The simulation effort for the basic plasma experiment TORPEX

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Why TORPEX?
How its dynamics can be approached?
What are the turbulent regimes?
How do simulations and experiments compare?
What are we really learning from TORPEX simulations?
Plasma turbulence in the edge

- Scrape-off Layer
- Complex magnetic geometry
- Coupling with core region
- Difficult diagnostics access
- Sheath physics & atomic physics

Need for basic plasma physics experiments
The TORPEX experiment, paradigm of edge turbulence

crpp.epfl.ch/torpex/

Fundamental elements of SOL turbulence

Fasoli et al., PoP 2006
High resolution diagnostics with full coverage

Measurements of all relevant plasma and field parameters
Properties of TORPEX turbulence

\[ n_{\text{fluc}} \sim n_{\text{eq}} \]

\[ L_{\text{eq}} \sim L_{\text{fluc}} \]

\[ L \gg \rho_i \]

\[ T_i \ll T_e \]

\[ \beta \ll 1 \]

\[ \omega \ll \Omega_{ci} \]

\[ L_{\parallel} \gg L_{\perp} \]

Collisional
Fluid model

Collisional Plasma → Braginskii model → Electrostatic Drift-reduced Bragiskii equations

\[
\frac{\partial n}{\partial t} + [\phi, n] = D_n \nabla^2 n + \frac{2}{R} \left( n \frac{\partial T_e}{\partial y} + T_e \frac{\partial n}{\partial y} - n \frac{\partial \phi}{\partial y} \right) - \nabla_{||} (n V_{||e}) + S
\]

\( T_e, \Omega \) (vorticity) → similar equations

\( V_{||e}, V_{||i} \) → parallel momentum balance

\( \nabla^2 \phi = \Omega \)
Global simulations

Evolve both equilibrium and fluctuations
The character of TORPEX turbulence

Depends on $N$, the number of B turns

Low $N$: $k_\parallel = 0$  &  Ideal interchange dominated
High $N$: $k_\parallel \neq 0$  &  Resistive interchange dominated

Example: $N=2$

Another instability regime – driftwaves – to be discussed later
(likely inaccessible to the experiments)
Ideal interchange mode

\[ k_{||} = 0 \]

**n + T_e eqs.**

\[ \frac{\partial p_e}{\partial t} = \frac{c}{B} [\phi, p_e] \]

**Vorticity eq.**

\[ \frac{\partial \nabla_\bot^2 \phi}{\partial t} = \frac{2B}{cm_i Rn} \frac{\partial p_e}{\partial y} \]

\[ \gamma = \gamma I \quad \gamma I = c_s \sqrt{\frac{2}{L_p R}} \]
Anatomy of a $k_{\parallel} = 0$ perturbation

$N = 2$

$\Delta = L_v / N$

$\lambda_v$ : longest possible vertical wavelength of a perturbation

If $k_{\parallel} = 0$ then $\lambda_v = \Delta = \frac{L_v}{N}$
TORPEX shows \( k_{\parallel} = 0 \) turbulence at low \( N \)

\[
k_{\parallel} = 0 \quad (\lambda_v = \frac{L_v}{N})
\]

Ideal interchange regime

\[
\frac{L_v}{\lambda_v} = N
\]

Resistive interchange regime – return to this later

Poli et al., PoP 2006, 2008
For $N \sim 1-6$, ideal $(k_{||} = 0)$ interchange modes dominant
Ideal interchange ($N=2$)
Simulations and TORPEX experiments dominated by $k_{||} \neq 0$ toroidally symmetric turbulence.

Resistive interchange $k_{||} \neq 0$


Poli et al., PoP 2006, 2008
At high $N>7$, toroidal $\lambda_v \sim L_v$ symmetric turbulence
Resistive interchange ($N=16$)
Resistive interchange modes

\[ n + T_e \text{ eqs.} \quad \frac{\partial p_e}{\partial t} = \frac{c}{B} [\phi, p_e] \]

Vorticity eq. \[ \frac{\partial \nabla^2 \phi}{\partial t} = \frac{2B}{cm_i Rn} \frac{\partial p_e}{\partial y} + \frac{4\pi V_A^2}{c^2} \frac{\partial j_\parallel}{\partial z} \]

Ohm’s law \[ \eta_\parallel j_\parallel = -\frac{\partial \phi}{\partial z} \]

\[ \gamma^2 = \frac{\gamma_i^2}{\eta_\parallel c^2 k_y^2} \quad \text{or} \quad \eta_\parallel > \frac{4\pi V_A^2 k_\parallel^2}{\gamma_i c^2 k_y^2} \]

Two cases:
- \[ k_\parallel = 0 \quad \text{Ideal interchange mode} \]
- \[ k_\parallel \neq 0 \quad \text{Resistive interchange mode (requires} \eta_\parallel \neq 0) \]
Parameters of the resistive interchange mode

\[ k_{\parallel} = \frac{k \cdot B}{B} = \left( \frac{k_v B_v + k_\varphi B_\varphi}{B} \right) \]

Define: \( k_v = 2\pi l / L_v \), \( k_\varphi = -n / R \) \((= -2\pi n/(2\pi R))\) :

\[ k_{\parallel} = \frac{l}{RN} - \frac{n}{R} \quad \text{and} \quad \frac{k_{\parallel}}{k_v} = \frac{L_v}{2\pi R} \left( \frac{1}{N} - \frac{n}{l} \right) \]

Since the RI needs \( k_{\parallel}^2 / k_v^2 < \gamma_I \eta_{\parallel} c^2 / (4\pi V_A^2) \)

The most unstable mode is for \( N \gg 1 \) is \( n = 0, \ l = 1 \)

In TORPEX the RI mode has \( k_{\parallel} = 1/(RN) \), \( k_v = 2\pi / L_v \)
and requires \( N^2 > V_A^2 L_v^2 / (\gamma_I \eta_{\parallel} c^2 \pi R^2) \)
Why does TORPEX transition from ideal to resistive interchange for large $N$?

Resistive interchange requires high $N$:
\[ N^2 > V_A^2 L_v^2 / (\pi \gamma I \eta || c^2 R^2) \]

Ideal interchange requires low $N$:
\[ \lambda_v = \frac{L_v}{N} \quad \text{thus} \quad k_v = \frac{2\pi N}{L_v} \]

stable: $k_v \rho_s > 0.3 \gamma_I / c_s \sim 0.2 \sqrt{R / L_p}$

Transport less effective at high $k$

Threshold: $N \sim 10$ TORPEX
Linear stability analysis: TORPEX

Resistive Interchange:
\[ l = 1 \]
\[ k_\perp = \frac{2\pi}{L_v} \]
\[ k_\parallel = \frac{1}{RN} \]

Ideal Interchange:
\[ l = N \]
\[ k_\perp = \frac{2\pi N}{L_v} \]
\[ k_\parallel = 0 \]

Ricci & Rogers, PRL 2010
Resistive Driftwaves

Neglecting the curvature terms, soundwaves, and $m_e/m_i$:

$$\nu_e k_y^2 \rho_s^2 \gamma^2 + k^2 || c_s^2 (1 + 2.94k_y^2 \rho_s^2) \gamma + ik^2 || c_s^2 \omega_\ast = 0 \quad (\nu_e = e^2 n \eta || / m_i)$$

Fastest mode:

$$\gamma_{dw} \simeq 0.1 c_s / L_p \quad \text{for} \quad k_\perp \rho_s \simeq 0.5, \quad k_\parallel \simeq 0.2 \sqrt{\nu_e / (c_s L_p)}$$

Define DW regime as:

$$\gamma_{dw} > \gamma_I$$

DW need $\frac{L_p}{R} < 0.01$
Linear stability analysis of TORPEX

Driftwaves (inaccessible)

Interchange transport prevents access to DW regime in TORPEX for realistic parameters (as in tokamak SOL)
Analysis of other devices: Helimak

Vertical mode number $l$
of fastest mode

Driftwaves and RI mode

Non-MHD drift-interchange
$k_{\perp} \rho_s \sim 1 \ (l \sim 30)$
$\eta_{||} = m_e = 0$
$k_{||} \neq 0$
Adiabatic electrons

Ideal interchange

Li et al., in preparation
Non-MHD drift-interchange mode

Braginskii equations with $\eta_\parallel = m_e = T_i = V_{\parallel i} = 0$:

$$\frac{\partial \nabla^2_{\perp} \phi}{\partial t} = \frac{2B}{cm_i R n} \frac{\partial p_e}{\partial y} + \frac{4\pi V_A^2}{c^2} \frac{\partial j_\parallel}{\partial z} \quad \text{(where } p_e = nT_e)$$

$$\frac{\partial n}{\partial t} = \frac{c}{B} [\phi, n] + \frac{2c}{eRB} \left( \frac{\partial p_e}{\partial y} - ne \frac{\partial \phi}{\partial y} \right) + \frac{1}{e} \frac{\partial j_\parallel}{\partial z}$$

$$\frac{\partial T_e}{\partial t} = \frac{c}{B} [\phi, T_e] + \frac{4cT_e}{3eRB} \left( \frac{7}{2} \frac{\partial T_e}{\partial y} + \frac{T_e}{n} \frac{\partial n}{\partial y} - e \frac{\partial \phi}{\partial y} \right) + \frac{2T_e}{3ne} \frac{\partial j_\parallel}{\partial z}$$

$$- \frac{\partial \phi}{\partial z} + \frac{1}{ne} \frac{\partial p_e}{\partial z} = \eta_\parallel j_\parallel \simeq 0 \rightarrow \tilde{p}_e \simeq n_0 e \tilde{\phi} \quad \text{(adiabatic electrons)}$$

These give $(\omega_{*n} = k_\perp \rho_s c_s / L_n$, $\omega_{*p} = k_\perp \rho_s c_s / L_p$, $\omega_d = k_\perp \rho_s c_s / R$):

$$\left(1 + \frac{5}{3} k_\perp^2 \rho_s^2 \right) \gamma^2 - i \gamma \left( \omega_{*p} + \frac{10}{3} \omega_d k_\perp^2 \rho_s^2 \right) - \frac{10}{3} \omega_d (\omega_{*n} - 2\omega_d) = 0$$

$\gamma \sim \sqrt{\omega_d \omega_{*n}} \sim c_s / \sqrt{RL_n}$ for $k_\perp \rho_s \sim 1$
What are we really learning from TORPEX simulations?

• How to characterize turbulence in a relatively simple system

• Need of global simulations
  Flux tube simulations are not appropriate to describe a certain set of instabilities

• Need of non-local simulations
  E.g., required by the saturation mechanism

• How to perform comparison between experiments and simulations
Need of global simulations

To describe instabilities like the resistive interchange mode

3D Flux tube simulations:

Ricci & Rogers, PoP 2009
Need of non-local simulations

Radial transport: analytical estimate
no shear flow, $\partial^2 \phi \sim 0$

$$\Gamma_n = \left\langle \frac{\delta n \partial \delta \phi}{\partial z} \right\rangle_{z,t}$$

$$\frac{\partial \delta n}{\partial r} \sim \frac{\partial n_0}{\partial r}$$
$$\frac{\partial n}{\partial t} + [\phi, n] \approx 0$$
$$\frac{\partial \delta \phi}{\partial z} \sim \gamma \delta n \frac{L_n}{n_0}$$

$$\Gamma_{n,A} = \Gamma_{n,A}(n_0, T_0, L_p, B_z)$$
Comparison of analytical and simulation results

\[ \Gamma_n (\text{simulations}) = \Gamma_{n,A} (\text{analytical}) \]

Ricci & Rogers, PRL 2008, PoP 2009
How to make experiment/simulation comparison

• Comparison performed using observables across different hierarchy levels.
• A composite metric that takes into account the agreement of each observable is introduced.
• The “quality” of the comparison has to be defined.

Ricci et al., PoP 2009, and to be submitted
Concluding remarks

What are we learning from TORPEX modeling?

- By using global simulations and evolving both plasma equilibrium and fluctuations, it is possible to interpret the experimental results.
- The turbulence is subject to a number of driving mechanisms, as a competition between ideal interchange, drift waves, and resistive interchange.
- The properties of plasma turbulence reflect the different linear drives.
- Similar analysis can be carried out in other basic plasma devices.
- TORPEX is providing an ideal test-bed to study techniques and assumptions to be used for edge plasma turbulence simulations.
What’s next?

SOL simulations
Some of the recent experimental results

Identification of transport mechanism, quantification of turbulent structures

Universal properties of turbulence

Blob generation mechanism and dynamics

Fast ion dynamics

Müller et al., PoP 2007
Podestà et al., PRL 2008

Labit et al., PRL 2007

Furno et al., PRL 2008
Theiler et al., PRL 2009
Diallo et al., PRL 2008
Concluding remarks

What are we learning from TORPEX modeling?

• By using global simulations and evolving both plasma equilibrium and fluctuations, it is possible to interpret the experimental results.
• The turbulence is subject to a number of driving mechanisms, as a competition between ideal interchange, drift waves, and resistive interchange.
• The properties of plasma turbulence reflect the different linear drives.
• Similar analysis can be carried out in other basic plasma devices.
• TORPEX is providing an ideal test-bed for a close comparison between experiments and simulations, in plasma edge conditions.
What needs to be done...

Better boundary conditions

Physics of neutrals

Better source modeling
Turbulence phase space

\[ m = l - nN \]

\[ k_\parallel = 0 \quad m = 0 \]

\[ l = nN \]

\[ n = 1 \]

\[ \frac{c_s}{\sqrt{RL_p}} \]

\[ \gamma \sim \frac{c_s}{L_p} \]

\[ \frac{c_s}{L_p} > \frac{c_s}{\sqrt{RL_p}} \]

\[ L_p < L_{p,\text{crit}} \]

Ideal interchange

Drift waves

Resistive interchange

low \( m \) \quad n = 0

\[ l = 1 \]
Turbulence phase space

\[ k_{||} = \mathbf{k} \cdot \mathbf{b} = k_v \frac{B_v}{B} + k_\varphi \frac{B_\varphi}{B} \]

- Parallel mode number
- Vertical mode number
- Toroidal mode number: \( = 0, 1, 2, 3, \ldots \)
Non-MHD drift-interchange mode

Braginskii equations with $\eta_{\parallel} = m_e = T_i = V_{\parallel i} = 0$

\[
\frac{\partial \nabla_\perp^2 \phi}{\partial t} = \frac{2B}{cm_i R n} \frac{\partial p_e}{\partial y} + \frac{4\pi V_A^2}{c^2} \frac{\partial j_{\parallel}}{\partial z} \quad \text{(where } p_e = nT_e) \\
\frac{\partial n}{\partial t} = \frac{c}{B} [\phi, n] + \frac{2c}{eRB} \left( \frac{\partial p_e}{\partial y} - ne \frac{\partial \phi}{\partial y} \right) + \frac{1}{e} \frac{\partial j_{\parallel}}{\partial z} \\
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- \frac{\partial \phi}{\partial z} + \frac{1}{ne} \frac{\partial p_e}{\partial z} = \eta_{\parallel} j_{\parallel} \simeq 0 \rightarrow \tilde{p}_e \simeq n_0 e \tilde{\phi} \quad \text{(adiabatic electrons)}
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\]

\[\gamma \sim \sqrt{\omega_d \omega_{*n}} \sim c_s / \sqrt{RL_n} \quad \text{for} \quad k_{\perp} \rho_s \sim 1\]
Field-aligned computational grid

Non-orthogonal field-aligned grid in the whole domain
Turbulence phase space

Ideal interchange

Drift waves

Resistive interchange
Outlook: methodology for comparison

1st level

Probe ➔ Model

2nd level

Probe ➔ Model

3rd, … levels

Transport, etc…
Analysis of other devices:

LAPD

\[
\frac{\partial n}{\partial t} + \left[ \phi, n \right] = D_n \nabla^2 n + 2 \left( \frac{n}{R} \frac{\partial T_e}{\partial y} + T_e \frac{\partial n}{\partial y} - n \frac{\partial \phi}{\partial y} \right) - \nabla_{\parallel} (n V_{\parallel e}) + S
\]

- Convection
- Diffusion
- Magnetic curvature
- Parallel dynamics
- Source

Plasma gradients

Drift waves
Kelvin-Helmholtz
Sheath mode
Analysis of other devices:
LAPD
Outline

– The TORPEX experiment (why? what can it do?)
– The simulation approach
  • The model used? 2D and 3D
  • The turbulent regimes?
    Low (L) and High (H) confinement regimes
– How do experimental and simulation result compare?
Code Validation

Definition of the observables for the comparison

Experimental data

Simulation results

Metric

Code validated

<table>
<thead>
<tr>
<th>Agreement</th>
<th>Disagreement</th>
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<tbody>
<tr>
<td>Code validated</td>
<td>Code improvements</td>
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I_{\text{sat}}(\text{time})