Gyrokinetic Phase-space Turbulence and Energy Flows

Review, Progress and Open Problems

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2 August, 2010, Newton Institute

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Energy flow in multiscale gyrokinetics

$$\tau_E^{-1} \ll \omega \ll \Omega_c, \rho \ll L, \text{etc.}$$ ⟵ Scale separation

Multiscale gyrokinetics*: Sugama, et al., (1996)*

Gyrokinetics as a theory of *turbulence (and transport)*

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Multiscale gyrokinetics: *Sugama, et al., (1996)*

$$f = F_0 + \delta f$$ Turbulent fluctuations

Instability drives:
$$\boldsymbol{\nabla} T_0, \boldsymbol{\nabla} n_0, \text{etc.}$$

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injection →

"Energies" of the fluctuations:

$$W = \int d^3\mathbf{v} d^3\mathbf{r}\, T_0 \frac{\delta f^2}{F_0},$$

$$E \sim \int d^3\mathbf{r}\, \varphi^2$$

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The importance of W ("Free Energy" or Entropy): *Krommes and Hu, (1994)*
Actually, E is part the "exact" GK invariant: Dubin and Krommes (1983)

Gyrokinetics as a theory of *turbulence (and transport)*

Why be interested?

- Simulations Questions:
  - Resolution: When is enough enough?
  - Are there basic (nonlinear) problems on which all codes can agree?
- Fusion Physics:
  - What are the kinetic nonlinear mechanisms which damp turbulence?
  - How can small scales and large scales interact?
  - Under what conditions are interactions local or nonlocal?
  - When and how does a cascade go inverse (i.e. zonal flows)
- Reduced fluid models:
  - How can we create simple physical models which are robust across parameter ranges, and drastically improve computational efficiency?
  - Gyrofluid Models
  - Sub-grid-scale models, large eddy simulations

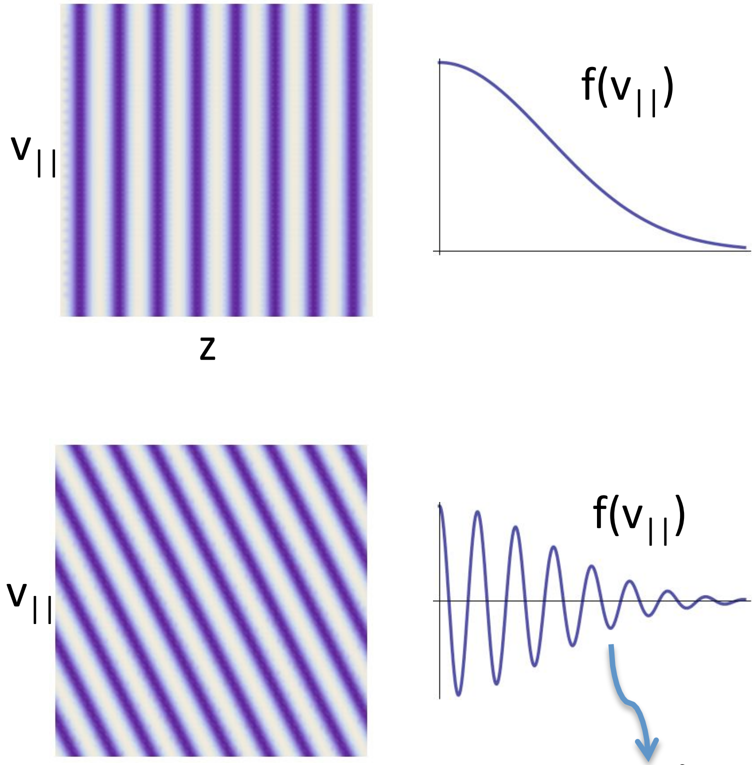
Outline

- Phase-mixing: Drive, Damping or Transfer?
- 2D Gyrokinetics: Navier-Stokes eqn for magnetized plasma turbulence
- Advertisement: Sub-Larmor cascade
  - Cascade through phase space
  - Fjortoft and the new flavors of the dual cascade
  - Linear + nonlinear phase mixing: a step forward?
- Free energy balance in a torus: preliminary results from Brussels

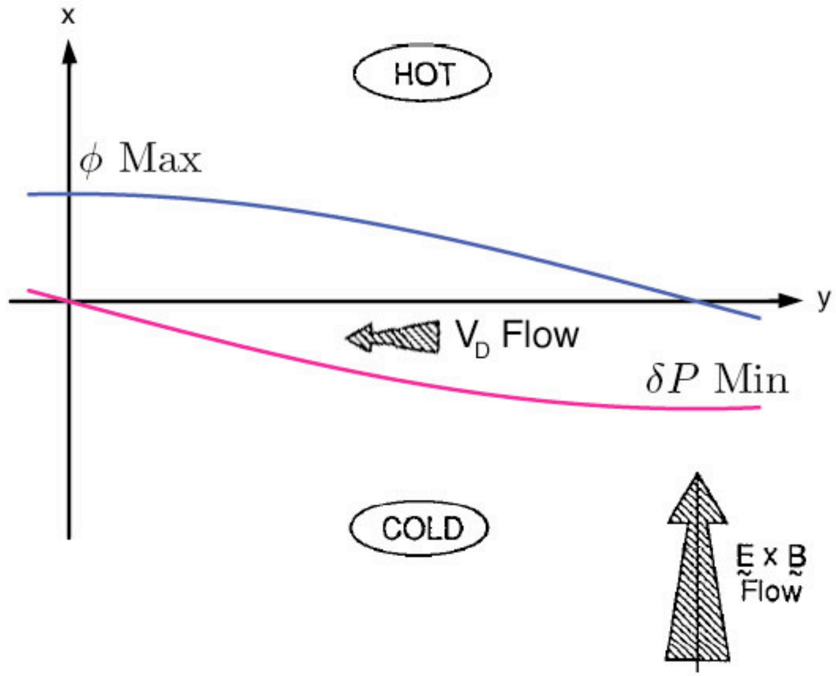
Linear phase mixing in gyrokinetics

Parallel phase mixing and linear Landau damping

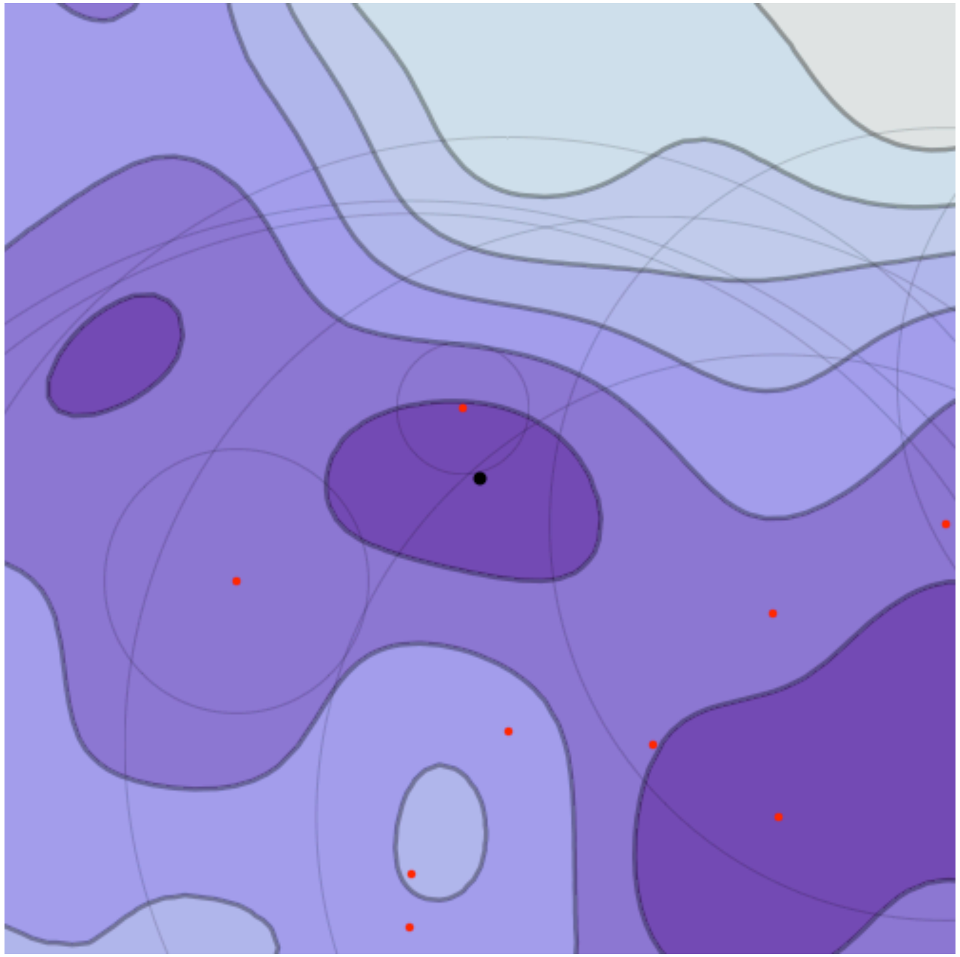
Perpendicular phase mixing and interchange instability

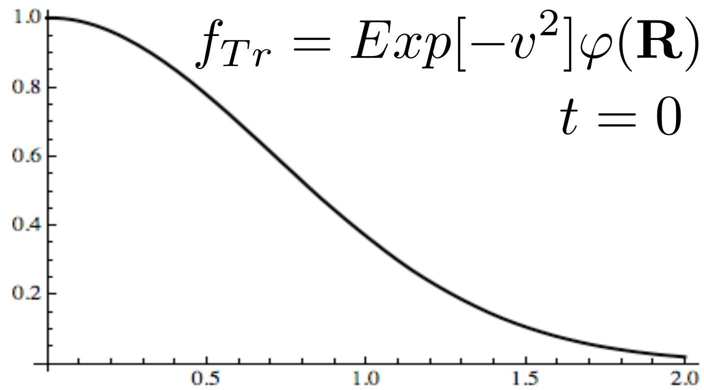


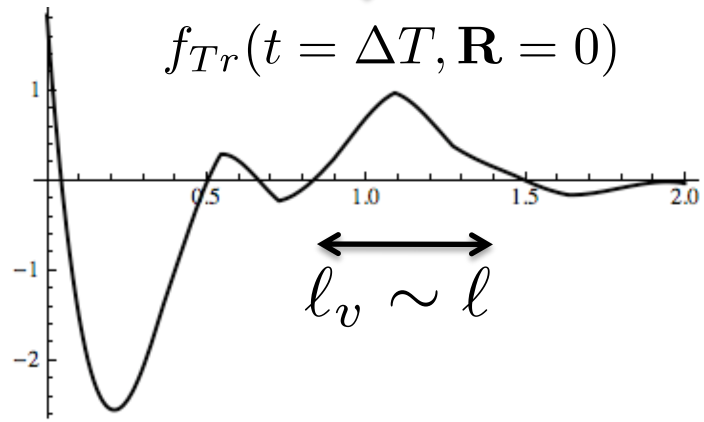
$$\varphi \sim \int dv_{\parallel} f$$



Physics of nonlinear phase mixing



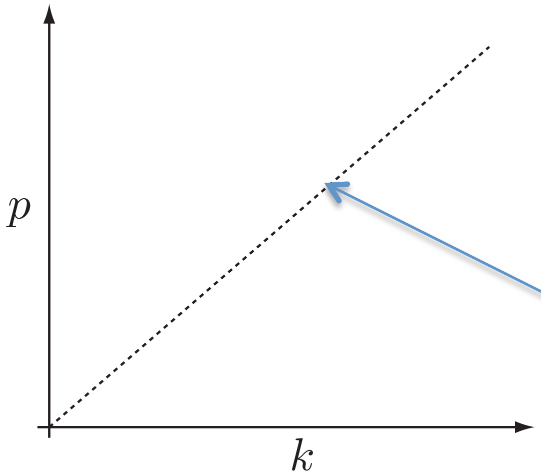




2D Gyrokinetics: A minimal model of magnetized plasma turbulence

$$\boxed{\frac{\partial g}{\partial t} + \{\langle\phi\rangle_{\mathbf{R}}\,,\ g\} = \langle C\rangle_{\mathbf{R}} \qquad\qquad \int d^3\mathbf{v}\,\langle g\rangle_{\mathbf{r}} = (1+\tau)\varphi - \Gamma_0\varphi}$$

"Gen. Free Energy": $W_g = 2\pi \int v dv \int \frac{d^2\mathbf{R}}{V}\frac{g^2}{2F_0}$

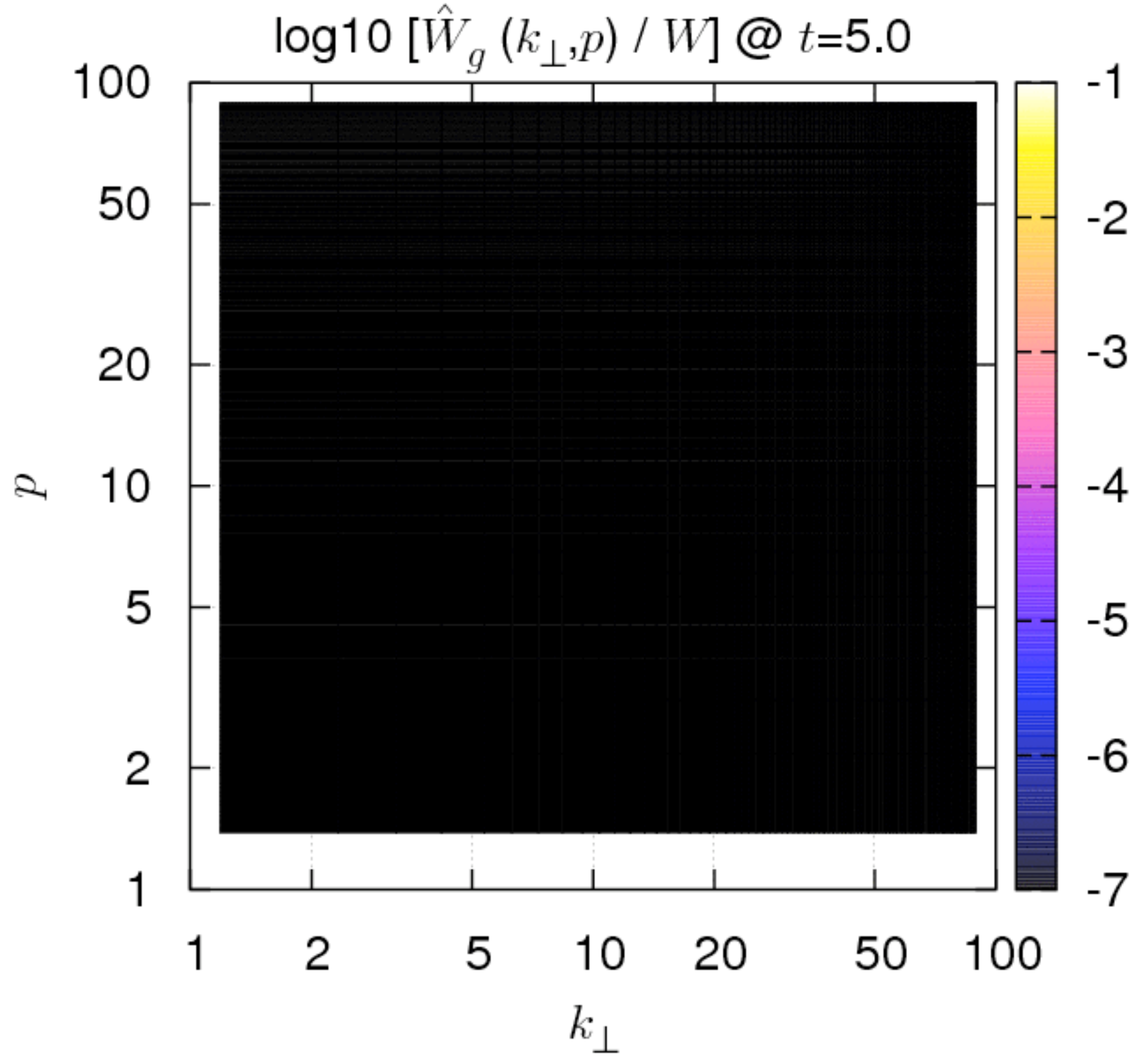


Hankel & Fourier Transform:

$$\hat{g}(\mathbf{k},p) \equiv \frac{1}{2\pi}\int_{\mathbb{R}} d^2\mathbf{R}\int_0^\infty v dv J_0(pv)\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{R}} g(\mathbf{R},v)$$

$$\hat{\varphi}(\mathbf{k}) = \beta(\mathbf{k})\hat{g}(\mathbf{k},k)$$

Sub-Larmor free energy cascade



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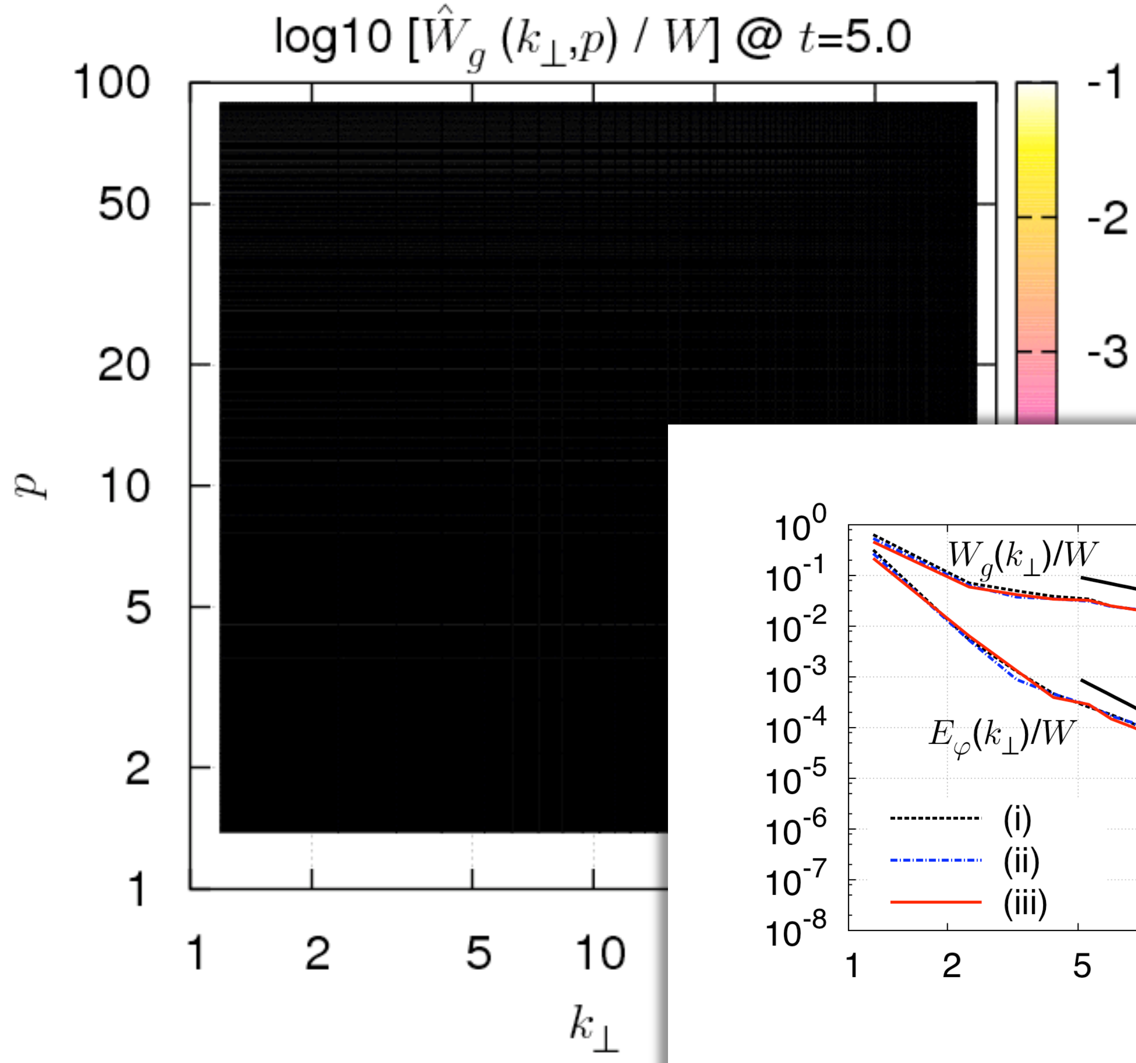
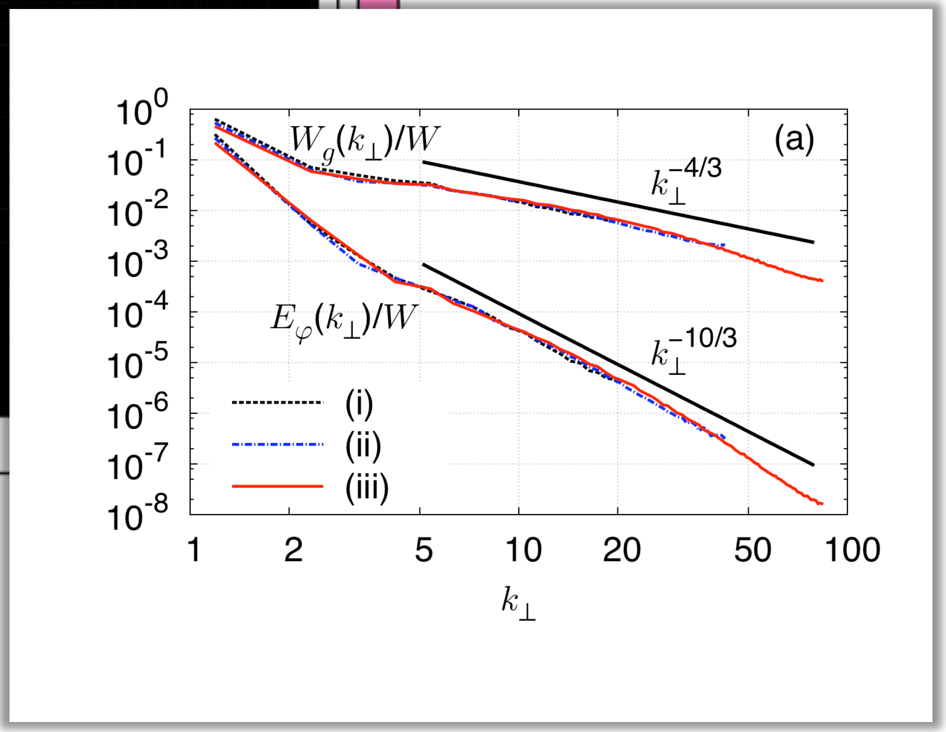
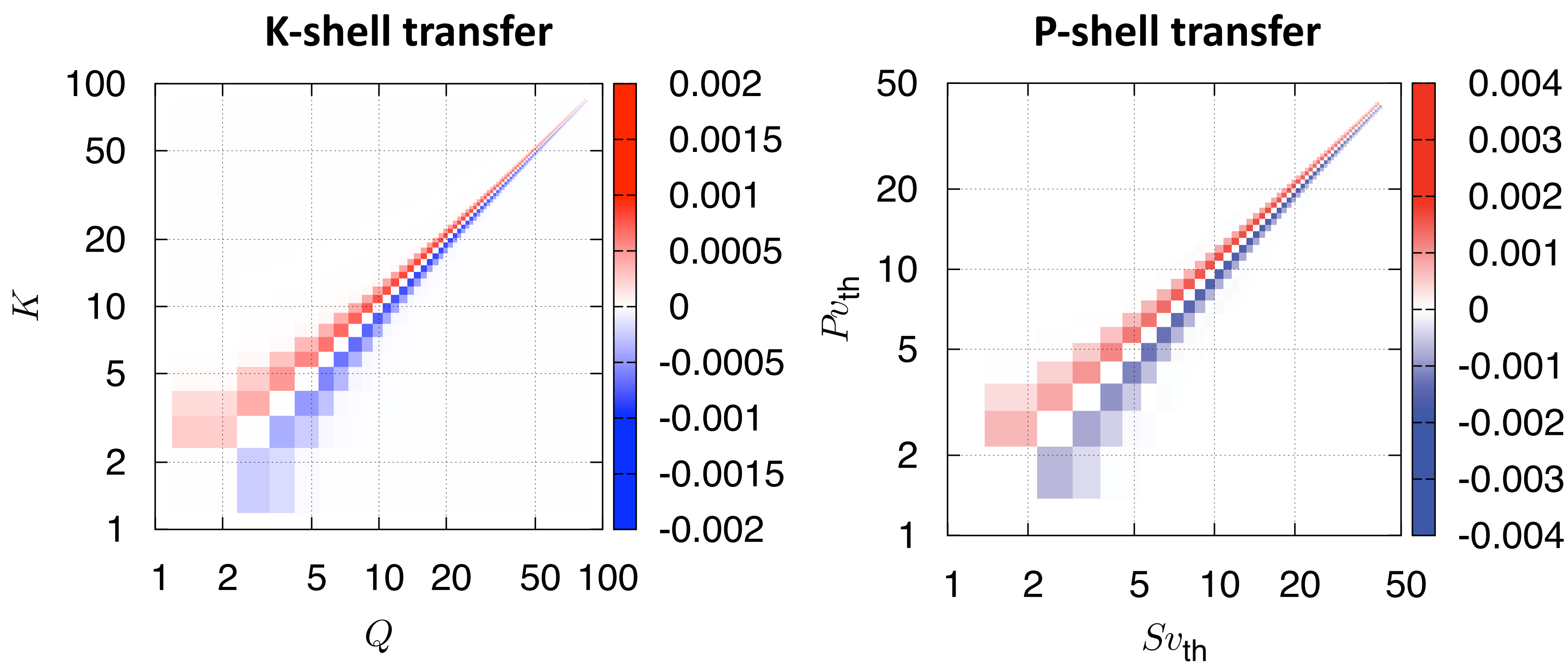




Nonlinear Free Energy Transfer: *Very local*



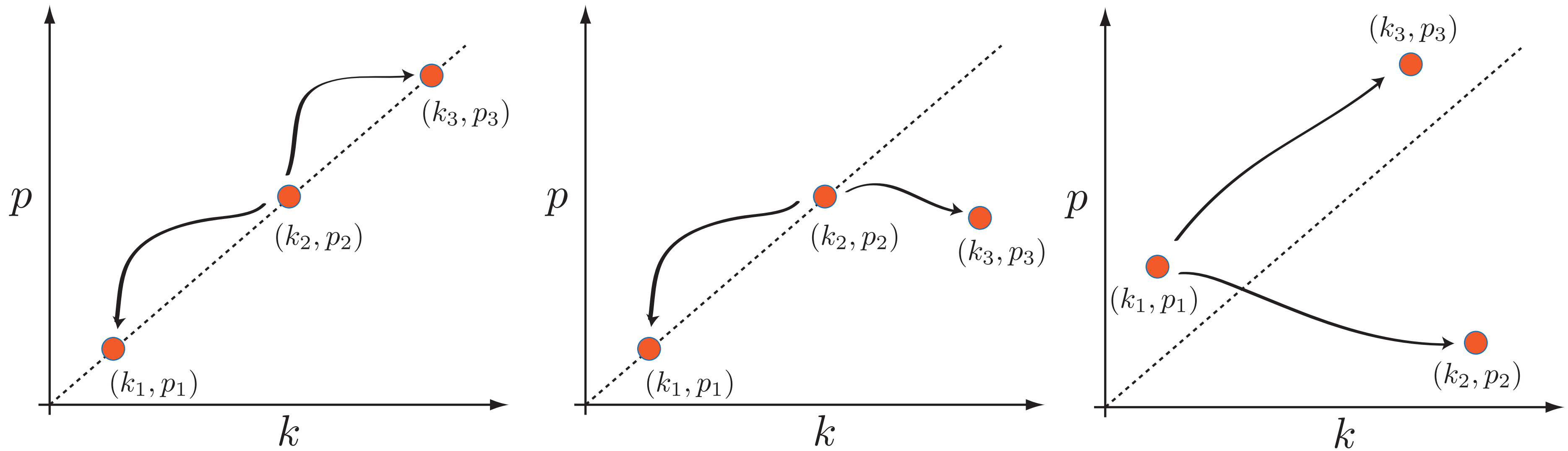
Tatsuno, et al., J. Plasma Fusion Res. SERIES (2010); arXiv:1003.3933

Dual cascade in phase-space

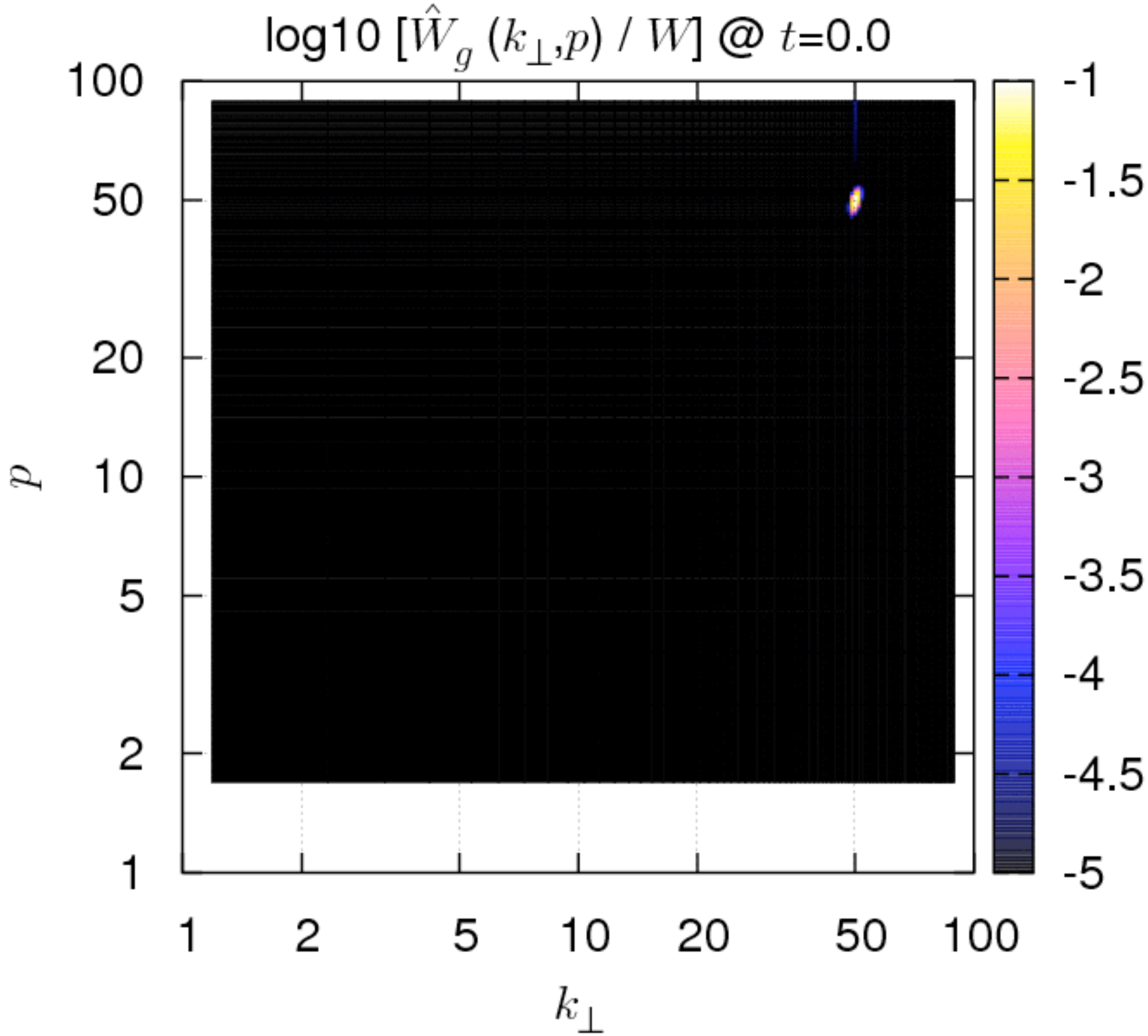
"Free Energy": $$W_g = 2\pi \int v dv \int \frac{d^2\mathbf{R}}{V} \frac{g^2}{2F_0}$$

"Electrostatic Energy": $$E = \frac{1}{2} \int \frac{d^2\mathbf{r}}{V} \left[(1+\tau)\varphi^2 - \varphi\Gamma_0\varphi\right]$$

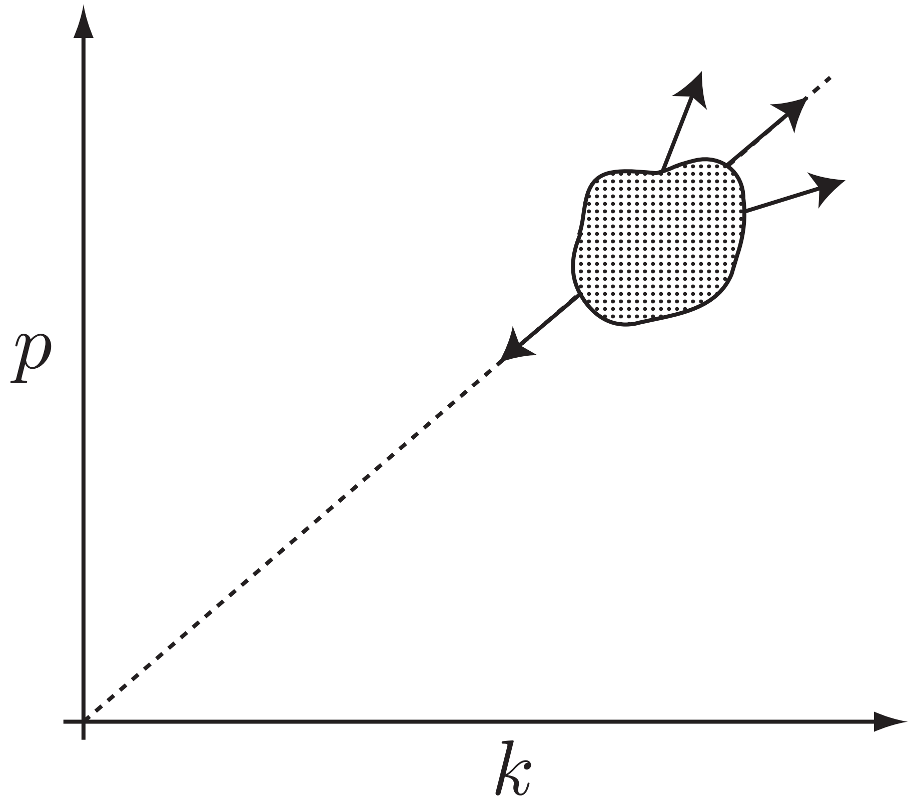
Constraint: $$E(k) = \frac{\beta(k)}{k} W_g(k,k) \sim W_g(k,k)/k$$



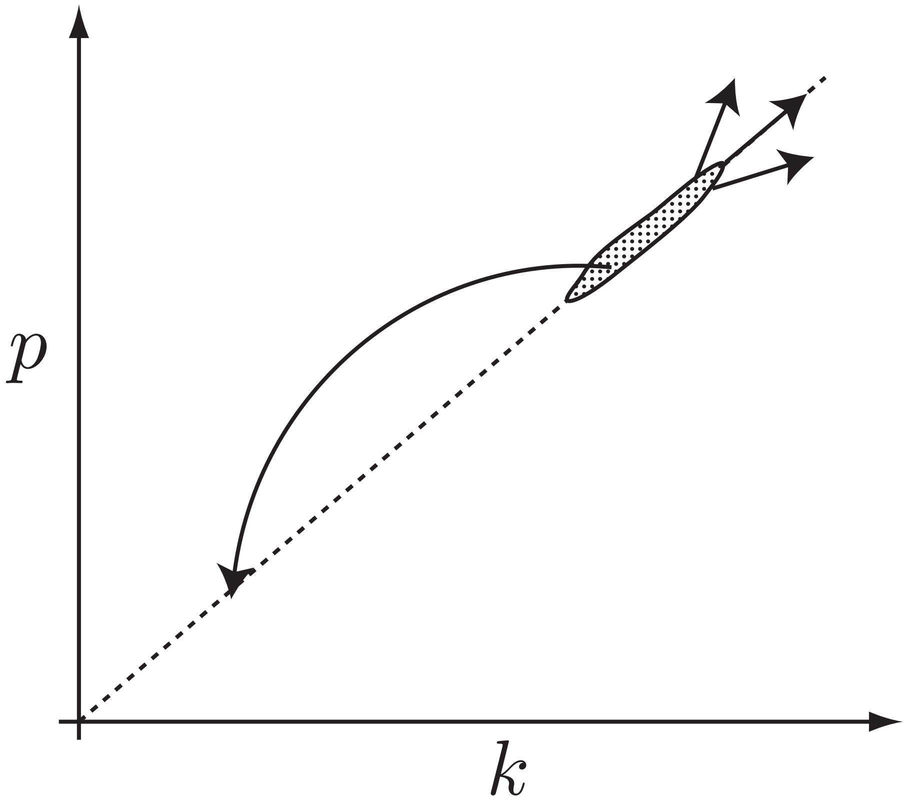
Inverse cascade of E



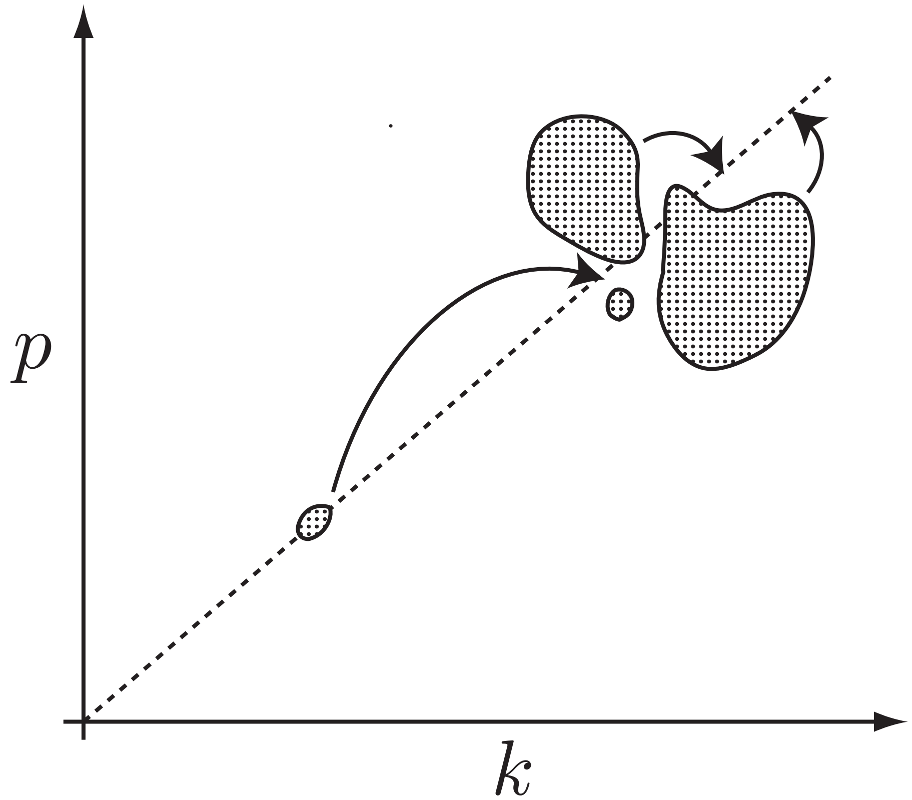
Flavors of dual cascade:
Local forward, Local inverse



Flavors of dual cascade:
Local forward, Nonlocal inverse



Flavors of dual cascade: Dual forward



"Sub-Larmor damping"

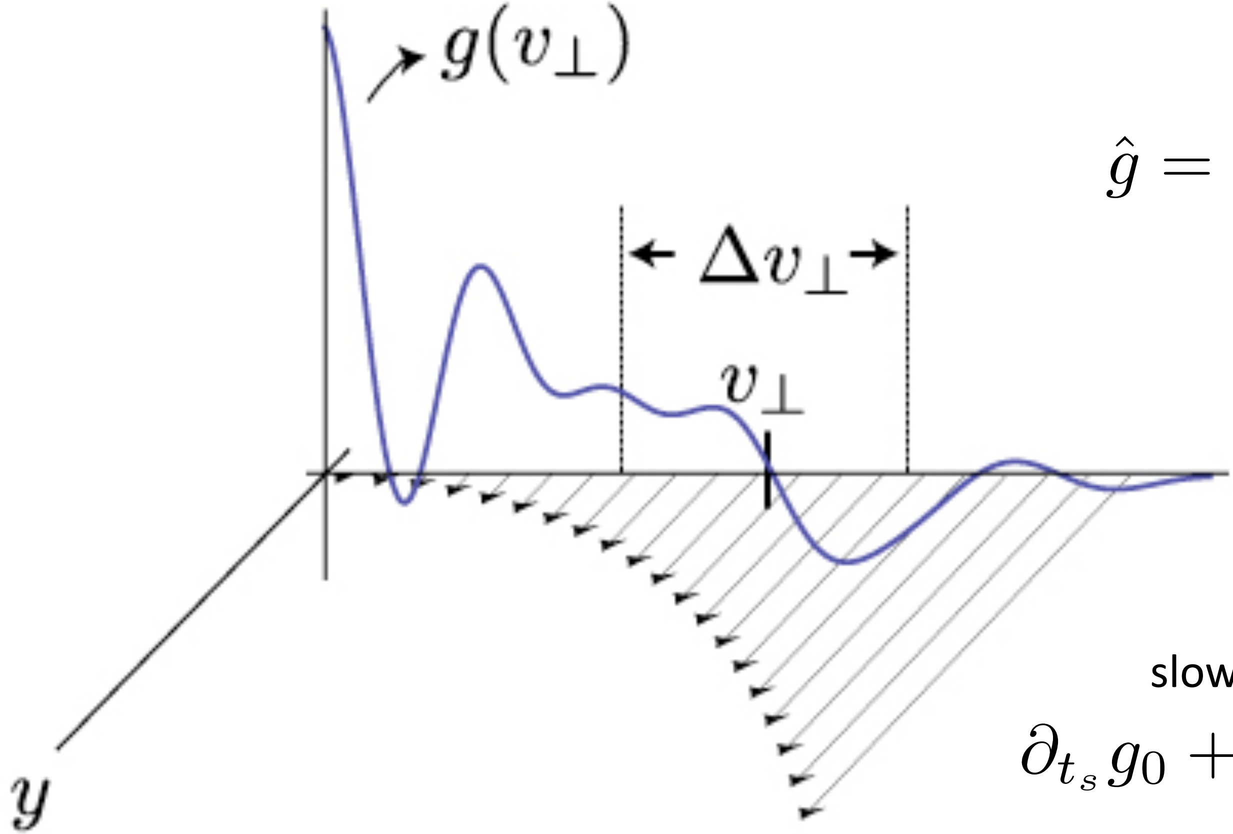
Nonlinear + linear phase mixing?

- Parallel phase mixing can be phenomenologically treated with a critical balance assumption – it is probably weak!
- Difficult to investigate numerically because it requires an additional dimension

- Idea: Include perpendicular phase mixing via magnetic drift:

$$\frac{\partial g}{\partial t} + \{\langle\phi\rangle_{\mathbf{R}}\,,\ g\} + \mathbf{v}_D \cdot \boldsymbol{\nabla} g = \langle C\rangle_{\mathbf{R}} \qquad \mathbf{v}_D = \frac{\hat{\mathbf{y}}}{L_B} v_\perp^2/2$$

Coexistence of “*fast-weak*” and “*slow-strong*” phase mixing



fast “wave timescale”:

$$\hat{g} = \hat{g}_0(t_s)exp(-i\omega_D t_f)$$

$$\omega_D = \frac{k_y v_\perp^2}{2L_B}$$

slow “turbulent timescale”:

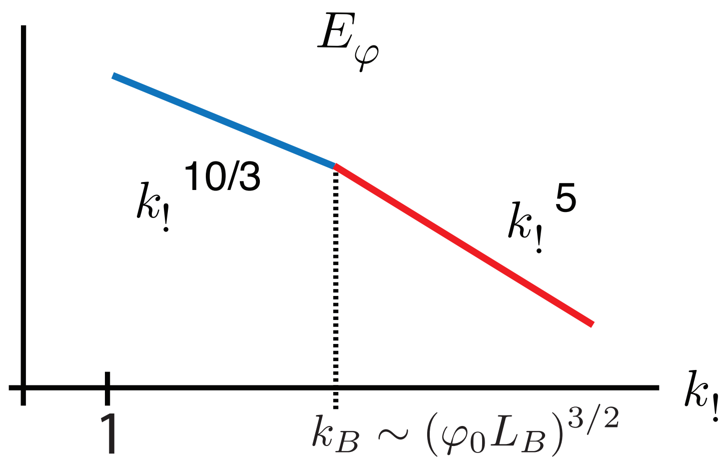
$$\partial_{t_s} g_0 + \{\langle \varphi_{eff} \rangle_{\mathbf{R}} \,,\ g_0\} = 0$$

$$\hat{\varphi}_{eff}(v_\perp) \sim \int_{v_\perp - \Delta v/2}^{v_\perp + \Delta v/2} v'_\perp dv'_\perp J_0(k_\perp v'_\perp)\hat{g}_0$$

Phenomenological Cascade Scaling

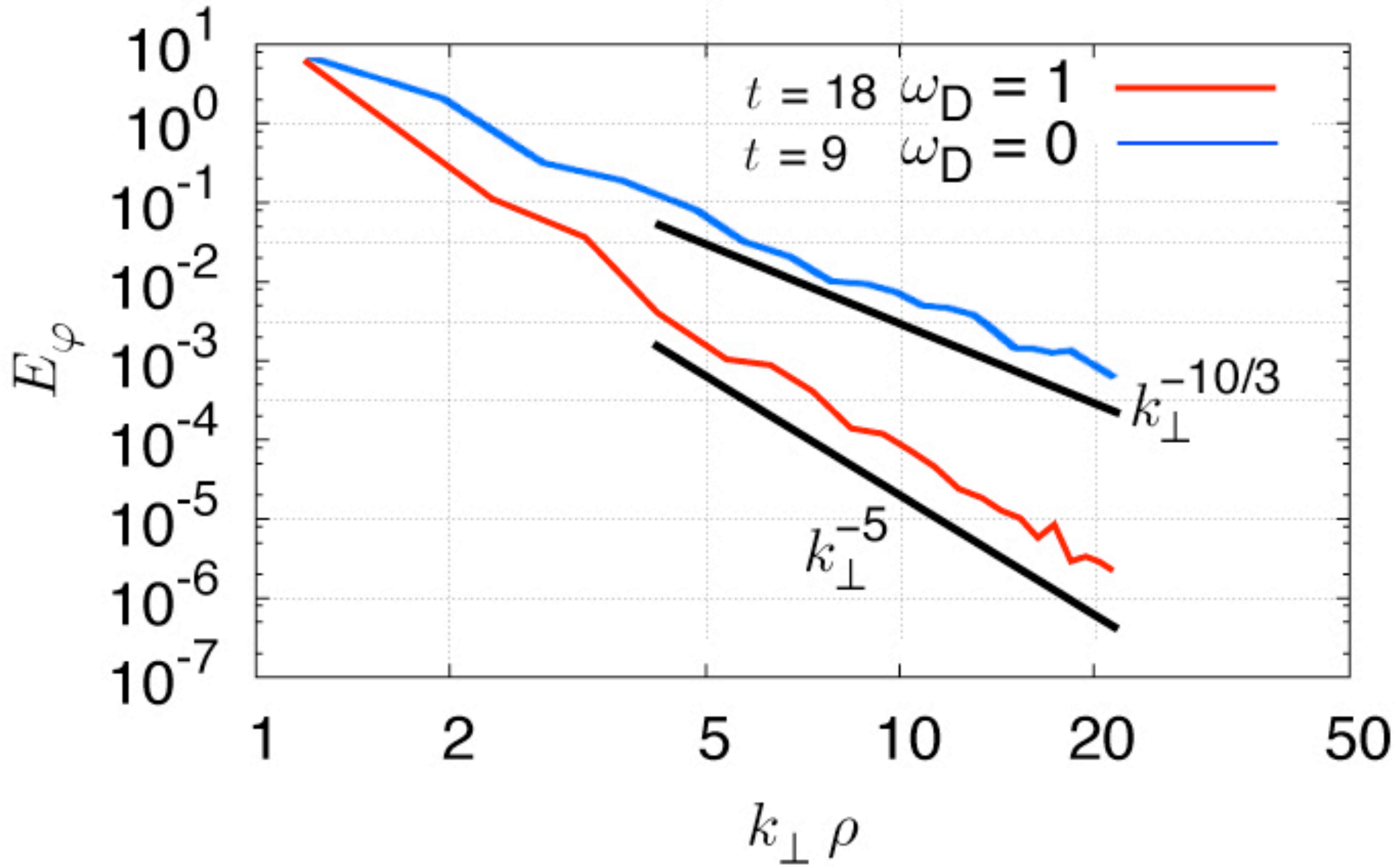
- Effective potential depends on NL turnover time
- The usual inertial range assumption of constant nonlinear flux gives:

$$E \sim k_\perp^{-5}$$



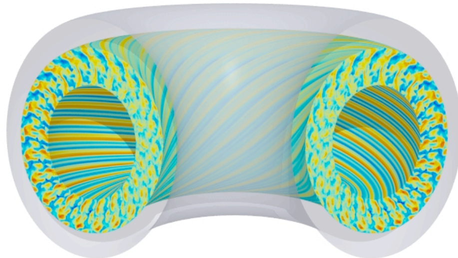
Preliminary Results

(Tomo Tatsuno)

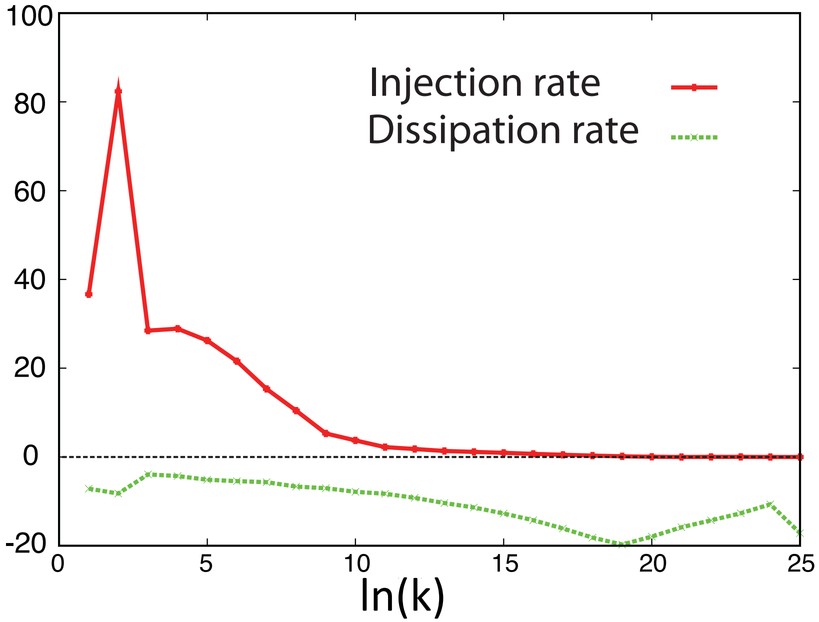


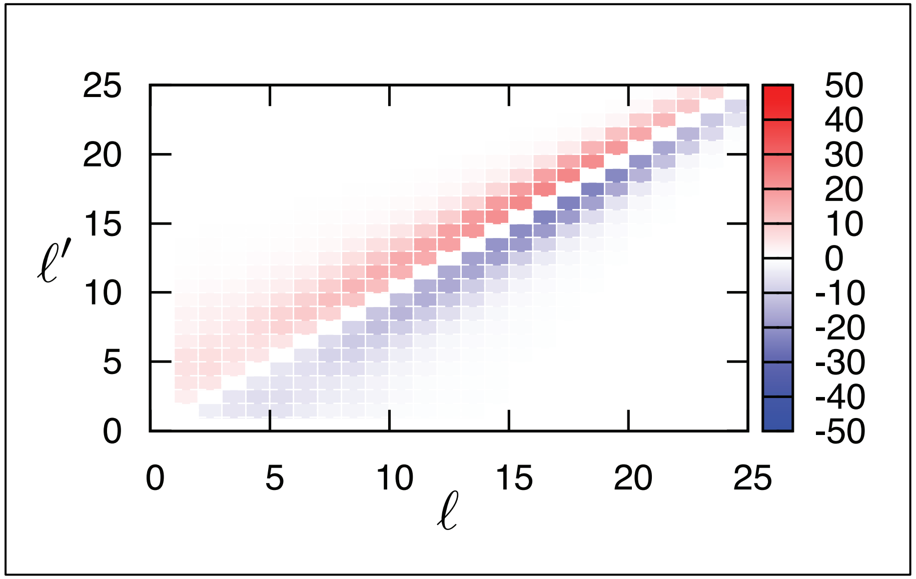
Free Energy Cascade in ITG turbulence

A. Bañon Navarro, et. al



- ITG Cyclone base case
- 32×8 grid points in $v_{||}$-μ
- $256 \times 64 \times 64$ spatial pts
- Hyperviscosity in z and $v_{||}$

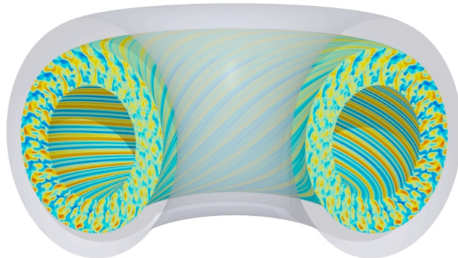




- Injection occurs at $k\rho_i < 1$
- "Self similar" from $k\rho_i \sim 1$ to $k\rho_i \sim 4$
- Local and directed forward (larger k)
- Stationary: $dW/dt = 0$

Electrostatic Energy in ITG turbulence

A. Bañon Navarro, et. al



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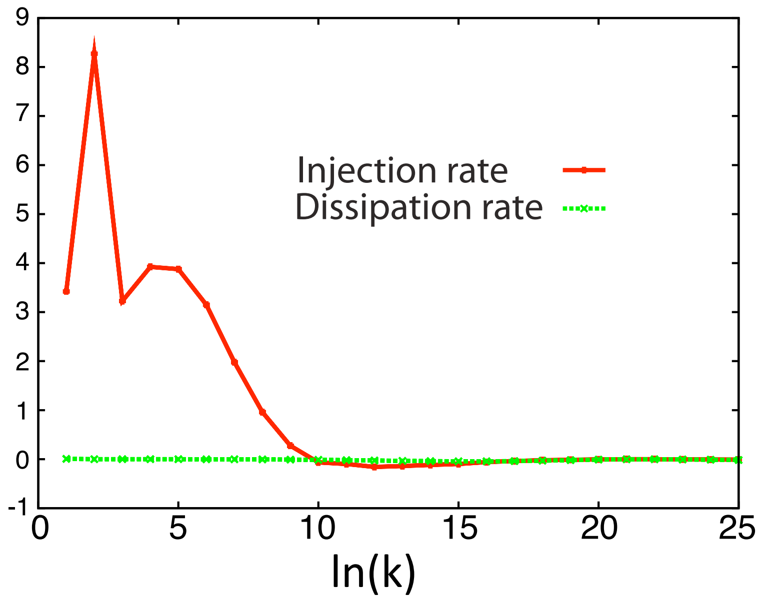

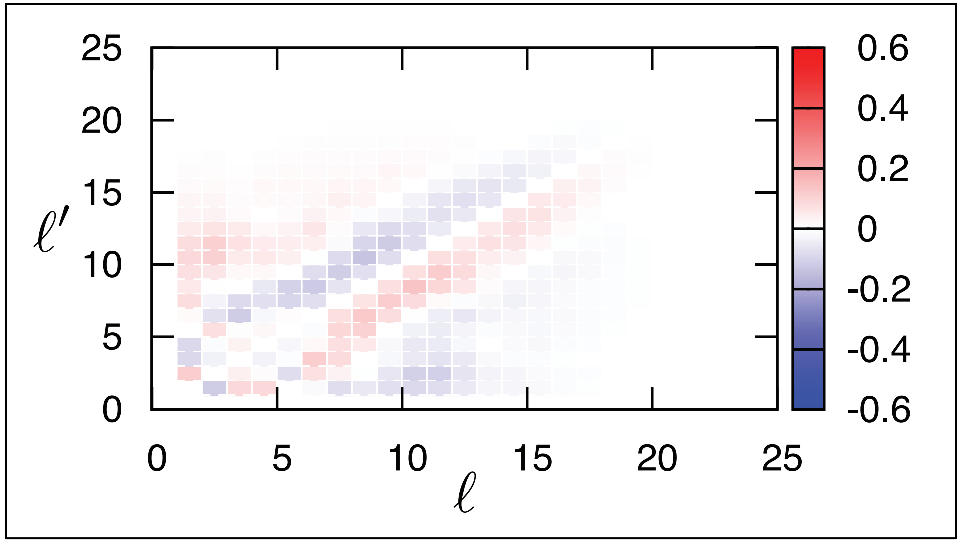

- Electrostatic Energy cascade is weak
- Not yet reached steady state
- Zero dissipation (looks "2D"!)

- Mixture of local, nonlocal, inverse and forward
- Energy re-circulating in k-space

Highlights

- Gyrokinetic turbulence “fits” the fluid-turbulence mold
  - In the absence of injection, damping, or dissipation, nonlinear interactions conserve two positive definite quantities
  - A local cascade transfers free energy from large scales to small scales to be dissipated
  - The nonlinear cascade of these quantities is constrained in the sense of 2D fluids and the dual cascade can take on different forms depending on the injection and damping.

Open Questions

- Mixture of linear and nonlinear phase mixing
  - What sets $k_{||}$?
  - Mixture of weak and strong processes?
- Physical regimes for different inverse cascade behavior
  - What determines injection rates of W/E
- Energy flows at the injection scale (zonal flows, fluid equations, phase-mixing channels)

Papers

[A. A. Schekochihin, S. C. Cowley, W. Dorland, G. W. Hammett, G. G. Howes, G. G. Plunk, E. Quataert, and T. Tatsuno,.Plasma Phys. Control. Fusion, 50:124024, (2008) arXiv:]

[T. Tatsuno, W. Dorland, A. A. Schekochihin, G. G. Plunk, M. Barnes, S. C. Cowley, and G. G. Howes, Phys. Rev. Lett. 103, 015003 (2009) arXiv:0811.2538]

[T. Tatsuno, M. Barnes, S. C. Cowley, W. Dorland, G. G. Howes, R. Numata, G. Plunk and A. A. Schekochihin, accpeted J. Plasma Fusion Res. (2010), arxiv:1003.3933]

[G. G. Plunk, S. C. Cowley, A. A. Schekochihin, T. Tatsuno, *accepted* JFM (2010) arXiv:0904.0243]

[G. G. Plunk, T. Tatsuno, submitted POP (2010) arXiv:1007.4787]

Gyrokinetic Equation

$$\frac{\partial h}{\partial t} + v_\parallel \frac{\partial h}{\partial z} + \mathbf{v}_E \cdot \nabla h + \mathbf{v}_\mathbf{D} \cdot \nabla h = \frac{q}{T} \frac{\partial \left\langle \varphi \right\rangle_\mathbf{R}}{\partial t} F_0 - \mathbf{v}_E \cdot \nabla F_0 + \left\langle C[h] \right\rangle_\mathbf{R}$$