# Turbulent Transport In LAPD

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Global 3D two-fluid Braginskii simulations
with particle and heat sources

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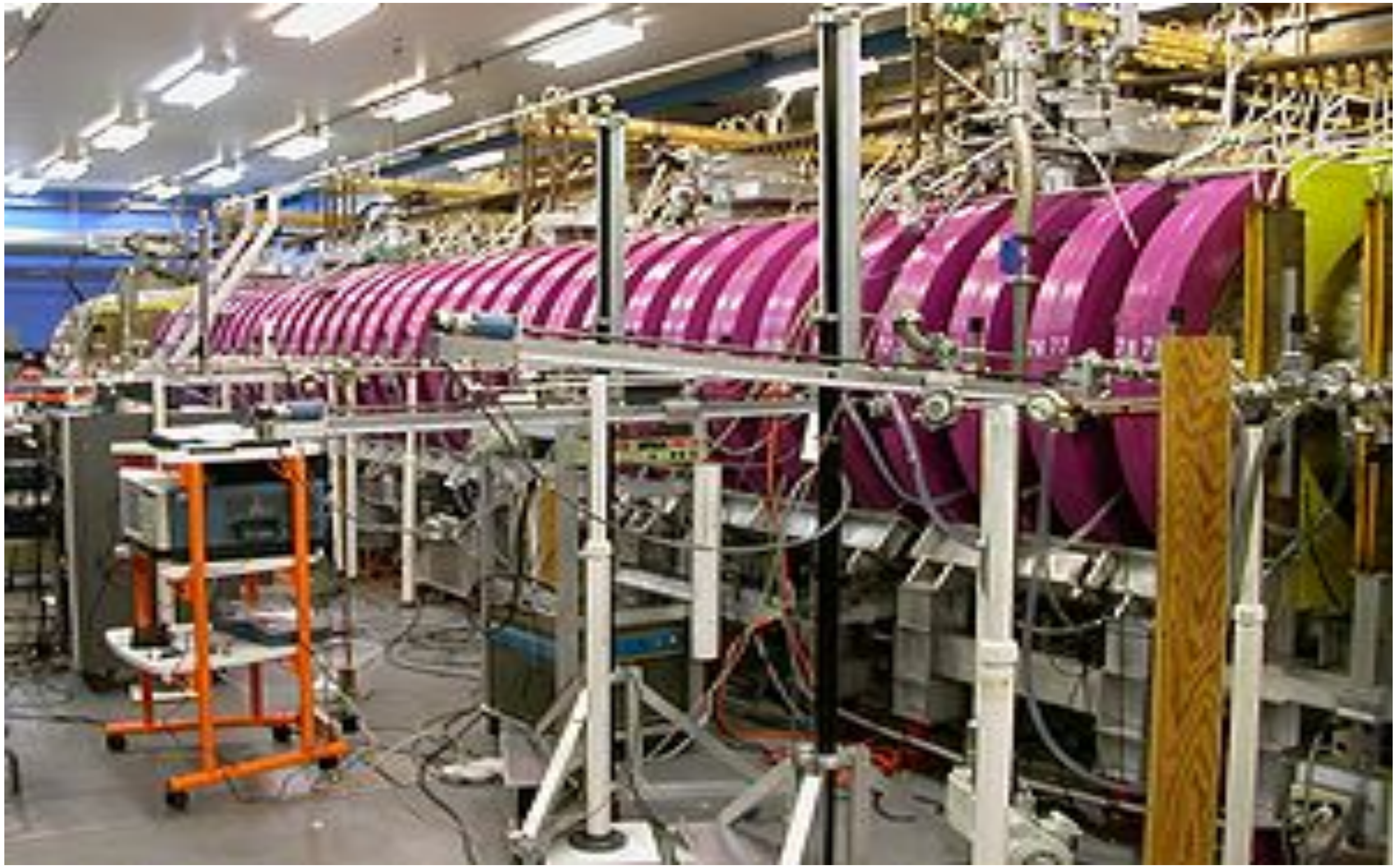
# LAPD: LArge Plasma Device



$$L \sim 17\ m,\ \ r \sim 0.5\ m,\ \ B \sim 1\ kG,$$

$$n \sim 10^{12}\ cm^{-3},\ T_e \sim 5\ eV,\ T_i \sim 1\ eV$$

# 3D Braginskii Equations (electrostatic, $T_i$=0)

Source terms

$$
\begin{aligned}
\frac{dn}{dt} &= -\frac{\partial(nV_{||e})}{\partial z} + S_n \\
\frac{dT_e}{dt} &= \frac{2}{3}\frac{T_e}{en}0.71\frac{\partial j_{||}}{\partial z} - \frac{2}{3}T_e\frac{\partial V_{||e}}{\partial z} - V_{||e}\frac{\partial T_e}{\partial z} + S_T \\
\frac{d\nabla_\perp^2\phi}{dt} &= -V_{||i}\frac{\partial \nabla_\perp^2\phi}{\partial z} + \frac{m_i\Omega_{ci}^2}{e^2 n}\frac{\partial j_{||}}{\partial z} \\
m_e\frac{dV_{||e}}{dt} &= -m_e V_{||e}\frac{\partial V_{||e}}{\partial z} - \frac{T_e}{n}\frac{\partial n}{\partial z} + e\frac{\partial\phi}{\partial z} - 1.71\frac{\partial T_e}{\partial z} + \frac{ej_{||}}{\sigma_{||}} \\
\frac{dV_{||i}}{dt} &= -V_{||i}\frac{\partial V_{||i}}{\partial z} - \frac{1}{n}\frac{\partial p_e}{\partial z}
\end{aligned}
$$

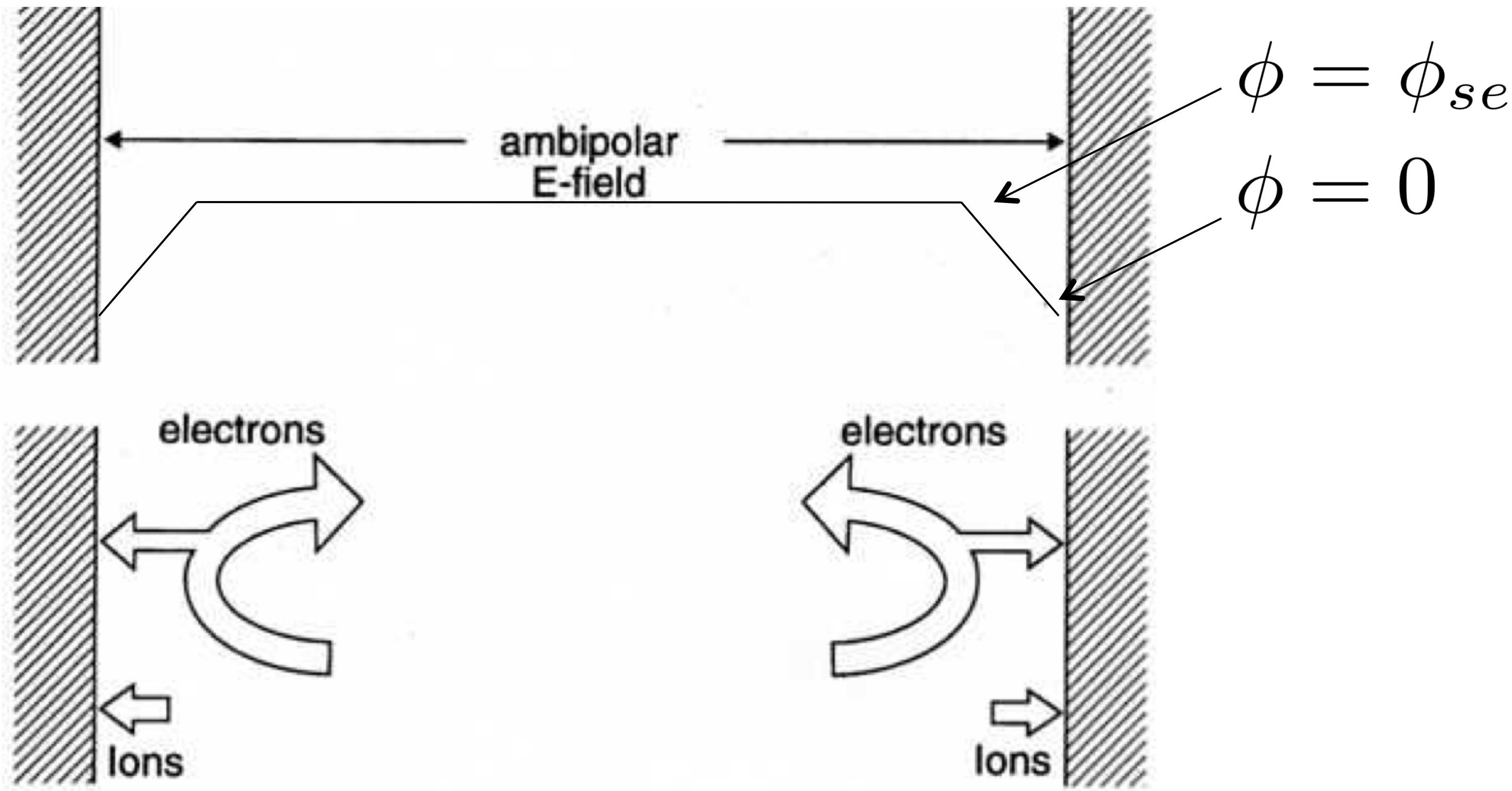
$$
Parallel\ BCs\ at\ sheath\ edges\ (z = \pm L_z/2):
$$

$$
V_{||i} = \pm c_s\ , \quad c_s = \sqrt{T_e/m_i}
$$

$$
V_{||e} = \pm c_s \exp\left(\Lambda - e\phi/T_e\right)\ , \quad \Lambda = \ln\sqrt{m_i/(2\pi m_e)} \simeq 3
$$

# Sheath Physics

In an open fieldline system, the plasma develops a positive potential relative to the wall until the electron and ion outflows along B balance:



# Electrons dynamics in sheath region

$$f_e = n_{se}\left[m_e/(2\pi T_e)\right]^{3/2}\exp\left\{-\left[(1/2)m_e v^2 - e(\phi-\phi_{se})\right]/T_e\right\}$$

$$n = \int f_e d^3v = n_{se}\exp\{e(\phi-\phi_{se})/T_e\} \quad (= n_{se} \text{ for } \phi=\phi_{se})$$

$$\Gamma_{\parallel e} = \int_{v_z>0} f_e v_z d^3v = n_{se}\sqrt{T_e/(2\pi m_e)}\exp\{e(\phi-\phi_{se})/T_e\}$$

$$= n_{se}c_s\exp\{\Lambda + e(\phi-\phi_{se})/T_e\}$$

$$\text{where } c_s = \sqrt{T_e/m_i}\ ,\ \Lambda = \ln\sqrt{m_i/(2\pi m_e)}$$

So electron flux to wall where $\phi = 0$ :

$$\Gamma_{\parallel e,wall} = n_{se}c_s\exp\{\Lambda - e\phi_{se}/T_e\}$$

Continuity across sheath :

$$n_{se}V_{\parallel e,se} = \Gamma_{\parallel e,wall} \quad \text{so} \quad \boxed{V_{\parallel e,se} = c_s\exp\{\Lambda - e\phi_{se}/T_e\}}$$

# Ion dynamics at edge of sheath

Continuity :

$$\partial_z \left(nV_{\|i}\right) = 0 \quad \text{so} \quad n\partial_z V_{\|i} = -V_{\|i}\partial_z n$$

Momentum (isothermal for simplicity) :

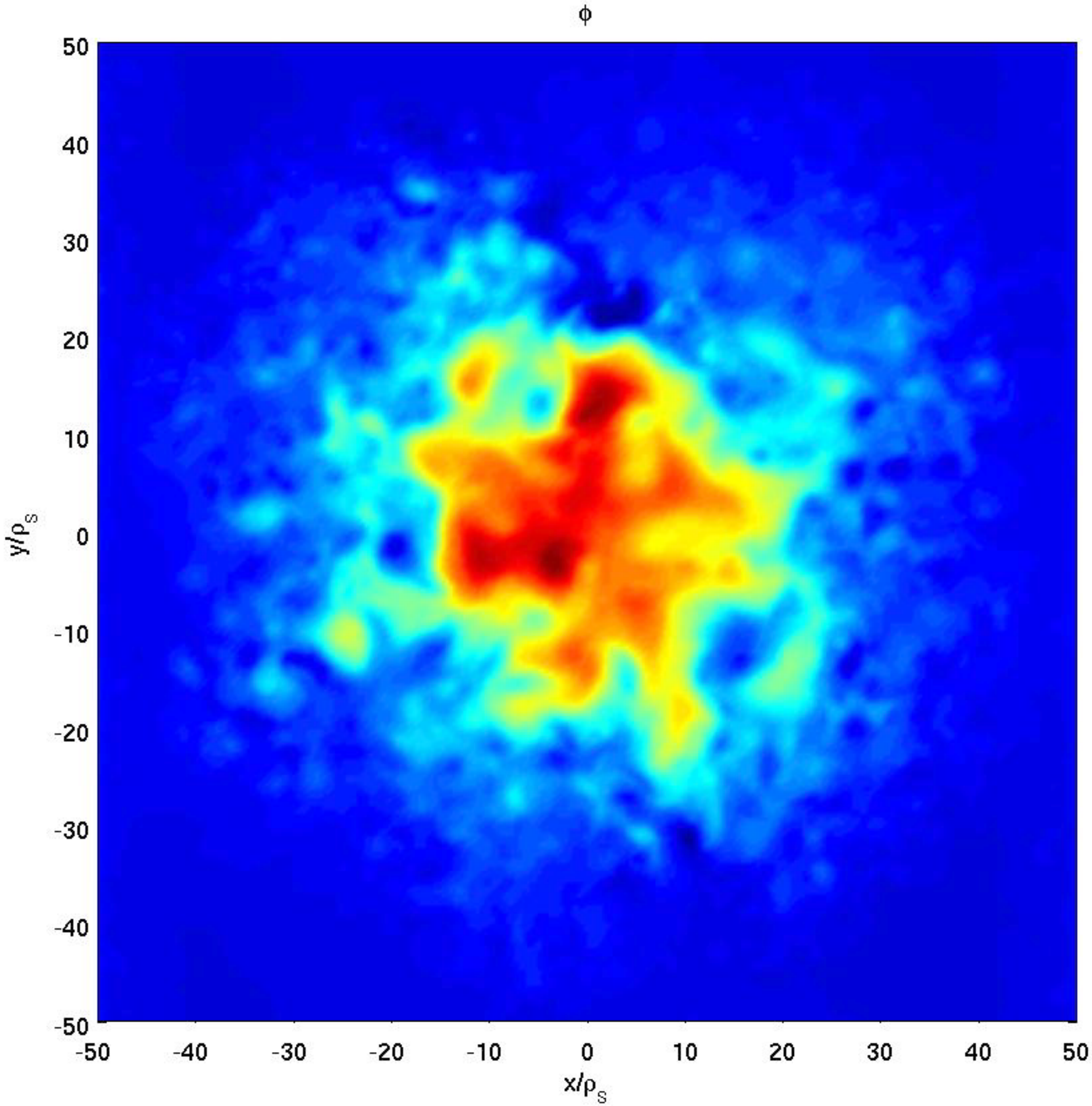
$$m_i n V_{\|i}\partial_z V_{\|i} = -T_e \partial_z n$$

Combining these gives :

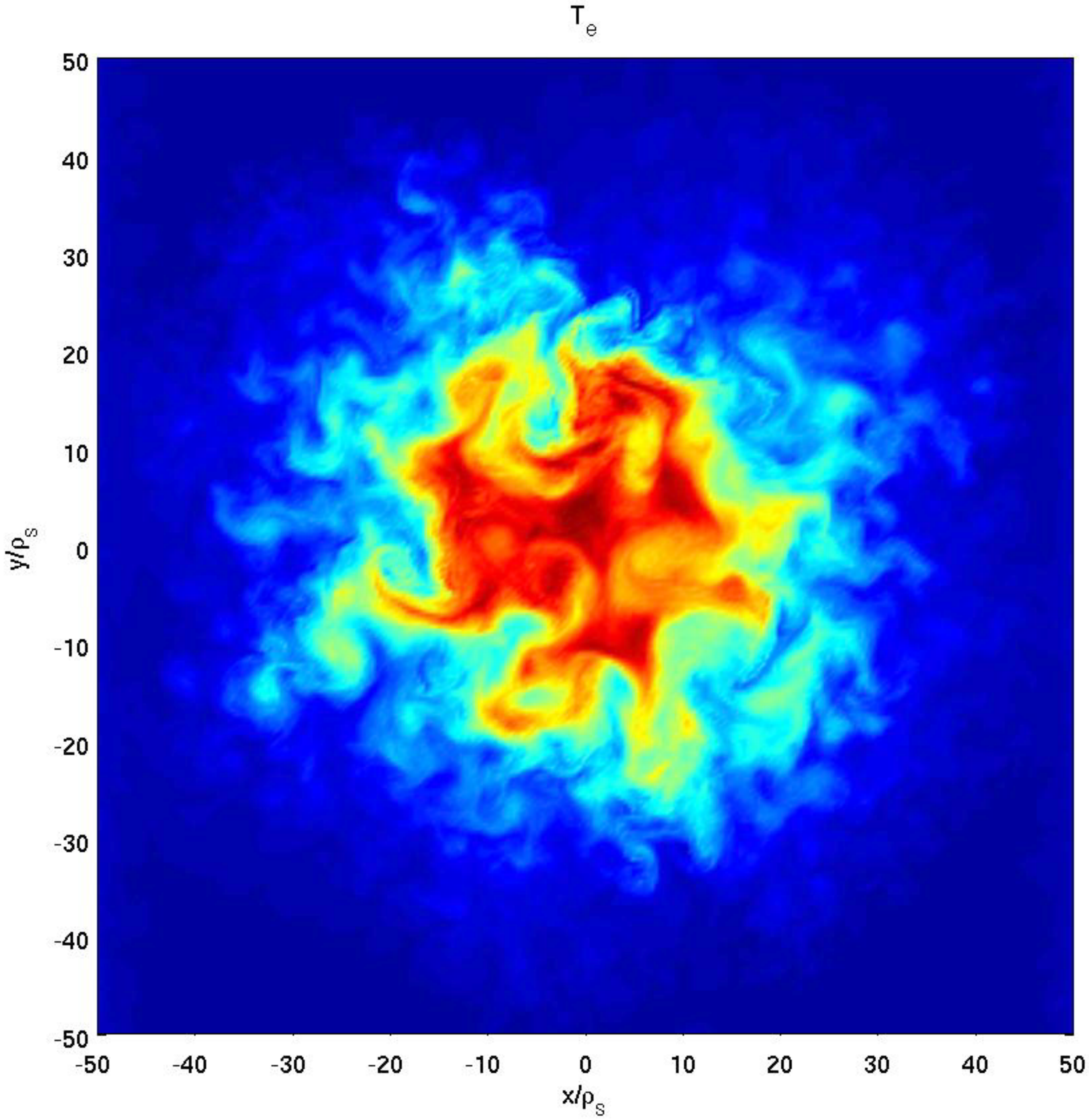
$$-m_i V_{\|i}^2 \partial_z n = -T_e \partial_z n \quad \text{or} \quad \boxed{V_{\|i,se} = c_s \ , \quad c_s = \sqrt{T_e/m_i}}$$

Electrons : $\quad V_{\|e,se} = c_s \exp\left\{\Lambda - e\phi_{se}/T_e\right\}$

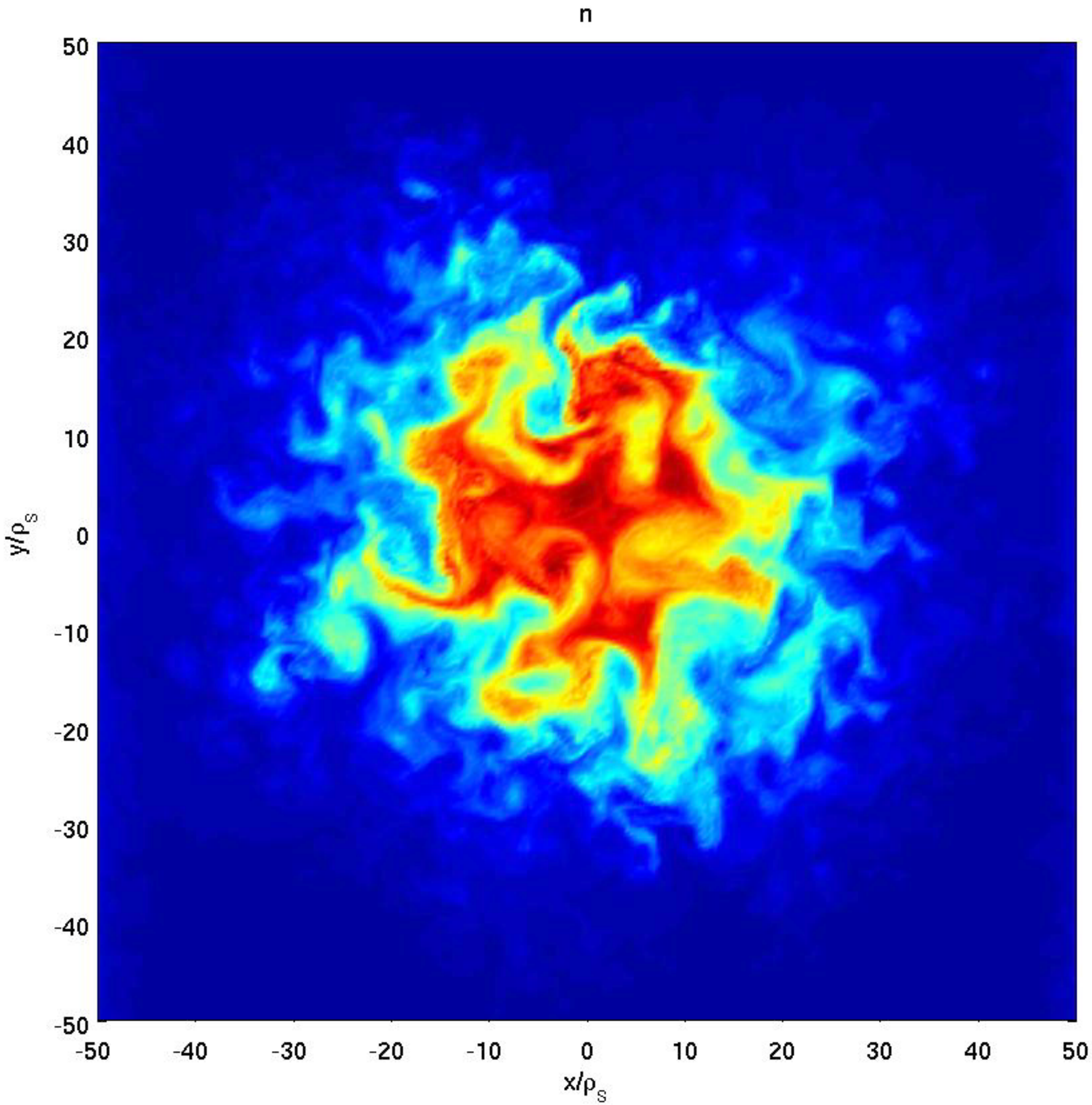
$$\boxed{\text{Bottom Line : in equilibrium need } V_{\|i} \simeq V_{\|e} \rightarrow \phi \simeq \Lambda T_e/e}$$



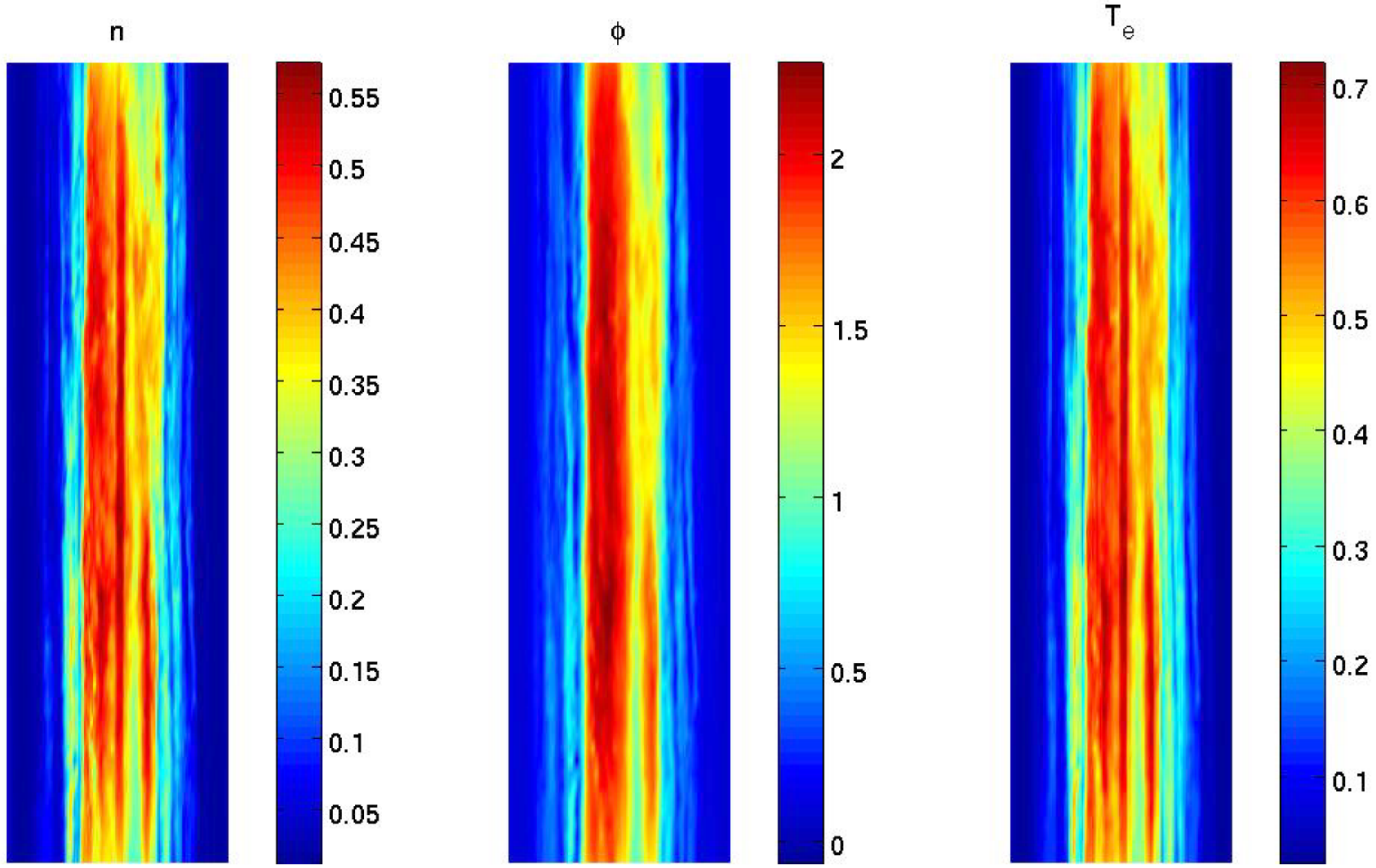
ϕ
50
40
30
20
10
0
-10
-20
-30
-40
-50
y/ρS
-50
-40
-30
-20
-10
0
10
20
30
40
50
x/ρS



T_e
50
40
30
20
10
0
-10
-20
-30
-40
-50
y/ρ_S
-50
-40
-30
-20
-10
0
10
20
30
40
50
x/ρ_S

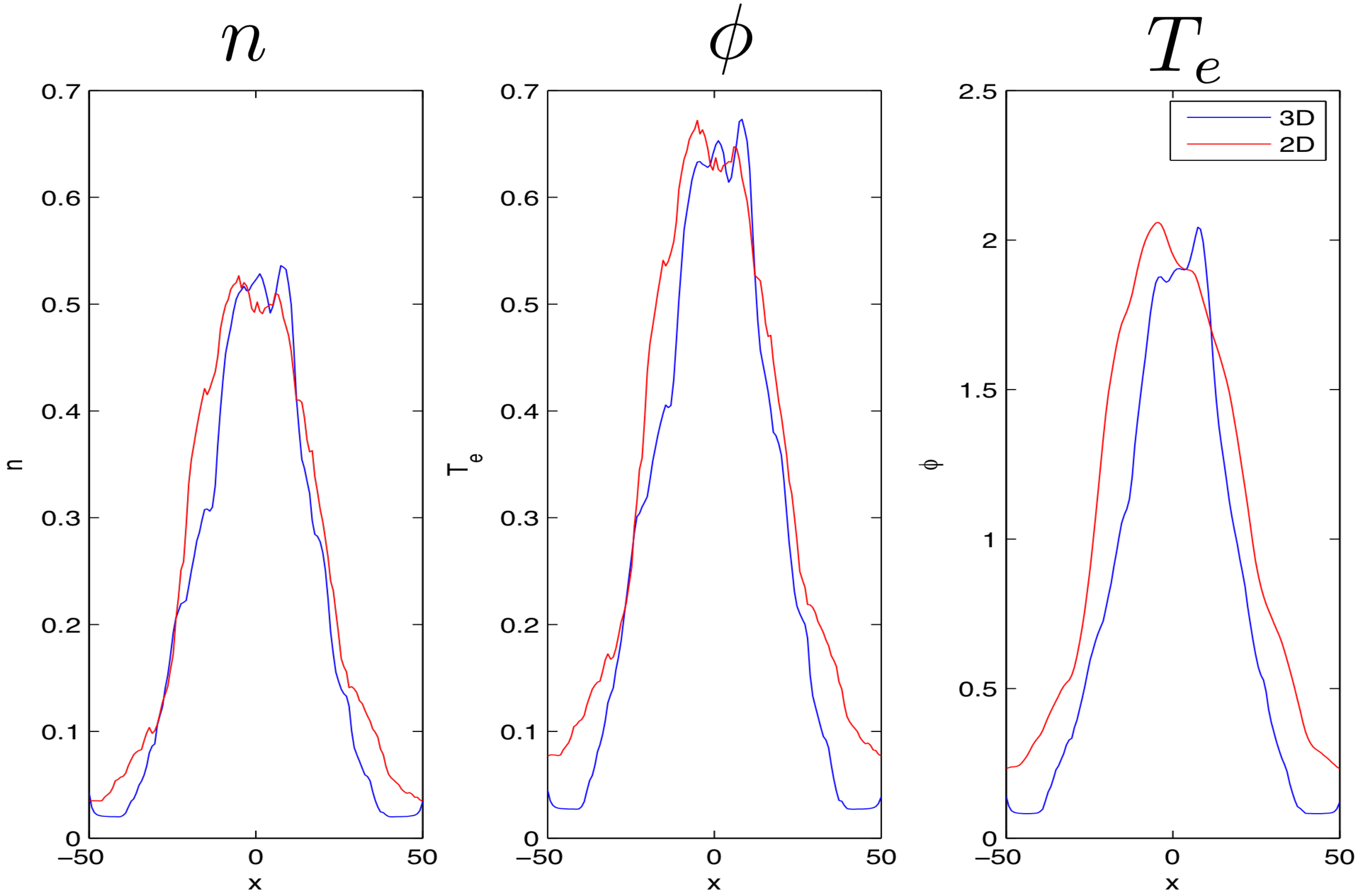

n
50
40
30
20
10
0
-10
-20
-30
-40
-50
y/ρS
-50
-40
-30
-20
-10
0
10
20
30
40
50
x/ρS

# Parallel to B structure



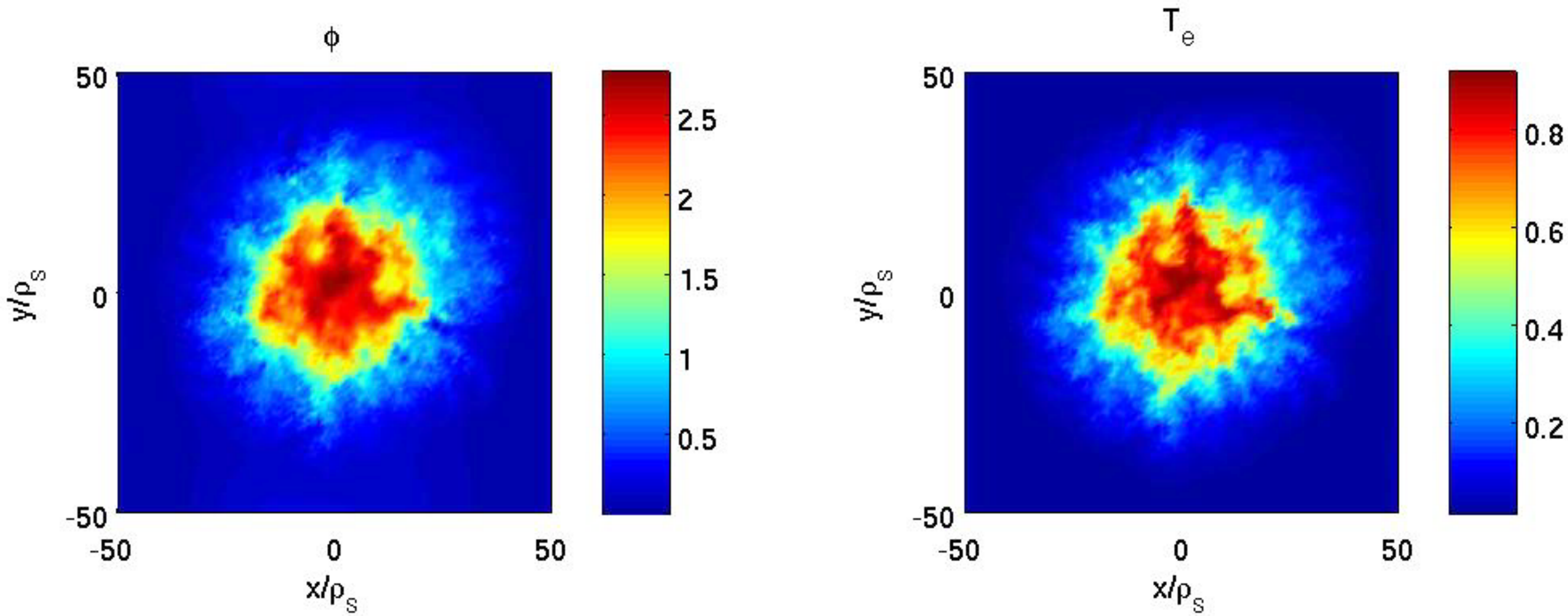

$n$ $\phi$ $T_e$

# Time-averaged profiles $(\perp \ B)$



# Restart of a simulation without the KH drive

$$\frac{\partial}{\partial t}\nabla^2\phi + [\phi, \nabla^2\phi] \rightarrow \frac{\partial}{\partial t}\nabla^2\phi + [\langle\phi\rangle, \nabla^2\phi]\ , \quad \langle\phi\rangle = \frac{1}{2\pi}\int \phi\ d\theta$$




Suggests that KH mode is the dominant transport channel

# Approximate 2D system assuming $k_\parallel = 0$

Assume $n(x,y,z) \simeq n(x,y)\ ,\ T_e \simeq T_e(x,y)\ ,\ \phi \simeq \phi(x,y)\ ,\ S_{n,T} \simeq S_{n,T}(x,y)\ ,$

$V_{\parallel i} \simeq (z/L_z)c_s\ ,\ V_{\parallel e} \simeq V_{\parallel i}\exp(\Lambda - e\phi/T_e)$

then : $\dfrac{dn}{dt} = -\dfrac{\partial(nV_{\parallel e})}{\partial z} \simeq -\dfrac{nc_s}{L_z}\exp(\Lambda - e\phi/T_e) + S_n$

*Similarly:*

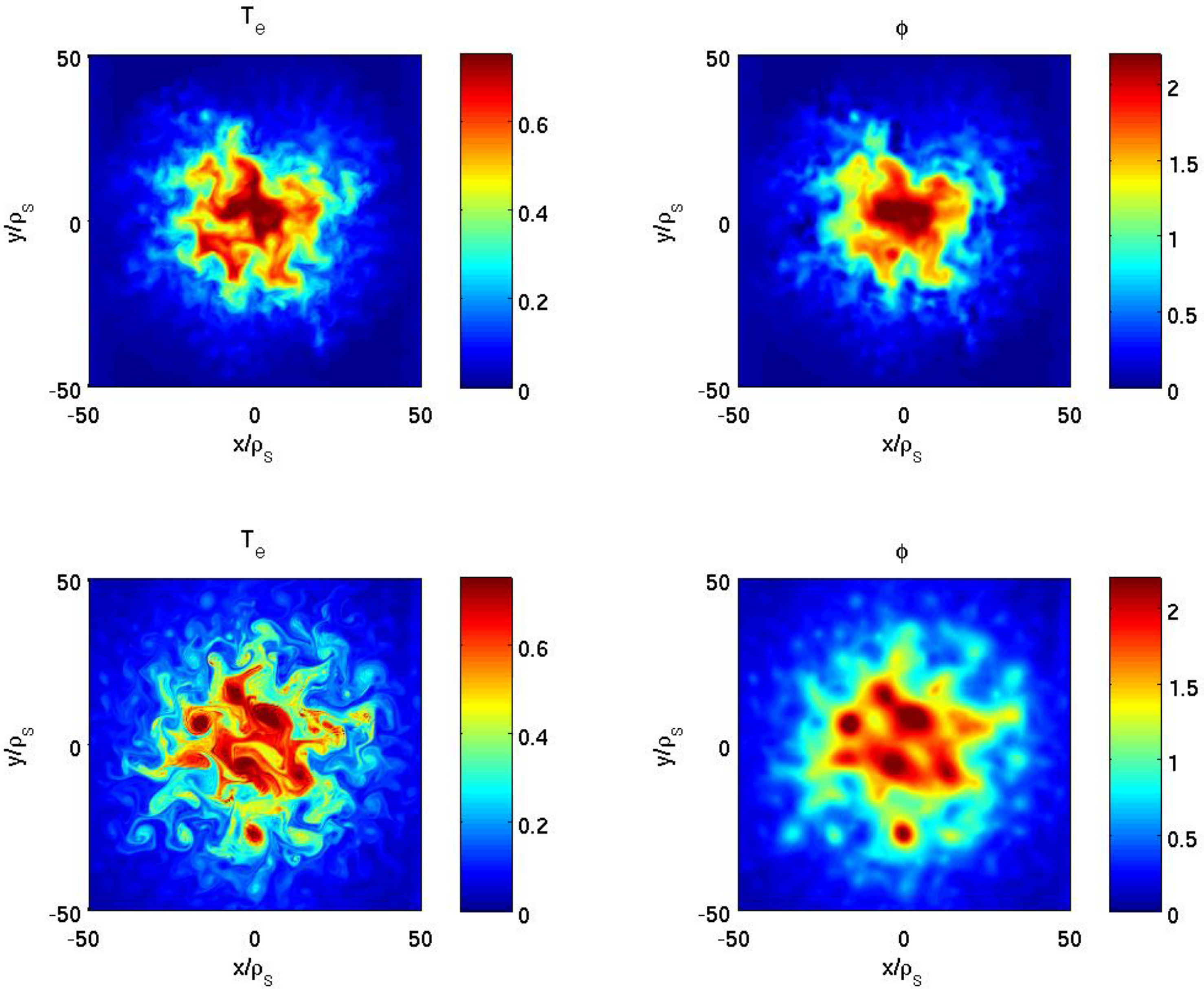
$$\begin{aligned}
\frac{dn}{dt} &= -\frac{nc_s}{L_z}\exp\left(\Lambda - e\phi/T_e\right) + S_n \\
\frac{dT_e}{dt} &= -\frac{2}{3}\frac{T_e c_s}{L_z}\left[1.71\exp\left(\Lambda - e\phi/T_e\right) - 0.71\right] + S_T \\
\frac{d\nabla^2\phi}{dt} &= \frac{c_s m_i \Omega_i^2}{eL_z}\left[1 - \exp\left(\Lambda - e\phi/T_e\right)\right]
\end{aligned}$$

No driftwaves in this system. Only KH and sheath modes.

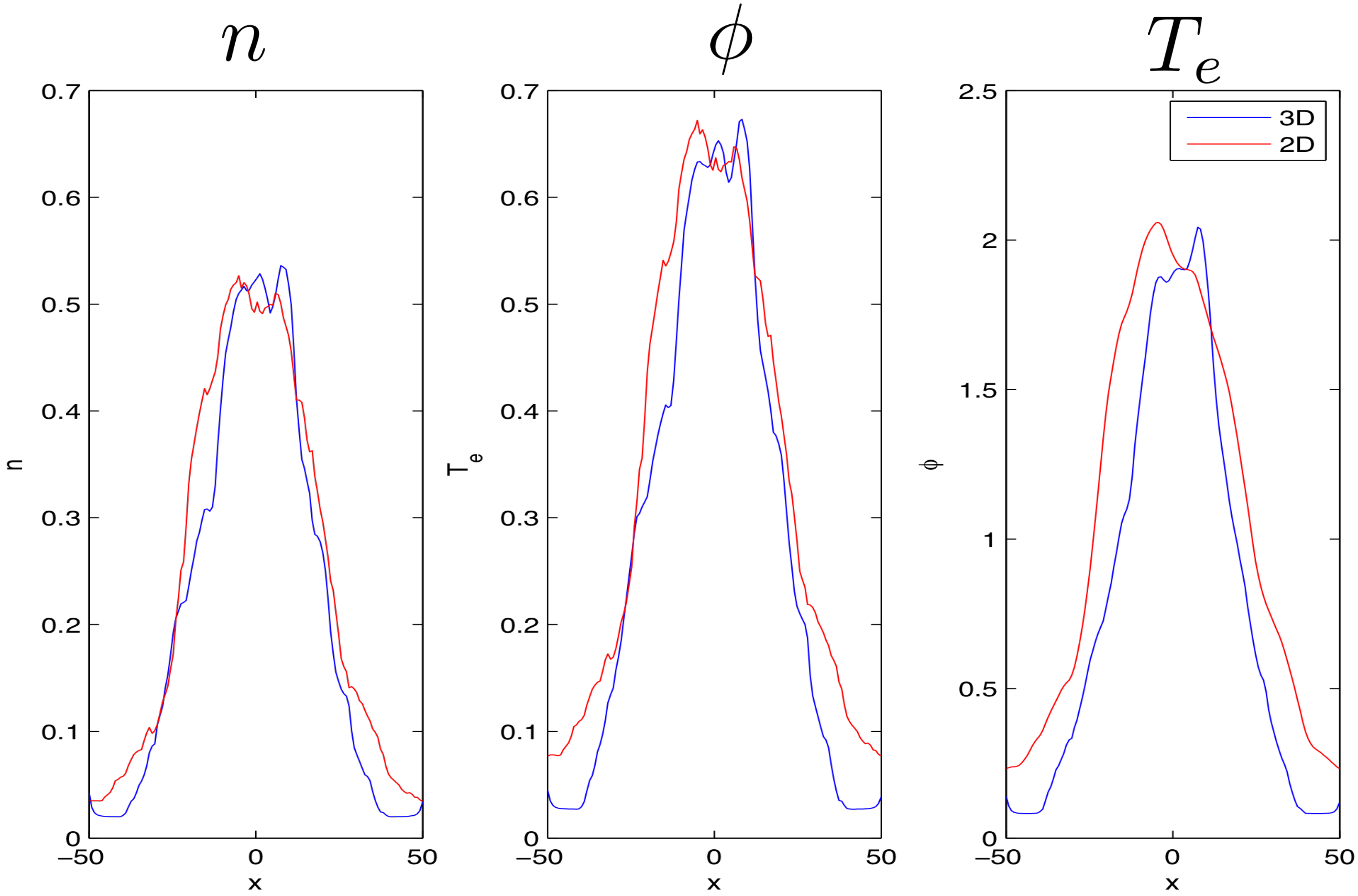
# 3D (top) vs 2D (bottom)

3D:

2D:



# Time-averaged profiles $(\perp \ B)$



# Summary

- For standard sheath BCs, KH is dominant cross-field transport channel
- Essential to distinguish the (linearly unstable) instantaneous profiles from the smoother (stable or nearly stable) time-averaged profiles
- In progress: study of more realistic cathode-anode parallel BCs and comparison to LAPD data.