Understanding the Effect of Sheared Flow on Microinstabilities

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Introduction

- Experiments indicate tokamak turbulence is strongly affected by sheared flows
- Extensive theoretical work has been undertaken
  - flow shear can tear apart turbulent eddies, suppressing turbulence
  - not all shear flows are stabilising *eg* Parallel Velocity Gradient (PVG) instability
  - beyond critical perpendicular shear PVG linearly stabilised: transient perturbations remain [1]
- Strong toroidal flows of tokamaks have both perpendicular and parallel shear
- *What is the optimum shear flow for confinement – do we want ever more toroidal flow shear?*
- Investigate effect of flow shear on Ion Temperature Gradient (ITG) and PVG instabilities in sheared slab via dissipative fluid model: simple system allows clear interpretation

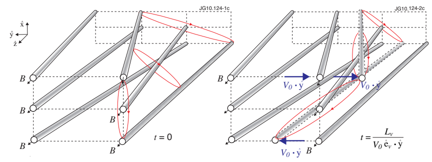
System Equations

- Analyse sheared slab, magnetic and flow fields: $\boldsymbol{B} = B_0\left(\hat{\mathbf{z}} + \frac{x}{l_s}\hat{\mathbf{y}}\right)$ $\quad \boldsymbol{V}_0 = V_0\frac{x}{L_v}\hat{\mathbf{e}}_v$, $\boldsymbol{V}_0\cdot\hat{\mathbf{x}} = 0$
- Work in **"twisting-shearing" representation** [2]
  - removes problems of shear and time dependence from background flow
  - aligns coordinate lines with characteristics of plasma response: *sound waves*

$$z' = z + u_f t\,;\quad y' = y - \frac{x}{l_s}\left(z + u_f t\right);\quad x' = x \qquad \left.\begin{array}{l}\boldsymbol{B}\cdot\nabla x'\\ \boldsymbol{B}\cdot\nabla y'\end{array}\right\} = 0\,;\quad \boldsymbol{B}\cdot\nabla = B_0\frac{\partial}{\partial z'}\,;\qquad \frac{\partial}{\partial t} + \boldsymbol{V}_0\cdot\hat{\mathbf{y}}\frac{\partial}{\partial y} = \frac{\partial}{\partial t'} + u_f\frac{\partial}{\partial z'}$$

$$M = \frac{u_f}{c_s} = \frac{V_0}{c_s}\frac{l_s}{L_v}\hat{\mathbf{e}}_v\cdot\hat{\mathbf{y}}$$

background variations in x; perpendicular shear flow advects flux tubes along y; eddy twists, drive-aligned position retreats at speed u_f



- Take Boltzmann electron response; ions described by **collisional fluid equations**
  - derived from gyro-kinetic equation in doubly sheared coordinates with flow
  - normalise: $(x', y') = \rho_s(x, y)$, $z' = l_s z$, $t' = (l_s/c_s)\,t$; $c_s = \sqrt{(\gamma_e + \gamma_i)T/m}$, $T_i = T_e$, $\gamma_e = 1$, $\gamma_i = 5/3$
  - usual orderings: $\omega/\Omega \sim O(k_\parallel/k_\perp) \sim O(\delta f/F_0) << 1$
  - additional collisional orderings: $\nu >> \omega,\ \omega^*,\ v_{th}k_\parallel,\ u_f k_\parallel,\ \nu k^2\rho^2$; $\quad k\rho \sim O\left(\sqrt{\omega/\nu}\right) << 1$
  - expand in ω/ν: lowest order is perturbed Maxwellian
  - closed equations for evolution of perturbed density δn, parallel velocity $\delta V_\parallel$, temperature δT from moment equations for particle, parallel momentum and energy conservation $\quad (n, V, T) = \frac{l_s}{\rho_s}\left(\frac{\delta n}{n_0}, \frac{\delta V_\parallel}{c_s}, \frac{\delta T}{T}\right)$
- Restrict to **linear** case, take fields $\propto \exp(iky)$, $(\nu_k, \chi_k) = k^2(\nu_\perp, \chi_\perp)$, $\nabla_\perp^2 = -k^2\left(1 + z^2\right)$
- Dissipation: $(\nu_\perp, \chi_\perp) \propto (0.9, 1)\,\nu_{ii}$
- Drift frequencies characterise driving *x*-gradients: $(\omega_n^*, \omega_v^*, \omega_T^*) = \frac{3k}{8}\left(\frac{l_s}{l_n}, \frac{l_s}{l_v}, \frac{l_s}{l_T}\right)$; $\quad \frac{1}{l_v} = \frac{1}{L_v}\frac{V_0}{c_s}\hat{\mathbf{e}}_v\cdot\hat{\mathbf{z}}$

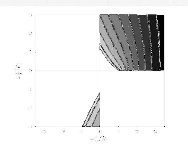
$$\left(\frac{\partial}{\partial t} + M\frac{\partial}{\partial z}\right)n + \frac{\partial V}{\partial z} = i\omega_n^* n \qquad M = u_f/c_s$$

$$\left(\frac{\partial}{\partial t} + M\frac{\partial}{\partial z}\right)V + \frac{3}{8}\frac{\partial}{\partial z}(2n + T) = i\omega_v^* n - \nu_k\left(1 + z^2\right)V \qquad \omega_v^* \propto k\ M\ \cot\theta_v$$

$$\left(\frac{\partial}{\partial t} + M\frac{\partial}{\partial z}\right)T + \frac{2}{3}\frac{\partial V}{\partial z} = i\omega_T^* n - \chi_k\left(1 + z^2\right)T$$

Parallel Flow Instability

- Formulate **local dispersion relation** neglecting dissipation
  - look for solution $\propto \exp\left(-i\omega t + ik_\parallel z\right)$
  - convective effect of perpendicular flow shear: $\omega' = \omega - k_\parallel M$
  - PVG mode $\omega_n^* = \omega_T^* = 0$: $\quad \omega'^2 = k_\parallel\left(k_\parallel - \omega_v^*\right)$
  - instability for: $\omega_v^* > k_\parallel$
  - effect of background parallel flow shear asymmetric

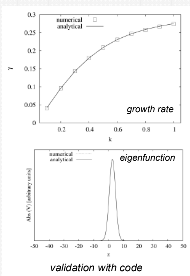


contours of highest growth rate, white stable, $\omega_v^*, \omega_T^* + ve, \omega_n^* = 0$

- **Dissipative terms** act to damp perturbations far along field lines: demonstrate for *M* = χ = *0*
  - solution has form: $V = s(z)\exp\left(i\omega_v^* z/2b\right)\exp(\gamma t)$

$$\frac{\partial^2 s}{\partial z^2} + \left[\frac{\omega_v^{*2}}{4b^2} - \frac{a}{b}\left(\gamma + \nu_k\right) - \frac{a}{b}\nu_k z^2\right]s = 0 \qquad \begin{array}{l} a = \gamma - i\omega_n^* \\ b = 1 + \frac{i}{4\gamma}\left(\frac{3}{2}\omega_T^* - \omega_n^*\right)\end{array}$$

  - simple harmonic oscillator equation: $s(z) = H_n\left[\sqrt[4]{a\nu_k/b}\ z\right]e^{-\sqrt{a\nu_k/b}\,(z^2/2)}$
  - eigenvalue condition: *n integer > 0* $\quad \frac{a}{b}\left(\gamma + \nu_k\right) + (2n + 1)\sqrt{\frac{a}{b}\nu_k} - \frac{\omega_v^{*2}}{4b^2} = 0$
  - large *k* modes dominated by viscosity: *FLR effects excluded*



validation with code $\quad l_s/l_n = 3, l_s/l_v = 9, l_s/l_T = 6$

- **Twisting mode localised where perpendicular gradients are small to minimise collisional dissipation**

Impact of Convection

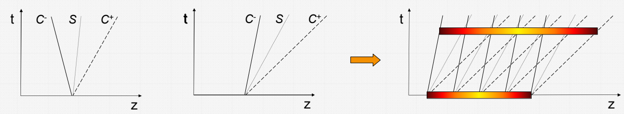
- Anticipate **rapid convection sweeps growing instabilities to dissipative region**: forces decay
- Motivate by looking at characteristics of linear system

$$\left[\frac{\partial}{\partial t} + M\frac{\partial}{\partial z}\right]S = f_s\left(\omega^*, \chi_k, S, C^\pm\right) \qquad \text{entropy wave} \qquad S = \frac{3}{2}T - n$$

$$\left[\frac{\partial}{\partial t} + (M \pm 1)\frac{\partial}{\partial z}\right]C^\pm = f_c^\pm\left(\omega^*, \chi_k, \nu_k, S, C^\pm\right) \qquad \text{forward and backward sound waves} \qquad C^\pm = V \pm \frac{3}{4}\left(n + \frac{T}{2}\right)$$

  - waves coupled by finite RHS: does not change propagation speed

$M < 1$ $\qquad M > 1$ *all characteristics forward going: eigenmodes cannot form*



perturbation initially localised in z swept into dissipative region

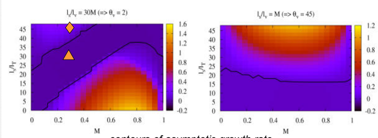
- Obtain analytic form of **transitory solution for *M* >> 1**, $\omega_n^* = \omega_T^* = \chi = 0$
  - work in moving frame $z = z_0 + Mt$, expand in large parameter *M*
  - look for solution with time dependent growth rate, $O\left(M^2\right)$ determines γ

$$V(z_0, t) = s(z_0, t)\exp\left(i\omega_v^* z_0/2\right)\exp\left(\int\gamma(t')\,dt'\right) \qquad \frac{\gamma}{\gamma_0} = -\tau^2 + \sqrt{\tau^4 + 1} \qquad \begin{array}{l}\tau = \sqrt{\nu_k/2\gamma_0}\,Mt \\ \gamma_0 = \omega_v^*/2\end{array}$$

  - instability grows exponentially for $t \sim \frac{\sqrt{\gamma_0}}{\sqrt{\nu_k}M} \propto \frac{1}{\sqrt{M}}$ $\Rightarrow$ **amplification factor large** $\sim \exp\left(c\sqrt{M}\right)$
  - *s* determined at $O\left(M^{3/2}\right)$
  - gives asymptotic form: $\quad V(z_0, t) = -\tilde{s}(z_0, t)\frac{\gamma_0}{\nu_k M^2}\left(\frac{1}{t^2}\right)\exp[\Phi]$; $\quad \tilde{s}(z_0, t)$ *slowly varying function*, $\exp[\Phi]$ *factor saturates*
- **Algebraic decay with time**: amplified perturbation can linger

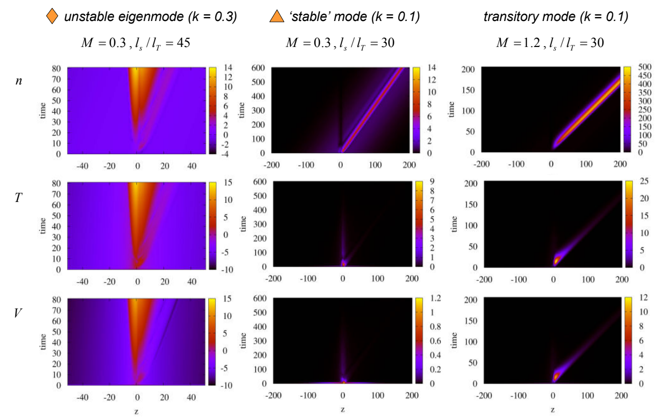
Numerical Results

- Linear system solved **numerically**: 2nd-order upwind scheme, typical resolution Δ*z* = 0.1
  - growth rate maximised over *k*, consistency with orderings for $\nu_\perp = 3.0$; here $l_s/l_n = 0$
- Vary flow, at fixed angle
  - **stability threshold seen at *M* = 1**
  - interplay of drives produces **regions of stability for *M* < 1**
  - sensitive to angle of flow
  - lines show zero growth rate



contours of asymptotic growth rate

- Evolution of **perturbed fields**
  - *density perturbation remains as particle transport is neglected*



Conclusions

- Investigated ITG and PVG stability in a sheared field with parallel and perpendicular flow shear
- Instabilities are twisting modes, convected along the field by perpendicular sheared flow
- Parallel flow shear drives instability, having complex interaction with ITG drive
  - sensitivity to angle of flow favourable for instability suppression in spherical tokamaks
- Modes convected faster than the sound speed, *M* > 1, are swept downstream to be damped
  - indicating suppression if flow shear convection faster than characteristic propagation speed of mode?
- Only transitory growth occurs for *M* > 1, but for large *M* growth can be substantial
  - mode decay slow; promotes subcritical turbulence?

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Particular References:

[1] FL Waelbroeck *et al* 1994 *Phys Plasmas* **1** 3742
[2] GG Howes *et al* 2001 *Astrophys J* **560** 617