Gyrokinetic Simulations of Microtearing Instability

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R. NUMATA[A,†], W. Dorland[A], N. F. Loureiro[B], B. N. Rogers[C], A. A. Schekochihin[D],
T. Tatsuno[A]

† `rnumata@umd.edu`

A) Center for Multiscale Plasma Dynamics, Univ. Maryland
B) Associação EURATOM/IST, Instituto de Plasmas e Fusão Nuclear
C) Dept. Phys. & Astron., Dartmouth College
D) Rudolf Peierls Centre for Theor. Phys., Univ. Oxford

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Microtearing Instability

- Tearing modes can be driven by electron temperature gradient with current density gradient [Hazeltine *et al.* (1975)].
- High k mode (Microtearing [MT]): unstable even normal tearing stable regime ($\Delta' < 0$).
- Collisional mode ($\nu_{\mathrm{e}}/\omega_{\mathrm{e}}^{*} > 1$) [Drake and Lee (1977)].
- Nonlinear theory by Drake *et al.* (1980) predicts saturation level of magnetic fluctuation $\tilde{B}/B_0 \sim \rho_{\mathrm{e}}/L_{T_{\mathrm{e}}}$.
- May account for anomalous electron transport in fusion experiments, but may not in (conventional) tokamaks because of weak collisionality.
- Trapped particle effect: Catto and Rosenbluth (1981), Conner (1990).
- Recent revival: MT may be relevant in Spherical Tokamaks (ST); Redi *et al.* (2003) – NSTX, Applegate (2004) – MAST.
- This study: simplified geom. using `AstroGK`, no curvature, no trapped particles.

Theory (Drake and Lee)

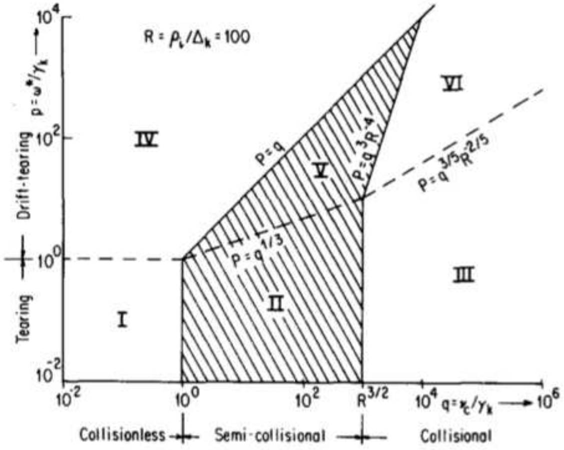
Drift-kinetic electron + Lorentz collision operator

TABLE II. Drift-tearing instability.

	Frequency/growth rate	Width layer	Restrictions
IV Collisionless	$\omega_1{}^* = \omega_n^* + \omega_T^*/2$ $\gamma_k = \frac{k_y v_e (\Delta' a)}{2\pi^{1/2} k_0^2 a l_s}$	$\Delta \simeq \left(1 + \frac{\omega^{*2}}{\gamma_k^2}\right)^{1/2} \Delta_k$	$\nu_c \ll \omega^*$ $\omega^* \gg \gamma_k$
V Semi-collisional	$\omega_2^* = \omega_n^* + 5\omega_T^*/4$ $\gamma = \boxed{\left[\frac{3\pi^{1/4}}{2^{1/2} 4 \Gamma(11/4)}\right] \frac{\gamma_k \nu_c^{1/2}}{\omega_2^{*1/2}}}$ $+ \boxed{\left[\frac{2\Gamma(17/4)}{\pi^{1/2}\Gamma(11/4)}\right] \frac{\omega_2^*}{\nu_c} \omega_T^*}$	$\Delta \simeq \Delta_k \frac{(\omega^* \nu_c)^{1/2}}{\gamma_k}$	$\omega^{*1/3} \gamma_k^{2/3} (\rho_i/\Delta_k)^{4/3}$ $\omega^* \gg (\nu_c \gamma_k^2)^{1/3}$
VI collisional	$\omega_3^* = \omega_n^* + 5\omega_T^*/2$ $\gamma = \left(\frac{315}{32}\right) \frac{\omega_3^*}{\nu_c} \omega_T^*$ $+ \left[\frac{\gamma_c^5}{\omega_3^*(\omega_3^* + \omega_i^*)}\right]^{1/3} \mathrm{Im}(i^{1/3})$	$\Delta \simeq (\omega^*/\gamma_c)^{2/3} \Delta_c$ $\simeq (\omega^*/\gamma_k)^{2/3} \rho_i^{2/3} \Delta_k^{1/3}$	$\nu_c \gg \omega^{*1/3} \gamma_k^{2/3} (\rho_i/\Delta$ $\omega^* \gg \gamma_k^{2/5} \nu_c^{3/5} (\Delta_k/\rho_i)^{2/5}$

Stabilizing normal tearing $\propto \Delta'$

Destabilizing $\propto \omega_T^*$



Classification of Drift-Tearing Mode

Simulation Setup

- `AstroGK` [Numata *et al.*: arXiv: 1004:0279 (2010)] is used. Full collision op.
- Electron and one ion species, both treated kinetically.
- Purely 2D: $k_z = 0$.
- Equilibrium (on top of f_{0s} and B_{z0}): Electron parallel flows $\delta f_{e0} \propto v_{\|} f_{0e}$ to generate B_{y0}:
    - cosh type (normalized to $|B_{y0}| \leq 1$; $B'_{y0}(x=0) \sim 2.6$)

      $$B_{y0} = B_{y00} \cosh^{-3}(x/a) \sinh(x/a)$$

      Critical $k_{y,\mathrm{crit}} a = \sqrt{5}$ for normal tearing
    - sin type

      $$B_{y0} = \sin(x/a)$$

      Critical $k_{y,\mathrm{crit}} a = 1$ for normal tearing
- $\phi_0 = \delta B_{\|0} = 0$, and $\delta f_{i0} = 0$.

Simulation Parameters

- L_s: Magnetic shear length $\boldsymbol{B} = B_{z0}\hat{z} + x/L_s\hat{y}$ (used in the theory).
- a: Magnetic shear length at $x = 0$ in our simulation (see previous slide).
  $a = \epsilon L_s$, where $\epsilon = \rho_0/a_0$ is the gyrokinetic ordering parameter, ρ_0 and a_0 are the reference length scale in the perpendicular and parallel direction, respectively.
- k_y: wavenumber of perturbation.
  If $k_y > k_{y,\mathrm{crit}}$, the normal tearing mode is stable $\Delta' < 0$.
- $L_{n_{0\mathrm{e}}} \equiv -\partial(\ln n_{0\mathrm{e}})/\partial x$, and $L_{T_{0\mathrm{e}}} \equiv -\partial(\ln T_{0\mathrm{e}})/\partial x$ (or $\eta_\mathrm{e} \equiv L_{T_{0\mathrm{e}}}/L_{n_{0\mathrm{e}}}$):
  Density and temperature gradients define the drift frequency $\omega^*_{n,T} = k_y U^\mathrm{d}_{n,T}$ with the drift velocity $U^\mathrm{d}_n = -T_0/(qB_0L_{n_0})$, $U^\mathrm{d}_T = -T_0/(qB_0L_{T_0})$.
  (Note: rigorously speaking the gradients must satisfy $\sum_s n_{0s}T_{0s}(L^{-1}_{n_{0s}} + L^{-1}_{T_{0s}}) = 0$)
- $m_\mathrm{e}/m_\mathrm{i} = 0.01$, $T_{0\mathrm{i}}/T_{0\mathrm{e}} = 1$, $n_{0\mathrm{i}}/n_{0\mathrm{e}} = 0$, $q_\mathrm{i}/q_\mathrm{e} = -1$.
- $\beta_\mathrm{tot} = 2\beta_\mathrm{e} = 0.01$.

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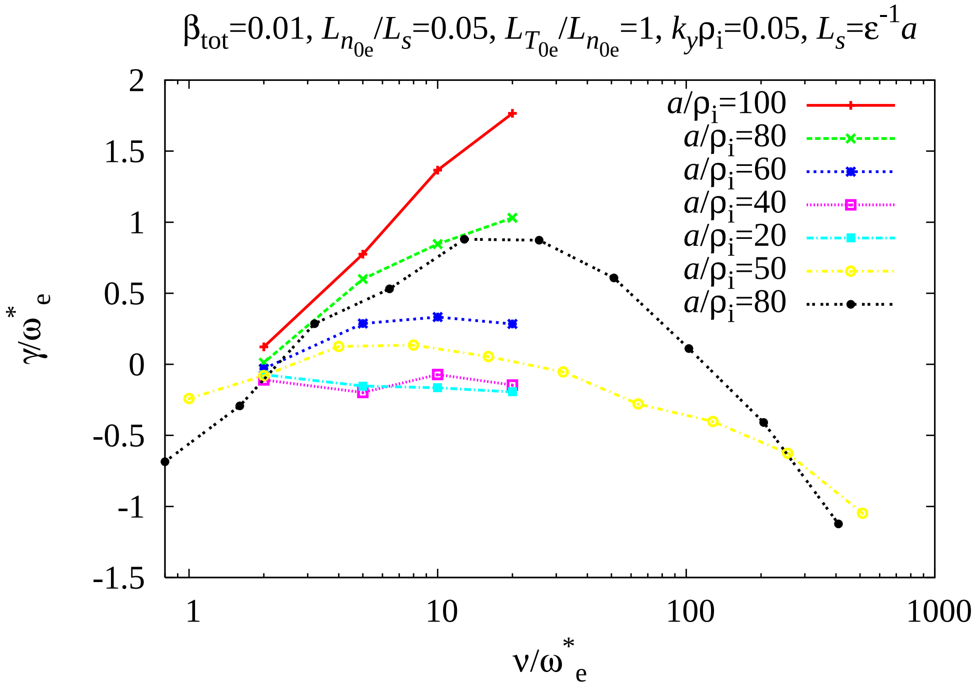
Comparison with Theory



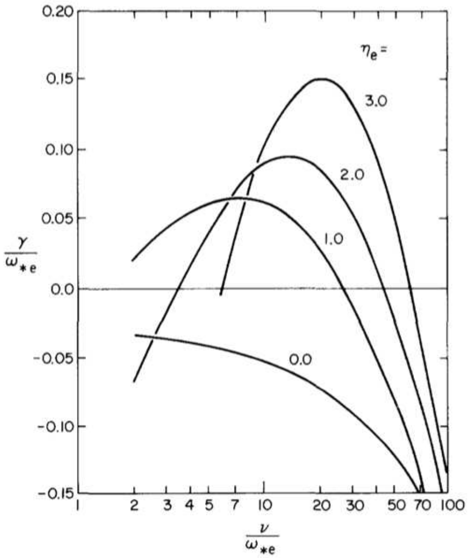


FIG. 1. Growth rate as a function of collisionality for different values of temperature gradient. Other relevant dimensionless parameters are $k_y\rho_i=0.05$, $\beta=0.01$, $L_n/L_s=0.05$, $m_i/m_e=1836$, and $T_e/T_i=1$. All further calculations will have the same values of m_i/m_e and T_e/T_i.

Gladd *et al.* (1980)

- Unstable if $\nu_{\rm e}/\omega^*_{\rm e} \gtrsim 1$ (where $\omega^*_{\rm e} = \omega^*_{{\rm e},n}$).
- Larger $a/\rho_{\rm i}$ is destabilizing. Note that parameter $a/\rho_{\rm i}$ is missing in the theory.

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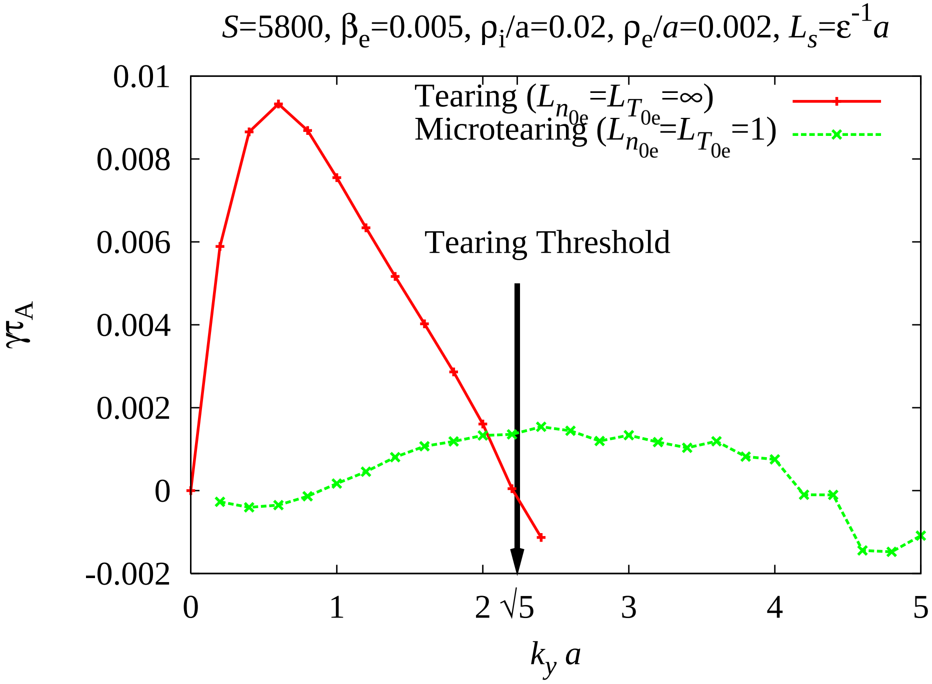
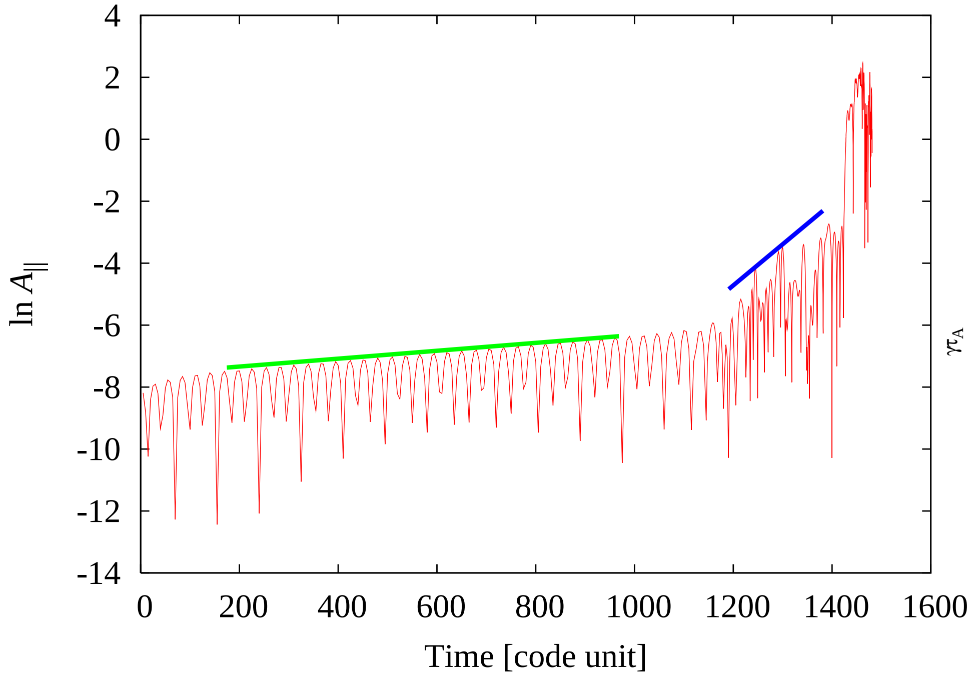
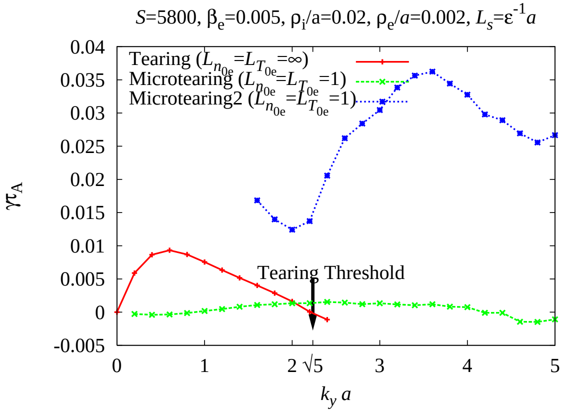
Comparison with Normal Tearing



- Slower than normal tearing mode (stabilization by drift).
- Broad spectrum (destabilization by drift).

Peculiar Behavior





- Growth rate jumps up at later stage for cosh type equilibrium.
- Probably because of existence of current sheet with longer shear length at $x = 0$.

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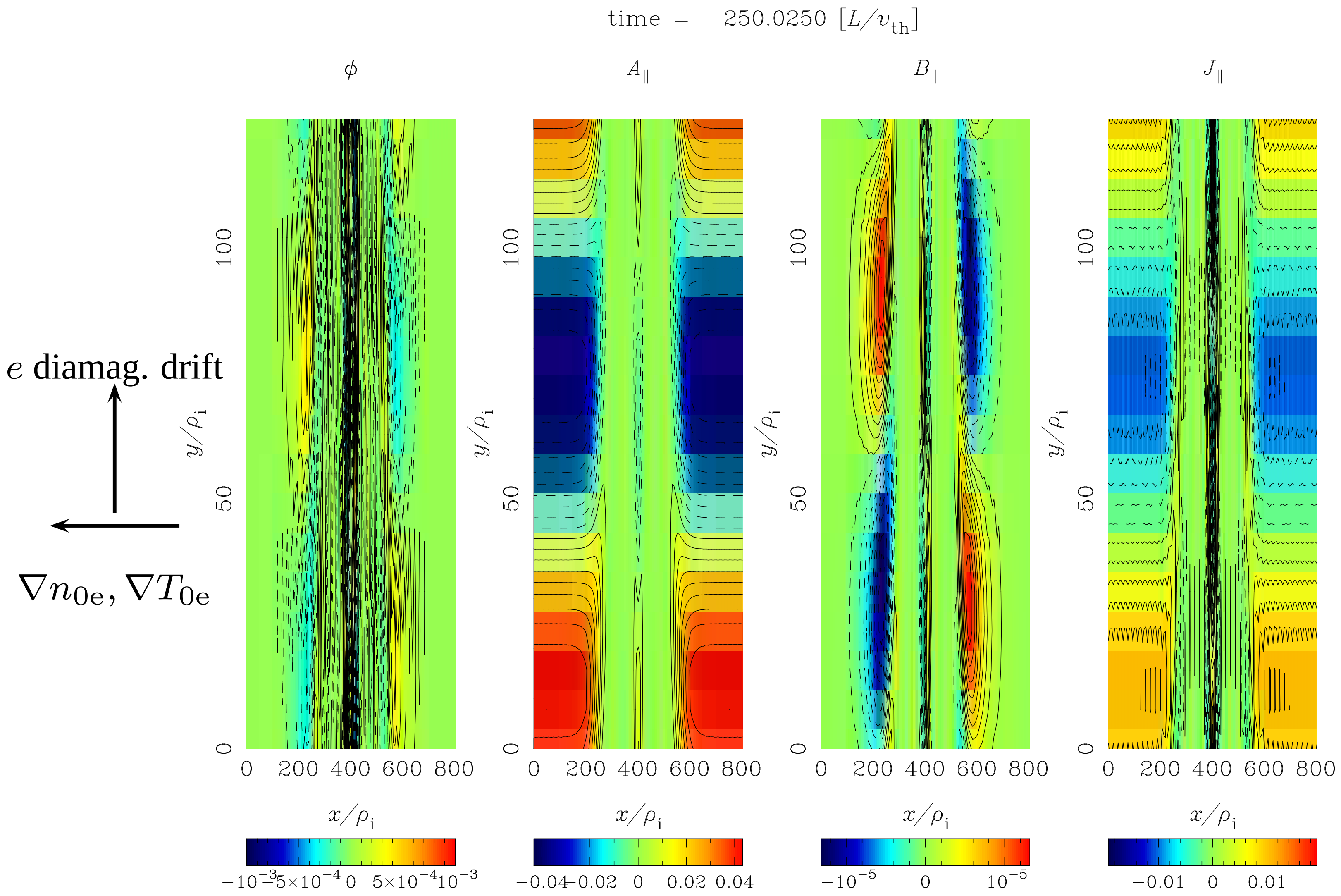
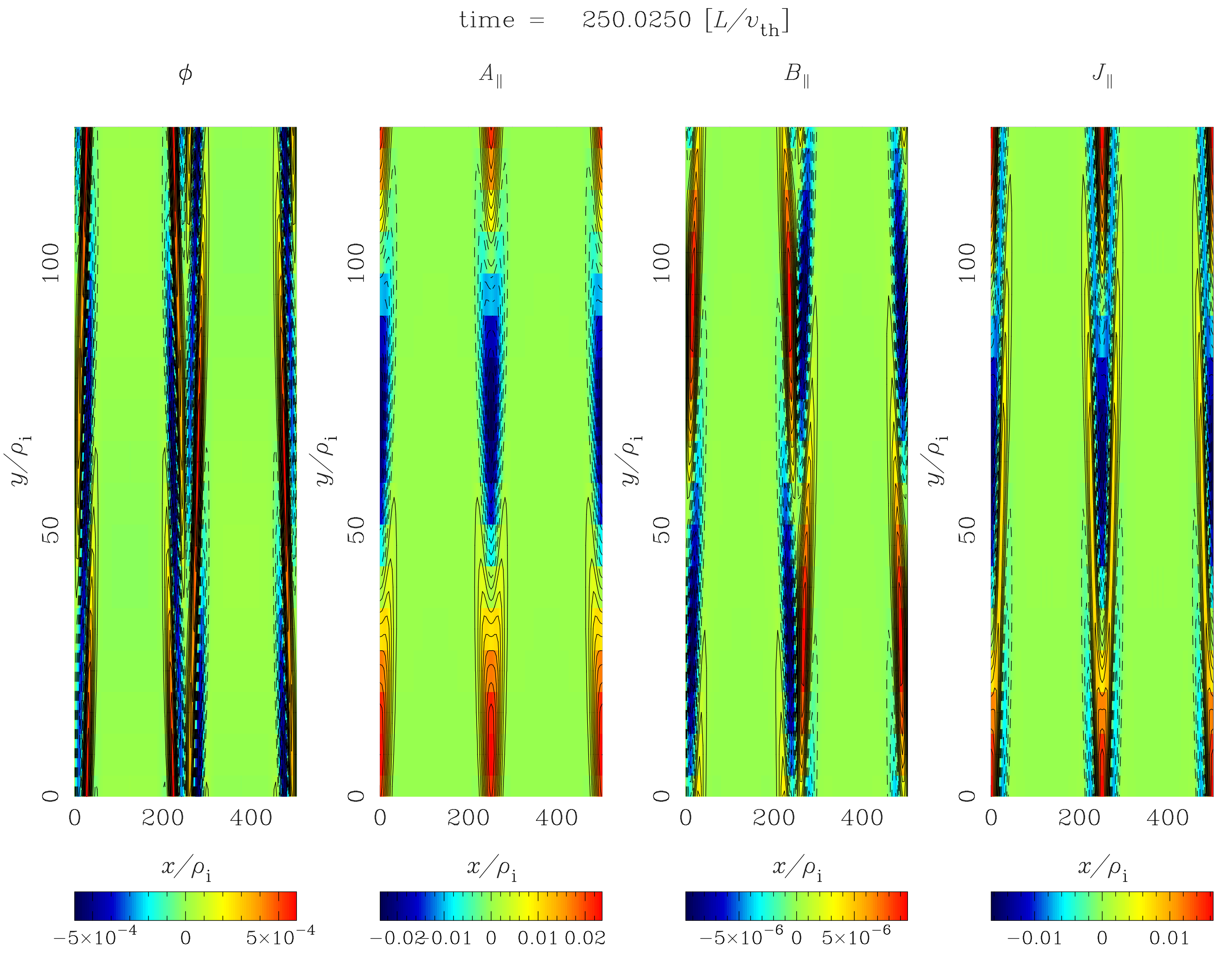
Eigenfunctions (cosh ***type eq.)***



e diamag. drift

$\nabla n_{0\rm e}, \nabla T_{0\rm e}$

Eigenfunctions (sin *type eq.)*



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Summary

- We have successfully demonstrated linear microtearing instability using `AstroGK`.
- One missing parameter a/ρ_i prevents direct comparison of simulation results with the theory. But, we have confirmed qualitative behavior is consistent with the theory.
- Large a/ρ_i enhances microtearing growth.
- Peculiar two-phase growth for $\cosh$ type equilibrium profile. This may be because two current sheet with different shear length.
- More detailed analysis are needed. Convergence test must also be performed.

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