbf{\Pi}\nabla\cdot\mathbf{V}+\left(\mathbf{\Pi}\cdot\nabla\mathbf{V}+\mathbf{Tr}-\frac{2}{3}\mathbf{I}\mathbf{\Pi}:\nabla\mathbf{V}\right)+\\
&+\left(p\nabla\mathbf{V}+\mathbf{Tr}-\frac{2}{3}p\mathbf{I}\nabla\cdot\mathbf{V}\right)+\frac{2}{5}\left(\nabla\mathbf{q}+\mathbf{Tr}-\frac{2}{3}\mathbf{I}\nabla\cdot\mathbf{q}\right)+\frac{1}{2p}\left(\nabla\cdot\boldsymbol{\tau}+\mathbf{Tr}-\frac{2}{3}\mathbf{I}\nabla\cdot\boldsymbol{\tau}:\mathbf{I}\right)\\
=\;&\omega_c\left(\mathbf{\Pi}\times\mathbf{e}_0-\mathbf{e}_0\times\mathbf{\Pi}\right)-\frac{6}{5}\nu\left(\mathbf{\Pi}+\frac{3}{4}\mathbf{\Pi}^*\right).
\end{aligned}
$$

Third order tensor

Strong magnetic field, $\omega_c >> \dfrac{d}{dt}$ $\quad \omega_c >> \nu$

Leading order terms: Gyroviscosity

$$
\begin{aligned}
&\left(p\nabla\mathbf{V}+\mathbf{Tr}-\frac{2}{3}p\mathbf{I}\nabla\cdot\mathbf{V}\right)+\frac{2}{5}\left(\nabla\mathbf{q}+\mathbf{Tr}-\frac{2}{3}\mathbf{I}\nabla\cdot\mathbf{q}\right)\\
=\;&\omega_c\left(\mathbf{\Pi}\times\mathbf{e}_0-\mathbf{e}_0\times\mathbf{\Pi}\right).
\end{aligned}
$$

Gyroviscosity

(Braginskii, + heat flux terms: Mikhailovskii-Tsypin, Herdan-Liley, Nemov, Moiseev, McMahon,...

$$\widehat{\mathbf{K}}\mathbf{\Pi} \equiv (\mathbf{\Pi}\times\mathbf{e}_0 - \mathbf{e}_0 \times \mathbf{\Pi})$$

$$\mathbf{\Pi}_g = \frac{1}{\omega_c}\widehat{\mathbf{K}}^{-1}\left[\left(p\nabla\mathbf{V} + \mathbf{Tr} - \frac{2}{3}p\mathbf{I}\nabla\cdot\mathbf{V}\right) + \frac{2}{5}\left(\nabla\mathbf{q} + \mathbf{Tr} - \frac{2}{3}\mathbf{I}\nabla\cdot\mathbf{q}\right)\right]$$

$$\mathbf{\Pi} = \frac{1}{\omega_c}\mathbf{K}^{-1}\left(p\nabla\mathbf{V} + \mathbf{Tr}\right) = \boldsymbol{\pi}_3 + \boldsymbol{\pi}_4.$$

Braginskii

$$\boldsymbol{\pi}_3 = -\eta_3\left[\left(\mathbf{e}_1\mathbf{e}_1 - \mathbf{e}_2\mathbf{e}_2\right)\beta - \left(\mathbf{e}_1\mathbf{e}_2 + \mathbf{e}_2\mathbf{e}_1\right)\gamma\right],$$

$$\boldsymbol{\pi}_4 = -\eta_4\left[\left(\mathbf{e}_0\mathbf{e}_1 + \mathbf{e}_1\mathbf{e}_0\right)\alpha_2 - \left(\mathbf{e}_0\mathbf{e}_2 + \mathbf{e}_2\mathbf{e}_0\right)\alpha_1\right]$$

$\eta_3 = p/2\omega_c$, and $\eta_4 = p/\omega_c$,

unit vectors $\mathbf{e}_0$, $\mathbf{e}_1$ and $\mathbf{e}_2$,

$$\mathbf{e}_0 = \mathbf{e}_1 \times \mathbf{e}_2,$$

$$\alpha_1 = \left(\mathbf{e}_0\mathbf{e}_1 + \mathbf{e}_1\mathbf{e}_0\right) : \nabla\mathbf{V},$$

$$\alpha_2 = \left(\mathbf{e}_0\mathbf{e}_2 + \mathbf{e}_2\mathbf{e}_0\right) : \nabla\mathbf{V},$$

$$\beta = \left(\mathbf{e}_1\mathbf{e}_2 + \mathbf{e}_2\mathbf{e}_1\right) : \nabla\mathbf{V},$$

$$\gamma = \left(\mathbf{e}_1\mathbf{e}_1 - \mathbf{e}_2\mathbf{e}_2\right) : \nabla\mathbf{V}.$$

Parallel (neoclassical) viscosity

$$\mathbf{K}\left[\mathbf{\Pi}_{\|}\right] \equiv 0.$$

Pressure anisotropy $\mathbf{\Pi}_{\|} = \frac{3}{2}\pi_{\|}\left(\mathrm{e}_0\mathrm{e}_0 - \frac{1}{3}\mathbf{I}\right).$ $3\pi_{\|}/2 = p_{\|} - p_{\perp}$

$$\frac{d}{dt}\boldsymbol{\pi}_{\|} + \left(\nabla\cdot\mathbf{V} + \frac{2}{5}\nabla\cdot\mathbf{q}\right) + \ ... = -\frac{6}{5}\nu\boldsymbol{\pi}_{\|}$$

No small parameter is available in the parallel component,

unless in the short-mean path regime, large ν, $\Pi \sim \frac{1}{\nu}(p\nabla V + ...)$

Kinetic equation has to be solved in banana and plateau regimes

Further expansion for the viscosity

$$\Pi = \Pi_{\|} + \Pi_g + O\left(\frac{1}{\omega_c}\frac{d}{dt}\Pi_g + \frac{1}{\omega_c}\nabla\bullet\tau\right) + O\frac{\nu}{\omega_c}\left(\Pi_g\right)$$

Parallel viscosity

Gyroviscosity

(oblique viscosity)

Inertial and FLR terms

Perpendicular viscosity

Contributions of diamagnetic effects into the parallel momentum balance

$$\mathbf{e}_0 \cdot (\mathbf{V} \cdot \nabla)\mathbf{V}$$
$$= V_\parallel \ \nabla_\parallel V_\parallel - V_\parallel (\mathbf{V}_E + \mathbf{V}_p + \mathbf{V}_\pi) \cdot \nabla \ \ln B$$
$$+ (\mathbf{V}_E + \mathbf{V}_p + \mathbf{V}_\pi) \cdot \nabla V_\parallel .$$

From Reynolds stress

$$\mathbf{\Pi}_g = \boldsymbol{\pi}_4 = \boldsymbol{\pi}_4^{'} + \boldsymbol{\pi}_4^{''} .$$

From gyroviscosity tensor

$$\mathbf{e}_0 \cdot \nabla \cdot \pi_4^{'} = -\frac{1}{\omega_c}\mathbf{e}_0 \times \nabla \left(p - \frac{\pi_\parallel}{2}\right) \cdot \nabla V_\parallel$$
$$+ \frac{V_\parallel}{\omega_c}\nabla \left(p - \frac{\pi_\parallel}{2}\right) \cdot \mathbf{e}_0 \times \nabla \ln B + \frac{4}{\omega_c}\left(p - \frac{\pi_\parallel}{2}\right)\mathbf{e}_0 \times \nabla \ln B \cdot \nabla V_\parallel .$$

$$\mathbf{e}_0 \cdot \nabla \cdot \boldsymbol{\pi}_4^{''} = 3\omega_c^{-1}\mathbf{e}_0 \times \kappa \cdot \nabla \left(\pi_\parallel V_\parallel\right) ,$$

$$\mathbf{V} = V_\parallel \mathbf{b} + \mathbf{V}_\perp^{(0)}$$

Final form of the momentum balance

ExB transport

$$mn\left(\frac{\partial V_\parallel}{\partial t} + V_\parallel \mathbf{e}_0 \cdot \nabla V_\parallel + V_E \cdot \nabla V_\parallel\right) + \frac{4p + 5\pi_\parallel/2}{\omega_c} \mathbf{e}_0 \times \nabla \ln B \cdot \nabla V_\parallel$$

$$- mnV_\parallel V_E \cdot \nabla \ln B - \frac{2V_\parallel}{\omega_c} \mathbf{e}_0 \times \nabla (p + \pi_\parallel) \cdot \nabla \ln B$$

$$= enE_\parallel - \nabla_\parallel p - \mathbf{e}_0 \cdot \nabla \cdot \Pi_\parallel .$$

Coupling between pressure and potential fluctuations

and parallel momentum transport

This equation is identical to the one obtained in gyrokinetic theory,

Hahm et al., PoP 2007,2008

Some of these terms can be interpreted as Coriolis force effects in the moving plasma reference frame, Peeters et al., PRL 2007, PoP 2009

Coupled dynamics of density, pressure and parallel momentum

$$\frac{\partial n}{\partial t} + \mathbf{V}_E \cdot \nabla n - 2\mathbf{V}_E \cdot \nabla \ln B + \frac{2}{m\omega_c} \nabla p \cdot \mathbf{b} \times \nabla \ln B + \nabla \cdot \left(n V_\parallel \mathbf{b} \right) = 0.$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p\right) + \nabla \cdot \left(\frac{3}{2}\, p V_\parallel \mathbf{b} + \frac{3}{2} p\, \mathbf{V}_E + q_\parallel \mathbf{b} + \frac{1}{\omega_c}\left(5\frac{pT}{m} + \frac{7}{4}\frac{T}{m}\pi_\parallel + \frac{1}{2}\frac{T}{m}\pi_\parallel^* + \frac{2}{3}\frac{T}{m}\chi\right) \mathbf{b} \times \nabla \ln B\right)$$
$$- \left(2p + \frac{1}{2}\pi_\parallel\right) \mathbf{V}_E \cdot \nabla \ln B + \left(p - \frac{1}{3}(p_\parallel - p_\perp)\right) V_\parallel \nabla \cdot \mathbf{b}$$
$$+ \left(p + \frac{2}{3}(p_\parallel - p_\perp)\right) \mathbf{b} \cdot \nabla V_\parallel \;= 0,$$

$$mn\left(\frac{\partial}{\partial t} + V_\parallel \mathbf{b} \cdot \nabla + V_E \cdot \nabla + \frac{4p}{mn\omega_c}\, \mathbf{e}_0 \times \nabla \ln B \;\cdot\; \nabla \right) V_\parallel$$
$$- mnV_\parallel\; V_E \cdot \nabla \ln B \;- 2mnV_\parallel\; V_p \cdot \nabla \ln B = enE_\parallel - \;\nabla_\parallel p - \nabla \cdot \Pi_\parallel$$

Ion temperature gradient mode + parallel shear gradient mode

Lagrange invariants and turbulent homogenization, or

Turbulent equipartition

(Yan'kov, Isichenko,Nycander, Garbet, Mischenko, ...)

Pressure and density only $V_{\parallel} = 0$

$$\frac{\partial}{\partial t}\ln n + \mathbf{V}_E \cdot \nabla \ln - 2\mathbf{V}_E \cdot \nabla \ln B + 2\mathbf{V}_D \cdot \frac{\nabla p}{p} = 0.$$

$$\frac{\partial}{\partial t}\frac{3}{2}\ln p + \frac{3}{2}\mathbf{V}_E \cdot \nabla \ln p - 5\mathbf{V}_E \cdot \nabla \ln B + 5\mathbf{V}_D \cdot \frac{\nabla (pT)}{pT} = 0,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right)\ln\left(\frac{n}{B^2}\right) + 2\mathbf{V}_D \cdot \frac{\nabla p}{p} = 0.$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right)\ln\left(\frac{p^{3/2}}{B^5}\right) + 5\mathbf{V}_D \cdot \frac{\nabla (pT)}{pT} = 0,$$

Suspects for invariants

$$r_1 = \ln\left(\frac{n}{B^2}\right) \qquad r_2 = \ln\left(\frac{p^{3/2}}{B^5}\right)$$

Lagrange invariants

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_1 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 \ = 0.$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_2 + 5\mathbf{V}_D \cdot \nabla \left(\frac{4}{3} r_2 - r_1\right) = 0,$$

$$r_1 = \ln\left(\frac{n}{B^2}\right)$$

$$r_2 = \ln\left(\frac{p^{3/2}}{B^5}\right)$$

$$\frac{\partial}{\partial t} r_j^{'} + V_{*j} \cdot \nabla r_j^{'} = 0$$

$$\mathbf{V}_{*j} = \mathbf{V}_E + \beta_j \mathbf{V}_D$$

$$r = r_1 + \alpha r_2$$

$$r_1^{'} = \ln \frac{nB^{10(1-\Delta)/3}}{p^{1-\Delta}}$$

$$r_2^{'} = \ln \frac{nB^{10(1+\Delta)/3}}{p^{1-\Delta}}$$

$$\alpha_{1,2} = -\frac{2}{3}(1 \pm \Delta)$$

$$\Delta = \sqrt{\frac{2}{5}}$$

In fully developed (and reconnecting) turbulence one can assume that r_1 and r_2 are uniform

Turbulent equipartition

•Full gomogenization (mixing) of the invariants requires

-strong turbulence regime

-reconnection of the flow fields

$$\mathbf{V}_{*j} = \mathbf{V}_E + \beta_j \mathbf{V}_D$$

-caveat of incomplete mixing/absence of reconnection,

e.g. adiabatic density constraint for density transport; but

trapped particle will be mixed

Generalization including pressure anisotropy is possible

$$r = \frac{p_\perp^{\ \alpha} p_\parallel^{\ \beta} n^\gamma}{B^\delta}$$

Homogenization of the parallel momentum

$$r_3 = \ln\left(\frac{V_\parallel}{B}\right)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_1 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 \; + V_\parallel \mathbf{b}\cdot\nabla r_3 + V_\parallel \mathbf{b}\cdot\nabla r_1 + 2V_\parallel \mathbf{b}\cdot\nabla \ln B = 0.$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_2 + 5\mathbf{V}_D \cdot \nabla\left(\frac{4}{3}r_2 - r_1\right) + \frac{5}{2}V_\parallel \mathbf{b}\cdot\nabla r_3 + V_\parallel \mathbf{b}\cdot\nabla r_2 + 5V_\parallel \mathbf{b}\cdot\nabla \ln B = 0,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_3 + 4\mathbf{V}_D \cdot \nabla r_3 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 + V_\parallel \mathbf{b}\cdot\nabla r_3 + V_\parallel \mathbf{b}\cdot\nabla \ln B = 0,$$

One can try, $r = r_1 + \beta_j r_2 + \gamma_j r_3$ $\quad \mathbf{V} = \mathbf{V}_E + \beta_j \mathbf{V}_D + \gamma_j V_\parallel \mathbf{e}_0$

however parallel mixing (within the magnetic surface) and source like terms do not allow the invariants

$$\langle \mathbf{b}\cdot\nabla(\ldots)\rangle_\Psi = 0$$

Ensemble and surface averaging

$$\langle\langle V_\parallel \mathbf{b}\cdot\nabla \ln B\rangle\rangle = 0$$

Coupling of parallel momentum and pressure fluctuations

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_1 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 \quad = 0.$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_2 + 5\mathbf{V}_D \cdot \nabla \left(\frac{4}{3} r_2 - r_1\right) \quad = 0,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_3 + 4\mathbf{V}_D \cdot \nabla r_3 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 \quad = 0,$$

Parallel flow invariant

$$r_3' = \frac{B^{6/5} n V_\parallel^{3/5}}{p^{1/2}}$$

$$r_3 = \ln\left(\frac{V_\parallel}{B}\right)$$

Turbulent equipartition

- Full gomogenization (mixing) of the invariants requires

  -strong turbulence regime

  -reconnection of the flow fields

$$\mathbf{V}_{*j} = \mathbf{V}_E + \beta_j \mathbf{V}_D$$

  -Complete vs partial mixing?

$$r_1' = \ln \frac{nB^{10(1-\Delta)/3}}{p^{1-\Delta}}$$

$$r_2' = \ln \frac{nB^{10(1+\Delta)/3}}{p^{1-\Delta}}$$

$$r_3' = \frac{B^{6/5} n V_\parallel^{3/5}}{p^{1/2}}$$

Quasilinear theory:

Slab ITG, toroidal ITG, parallel shear instability (TEP effects are included)

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_1 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 + V_\parallel \mathbf{b}\cdot\nabla r_3 + V_\parallel \mathbf{b}\cdot\nabla r_1 + 2V_\parallel \mathbf{b}\cdot\nabla \ln B = 0.$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_2 + 5\mathbf{V}_D \cdot \nabla \left(\frac{4}{3} r_2 - r_1\right) + \frac{5}{2} V_\parallel \mathbf{b}\cdot\nabla r_3 + V_\parallel \mathbf{b}\cdot\nabla r_2 + 5V_\parallel \mathbf{b}\cdot\nabla \ln B = 0,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right) r_3 + 4\mathbf{V}_D \cdot \nabla r_3 + \frac{4}{3}\mathbf{V}_D \cdot \nabla r_2 + V_\parallel \mathbf{b}\cdot\nabla r_3 + V_\parallel \mathbf{b}\cdot\nabla \ln B = 0,$$

Turbulent equipartition pinch and residuals in quasilinear theory

Residual flux

$$\boldsymbol{\Gamma}_{\boldsymbol{\varphi}} = mnV_{\varphi}V_r + \Pi_g$$

$$\Gamma_X = -\chi \frac{\partial}{\partial r} X + VX + R$$

Pinch flux

$$\begin{pmatrix} \mathbf{Q} \\ \boldsymbol{\Gamma}_{\boldsymbol{\varphi}} \end{pmatrix} = \begin{pmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{pmatrix} (\nabla \mathbf{r}_1, \nabla \mathbf{r}_2, \nabla \mathbf{r}_3)$$

$$r_2 = \ln\left(\frac{p^{3/2}}{B^5}\right) \qquad r_1 = \ln\left(\frac{n}{B^2}\right) \qquad r_3 = \ln\left(\frac{V_\parallel}{B}\right)$$

Turbulent equpartition pinch

$$\Gamma_V \sim (\quad)\left(\frac{V_\parallel}{B}\right)' \sim (\quad)\left(\frac{V_\parallel'}{B} - \frac{V_\parallel}{B^2}B'\right)$$

Fluxes are diffusive in phase space (action variables)

In general, pinch and residual fluxes originate from compressibility in phase space, which is inhomogeneous due to e.g. magnetic field non-uniformities

Summary comments

- Gyroviscosity is an important contribution to the evolution of the parallel momentum
  - Define the coupling of pressure fluctuations to parallel dynamics (mediated by curvature effects)
  - Define the turbulent equipartition pinch and residual fluxes
  - Quasilinear transport fluxes are defined in terms of thermodynamics forces, which are gradients of $\left(\nabla \mathbf{r_1}, \nabla \mathbf{r_2}, \nabla \mathbf{r_3}\right)$

- Gyroviscosity contribution define the coupling to parallel heat flux, has important consequences for closure problem and for neoclassical like (collisional) transport of the momentum $\Pi = \frac{1}{\omega_c}\mathbf{K}^{-1}\left(p\nabla\mathbf{V} + \frac{2}{5}\nabla\mathbf{q} + \mathbf{Tr}\right)$