Pinch regimes in gyrokinetic simulations of closed fieldline systems

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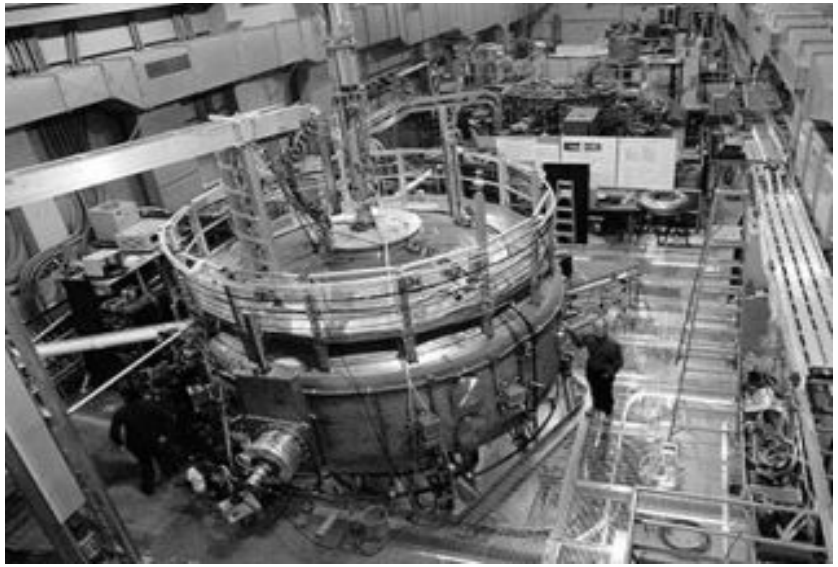
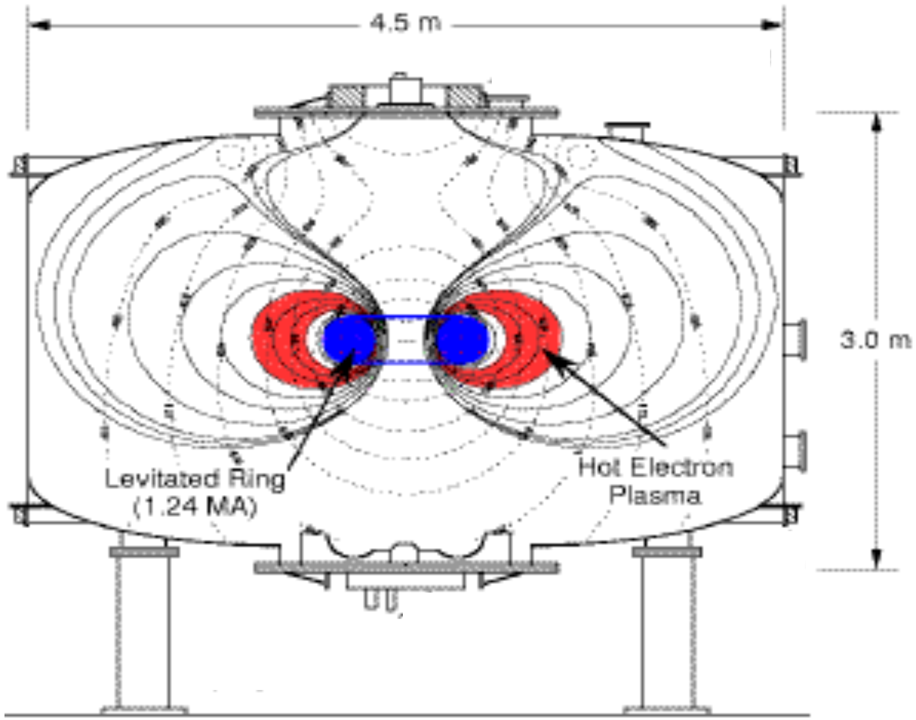
In most systems, the net result of microturbulence is to drive the pressure gradient away from ideal boundary.

In a dipolar geometry, the opposite is true.

MIT-Columbia Levitated Dipole Experiment (LDX)





Dipolar B-field created by a superconducting magnetically levitated current ring

$n \sim 10^{13}\ cm^{-3}$

$B \sim 2\ T$ (at ring)

$T_e \sim 160\ eV\ ,\ T_i < T_e$

$\beta_e \sim 0.1$

$I \sim 3200\ A$

$\rho_i \sim 0.02 - 10\ cm$

$\omega_* \sim 10^3\ s^{-1}$

$\Omega_{ci} \sim 10^5 - 10^6\ s^{-1}$

$V_{thi} \sim 10^4 - 10^7\ cm/s$

Simulation Model: Gyrokinetic GS2 Code

- Solves nonlinear gyrokinetic eqns for ions and electrons
- 5 dimensions $(\vec{r}, v_{\|}, v_{\perp})$
- Valid for arbitrary, $k_{\perp}\rho_i$, $k_{\perp}\rho_e$
- Includes: FLR, Landau damping, trapped particle effects, …
- Flux tube simulation domain ($k_{\perp} \sim 1 / \rho_s >> 1/L$)
- requires: $\omega << \Omega_{ci}$, $k_{||} << k_{\perp}$

Our simulations: Electrostatic ($\beta << 1$)

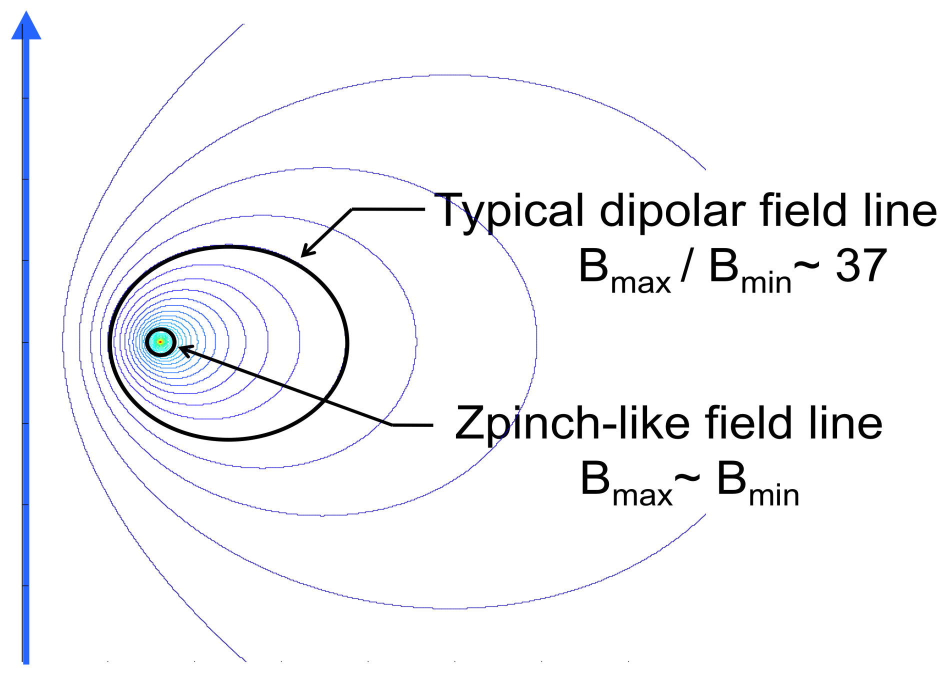
Two geometries: Ring-dipole and Z-pinch

(i) Ring-dipole

- near center of LDX plasma
- High mirror ratio $B_{\max} / B_{\min} \sim 37$
- High trapped particle fraction

(ii) Z-pinch

- Limiting case close to the ring
- B-field is circular, uniform along B
- trapped particles negligible



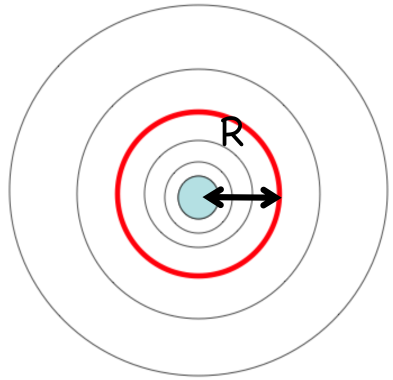
The transport levels are comparable with an outboard-midplane normalization in dipole case

Two main instabilities: Ideal interchange and Entropy modes

Ideal MHD Interchange mode:

- Growth rate (Z-pinch limit, β<<1)

$$\gamma^2 = \frac{2C_s^2}{RL_p}\left[1 - \frac{10L_p}{3R}\right] \qquad (k_{\|} = 0\ ,\ \ k_{\perp}L_p \gg 1)$$

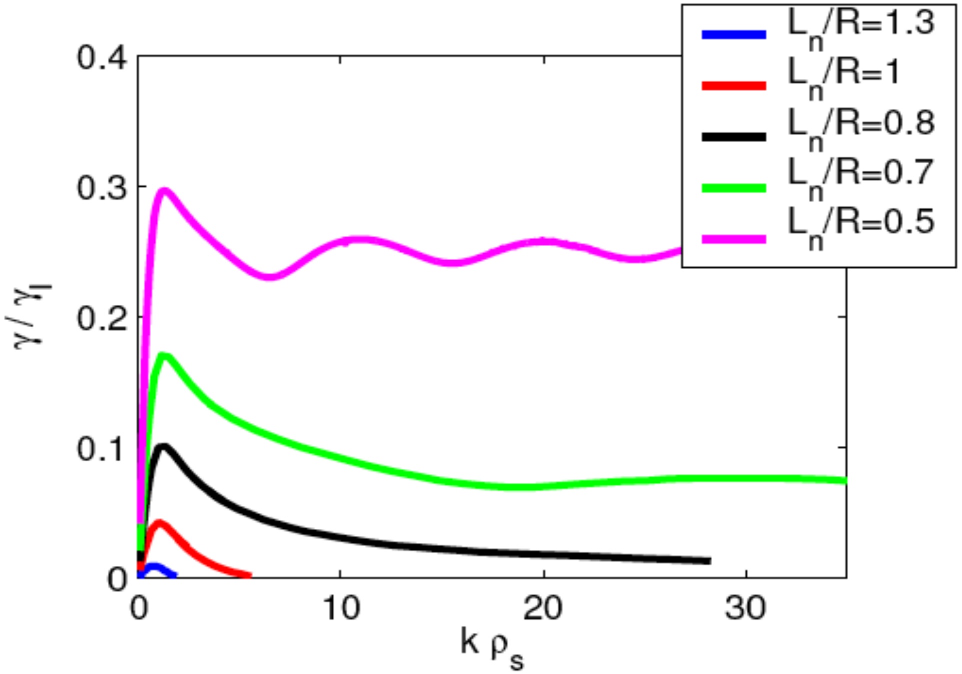


Zpinch

- Stable for L_p > (3/10)R
- Similar expression for ring dipole

We consider ideally stable plasmas and explore the transport due to small scale (non-MHD) entropy modes

Entropy mode (non-MHD)



- Small Scale: max growth at $k_\perp \rho_s \sim 1$ with $\gamma \sim \gamma_I$
- Driven by a gradient in the specific entropy $S=p/n^{5/3}=T/n^{2/3}$ (more later)

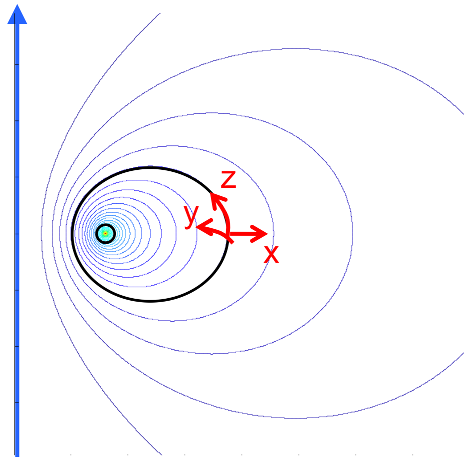
- Our simulations show these modes drive experimentally important levels of transport in ideally stable plasmas:

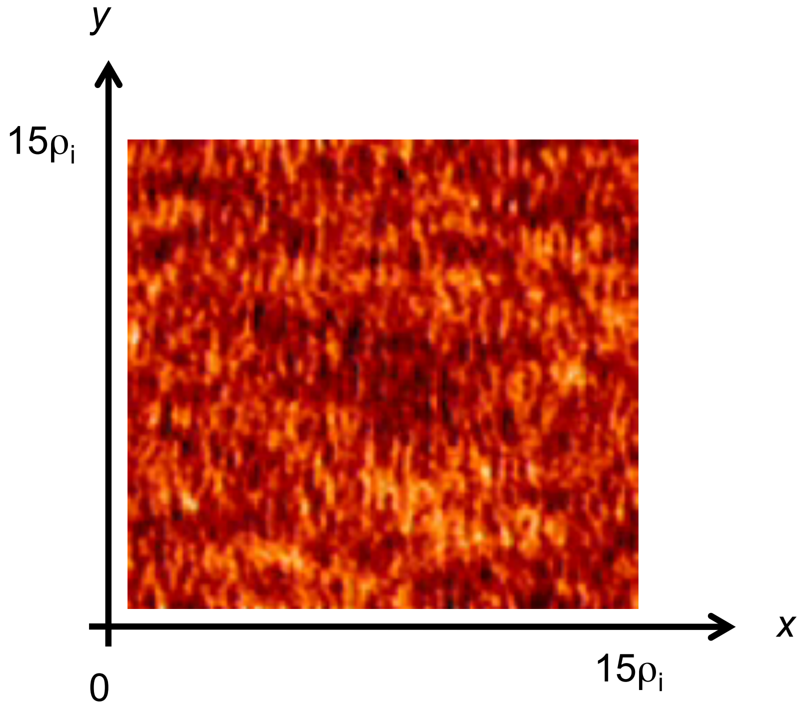
$$\Gamma = nV_{avg} \quad \text{with} \quad V_{avg} \sim 50 \; m/s \quad \text{or more}$$

- Observed in LDX but role in transport still unclear

Typical Simulation:

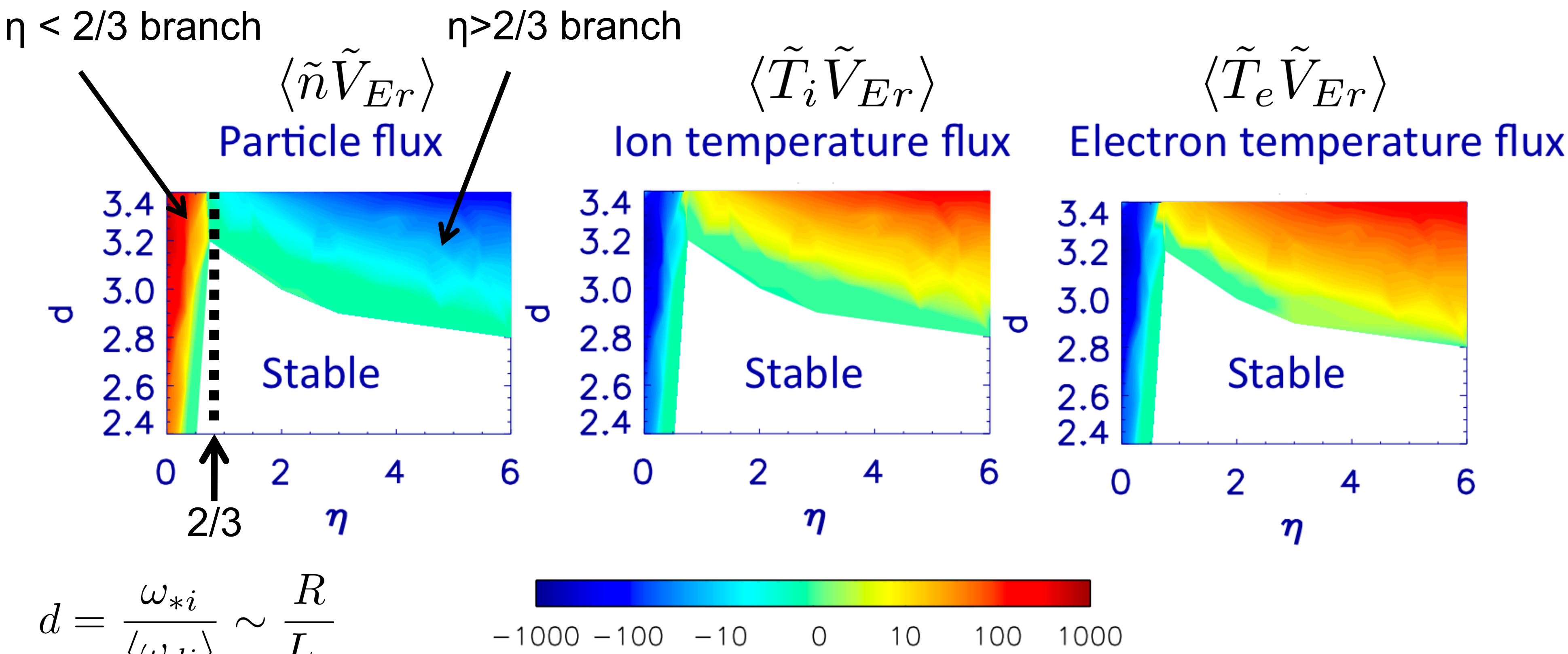
– Electrostatic potential Φ generated by entropy modes





- Three phases:
  (1) Linear growth
  (2) Nonlinear onset of KH
  (3) Nonlinear turbulent state

Two Entropy mode branches: $\eta>2/3$ and $\eta<2/3$ ($\eta=L_n/L_T$)



$$d = \frac{\omega_{*i}}{\langle \omega_{di} \rangle} \sim \frac{R}{L_p}$$

$$\eta = \frac{L_n}{L_T}$$

- $\eta>2/3$ branch driven by T′ : Particle flux negative
  Temp fluxes positive

- $\eta<2/3$ branch driven by N′: Particle flux positive
  Temp fluxes negative

Why two branches?

Consider the specific entropy: $S \propto \frac{p}{n^{5/3}} \propto \frac{T}{n^{2/3}}$

$$\frac{1}{S}\frac{dS}{dr} = -\frac{2}{3}\frac{1}{n}\frac{dn}{dr} + \frac{1}{T}\frac{dT}{dr} = \frac{2}{3}\frac{1}{L_n} - \frac{1}{L_T}$$

Thus since $\eta = \frac{L_n}{L_T}$:

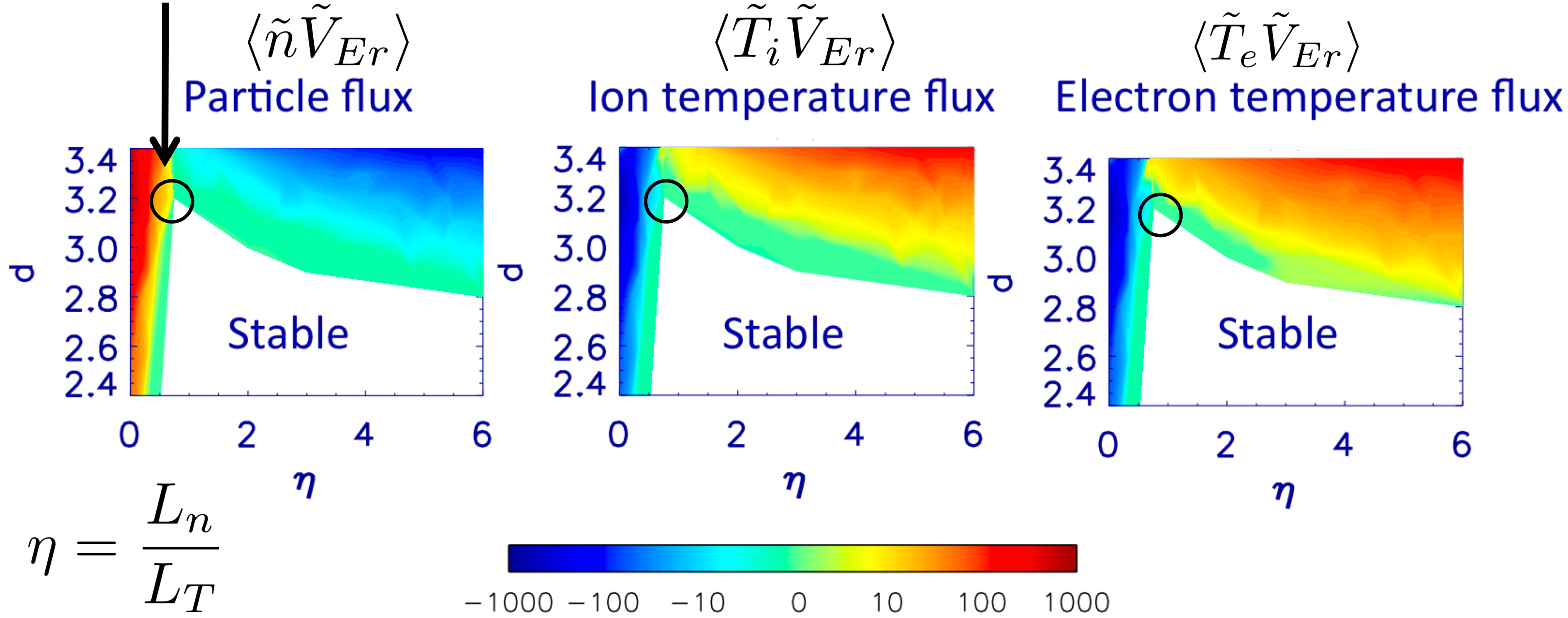
Result of transport:

If $\eta < \frac{2}{3}$ then $\frac{1}{S}\frac{dS}{dr} > 0$ — L_n increases, L_T decreases so $\frac{1}{S}\frac{dS}{dr} \downarrow$

If $\eta > \frac{2}{3}$ then $\frac{1}{S}\frac{dS}{dr} < 0$ — L_n decreases, L_T increases so $\frac{1}{S}\frac{dS}{dr} \uparrow$

Transport removes entropy gradient

Entropy mode transport expected to drive LDX profiles toward $\eta = 2/3$ and marginal interchange stability



$$\eta = \frac{L_n}{L_T}$$

- η>2/3 branch: Particle flux negative
  Temp fluxes positive

-> Transport decreases η

- η<2/3 branch: Particle flux positive
  Temp fluxes negative

-> Transport increases η

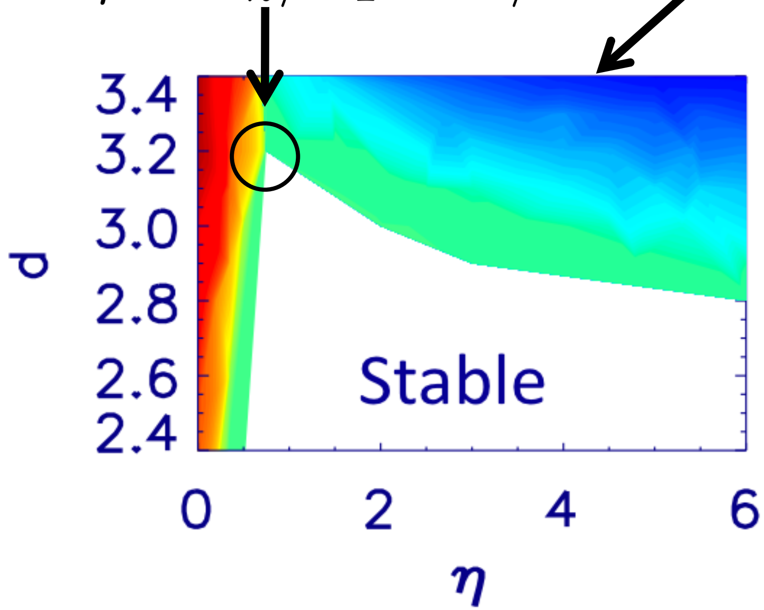
Observed LDX profiles consistent with $\eta \simeq 2/3$ and marginal interchange stability

[Boxer et al, Nature Physics, 2010]

“notch” region

$\eta = L_n/L_T = 2/3$

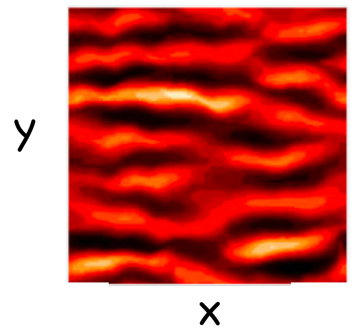
marginal MHD stability



As argued in Boxer et al, may also result from ideal interchange turbulence - more observations needed

Physical origin of particle pinch

- At high-η , main terms contributing are...



$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\mathbf{v_E} + ...)$$

$$\gamma \tilde{n} = -\nabla \cdot \left[n\frac{\mathbf{b}}{B} \times \nabla\tilde{\phi} + ... \right]$$

$$= -\nabla\tilde{\phi} \cdot (n\nabla \times \frac{\mathbf{b}}{B}) + ...$$

$$= -in_0\mathbf{k} \cdot \nabla \times \left(\frac{\mathbf{b}}{B}\right)\tilde{\phi} + ...$$

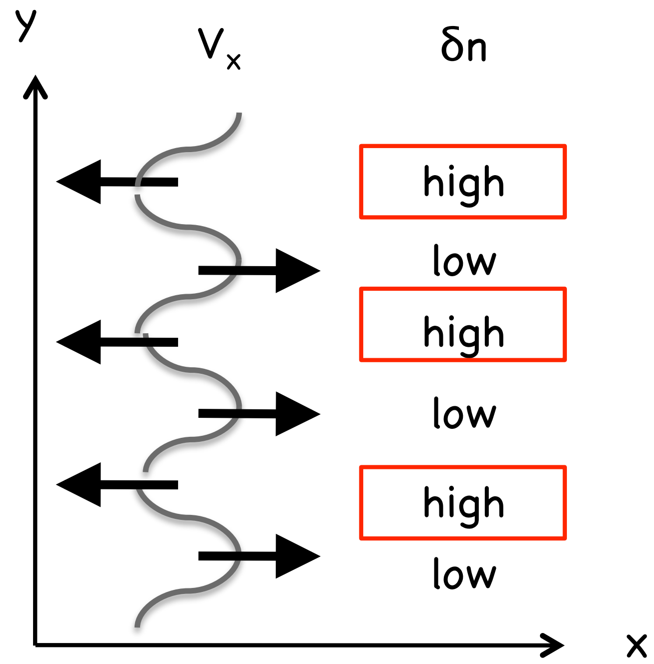
Using, $\nabla \times \frac{\mathbf{b}}{B} = \frac{2}{B}\mathbf{b} \times \vec{\kappa_m}$

and for Zpinch, $\vec{\kappa_m} = -\frac{\hat{r}}{R}$, and $\omega_{di} = k_\perp \rho_i \frac{v_{thi}}{R}$

$$\gamma \tilde{n} = -2in_0\omega_{di}\left(\frac{e\tilde{\phi}}{T_{e0}}\right) + ...$$

$$\gamma \bar{n} = -2\omega_{di} i\bar{\phi} + ...$$

$$\tilde{n} = -(const)\tilde{v_x} + ...$$



δn and δV_x are π out of phase

$$\Gamma = <\tilde{n}\tilde{v_r}> = <Re(\tilde{n})Re(ik\tilde{\phi})> = -\frac{2\omega_{di}k}{\gamma}\tilde{\phi_i}^2 = -\frac{2\rho_i v_{th}}{R}k^2\tilde{\phi_i}^2 < 0$$

Good agreement with GK quasi-linear theory

- Gyrokinetic Quasi-linear Theory

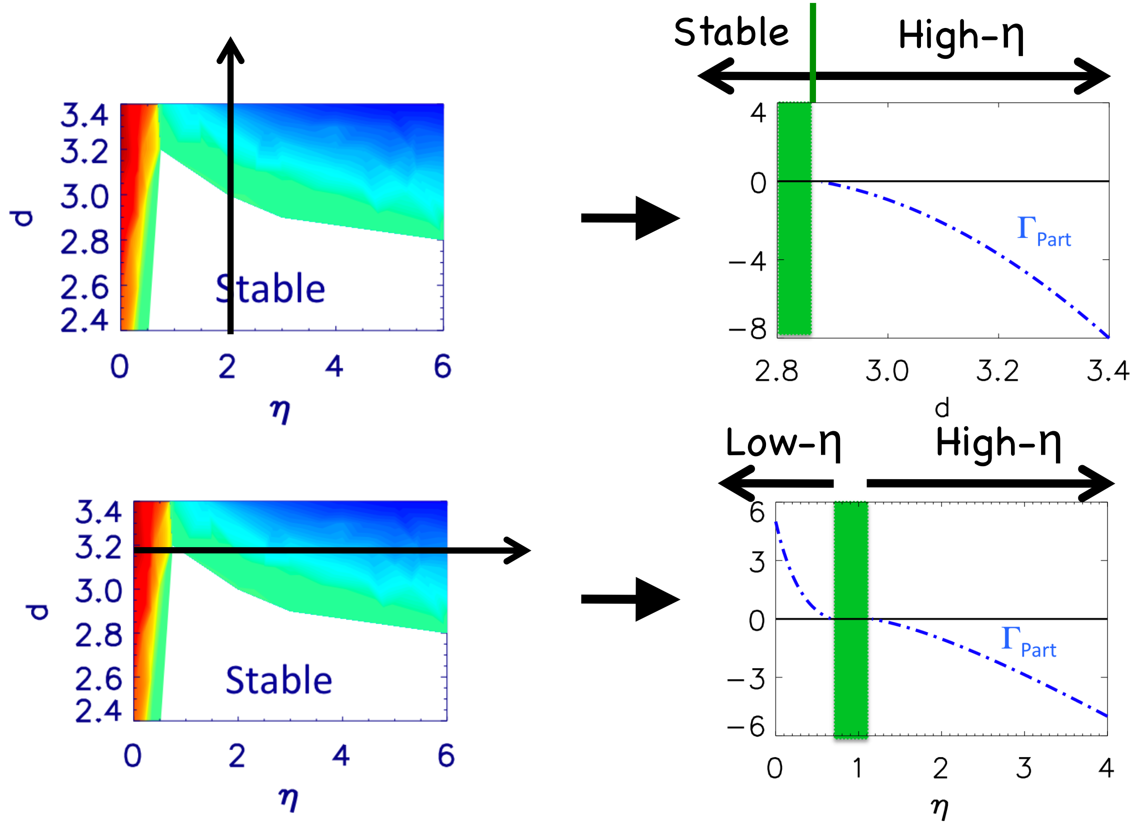
$$\Gamma_{part} = Im[\int \frac{\omega - \Omega_{*e}}{\omega - \Omega_{de}} \frac{\partial F_0}{\partial \epsilon} d^3v] k_z {\phi_0}^2 e^{2\gamma t} \frac{ec}{2B}$$

$$F_{0e} = n_0(\frac{m_e}{2\pi T_{0e}})^{3/2} \exp(-\frac{\epsilon}{T_{0e}})$$

$$\Omega_{*e} = -\omega_{*i}[1+(\frac{\epsilon}{T_{0e}} - \frac{3}{2})\eta]$$

$$\Omega_{de} = -\omega_{di}(\frac{{v_\parallel}^2 + {v_\perp}^2/2}{{v_{the}}^2})$$

$$\epsilon = \frac{m_e({v_\parallel}^2 + {v_\perp}^2)}{2{v_{the}}^2}$$



- Quasi-linear theory is consistent with GS2 simulations

Summary

- Entropy mode transport expected to drive LDX toward marginal ideal-interchange stability and η=2/3. In a dipolar field ($B \sim 1/r^3$) this means:

$$pV^{5/3} = const\ , \quad V \propto r^4 \ \Rightarrow \ p \propto r^{-20/3}$$
$$p/n^{5/3} = const \ \Rightarrow \ n \propto 1/V \propto r^{-4}$$
$$T/n^{2/3} = const \ \Rightarrow \ T \propto r^{-8/3} \quad (\eta = 2/3)$$

  - Consistent with Earth's magnetosphere and recent LDX observations [Boxer et al, Nature Physics, 2010]
  - May also result from ideal interchange turbulence

- Further work needed to determine role of entropy mode transport in LDX