Gyrokinetic Simulations of Nonlinear Tearing Instability

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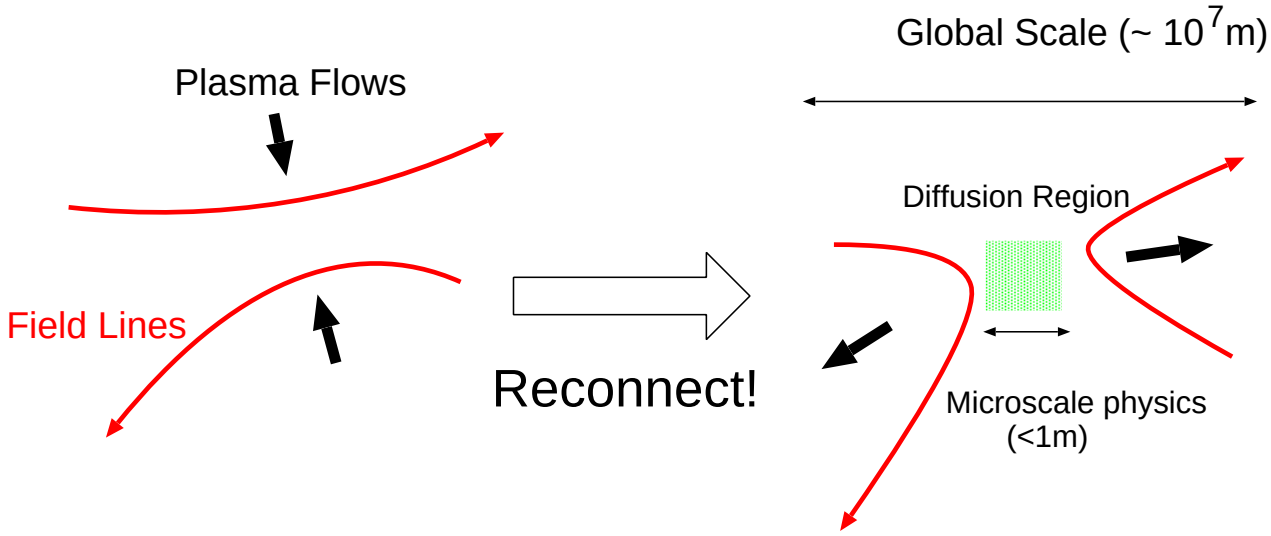
Ubiquity of Magnetic Reconnection in Plasmas



- Magnetic reconnection converts magnetic field energy into high speed plasma flows, heating of plasmas, and energetic particles.
- Sawtooth oscillations, island growth due to tearing instabilities, disruptions in fusion experimental devices (Degradation of confinement).
- Magnetospheric substorms, solar, stellar flares in astrophysical situation.

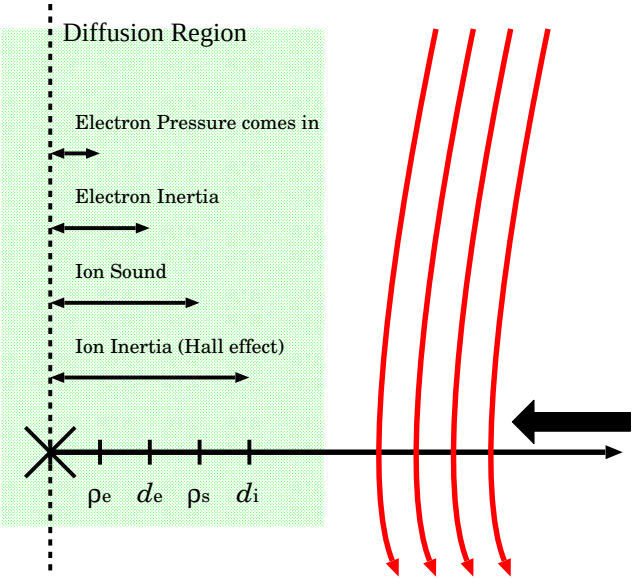
Magnetic Reconnection is Multiscale Problem

Magnetic reconnection is a classic example where multiple physics (scales) are involved

- Physics to break flux-freezing is necessary for field lines to change its topology: primarily by collisions (resistivity).
- In collisionless environments, time scale of reconnection based on resistivity is far too slow to explain realistic explosive phenomena.
- Resistive spatial scale falls below kinetic scales (MHD theory is not valid).
- Global structure drastically changes depending on microscopic processes.

Questions

- What determines time scale of reconnection?
- What provides mechanism for field lines to reconnect?
- What results? – Heating of electrons and/or ions? Electron jet flows? Islands? Transport in fusion devices?



Tearing Instability – Linear Theory

- Tearing instability is a resistive instability of current sheet configuration
- Spontaneous onset of reconnection process
- Standard boundary layer or singular perturbation problem

Since the pioneering work by Furth, Killeen, and Rosenbluth [PoF (1963)] based on MHD model, in which they derived the growth rate scaling $\gamma\tau_{\rm A} \propto S^{-3/5}$, the linear tearing instability theory has been extended by including various non-MHD (kinetic) effects.

- Hall effect (Ion inertia) [a] $\sim d_{\rm i}$ – Alfvén wave dispersion (Whistler)
- Pressure effect [b] (Ion Sound) $\sim \rho_s$ – Alfvén wave couples to sound wave
- Finite Larmor Radius (FLR) [c] $\sim \rho_{\rm i(e)}$ – Alfvén wave dispersion (KAW)
- Tensorial pressure $\sim \rho_{\rm e(i)}$ – break flux-freezing
- Electron inertia [d] $\sim d_{\rm e}$ – break flux-freezing

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[a] e.g. Terasawa, GRL (1983), Fitzpatrick & Porcelli, PoP (2004)

[b] Coppi *et al.*, NF (1966).

[c] Porcelli, PRL (1991)

[d] Schep *et al.*, PoP (1994)

Nonlinear Evolution of Tearing Mode

- Slow (algebraic) growth of island [Rutherford, PoF (1973)]: $\mathrm{d}W/\mathrm{d}t = \eta\Delta'$ where $W = 4\sqrt{\psi/\psi_0''}$ is the island width.
- X-point collapse and Sweet-Parker reconnection for $W > W_c \simeq 25/\Delta'$ [Waelbroeck, PRL (1993)]

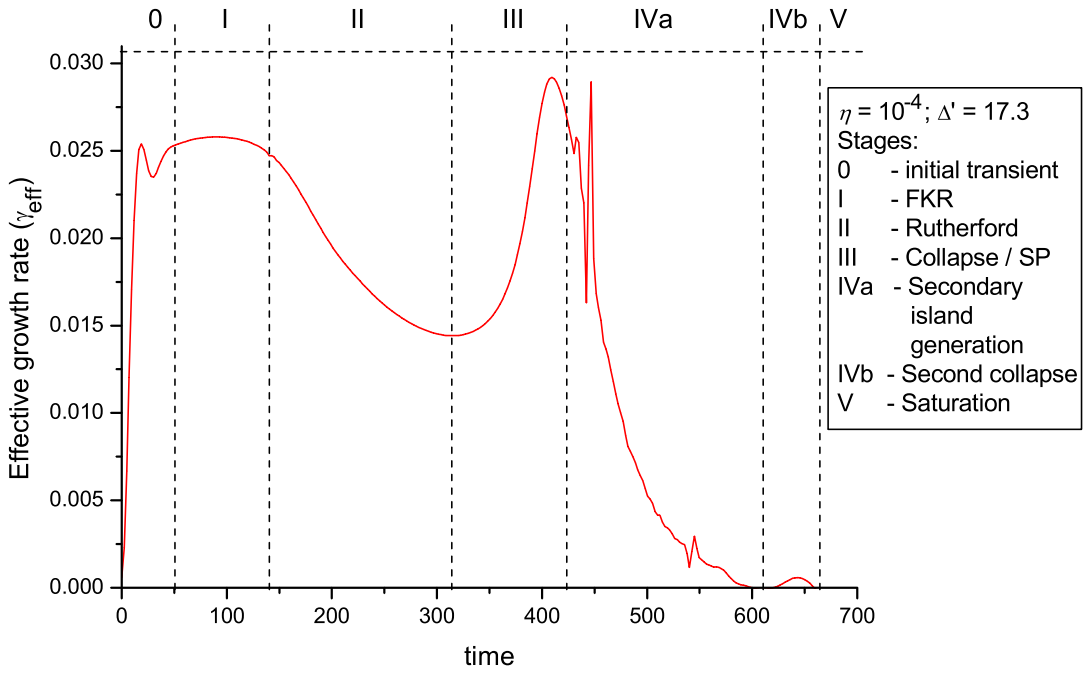
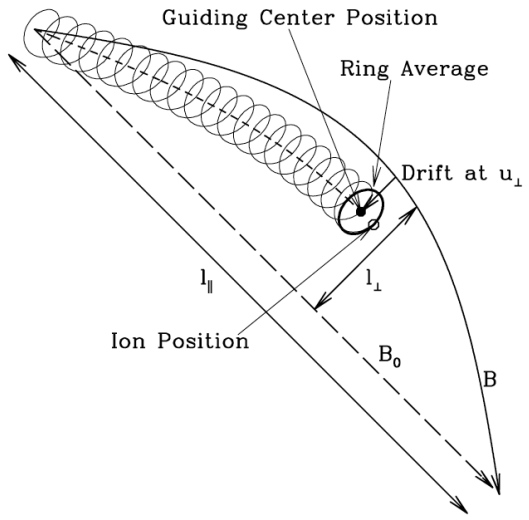


Figure illustrates regimes of tearing mode in MHD [image taken from Loureiro *et al.*, PRL (2005).]

Reduced Kinetic Approach to Multiscale Problem – Gyrokinetics



- Reduced kinetic model – 5 dimensional phase space.
- Existence of mean field (B_0) allows to separate out fast cyclotron motion – only low-frequency dynamics $\omega < \Omega_c$.
- Multiscale – distinct parallel and perpendicular scales ($k_{||} \gg k_\perp$).
- Retains finite Larmor radius (FLR) effects, wave-particle interactions; orders out fast MHD waves, cyclotron resonance.
- Note: two-dimensional magnetic reconnection or tearing instability dynamics primarily independent of the guide magnetic field. (In the sense of simplest incompressible model.)

Gyrokinetics: Basic equations

The distribution function of particles is given by $f = \left(1 - \frac{q\phi}{T_0}\right) f_0 + h$, where $f_0 = n_0/(\sqrt{\pi} v_{\rm th})^3 \exp(-v^2/v_{\rm th}^2)$ is the Maxwellian, and the thermal velocity is given by $v_{\rm th} = \sqrt{2T_0/m}$. The equations to solve are the gyrokinetic equation for $h = h(\boldsymbol{R}, V_\perp, V_\parallel)$,

$$\frac{\partial h}{\partial t} + V_\parallel \frac{\partial h}{\partial Z} + \frac{1}{B_0} \{\langle \chi \rangle_{\boldsymbol{R}}, h\} - \langle C(h) \rangle_{\boldsymbol{R}} = q \frac{f_0}{T_0} \frac{\partial \langle \chi \rangle_{\boldsymbol{R}}}{\partial t}, \tag{1}$$

$\chi = \phi - \boldsymbol{v} \cdot \boldsymbol{A}$ and the field equations for $\phi(\boldsymbol{r})$, $A_\parallel(\boldsymbol{r})$, and $\delta B_\parallel(\boldsymbol{r})$,

$$n_{\rm i} = n_{\rm e} \quad \Leftrightarrow \quad \sum_s \left[-\frac{q_s^2 n_{0s} \phi}{T_{0s}} + q_s \int \langle h_s \rangle_{\boldsymbol{r}} \, {\rm d}\boldsymbol{v} \right] = 0,$$

$$(\nabla \times \boldsymbol{B})|_\parallel = \mu_0 j_\parallel \quad \Leftrightarrow \quad \nabla_\perp^2 A_\parallel = -\mu_0 \sum_s q_s \int \langle h_s \rangle_{\boldsymbol{r}} v_\parallel \, {\rm d}\boldsymbol{v}$$

$$\nabla \cdot \left(\mathtt{I} \frac{B_0 \delta B_\parallel}{\mu_0} + \mathtt{P} \right) = 0 \quad \Leftrightarrow \quad \frac{B_0}{\mu_0} \nabla_\perp \delta B_\parallel = -\nabla_\perp \cdot \sum_s \int \langle m \boldsymbol{V}_\perp \boldsymbol{V}_\perp h_s \rangle_{\boldsymbol{r}} \, {\rm d}\boldsymbol{v}.$$

Gyrokinetics: Collision Operator

Recently, linearized collision operators for gyrokinetic simulations, which satisfies physical requirements are established and implemented in `AstroGK` [a].

The operators are the pitch-angle scattering (Lorentz), the energy diffusion, and moments conserving corrections to those operators for like-particle collisions. Electron-ion collisions consists of pitch angle scattering by background ions and ion drag are also included.

We, here, mainly discuss the electron-ion collisions since it contributes to resistivity. The operator is given by (in Fourier space)

$$C_{\rm ei}(h_{{\rm e},\boldsymbol{k}}) = \nu_{\rm ei}\left(\frac{v_{\rm th,e}}{V}\right)^3\left(\frac{1}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial h_{{\rm e},\boldsymbol{k}}}{\partial\xi} - \frac{1}{4}(1+\xi^2)\frac{V^2}{v_{\rm th,e}^2}k_\perp^2\rho_{\rm e}^2 h_{{\rm e},\boldsymbol{k}} + \frac{2V_\parallel J_0(\alpha_{\rm e})u_{\parallel,{\rm i},\boldsymbol{k}}}{v_{\rm th,e}^2}f_{0{\rm e}}\right) \qquad (2)$$

We examine how this collision operator relates with resistivity which decays the current.

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[a] Abel *et al*, Phys. Plasmas **15**, 122509 (2008), arXiv:0808.1300; Barnes *et al*, Phys. Plasmas **16**, 072107 (2009), arXiv:0809.3945.

AstroGK *reproduces Spitzer Resistivity*

From the fluid picture current decays due to collisional resistivity as

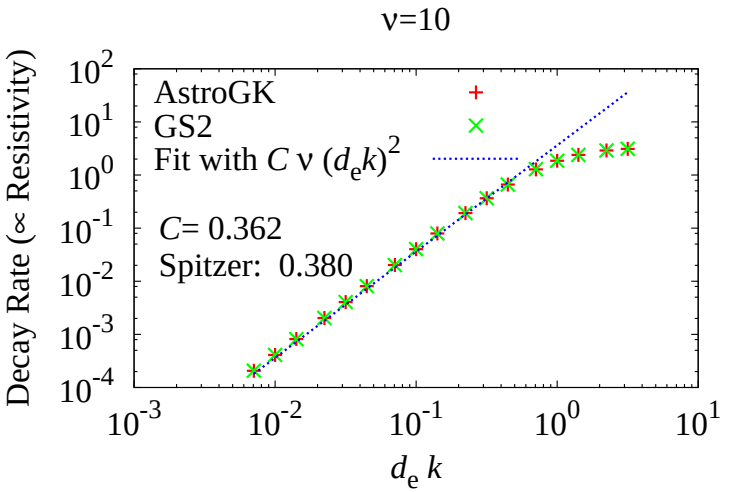
$$\frac{\partial j}{\partial t} = -\frac{\eta}{\mu_0} k^2 j, \tag{3}$$

and the decay rate is $\tau_{\rm decay}^{-1} = (\eta/\mu_0)k^2$. Using the Spitzer resistivity given by $\eta = m_{\rm e}/(1.98\tau_{\rm e} n_{\rm e} e^2)$ where $\tau_{\rm e} = 3\sqrt{\pi}/(4\nu_{\rm ei})$, the decay rate is casted into the following form,

$$\tau_{\rm decay}^{-1} = C\nu_{\rm ei}(d_{\rm e}k)^2 \tag{4}$$

where the constant $C = 4/(1.98 \times 3\sqrt{\pi}) \approx 0.380$.

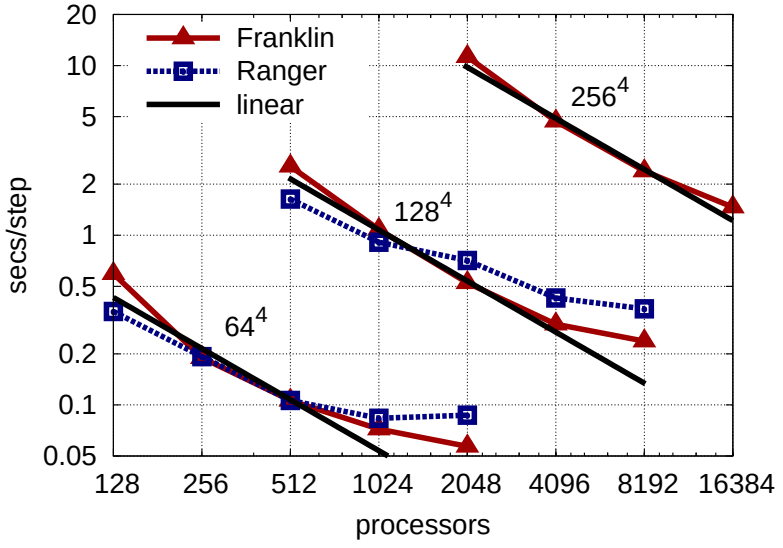
Figures show dependence of decay rate on ν and $d_{\rm e}k$. Numerical estimated proportionality constant agrees with Spitzer's value within 5% error for $d_{\rm e}k < 1$.



AstroGK *Code Details*

AstroGK is derived from GS2 by removing magnetic geometry effects to study fundamental aspects of kinetic plasmas mainly focusing on astrophysical problems. The code is publicly available at https://sourceforge.net/projects/gyrokinetics/. [E-Print: arXiv:1004.0279, submitted to JCP.]

- Eulerian continuum, local flux tube, δf, electromagnetic code.
- Fourier spectral in x and y.
- 2nd-order compact finite difference in z.
- Periodic boundary in x, y, and z.
- Gaussian quadrature for velocity space integral.
- Time integral:
  - 1st-order implicit scheme for all linear terms (including collisions).
  - 3rd-order Adams-Bashforth for nonlinear terms.
- Parallelized using MPI library – Scales up to $\sim$ 10000 procs.
- Easy to port: works on laptop PCs to major scalar supercomputers, e.g. Jaguar @ NCCS (Cray XT4, 5), Ranger @ TACC (Sun Constellation Linux Cluster).



Simulation Setup

- Electron and one ion species, both treated kinetically.
- Purely 2D: $k_z = 0$.
- Equilibrium (on top of f_{0s} and B_{z0}): $\delta f_{\mathrm{e}0} \propto v_{\parallel} f_{0\mathrm{e}}$ such that

$$A_{\parallel 0}(x) = A_{\parallel 00} \cosh^{-2}\left(\frac{x - L_x/2}{a}\right) S_{\mathrm{h}}(x), \quad B_{y0} = \partial_x A_{\parallel 0} \tag{5}$$

  where $A_{\parallel 00}$ is a constant, a defines the typical scale length of the system, L_x is the box size, S_{h} is a shape function to enforce periodicity. ($S_{\mathrm{h}} = 1$ in most of the region, and quickly falls to zero near the boundaries.) $\phi_0 = \delta B_{\parallel 0} = 0$, and $\delta f_{\mathrm{i}0} = 0$.

- Perturbation: $k_y a = 0.8$ gives $\Delta' a \approx 23$.
- Energy source: We maintain the equilibrium $A_{\parallel 0}$ throughout the simulation. It is interpreted physically that energy is injected into the system to compensate the collisional energy loss.n

Linear Tearing Instability: Problem Setup

Parameters

$$r \equiv \rho_S/a, \qquad \sigma \equiv m_e/m_i, \tag{6}$$

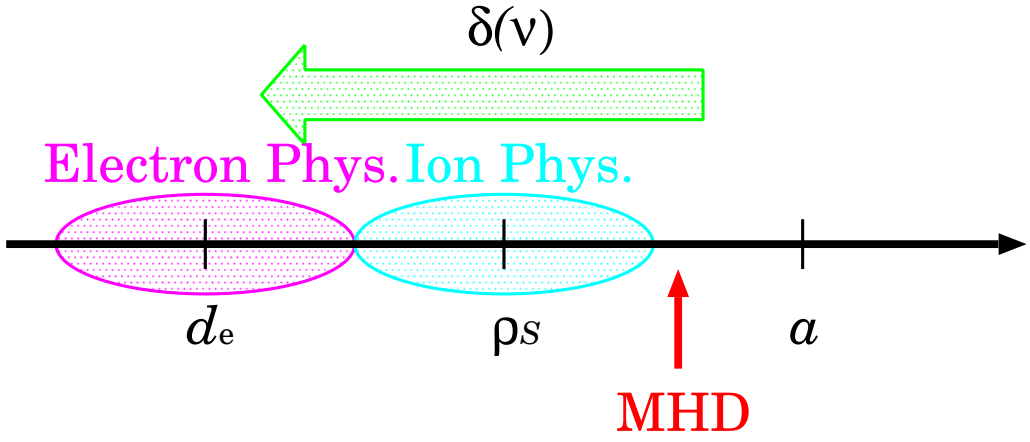
$$\tau \equiv T_{0i}/T_{0e}, \qquad \beta_e \equiv n_0 T_{0e}/(B_0^2/2\mu_0) \tag{7}$$

$\rho_S \equiv \sqrt{T_{0e}/m_i}/\Omega_{ci}$ is the ion sound Larmor radius, B_0 is the guide magnetic field

$$\rho_i/\rho_S = \tau^{1/2}, \qquad d_i/\rho_S = \beta_e^{-1/2}, \tag{8}$$

$$\rho_e/\rho_S = \sigma^{1/2}, \qquad d_e/\rho_S = \beta_e^{-1/2}\sigma^{1/2}, \tag{9}$$

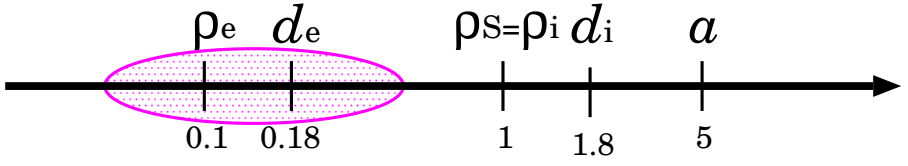
- Change collisionality ν to vary current layer width δ

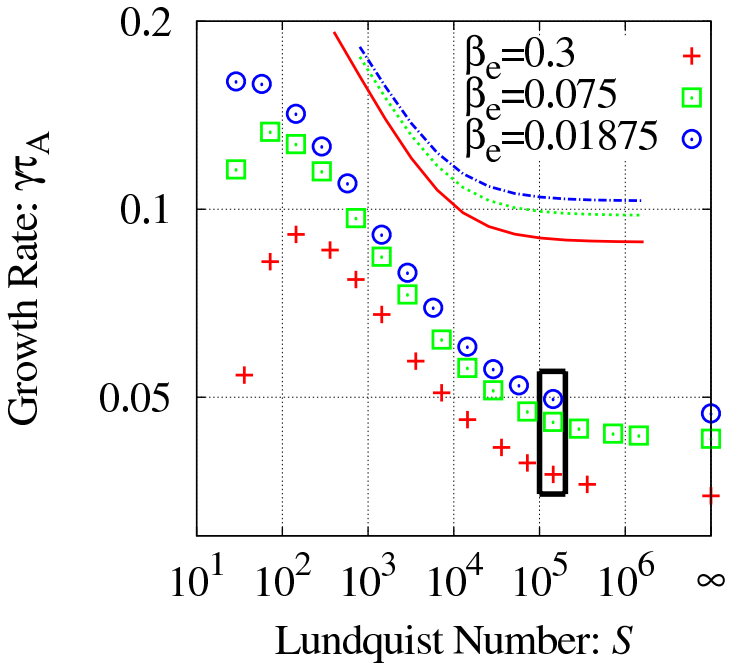


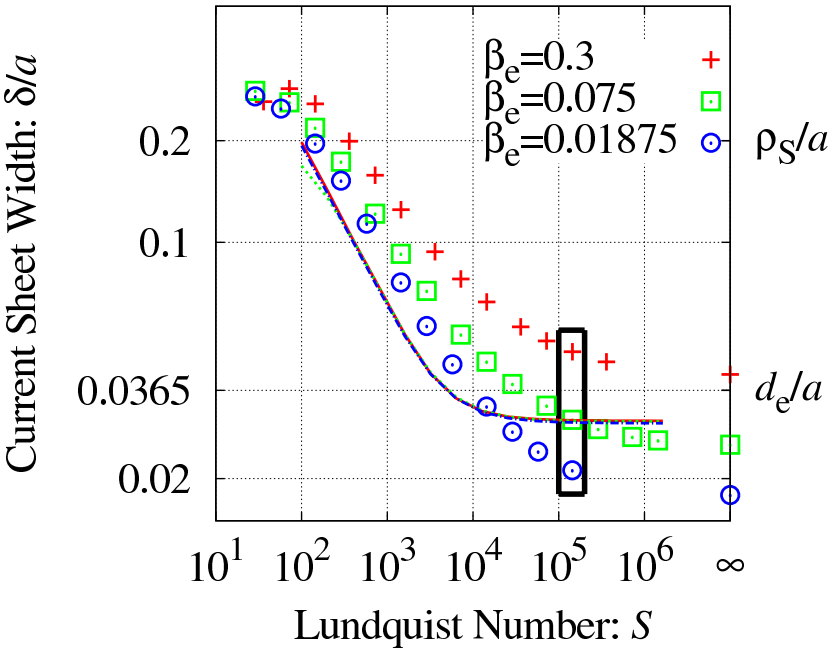
Collisional-Collisionless Transition

β_e and σ varied with fixed d_e

$$r = 0.2, \quad \tau = 1, \quad \sigma = 0.01, \quad \beta_e = 0.3 \tag{10}$$



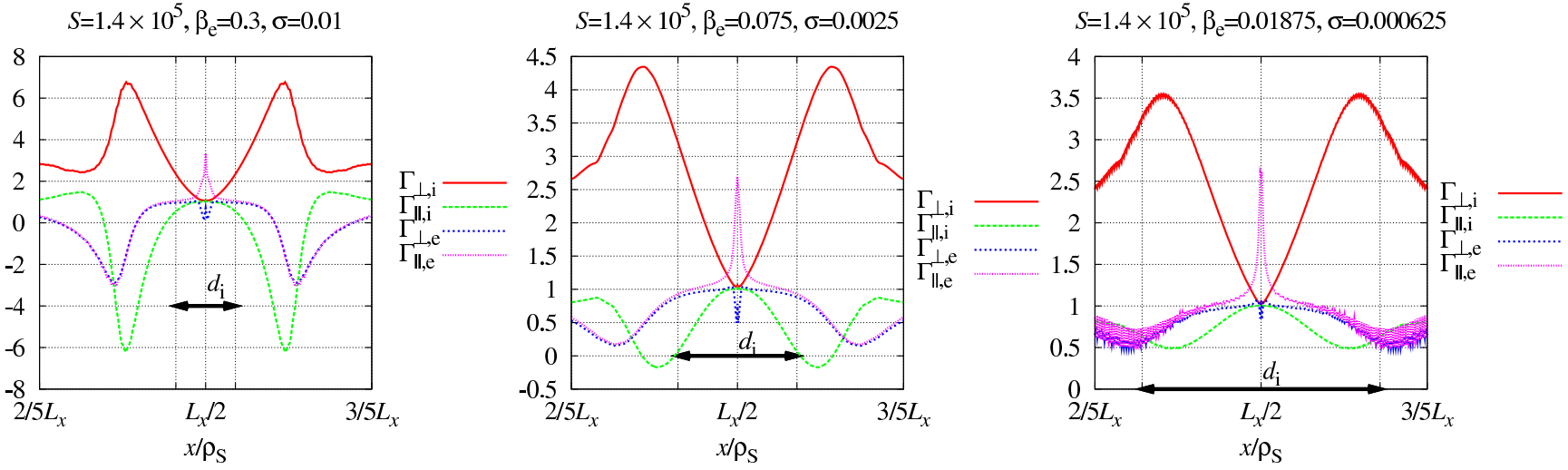




- Transition to collisionless reconnection – independent of collisions
- Electron inertia mediated – d_e sets lower bound of scale
- Difference with Hall RMHD (valid when $\tau \ll 1$) because of pressure treatment

Polytropic Eqn. of State is not Valid

Polytropic Eqn. of State $p \propto (n_0 m)^{\Gamma}$ leads to $\tilde{p} = \Gamma T_0 \tilde{n}$ or $\tilde{T}/T_0 = (\Gamma - 1)\tilde{n}/n_0$



- Direct measurements of $\Gamma_{\perp,\parallel}$ shows non-isotropy & non-polytropy
- Electrons temperature fluctuations are large inside the electron layer ($\sim d_e$) and zero outside the layer (far outside the layer both density and temperature is zero)

Energy Partition Diagnostics to Measure Linear Phase Mixing

FLR effects brings about a phase mixing effect which creates velocity space structures and then enhances collisional dissipation. To see this effect, we define the following energy quantities, and observe collisional dissipation.

Generalized gyrokinetic energy

$$W = \int \left[\sum_s \int \frac{T_{0s} \delta f_s^2}{2 f_{0s}} \mathrm{d}\boldsymbol{v} + \frac{|\nabla_\perp A_\parallel|^2 + \delta B_\parallel^2}{2\mu_0} \right] \mathrm{d}\boldsymbol{r} \tag{11}$$

Evolutions of the perturbed energy ($\tilde{W} = W - W^{\mathrm{eq}}$) is given by

$$\frac{\mathrm{d}\tilde{W}}{\mathrm{d}t} = P - D \tag{12}$$

where the input power P and dissipation D are given by

$$P = \int \frac{T_{0\mathrm{e}} h_\mathrm{e}}{f_{0\mathrm{e}}} C(h_{\mathrm{e},0}) \mathrm{d}\boldsymbol{v}, \qquad D = \int \frac{T_{0\mathrm{e}} h_\mathrm{e}}{f_{0\mathrm{e}}} C(h_\mathrm{e} - h_{\mathrm{e},0}) \mathrm{d}\boldsymbol{v}. \tag{13}$$

Is There Linear Phase Mixing? – No!

Fraction [%] of energy components and dissipation to P
Case A,B,C are collisionless $S = 1.4 \times 10^5$, varied $\beta_{\rm e}$: A: $\beta_{\rm e} = 0.3$, B: $\beta_{\rm e} = 0.075$, C: $\beta_{\rm e} = 0.01875$.
Case D, E are collisional with fixed $\beta_{\rm e} = 0.3$: D: $S = 3.6 \times 10^3$, E: $S = 1.4 \times 10^2$, $\beta_{\rm e} = 0.3$.

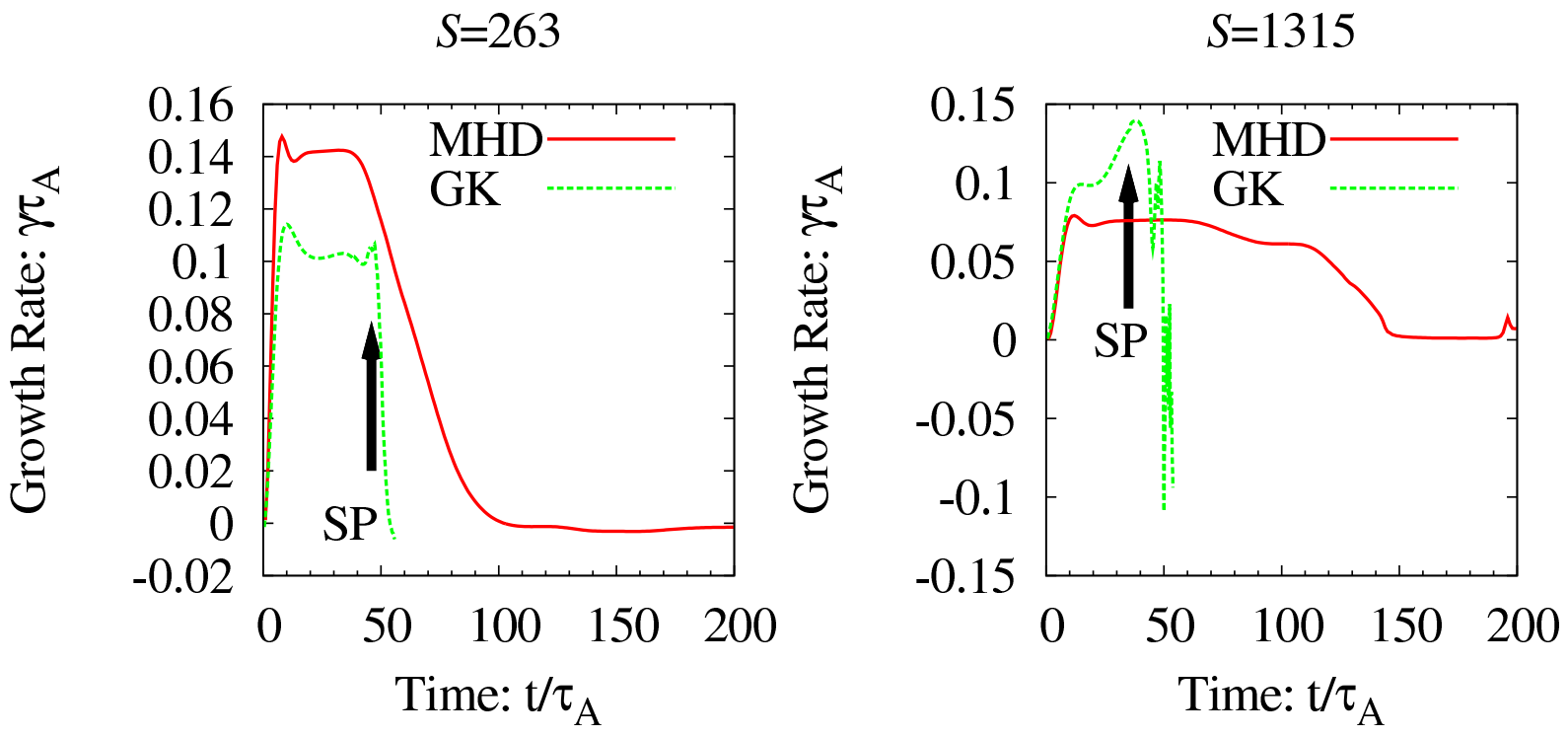
Case	$E_{\rm K\perp,i}$	$E_{\rm K\perp,e}$	$E_{\rm K\parallel,i}$	$E_{\rm K\parallel,e}$	$E_{\rm T,i}$	$E_{\rm T,e}$	$E_{\rm M,\perp}$	$E_{\rm M,\parallel}$	D
A	1.9	1.6	1.1	5.3	5.6	6.7	73.4	1.1	3.2
B	2.0	1.3	0.7	7.5	4.9	6.8	74.3	0.5	1.9
C	2.0	0.7	0.3	8.8	5.4	6.8	74.4	0.2	1.4
D	4.1	0.7	2.0	2.6	7.0	2.7	70.0	1.5	9.3
E	9.5	0.2	6.0	0.4	10.1	2.0	46.5	1.2	24.1

- Dissipation is weak in collisionless case.
- Fine velocity space structure does not develop. (As a result, the numbers of velocity grids of ~ 10 are sufficient.)
- Ballistic term (parallel phase mixing along the perturbed [in gyrokinetic sense] magnetic field) yields $\propto e^{{\rm i}J_0(k_\perp \rho_{\rm i}) k_y B_{y0} v_\parallel t}$ dependence, whose time scale is slower than the tearing growth rate.

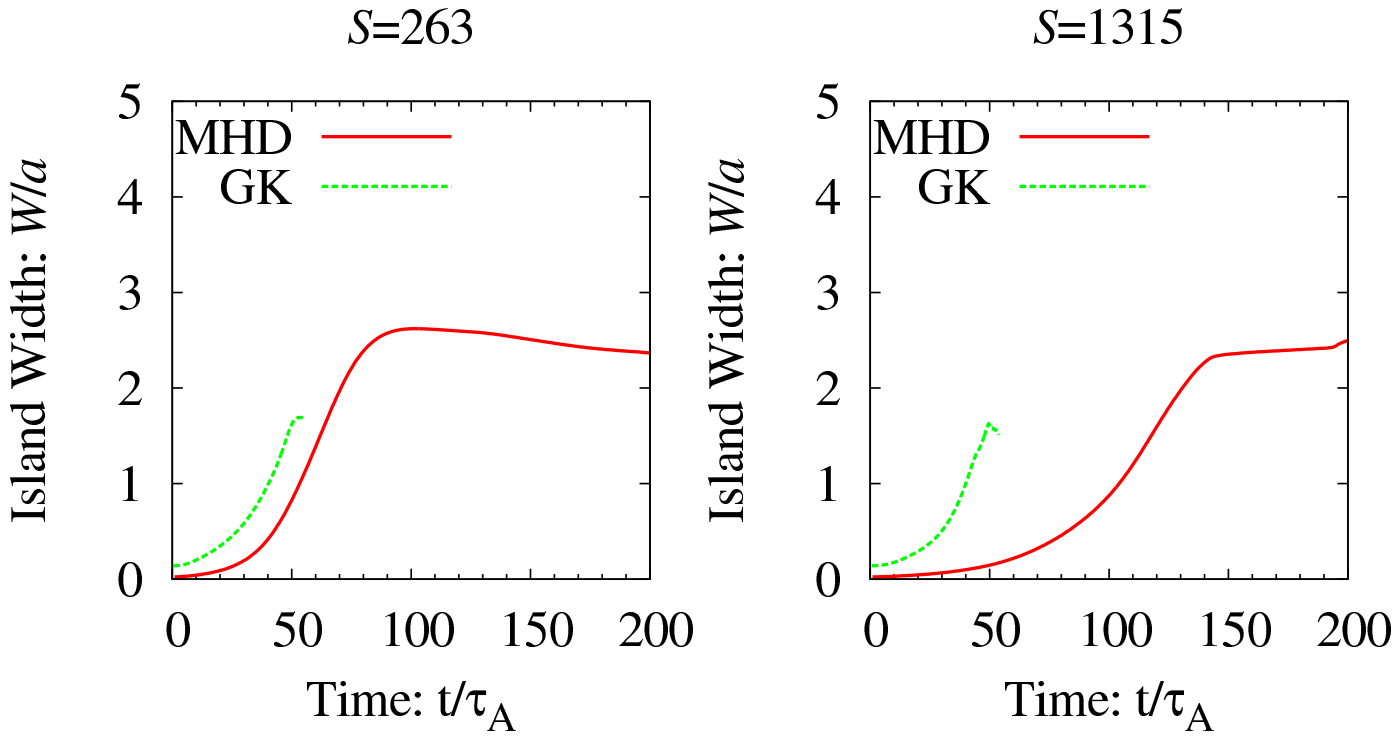
Nonlinear Stage: Only Low S So Far

$$r = \rho_S/a = 0.25, \quad \beta_e = 0.25, \quad m_i/m_e = 0.01, \quad T_{0i}/T_{0e} = 1 \tag{14}$$

$$\rho_i/a = \rho_S/a = 0.25, \quad d_i/a = 0.5, \quad d_e/a = 0.05, \quad \rho_e/a = 0.025. \tag{15}$$

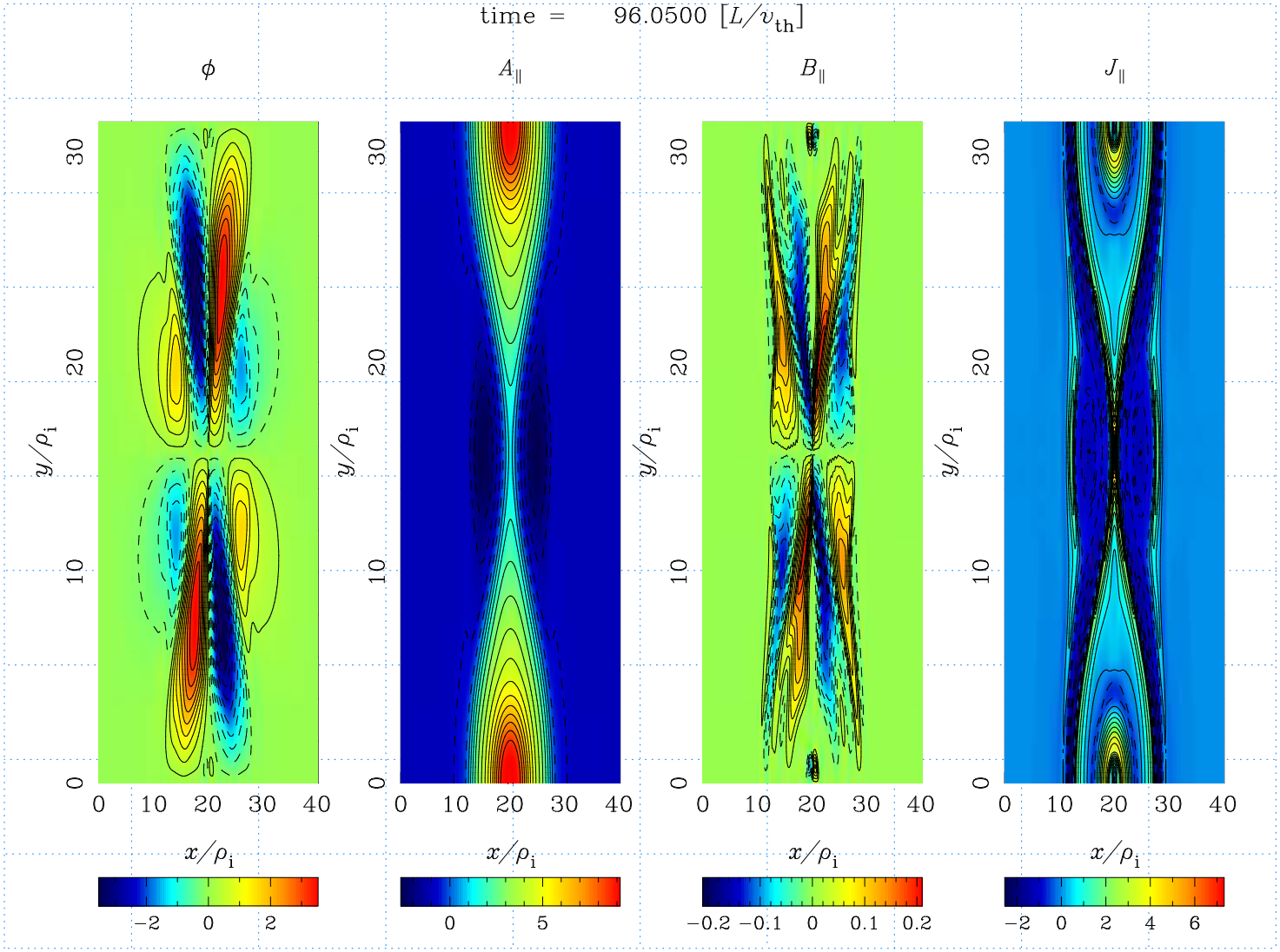


Nonlinear Stage: Island Growth and Saturation



- Smaller island width saturation level in GK compared with MHD.
- Critical island width for X point collapse is smaller in GK.
- ($S = 1315$ case may not be well resolved. Needs higher resolution.)

Nonlinear Stage



Summary

- Magnetic reconnection and tearing instability are very good example of multiscale physics.
- We have performed collisionless and collisional linear tearing instability simulations, and have scanned for $\nu_{\rm ei}$.
- We have observed transition from collisional regime (collision dependent growth rate) to collisionless regime (collision independent growth rate).
- Equation of state need to be considered carefully – not simply $p \propto \rho^{\Gamma}$
- Collisional dissipation and heating does not matter in linear regime
- We have also performed nonlinear simulations. Only small S so far. (Electron kinetic effects is minor.)
- Nonlinear tearing mode growth scenario has been confirmed in gyrokinetic simulations. X point collapse occurs even for small S case.
- Smaller island in gyrokinetic case. There may be other channels for energies to go: heating, kinetic energies.
- Further investigations are necessary: Detailed analysis for the stage after the collapse, e.g. scaling; What happenes for collisionless case?

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