Gyrokinetic Microtearing Studies

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summarising work involving many collaborators:

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Outline of the Talk

Summary of mostly old gyrokinetic simulations of microtearing modes in STs, using GS2.

(1) Tearing Parity Modes and Simulation Literature
(2) Microtearing Mode in MAST
(3) Contact with Analytic Theory
(4) Nonlinear Simulations
(5) Key Questions
Local ballooning space represents physical quantities as twisting slices:

\[
F(x, y, \theta) = e^{ik_y(y+s(\theta-\theta_0)x)} \sum_{p=-\infty}^{\infty} \hat{F}(\theta - \theta_0 - 2\pi p) e^{inq(x)2\pi p}
\]

**fast \perp variation** \quad \text{and} \quad \text{slow \parallel variation}

- \(x\) is equ’m flux surface label, \(x=0\) at \(q(x)=m/n\)
- \(y\) equ’m field line label, \(\perp\) to \(b\), lying in the flux surface
- \(\theta\) is \(\parallel\) to \(b\)

\(\hat{F}\) is defined on infinite domain in the ballooning angle \(\eta\),

\(\theta_0\) is the ballooning parameter.

\[
\hat{F}(\eta) \to 0 \quad \text{as} \quad \eta \to \pm \infty
\]

\(\hat{F}\) eigenfunctions are either even or odd in \(\eta\), about \(\eta = \theta_0\)
At $x=0$, the parity of $\hat{F}(\eta)$ about $\eta = \theta_0$ in ballooning space determines the symmetry of $F$ along the field line in real space.

Perturbed magnetic field comes from $\delta B = \nabla \times \delta A$

$\Rightarrow$ radial component: $\delta B_x = \partial A_\parallel / \partial y = ik_y A_\parallel$

$A_\parallel$ even, conclude for $x=0$ that

$\Rightarrow \delta B_x$ same sign along equ’m field line
$\Rightarrow \delta B_x$ sinusoidal in $y$ at fixed $\theta$
$\Rightarrow$ equilibrium field lines are torn!

Even $A_\parallel$ implies tearing of magnetic flux surface $x=0$
Some Gyrokinetic Microtearing Mode Simulations in the Literature

Microtearing found in study high $\beta$ and high performance plasmas:

Often dominant instabilities for $k_y\rho_i < 1$ at mid-radius in MAST plasmas:

Microtearing found to dominate ST Power Plant equilibrium:

Detailed numerical study of microtearing, ST reference, includes scan in R/a:
- D J Applegate et al, PPCF 49, 1113 (2007)

Nonlinear analytic theory of $\mu$-tearing may explain electron transport in NSTX

Edge plasmas in ASDEX-Upgrade have $\mu$-tearing modes
Linear Microstability Analysis at Mid-Radius in MAST

MAST equilibrium from ELMy H-Mode #6252

At mid-radius surface $\Psi_n = 0.4$, $\beta_e = 0.05$, $q \sim 1.35 T_i \sim T_e$, $a \sim 0.3m$, $R \sim 0.9m \Rightarrow R/a \sim 3$

Tearing Parity Modes at $\rho_i$ scale

Fastest growing modes in STs often found to have tearing parity:
- MAST [1], and NSTX [2]
- conceptual burning STs [3,4]


MAST tearing parity modes rotate in electron diamagnetic drift direction
Poincaré plot shows perturbed magnetic field at intersection of GS2 flux-tube with the outboard mid-plane.

Magnetic island on rational surface at $x=0$.

Microtearing mode is candidate to explain electron transport.

**Two Major Questions:**
What is the linear physics mechanism underlying these modes?
How much anomalous transport is generated at nonlinear saturation?
\[ \nabla T_e \] microtearing drive discovered in cylinder

- *Hazeltine Dobrott and Wang (1975):* kinetic, collisions key, any \( \nu_e/\omega \)

Further slab calculations confirm \( \nabla T_e \) drive at high \( \nu_e/\omega \)


\[ \Rightarrow \] collisional slab drive requires energy dependent \( \nu_e(E) \)

Kinetic calculations in toroidal geometry (large \( R/a \)), for low \( \nu_e/\omega \)

- *Catto and Rosenbluth (1981), Connor, Cowley and Hastie (1990)\n
\[ \Rightarrow \] low collisionality drive from trapped particle collisions on passing particles also requires energy dependent \( \nu_e(E) \)

MAST has small \( R/a \) and \( \nu_e/\omega \sim 0.5 \) so analytic theories should be poor.

Catto-Rosenbluth trapped particle drive mechanism, nevertheless, predicts growth with MAST parameters!

....Connor, Cowley, Hastie does not!

Analytic Theories of Microtearing Drives and Properties of the GS2 Modes

Two classes of linear drive in analytic theory literature:
- time dependent thermal force (high collisionality, $\nu_{ei} > \omega$)
- collisions close to the trapped-passing boundary ($\nu_{ei} < \omega$)

Both drives require
- finite $dT_e/dr$
- energy dependent collision frequency $\nu_{ei}(v)$

Some properties of the GS2 mode:
- **sensitive** to electron physics $v_e$, $\nabla T_e$ and $\nabla n_e$
- **sensitive** to $\beta$, $\nabla p$, $s$
- **insensitive** to ion parameters $v_i$, and $\nabla T_i$ and $\delta B_\parallel$
- current layer width $\sim O(\rho_i)$

GS2 Lorentz collision operator can capture boundary layers. Removed energy dependent collisions by setting $\nu_e(E) =$ constant

Modest affect on tearing $\gamma$

★ not consistent with analytic drive models!
Experiments Using s-α Model Equilibrium: Scan Aspect Ratio by varying $R_0$ at Fixed $r$  

DJ Applegate et al, PPCF 49, 1113 (2007)

Fit MAST mid-radius surface with s-α model for fixed $\beta$, $a/L_T$, $a/L_n$, $q$, $s$

Scan $r/R_0$ by varying $R_0$ and fixing $r$ and other parameters, varies drifts + $f_t$

- MAST instability in s- α too  
  $\Rightarrow$ shaping not essential
- $\gamma \downarrow$ as $r/R_0 \downarrow$
- still unstable at $r/R_0=0.118$
  $\Rightarrow$ μtearing may appear at conventional aspect ratio
Now scan in $R_0$ at fixed $r/R_0$ with other parameters constant

Mode survives at $R_0 = \infty$ i.e. zero drift

- mode has slab drive
Now scan $r/R_0$ to vary $f_t$ at fixed $R_0$ and other parameters.

High $f_t$:
- $\gamma \uparrow$ at low $\nu_e$
- $\gamma \downarrow$ at high $\nu_e$ (fewer passing $e$)

Low $f_t$:
- $\gamma$ more sensitive to energy dependent collision rate $\nu_e(E)$

Experiments Using s-α Model Equilibrium:
Scan in Trapped Particle Fraction, $f_t$

DJ Applegate et al, PPCF 49, 1113 (2007)
Microtearing mode is driven by $dT_e/dr$ as expected.

Mode is complicated and in awkward regime for analytic theory:
- unstable over broad range of collisionality $0.05 < \nu_{ei}/\omega < 1.2$
- current layer width $\sim O(\rho_i)$, so need ion FLR effects

Regimes where mode robust to energy independent collisions $\Rightarrow$ puzzle

Mode not only unstable in ST
- unstable in large aspect ratio $s-\alpha$ model equilibria

Gyrokinetic microtearing also at $r/R \sim 0.3$ ($\sim$ MAST mid-radius) in conventional aspect ratio: D Told et al, Phys. Plasmas 15, 102306 (2008)
Very High $\beta$: Microstability in STPP

Conceptual Culham ST Power Plant (STPP), 1GW electrical, $\beta=0.59$

GS2 used for microstability analysis of mid-radius flux-surface, $\Psi_n=0.35$.

Equilibrium features:

- striking variation in $|B|$ around the magnetic flux surface
- magnetic drift reversal owing to high pressure gradient
- diamagnetic $\omega_{se}$ strongly peaked on outboard midplane

see H R Wilson et al, Nuc Fus 44, 917 (2004)
STPP surface $\Psi_n=0.35$

- no electrostatic instabilities, $\alpha$ stabilisation giving drift reversal
- including EM gives tearing parity modes at ion and electron scales

$k_y\rho_i=0.4$

$k_y\rho_i=6$
STPP surface $\Psi_n = 0.35$

- no electrostatic instabilities (α stabilization from drift reversal)
- EM effects gives tearing parity modes at ion and electron scales, all propagating in electron drift direction
- Mixing length $\chi \sim 4 m^2 s^{-1}$ (no $\omega_{se}$)

*Microstability Results for Mid-radius Surface in STPP*
Nonlinear Microtearing Simulations with GS2

First nonlinear GK simulations with GS2 [1,2]:

- modified mid-radius MAST equilibrium for increased tractability

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MAST Equilibrium</th>
<th>Nonlinear Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>1.3463</td>
<td>1.3463</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.286</td>
<td>1.4</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.0495</td>
<td>0.12</td>
</tr>
<tr>
<td>( a/L_{ne} )</td>
<td>-0.1766</td>
<td>2.4</td>
</tr>
<tr>
<td>( a/L_{Te} )</td>
<td>2.0433</td>
<td>2.0433</td>
</tr>
<tr>
<td>( a/L_{Pe} )</td>
<td>1.8667</td>
<td>4.4433</td>
</tr>
<tr>
<td>( a/L_{ni} )</td>
<td>-0.1766</td>
<td>2.4</td>
</tr>
<tr>
<td>( a/L_{Ti} )</td>
<td>2.0433</td>
<td>2.0433</td>
</tr>
<tr>
<td>( a/L_{Pi} )</td>
<td>1.8667</td>
<td>4.4433</td>
</tr>
</tbody>
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Few \( k_y \) modes: \( n_{ky}=4 \), \( n_{kx}=47 \), \( n_\theta=32 \), \( n_E=8 \), \( n_\lambda \approx 20 \)

- “pseudo-saturation” with low transport, blows up later at high \( k_x \)
- small timesteps imposed by the CFL condition

Impact of Adding Dissipation at High $k$

R J Akers et al, IAEA FEC, Geneva, October 2008 EX/2-2

Use hyperviscosity for high $k$ dissipation, parameterised by $D$:
- no impact on linear physics
- improves convergence
- “saturation” insensitive to $D$

but what are we throwing away?

$D = 0, 0.5, 1, 2$

Diagram showing $\langle |A|^2 \rangle_v$ vs $t$.
Nonlinear Electron Heat Flux

Hyperviscosity smoothes high $k_x$
- spike events reappear at $nky=8$

$A_\parallel$ contribution dominates $q_e$
- low heat fluxes at “saturation”

![Graph 1](image1.png)
![Graph 2](image2.png)
Poincaré Plot and $\delta j_\parallel$ contours at $\theta=0$

before spike event, $t=532$
Poincaré Plot and $\delta j_{||}$ contours at $\theta=0$, $t=598$.  

D J Applegate

after spike event perturbed field wanders further, transport $\uparrow$
A_∥ Spectra for nky=8 Simulation

Spikes most evident at high k, but are controlled by D

highest k
middle k
lowest finite k_x, or k_y=0

steady growth in zonal modes
* $\Phi$ Spectra for nky=8 Simulation

Spikes most evident at high $k$, but suppressed by $D$.
Fidelity Issues

Convergence?
• saturation sensitive to $\text{Min}(k_y)$, and we need to go lower in $k_y$!
• what causes the high $k$ spikes? are we dissipating important physics?

Flux-Tube equilibrium?
• as reduce $\text{Min}(k_y \rho_i)$, we go to low $n$
• $s^{\text{SIM}} = 5 \ s^{\text{MAST}}$ so $L_x$ artificially small
• at lower $k_y$ and $s$, flux-tube gets fatter, to challenge local approximations

More work needed!
Impact of FLOW SHEAR on microtearing modes?

- $\gamma_E > \gamma_{\text{lin}}$ so will they be suppressed?
- slab drive may make suppression more difficult
- almost done
Conclusions

Microtearing modes from GS2 simulations of MAST are complicated!
- trapped and passing particles contribute drive with $dT_e/dr$
- insensitivity of $\gamma$ to energy dependent collision frequency is puzzling
- $\mu$tearing specific neither to ST geometry nor to GS2!
  - linear benchmark?
  - map out where $\mu$tearing important

Limited comparisons with analytic theory so far.
- do better in easier limits?

Preliminary nonlinear simulations for MAST mid-radius are interesting, but:
- more work needed to test convergence
- what is happening at high $k$?
- local flux-tube equilibrium is challenged if $n$ gets too small!
  - easier equilibria?
  - impact of FLOW SHEAR?