

# LONG-WAVELENGTH TURBULENCE CHARACTERISTICS, DYNAMICS, AND FLOWS IN TOKAMAK PLASMAS

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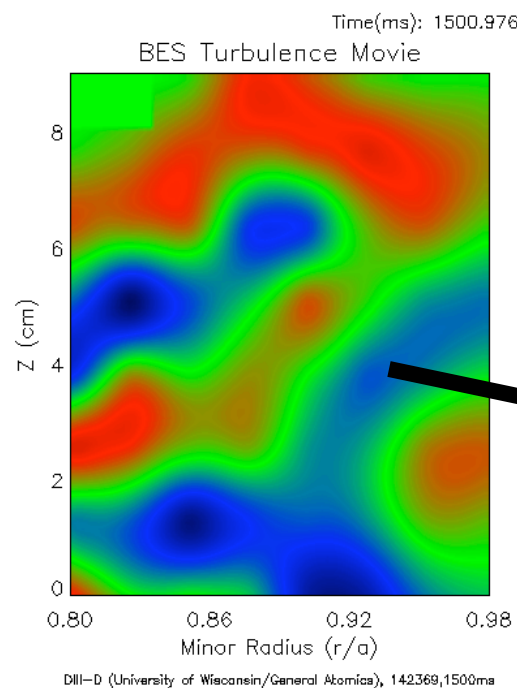
**Gyrokinetics In Laboratory and Astrophysical Plasmas**  
**Isaac Newton Institute for Mathematical Sciences**  
**Cambridge, United Kingdom**  
**19 July 2010**



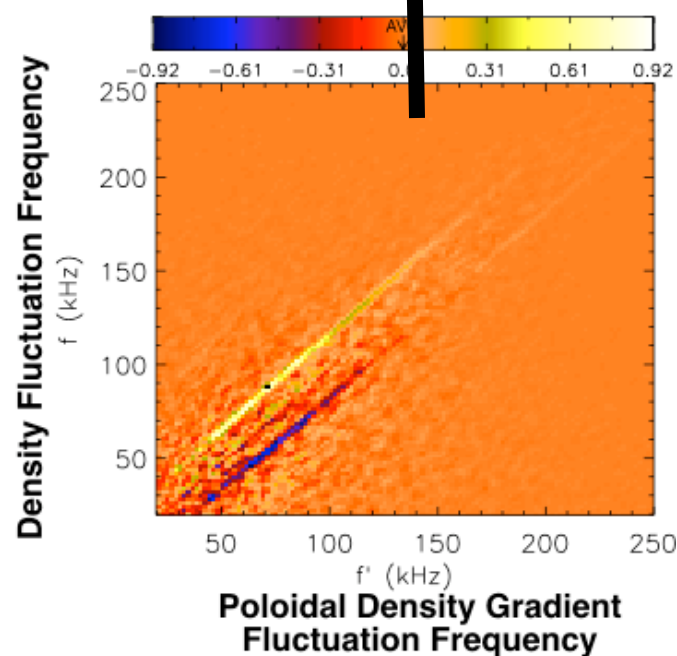
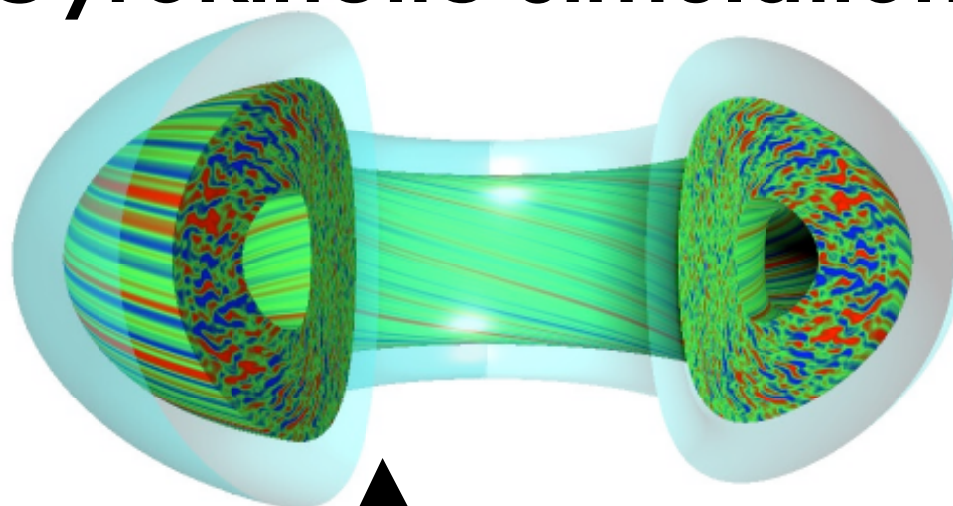
THE UNIVERSITY  
of  
**WISCONSIN**  
MADISON

# Turbulence plays a Central Role in Behavior and Performance of Magnetically-Confined Plasmas

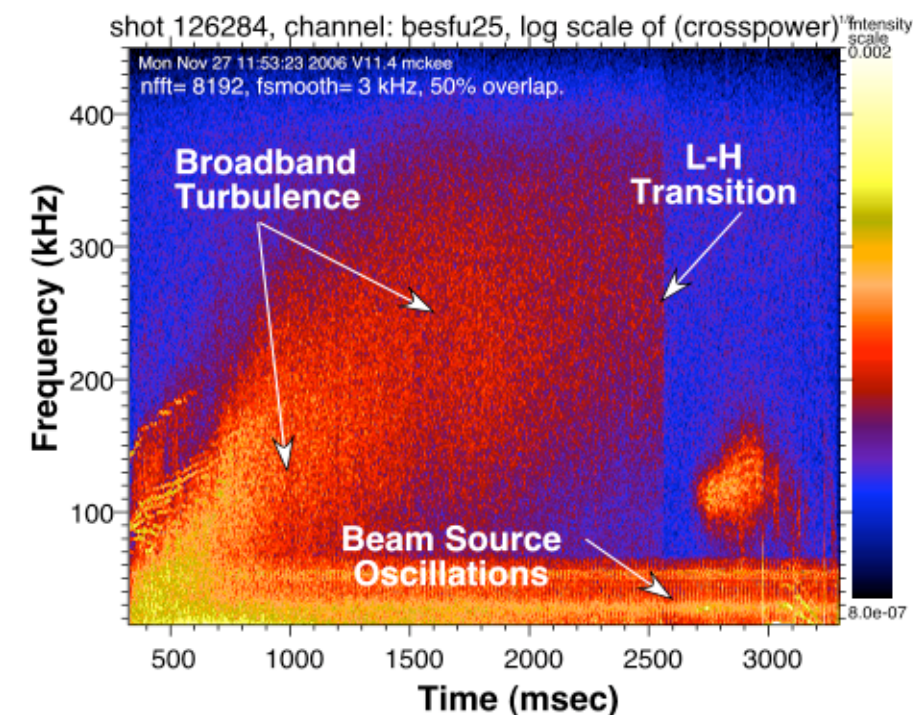
## Turbulence Measurement



## Validation of Gyrokinetic Simulations



## Confinement Bifurcations



Measurements helping to validate models and understand transport behavior

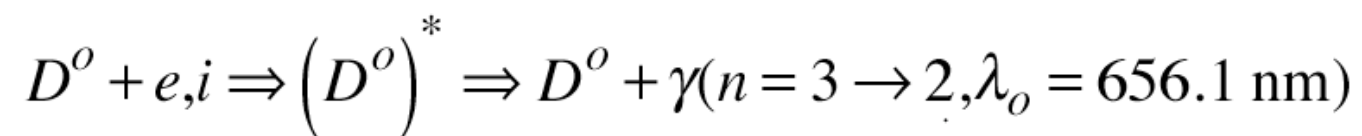
## Nonlinear Dynamics

# Outline

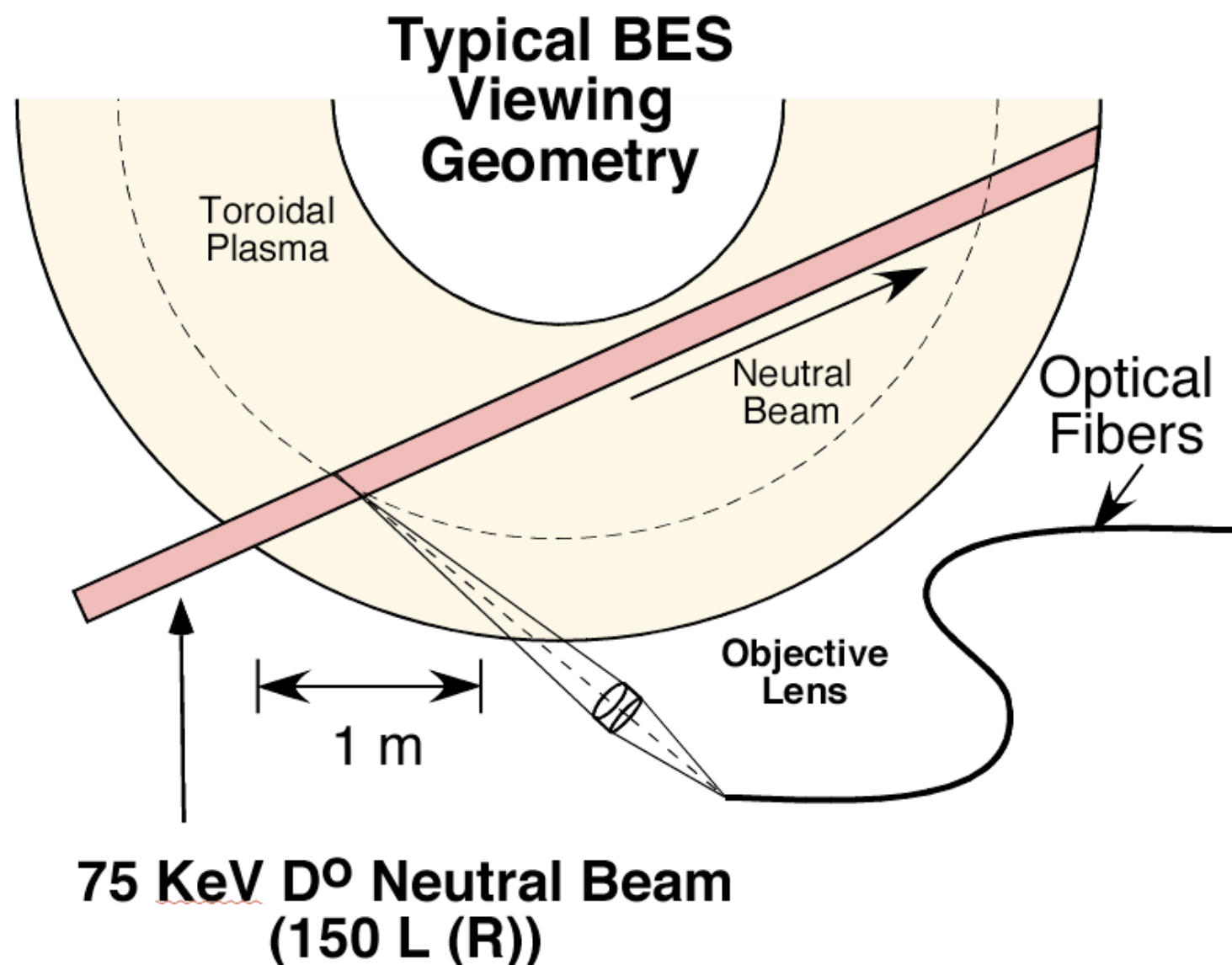
- **Diagnostic capability for measuring 2D long-wavelength density turbulence with Beam Emission Spectroscopy (BES)**
- **Turbulence imaging and visualization**
- **Turbulence characteristics and parameters**
- **Spatiotemporal correlation and spectral characteristics**
- **Time-dependent zonal flows**
  - Zonal Flow/Geodesic Acoustic Modes
  - Nonlinear energy transfer mediated by zonal flows
- **Comparisons with nonlinear 3D gyrokinetic simulations**

# Beam Emission Spectroscopy Measures Fluorescence of Heating Neutral Beam to Detect Local Density Fluctuations

**Collisionally-excited, Doppler-shifted neutral beam fluorescence**

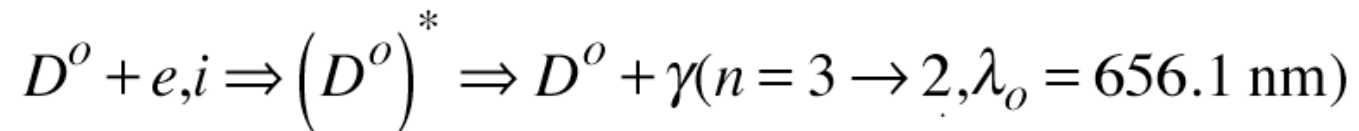


**Exploits Neutral Beam Heating Systems**

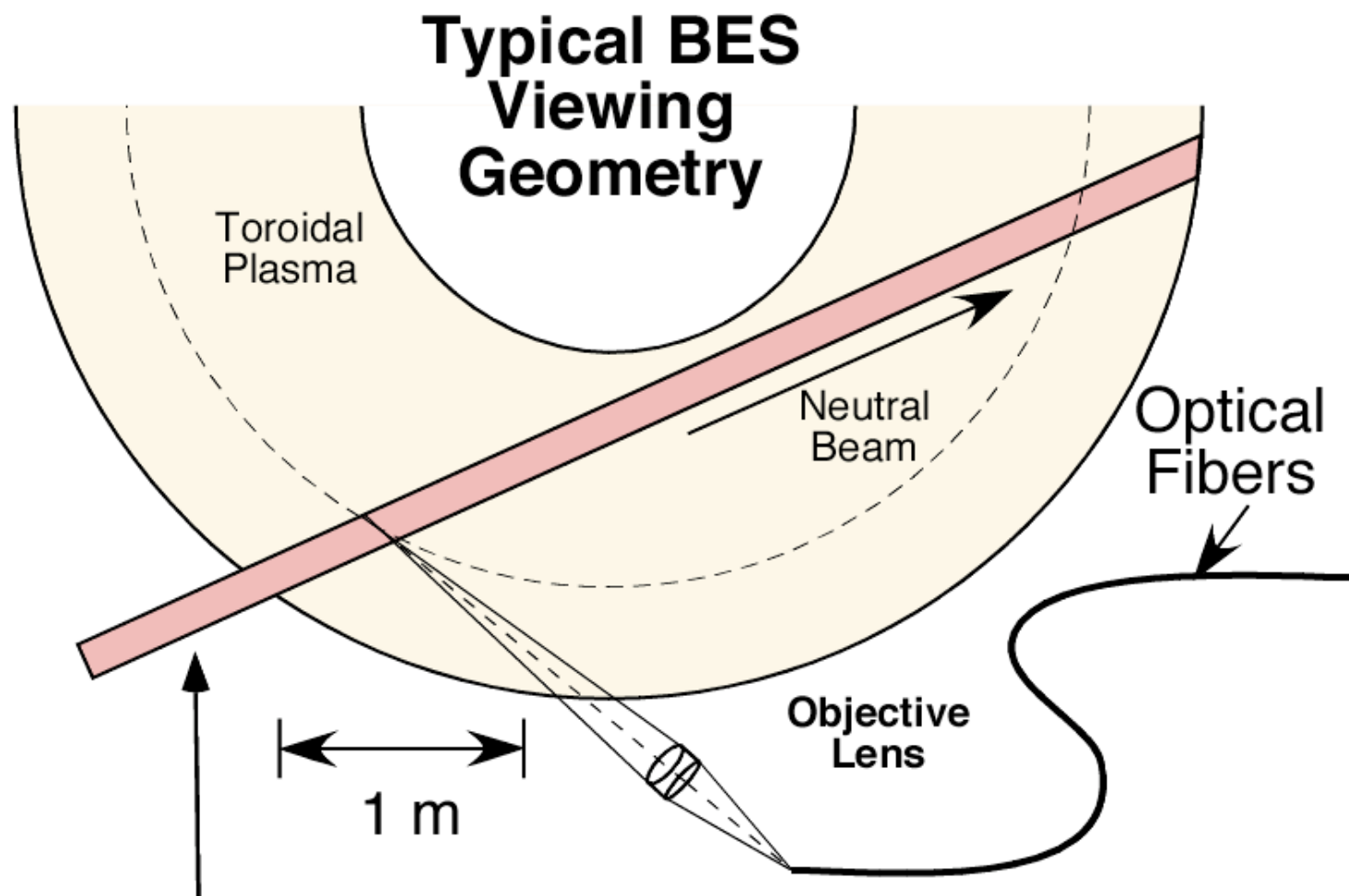


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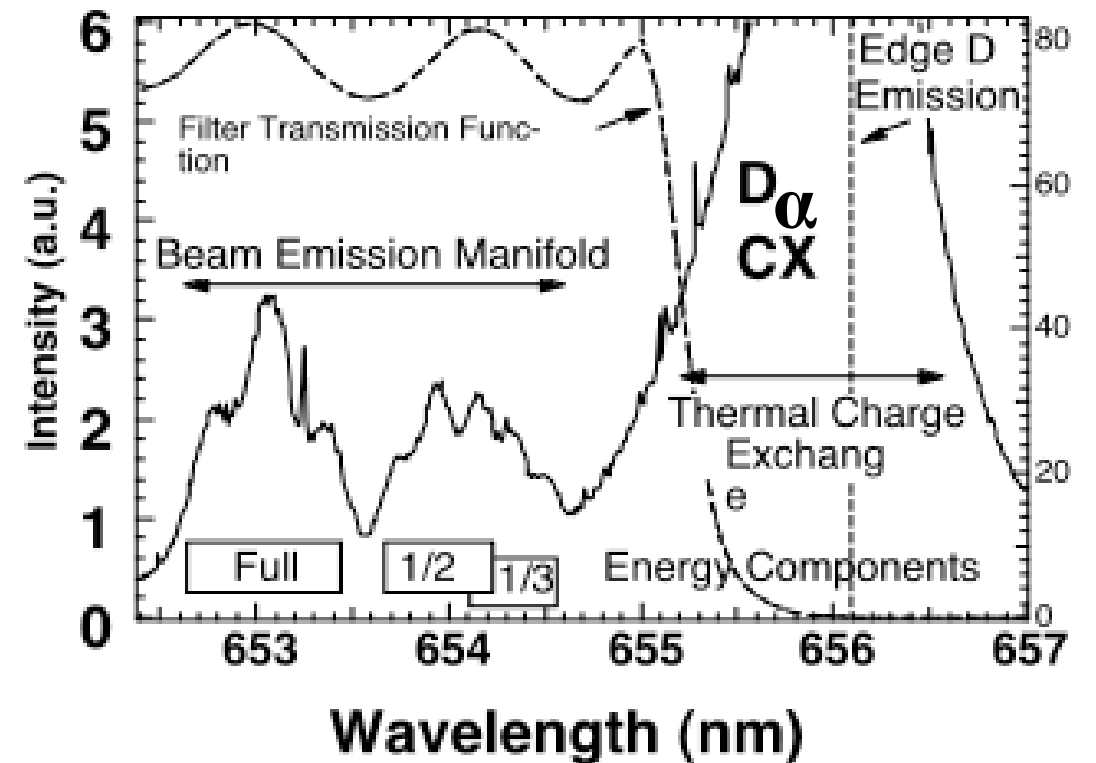
Collisionally-excited, Doppler-shifted neutral beam fluorescence



Exploits Neutral Beam Heating Systems

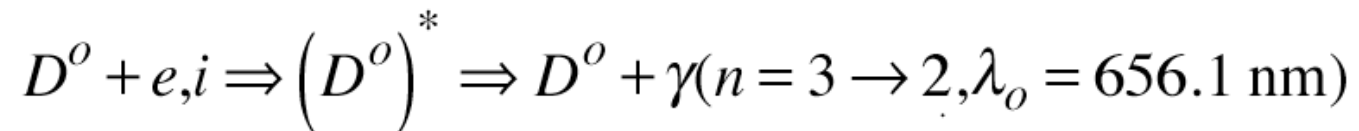


75 KeV  $D^0$  Neutral Beam  
(150 L (R))

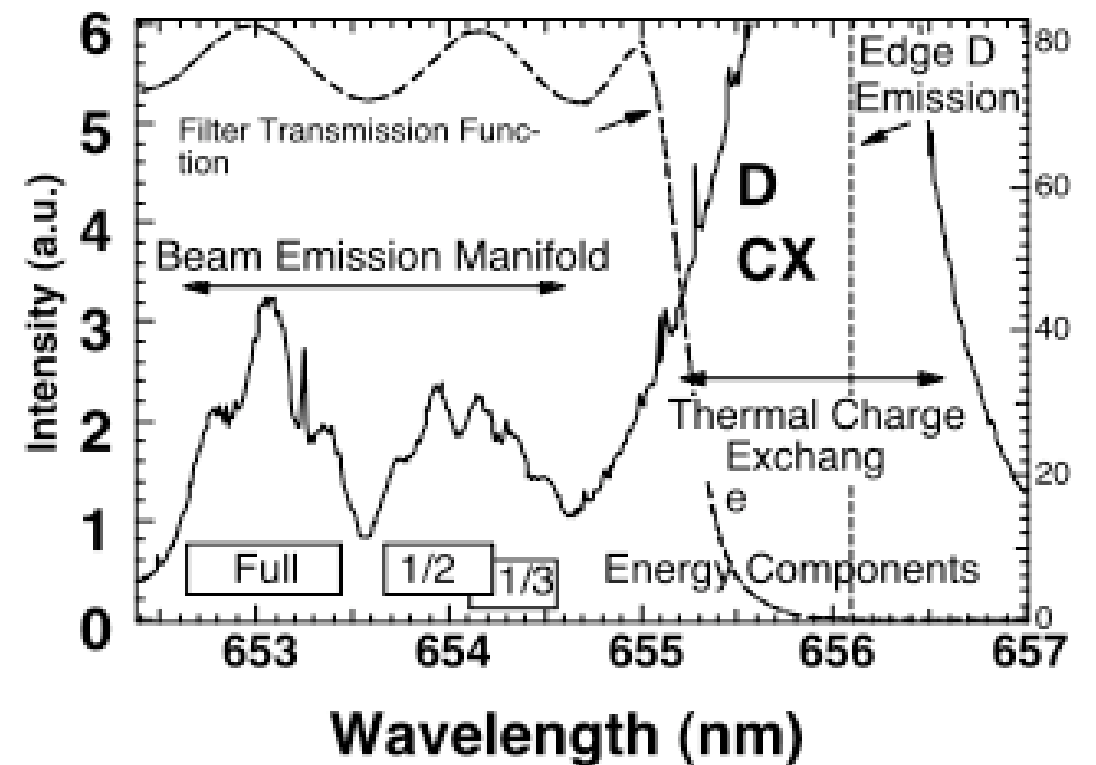
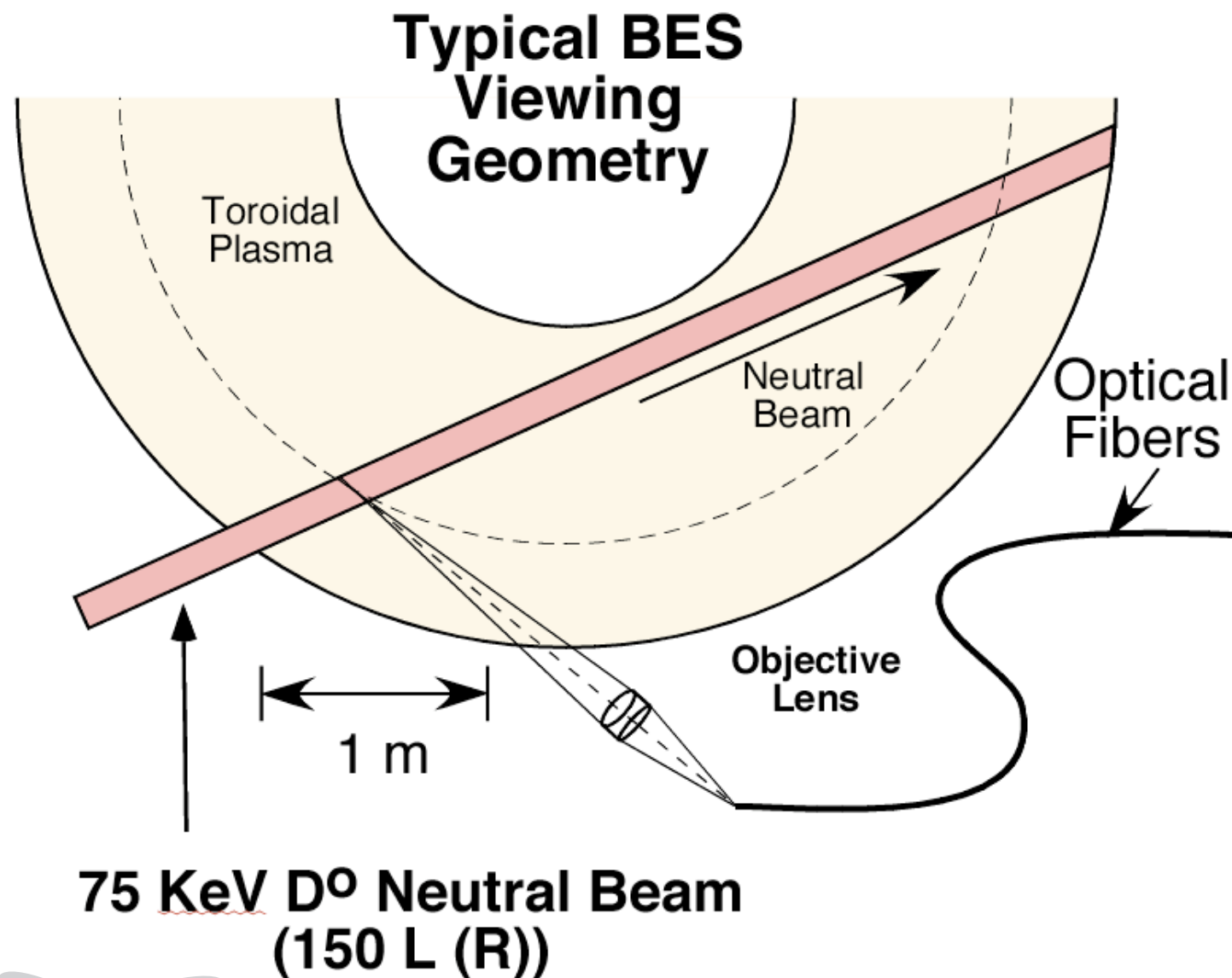


# Beam Emission Spectroscopy Measures Fluorescence of Heating Neutral Beam to Detect Local Density Fluctuations

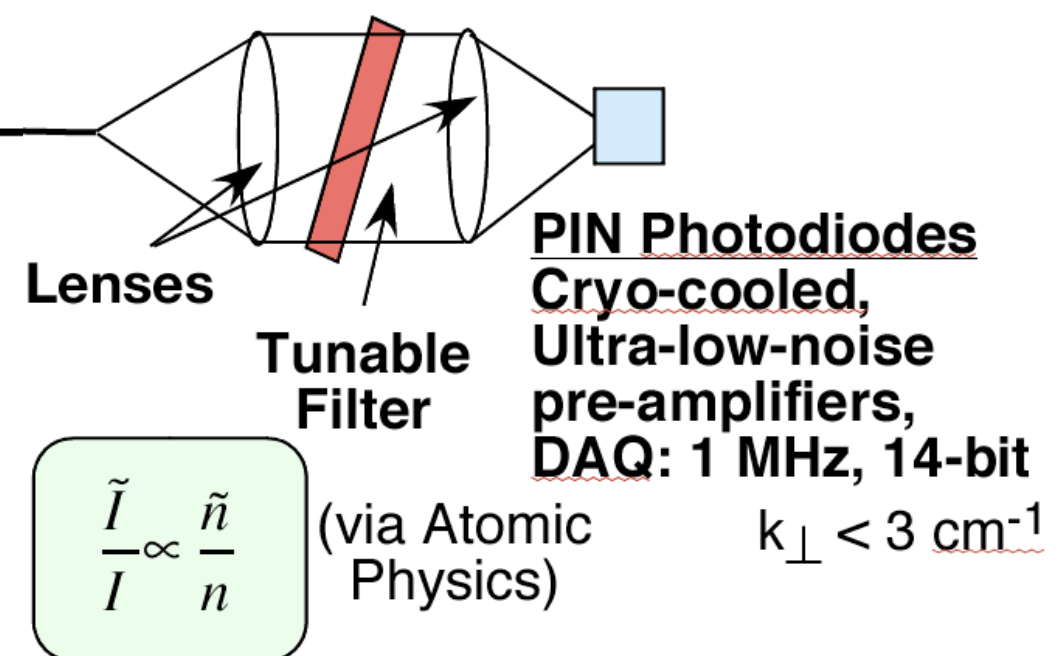
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Exploits Neutral Beam Heating Systems

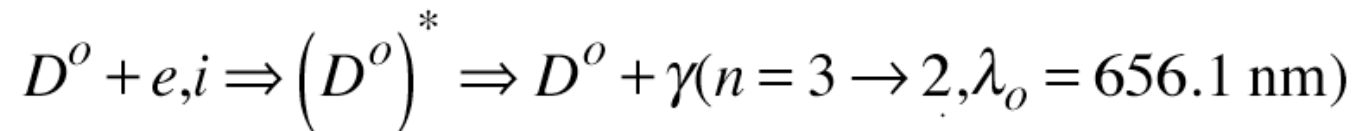


Remotely Located Spectrometers

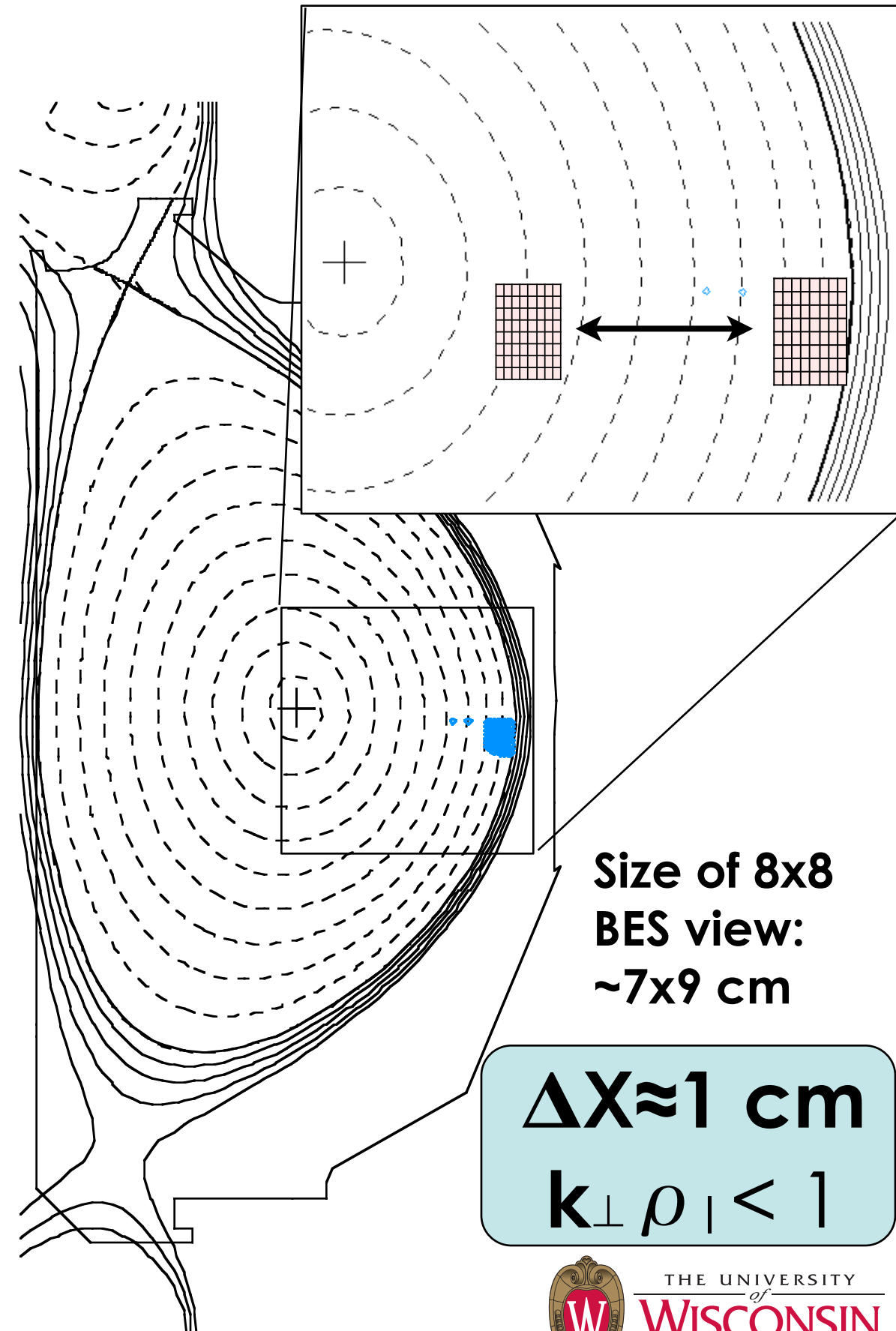
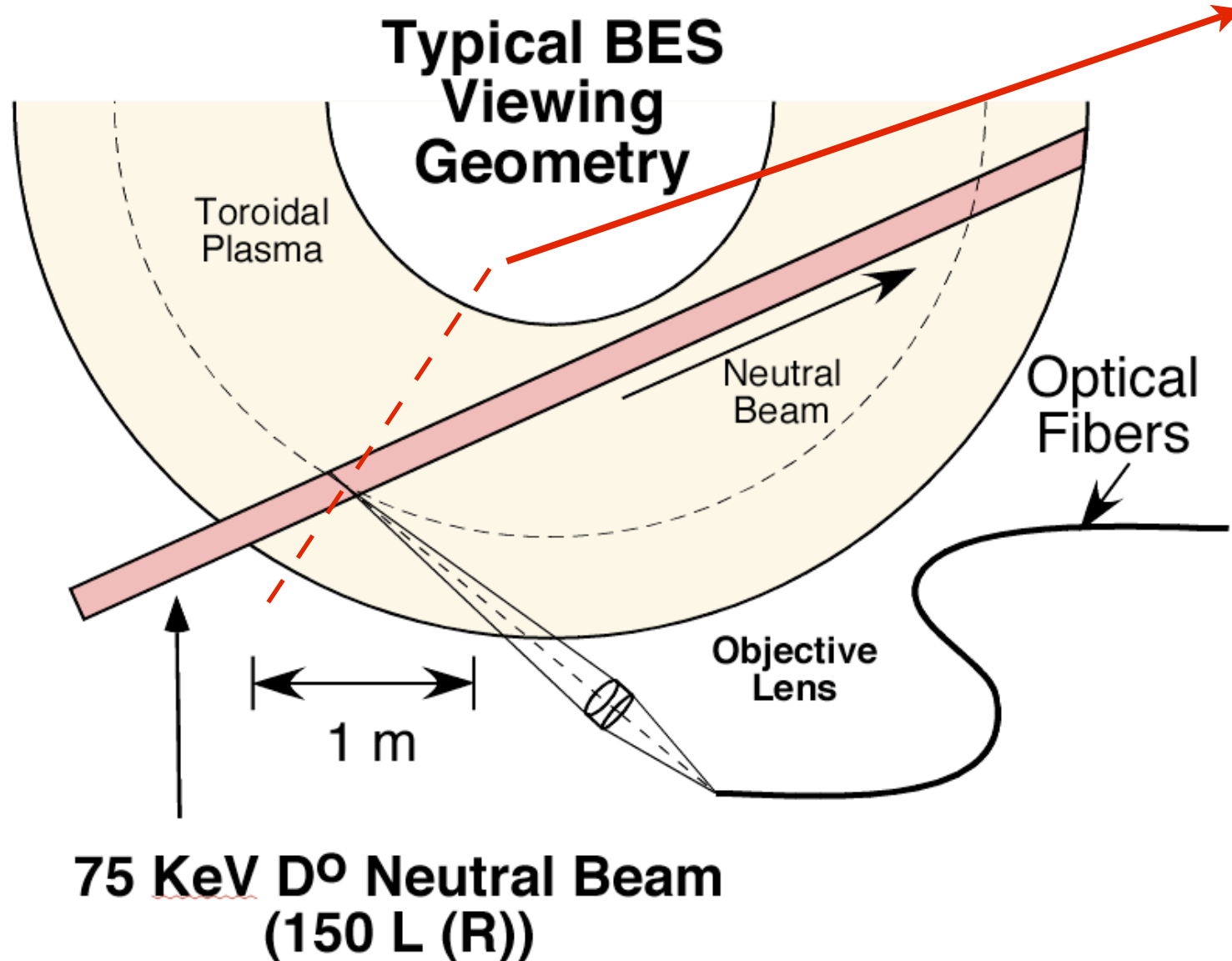


# Beam Emission Spectroscopy Measures Fluorescence of Heating Neutral Beam to Detect Local Density Fluctuations

Collisionally-excited, Doppler-shifted neutral beam fluorescence



Exploits Neutral Beam Heating Systems



# Turbulence Spectrum Evolves Dynamically During Discharge

- Measurements obtained in **Low Confinement-mode Discharges:**

$$I_p = 1 \text{ MA}, B_T = -2.0 \text{ T}$$

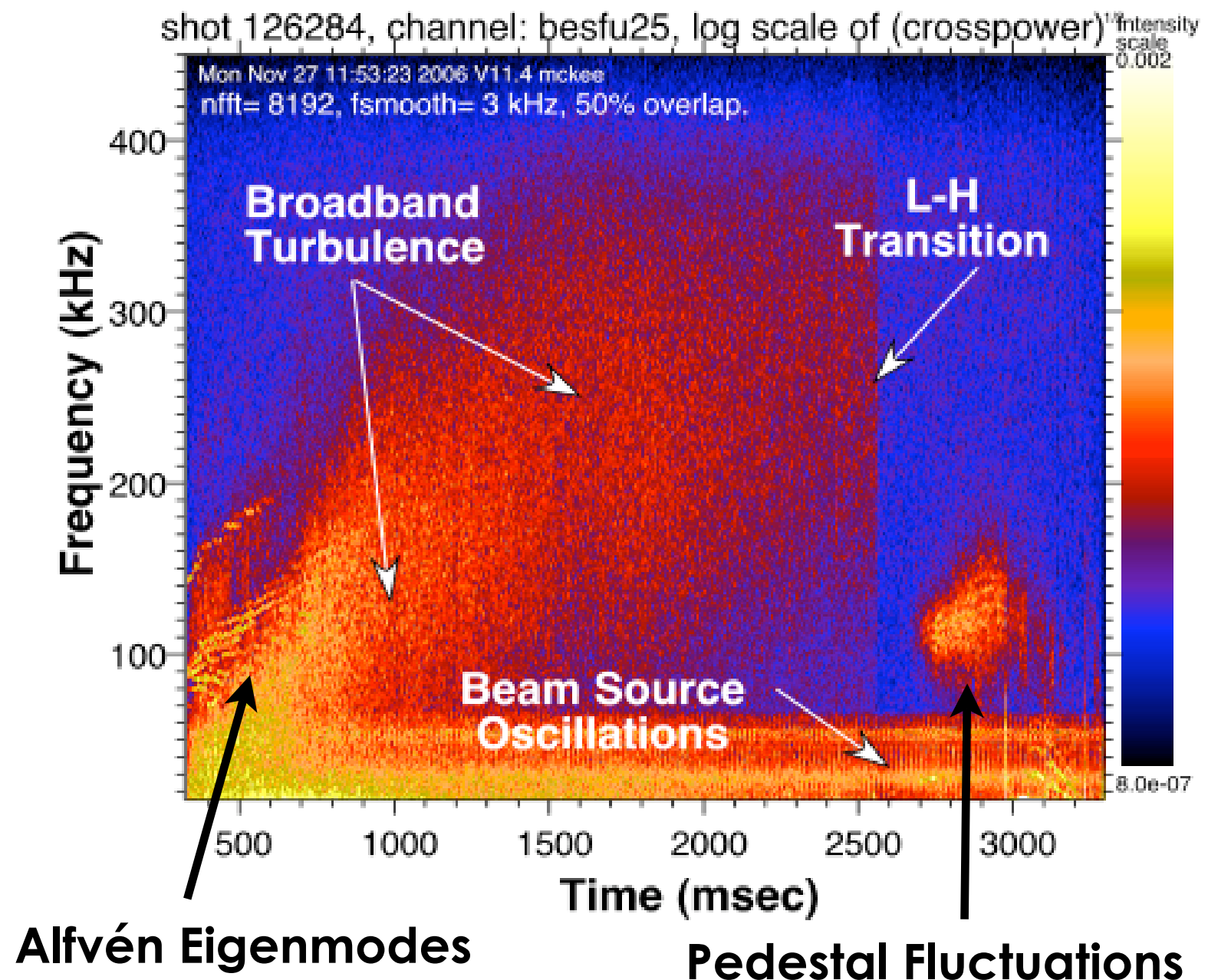
$$P_{inj} = 5 \text{ MW},$$

$$n_{e,o} = 3 \times 10^{19} \text{ m}^{-3},$$

$$T_{e,o} = 2.2 \text{ keV}, T_{i,o} = 2.7 \text{ keV}$$

- **Coherent energetic particle driven-modes observed early in plasma**
- **Broadband turbulence evolves to quasi-steady phase**
- **Fluctuations markedly reduced in core region at LH transition**
- **Beam source oscillations can be isolated and subtracted**

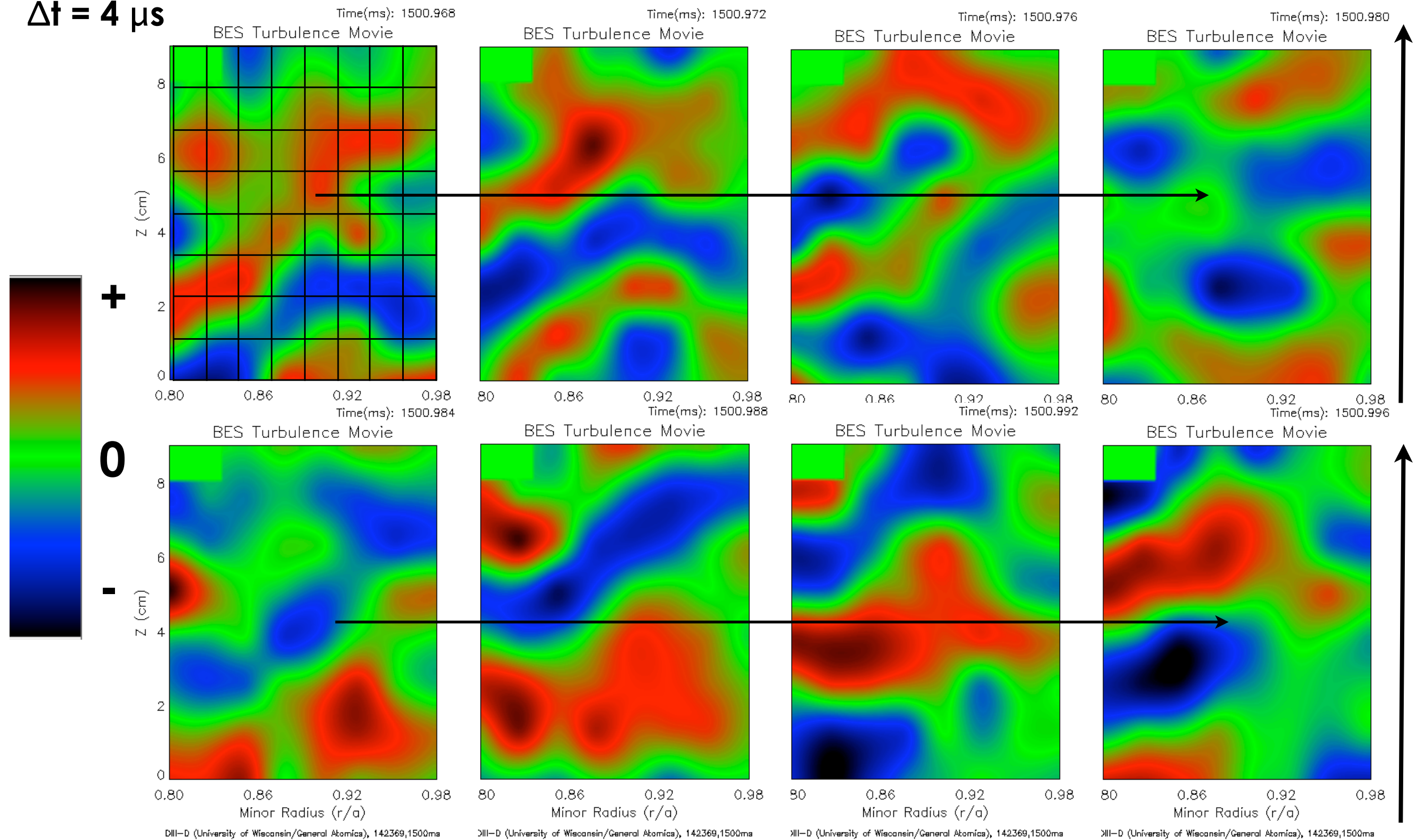
## Cross Power Spectrum at $r/a=0.64$ ( $\Delta Z=1.2$ cm)





# Example Turbulence Images from 8x8 BES Array

$\Delta t = 4 \mu s$



DIII-D (University of Wisconsin/General Atomics), 142369,1500ms

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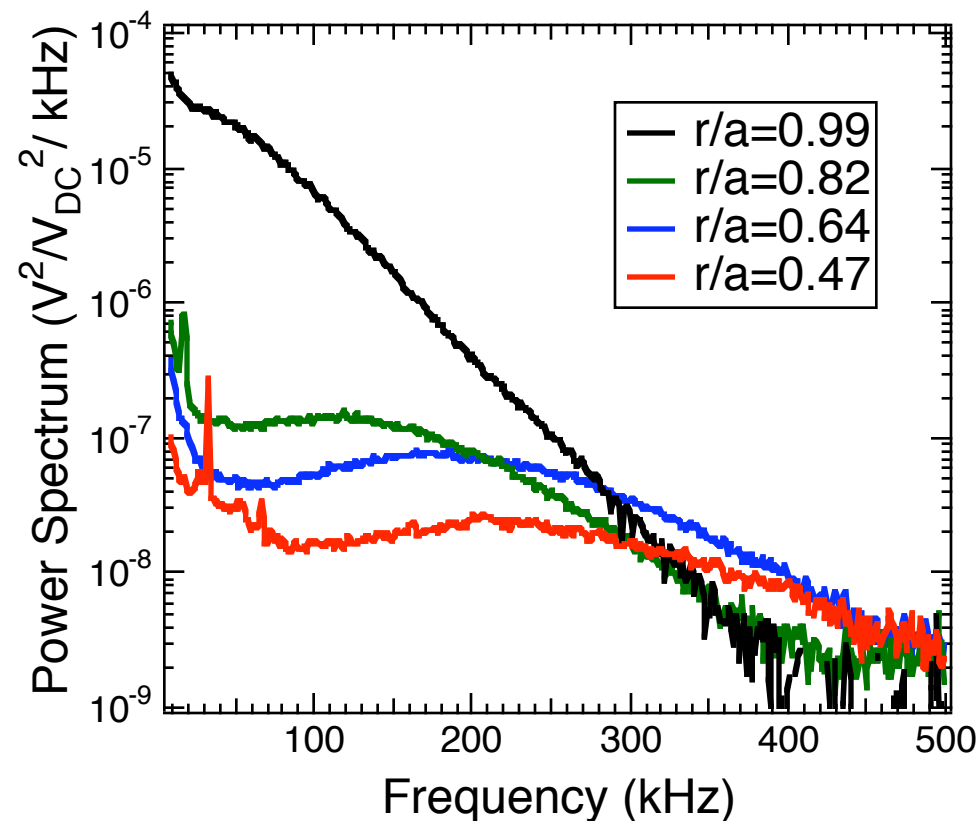
DIII-D (University of Wisconsin/General Atomics), 142369,1500ms

# Features Observed on Visualization

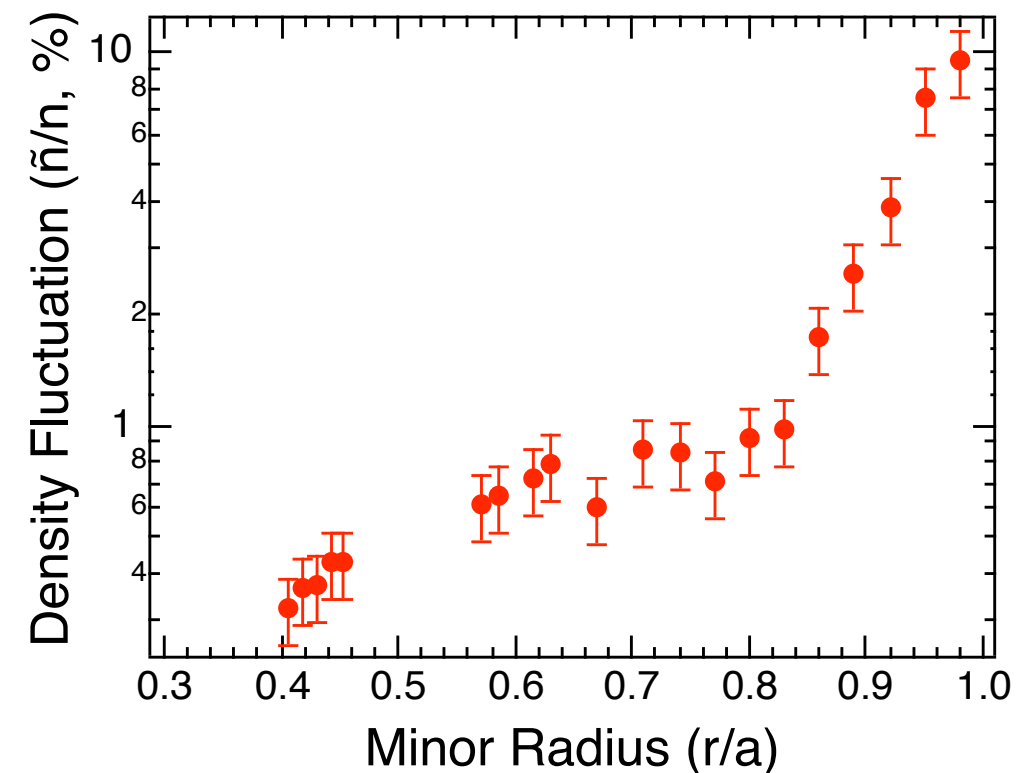
- **Eddy structures exhibit ~1-2 cm correlation lengths ( $\sim 10\rho_i$ )**
  - consistent with ensemble-averaged correlation functions
  - Periodically, significantly larger structures appear
- **Poloidal drift (“upwards” in visualization)**
  - Consistent with ExB drift in ion diamagnetic direction (co-current plasma rotation from neutral beams)
  - Fluctuating radial and poloidal motion
- **Turbulent eddy lifetimes  $\sim 10\text{-}20 \mu\text{s}$**
- **Significant interaction of smaller and larger eddy structures**
  - Evidence of nonlinear interactions; internal energy transfer
- **Shearing of eddies from background sheared flow**
- **Features are not observed in time (ensemble) averaged correlations**

# Fluctuation Spectra and Amplitude Vary Strongly with Radius

## Fluctuation Spectra at Several Radii

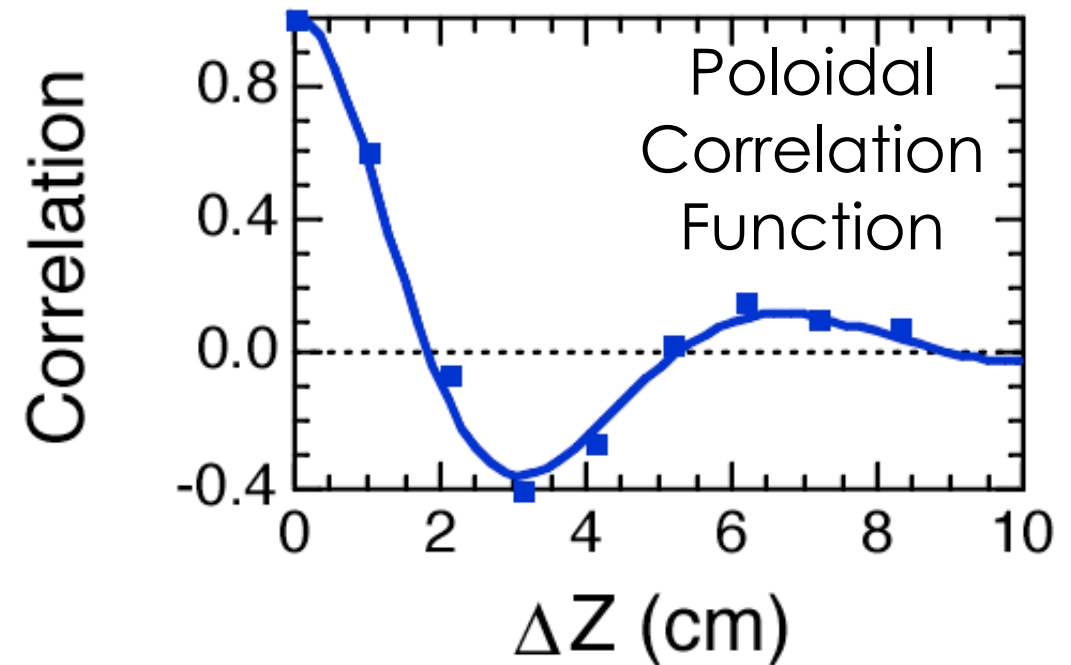
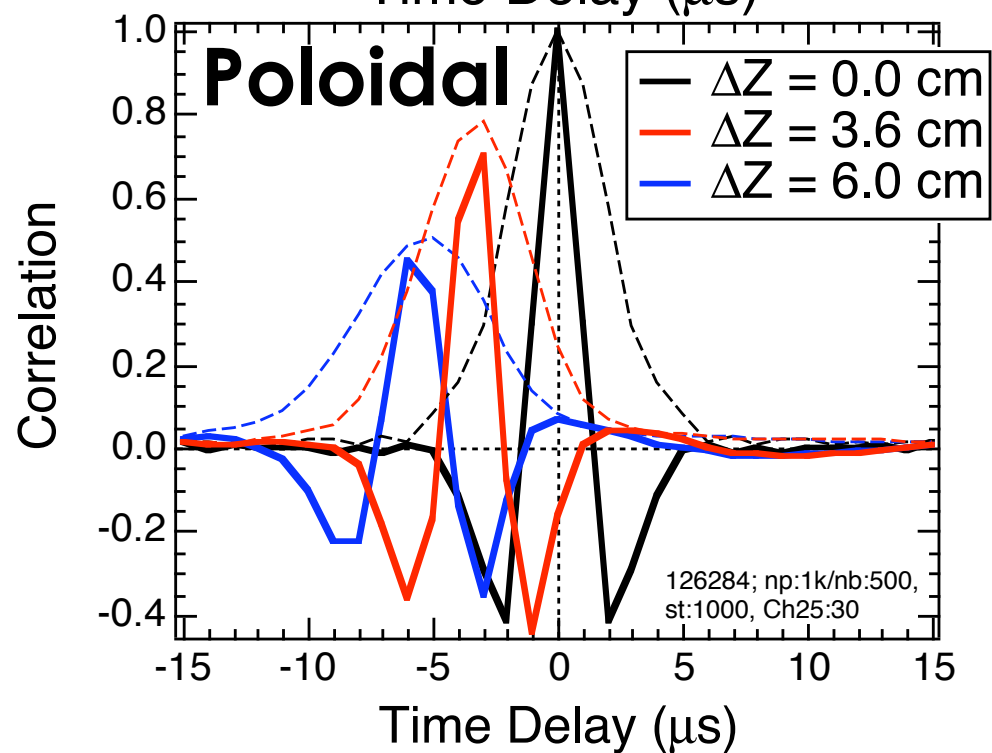
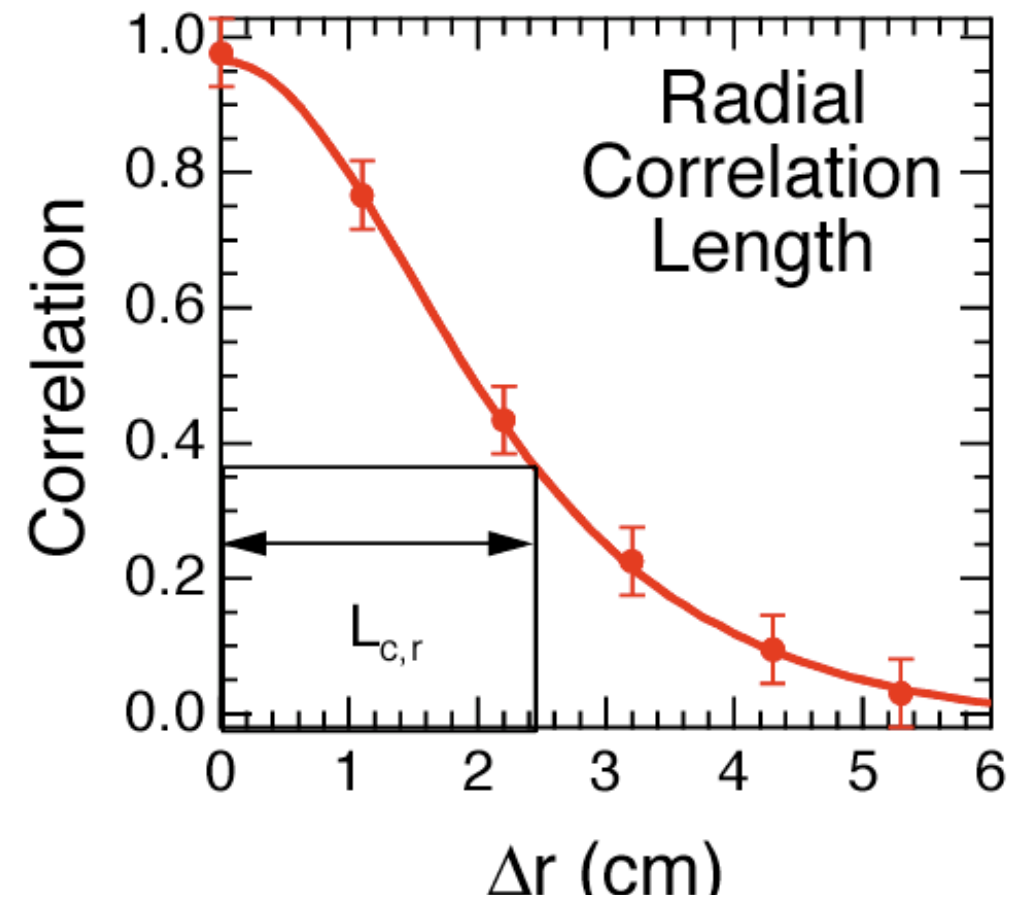
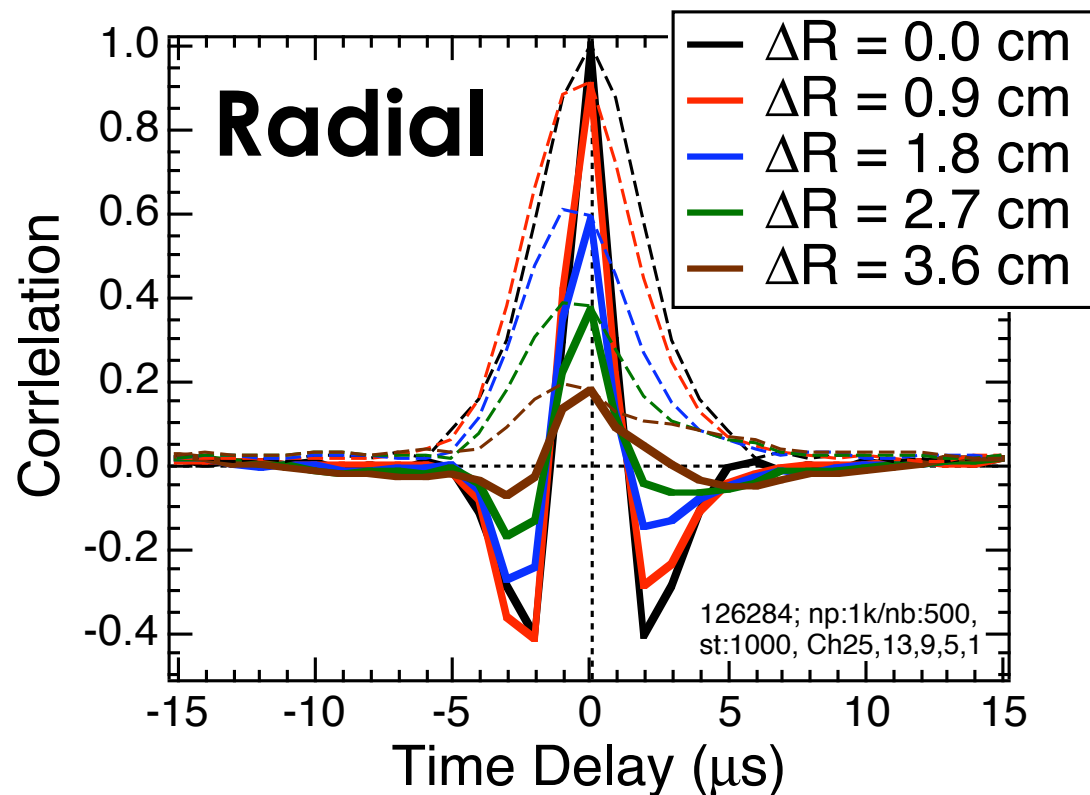


## Density Fluctuation Amplitude Profile



- **Density fluctuation amplitude in L-mode discharges shows wide dynamic range across plasma radius**
  - *Intense edge fluctuations routinely observed in L-mode plasmas*
- **Spectra Doppler-shifted to higher frequency towards core**

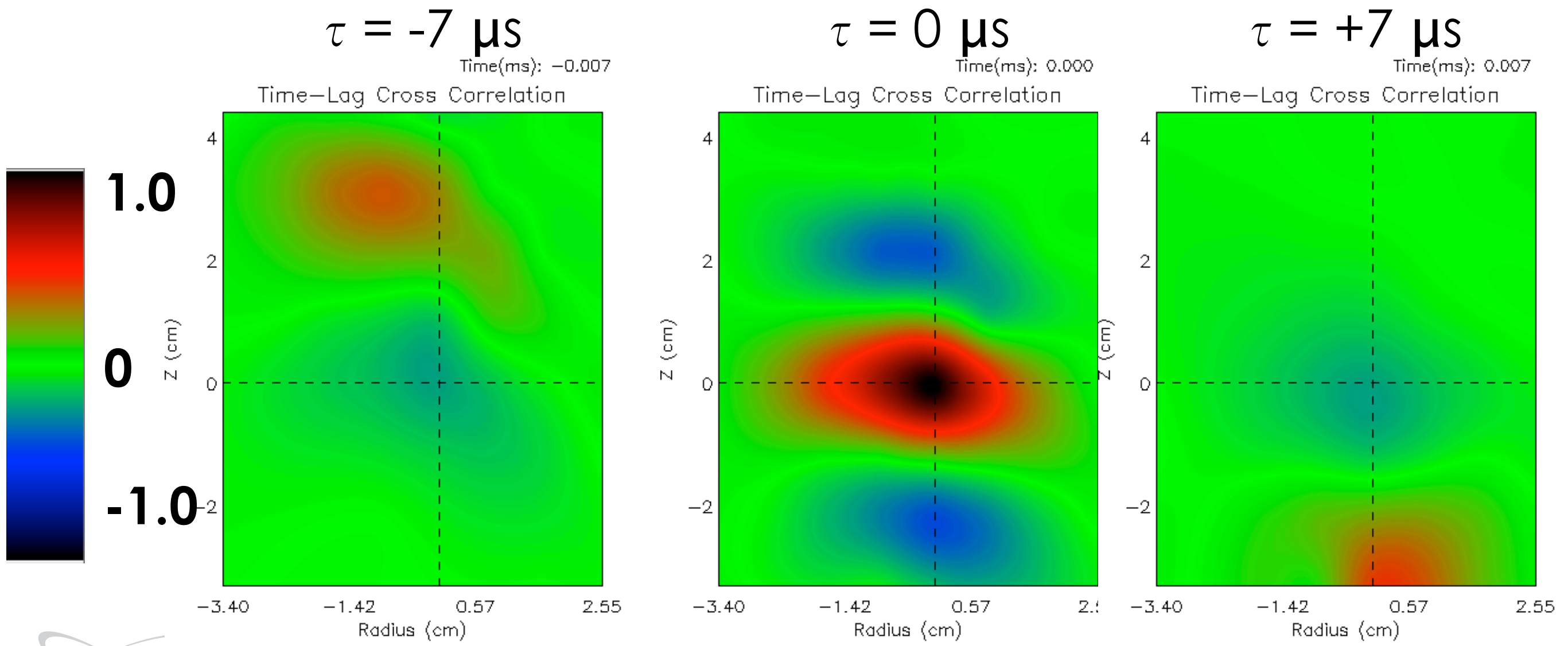
# Turbulence Correlation Lengths $\sim 10$ Ion Gyroradii, consistent with gyrokinetic predictions



$L_{c,r} \sim 2.4$  cm,  $L_{c,\theta} \sim 4$  cm,  $\sim 10 \rho_i$

# 2D Array Observes Full $\tau = 0$ Spatial Cross Correlation Function

- Ensemble-averaged, time-resolved 2D cross correlation function assembled from individual 2-point cross-correlations
- Illustrates poloidal advection, and alignment of 2D grid to flux surfaces
- Point-Spread-Function ( $\Delta X \sim 1$  cm) NOT deconvolved from data



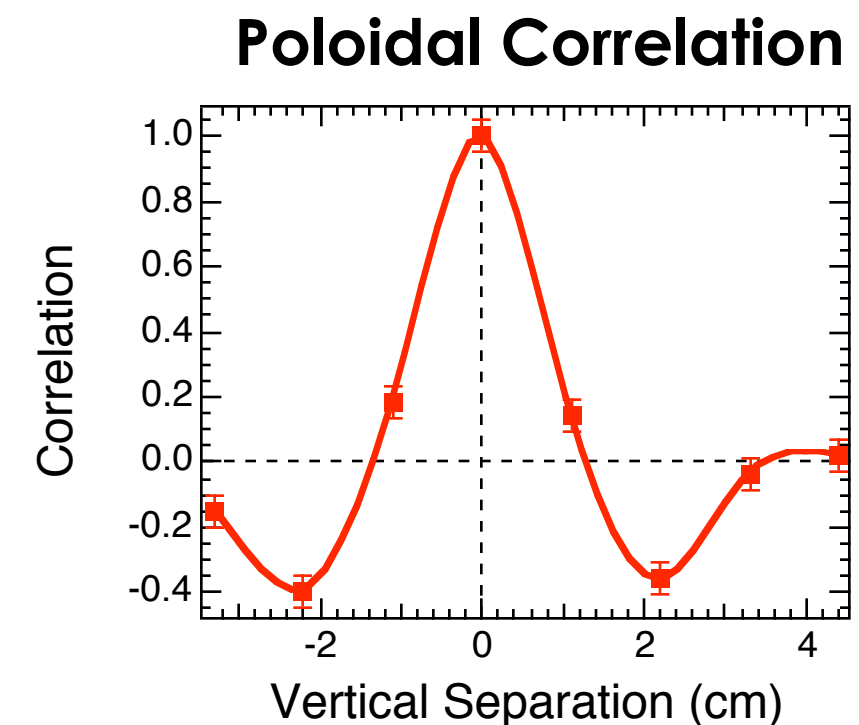
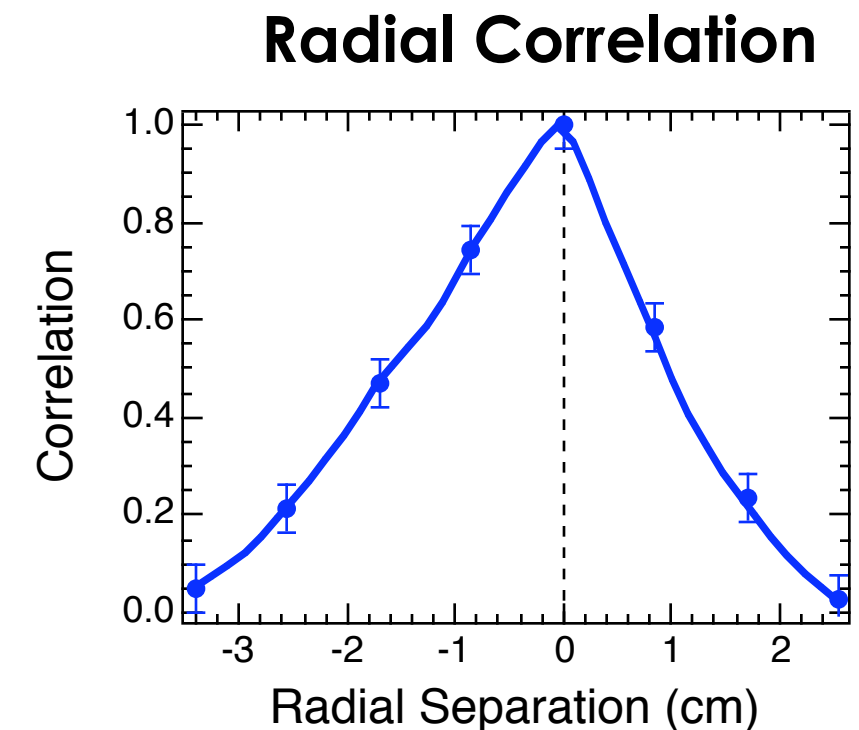
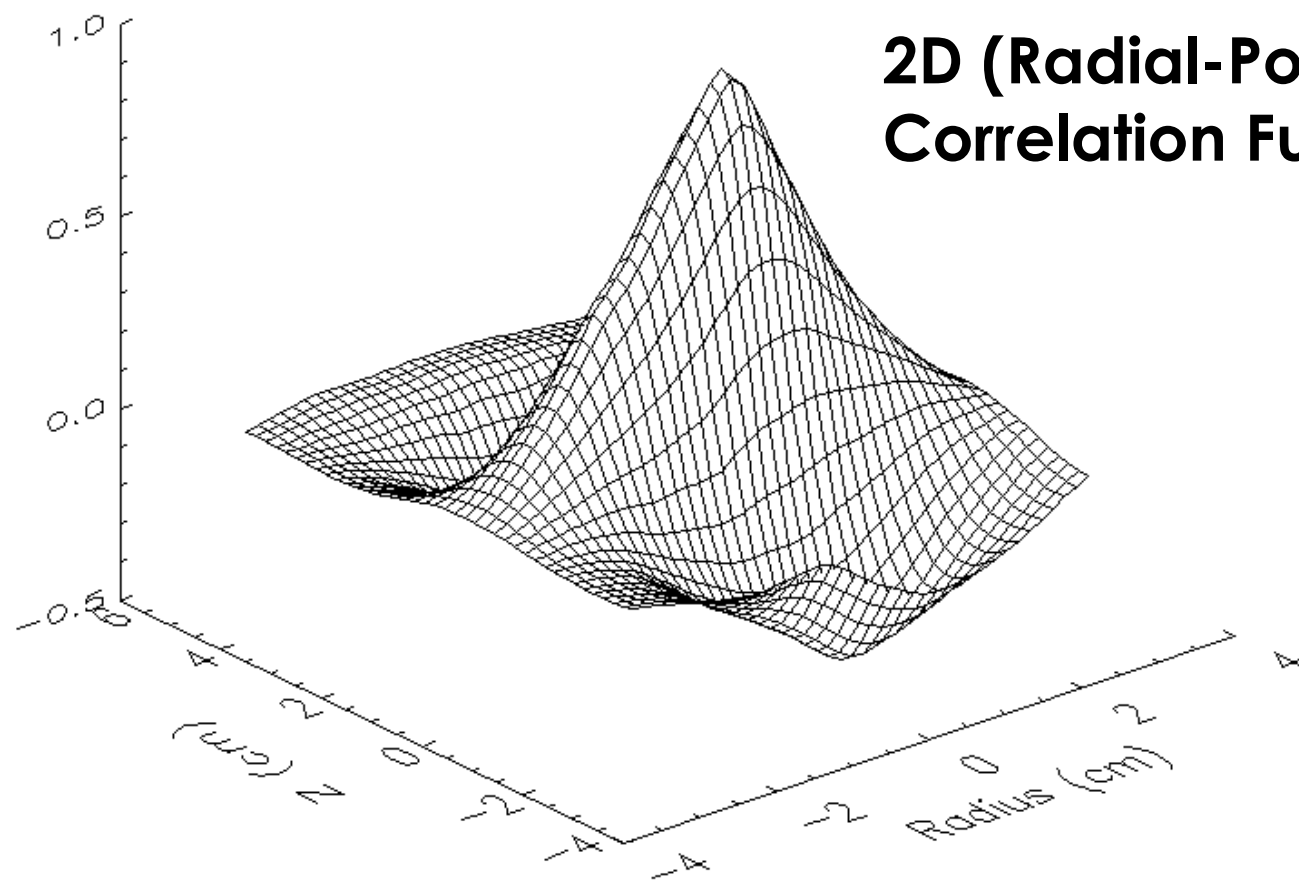
DIII-D (University of Wisconsin/General Atomics), 142369,1500ms

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# Full $\tau = 0$ Spatial Cross Correlation Function, $S(k_r, k_\theta)$ spectra measured with 2D 8x8 Array

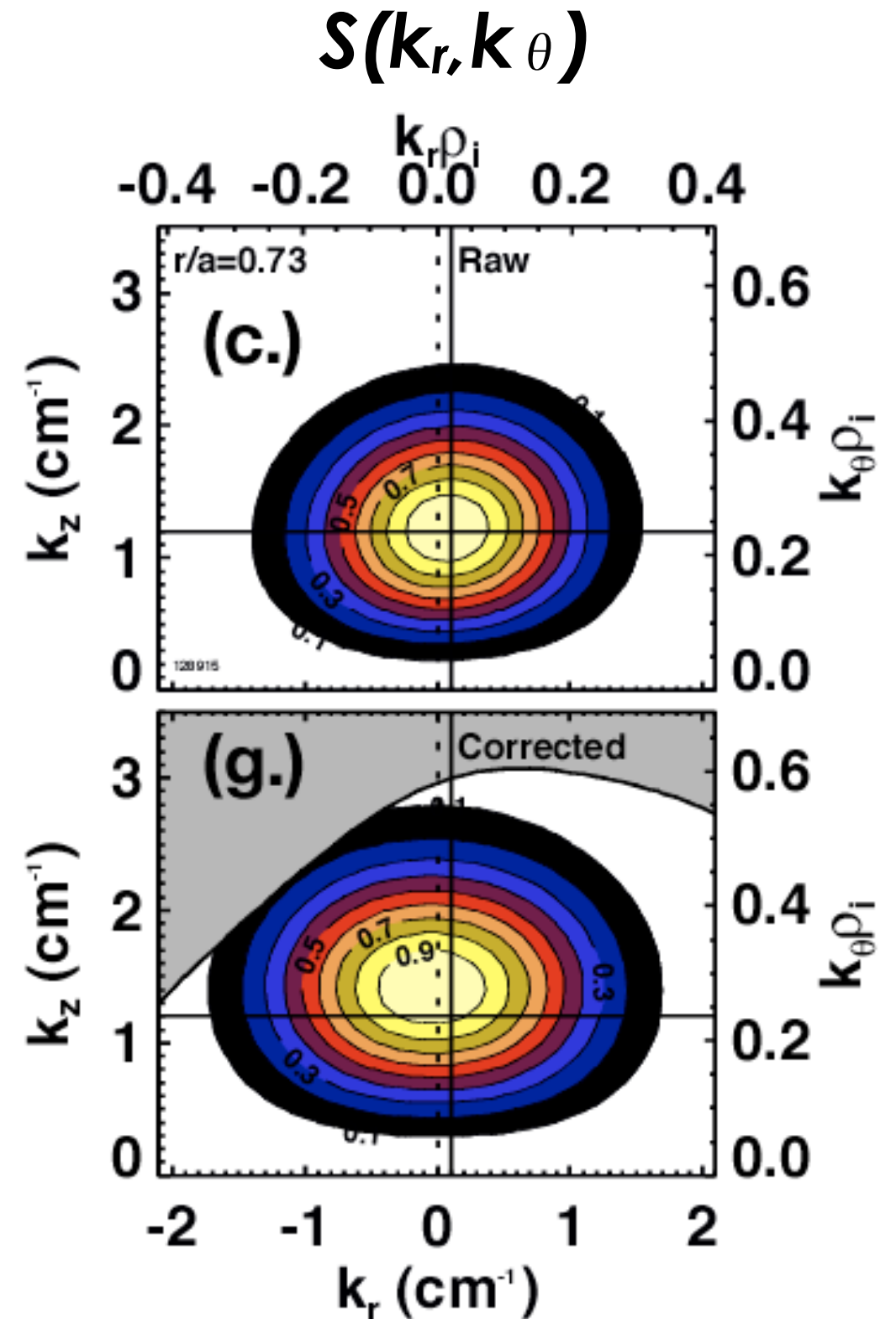
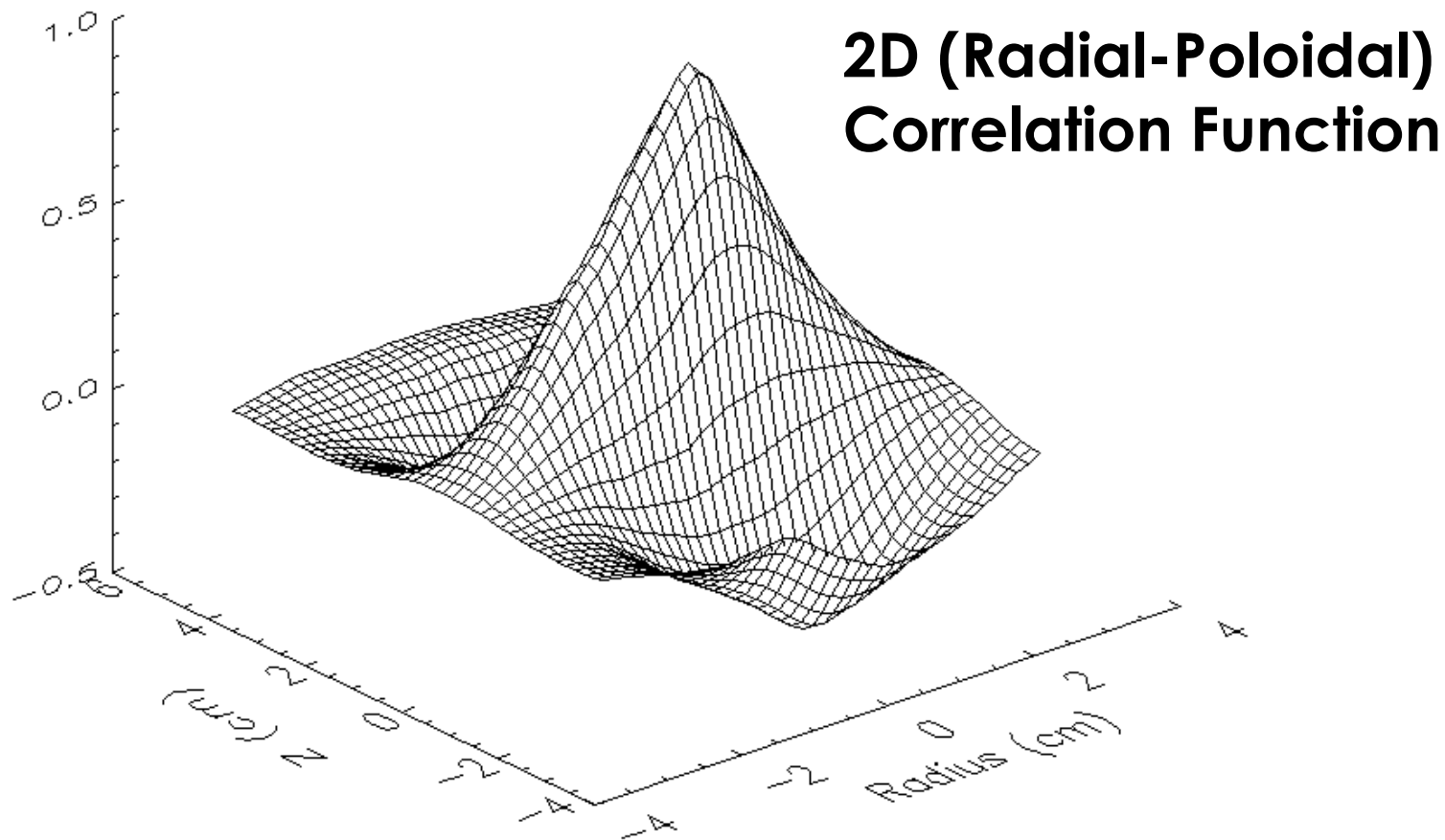
- 2D correlation function exhibits wave-like poloidal structure and decaying radial structure
  - Point Spread Function not yet applied
  - Radially asymmetric function



# Full $\tau = 0$ Spatial Cross Correlation Function, $S(k_r, k_\theta)$ spectra measured with 2D 8x8 Array

- 2D correlation function exhibits wave-like poloidal structure and decaying radial structure

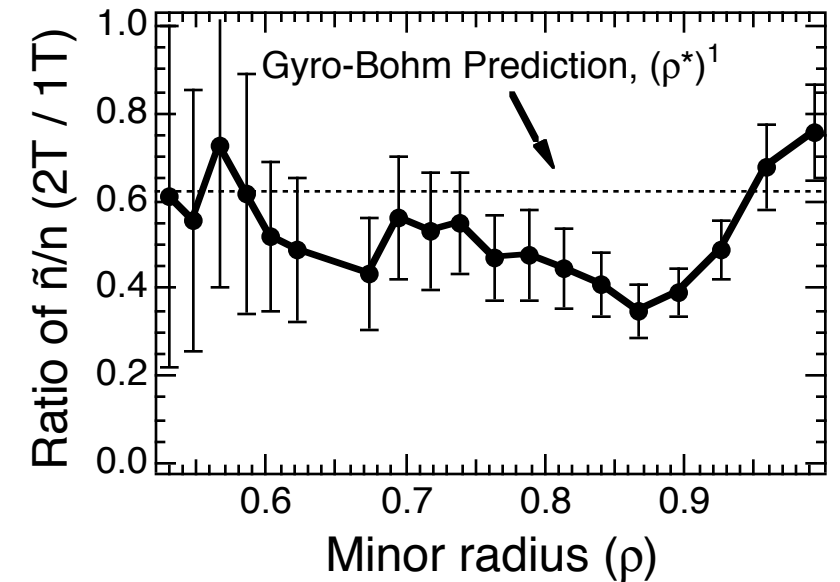
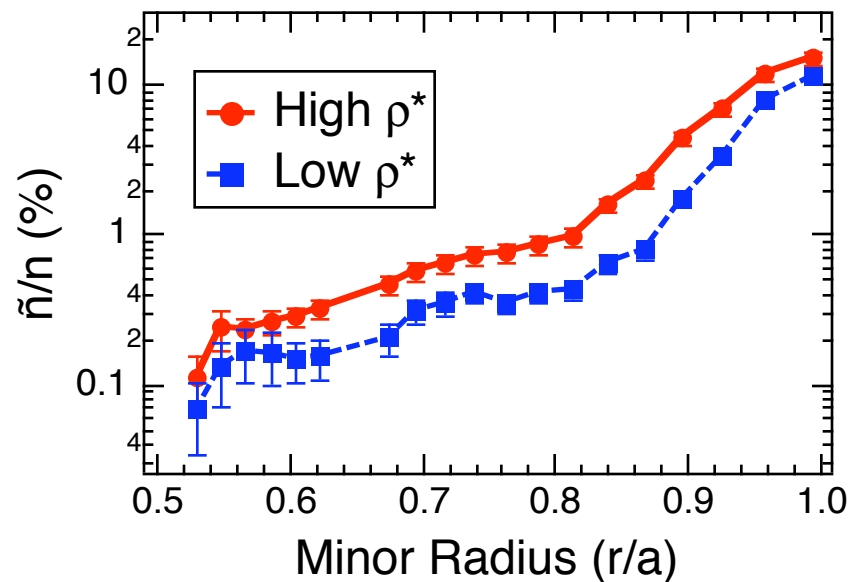
- Point Spread Function Deconvolved
- Radially asymmetric function



# Spatiotemporal Features of Turbulence Scale with $\rho_i^*$ , as Expected from Gyrokinetic Equations

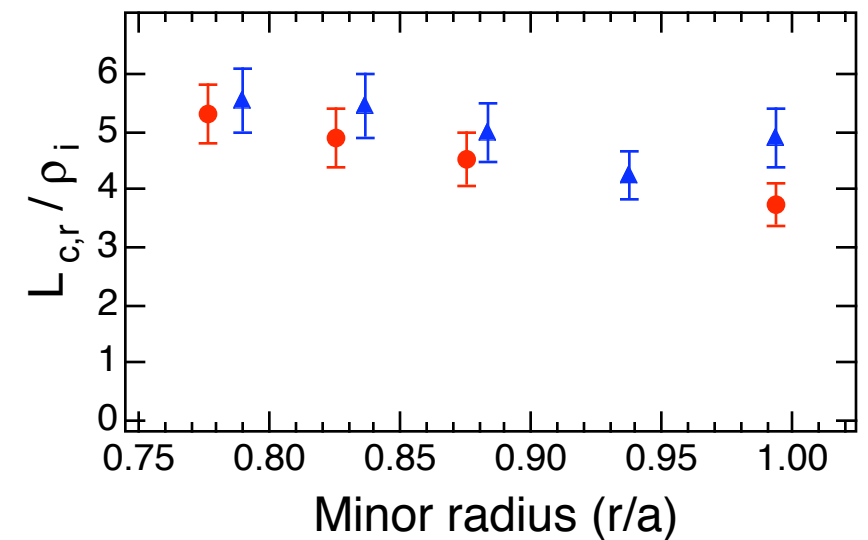
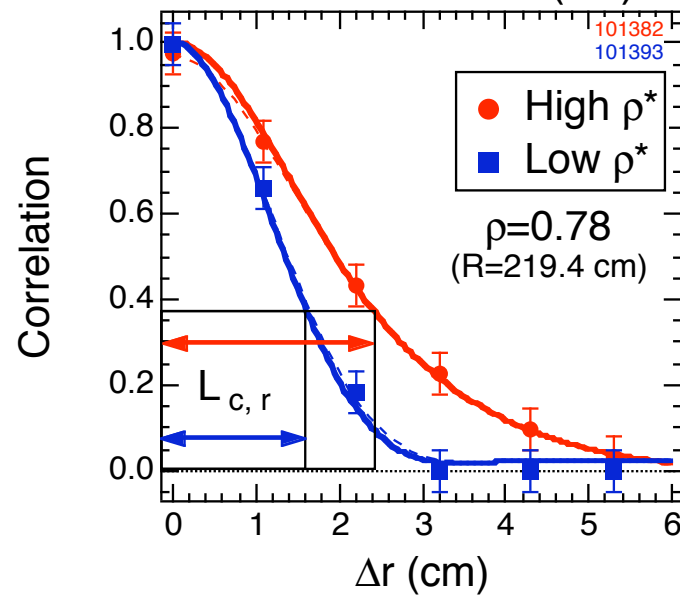
- **Fluctuation amplitude**

- Scales as  $\tilde{n}/n \sim \rho_i^*$



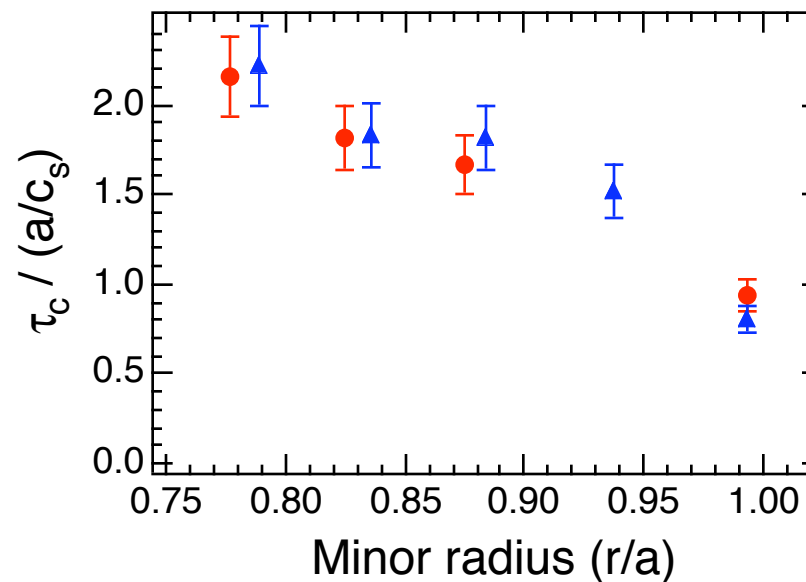
- **Radial Correlation Length**

- Scales with ion gyroradius,  $\rho_i$



- **Decorrelation time**

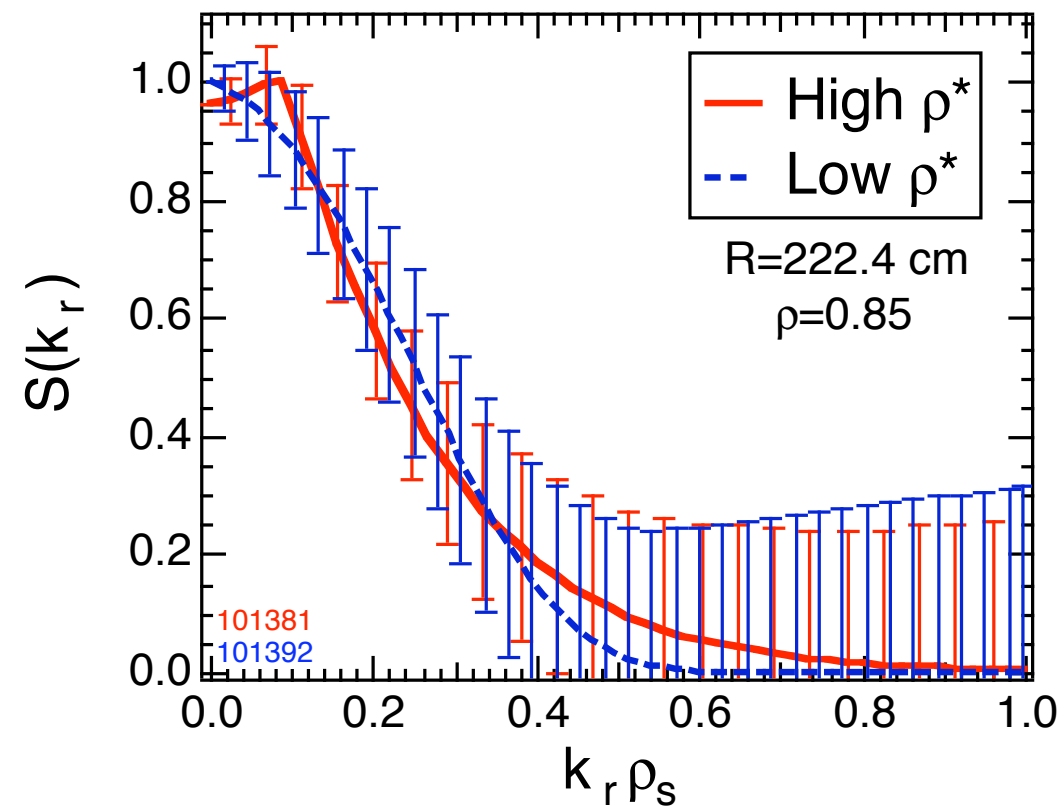
- $\tau_c \sim a/c_s$



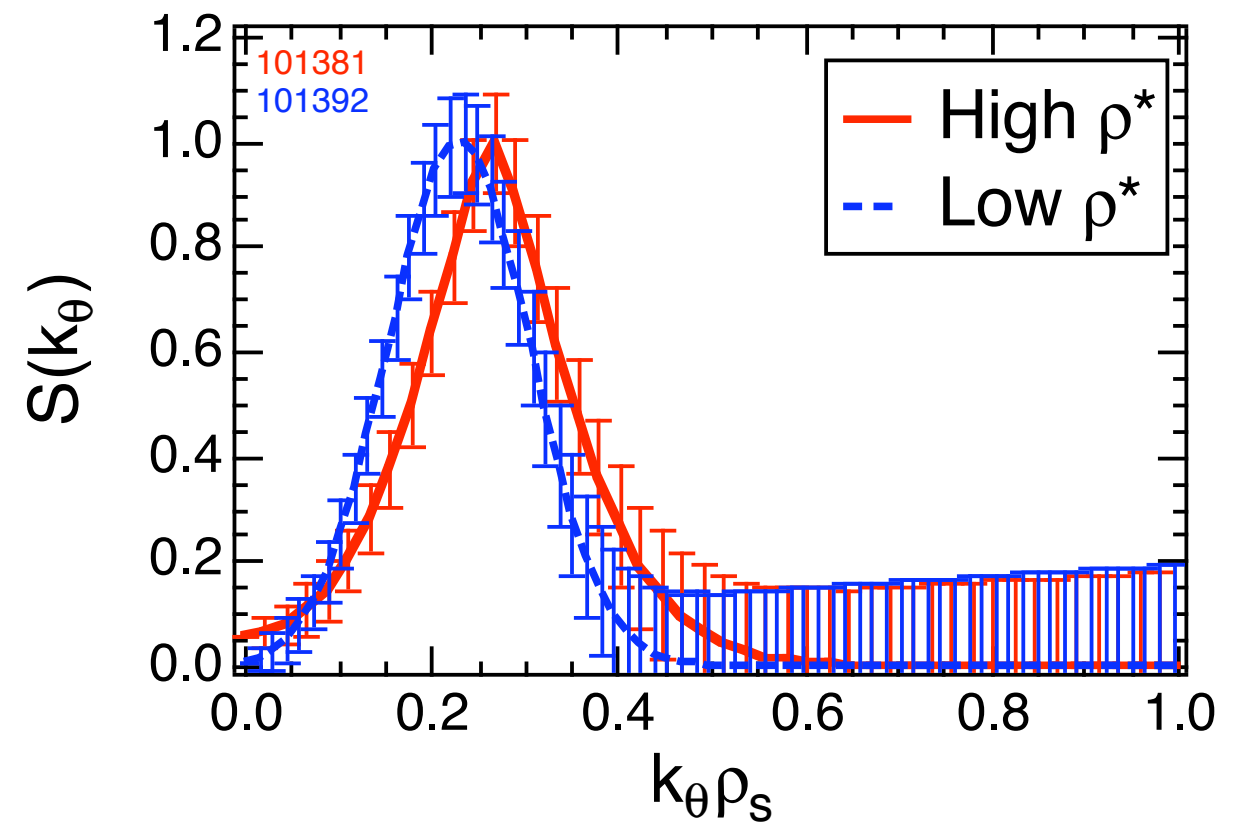


# Measured Wavenumber Spectra are Matched as $\rho_i^*$ Varied

## $S(k_r)$ Spectrum



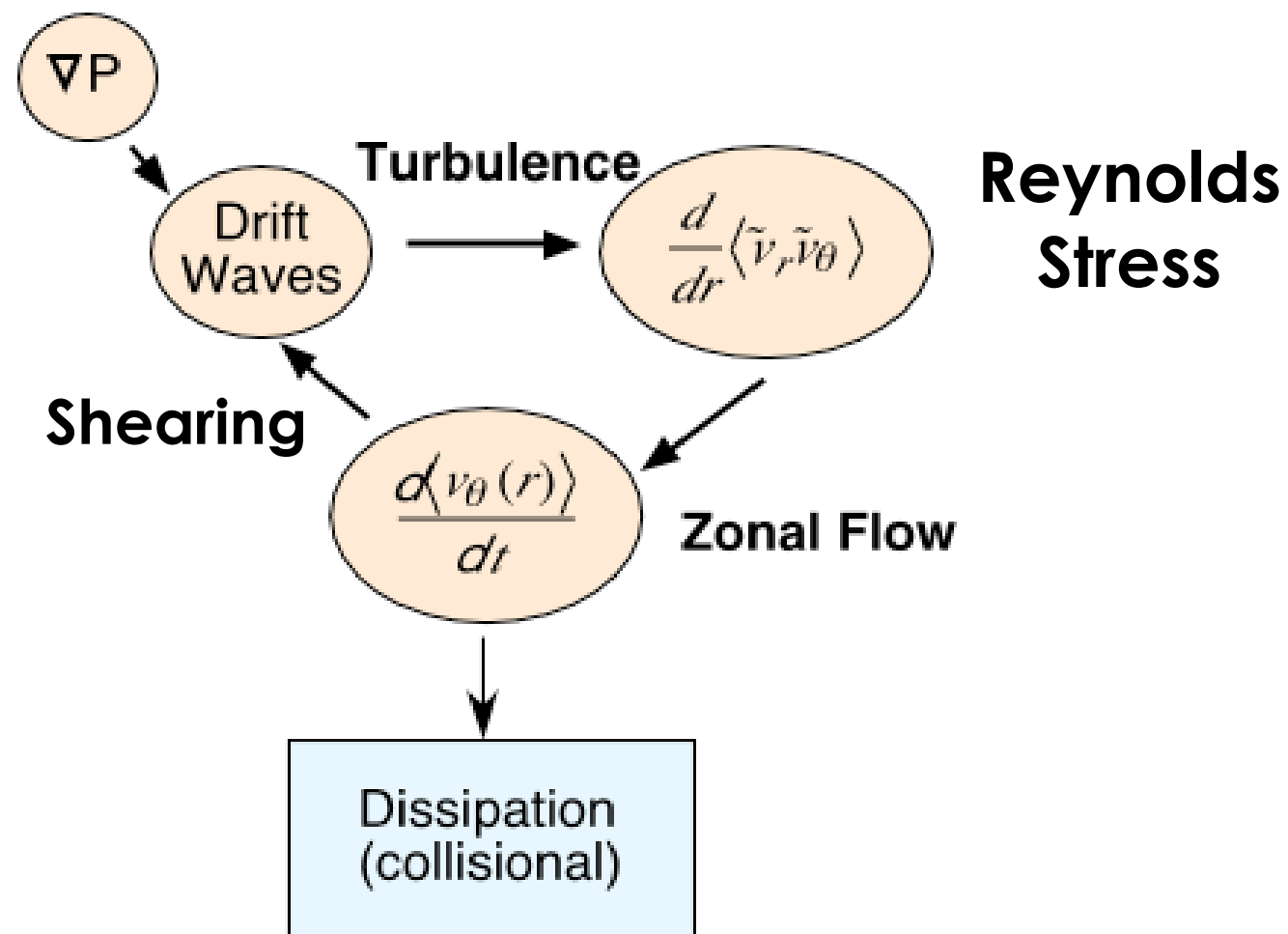
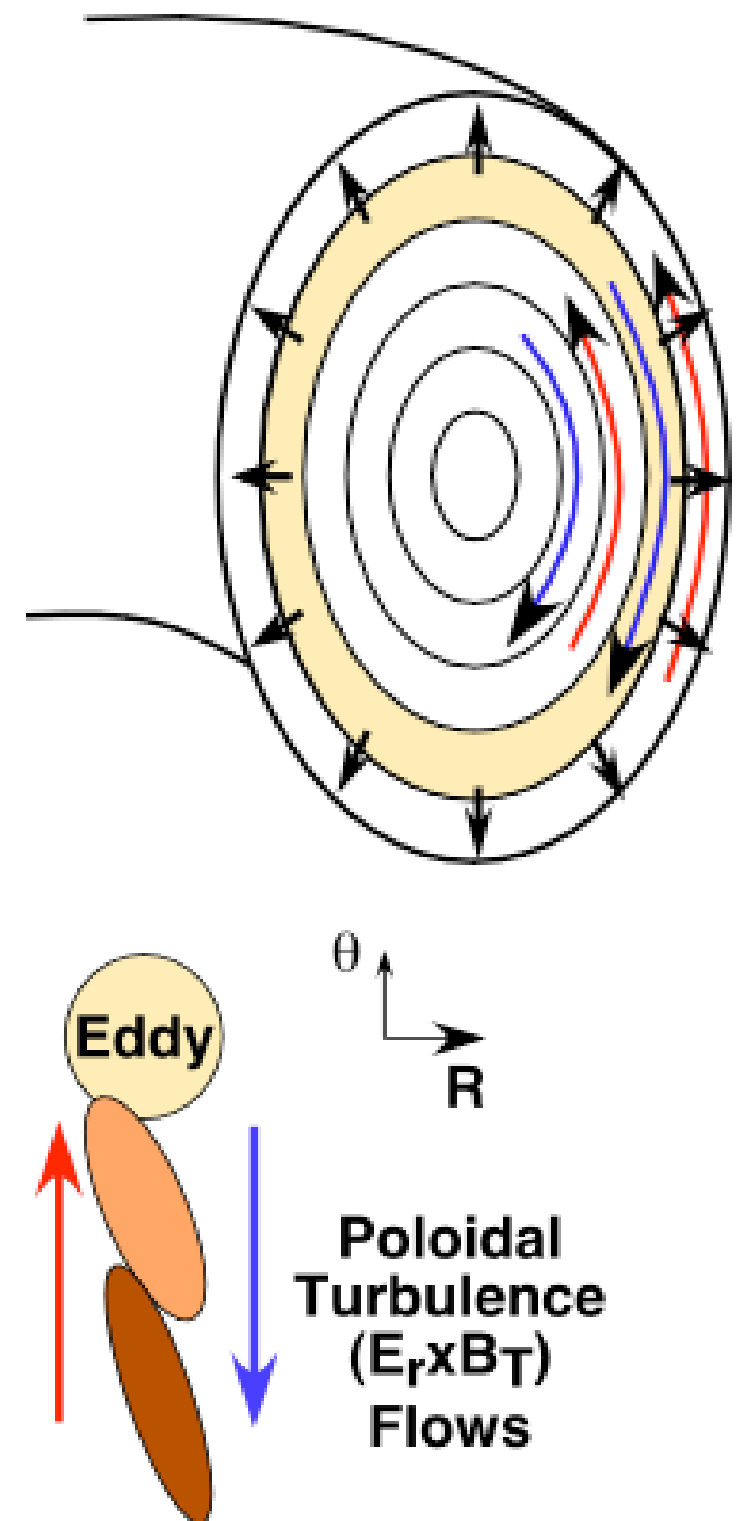
## $S(k_\theta)$ Spectrum



- Normalized spectra are nearly self-similar

# Theoretically-predicted Zonal Flows Thought Crucial to Mediating Fully Saturated Turbulence in Plasmas

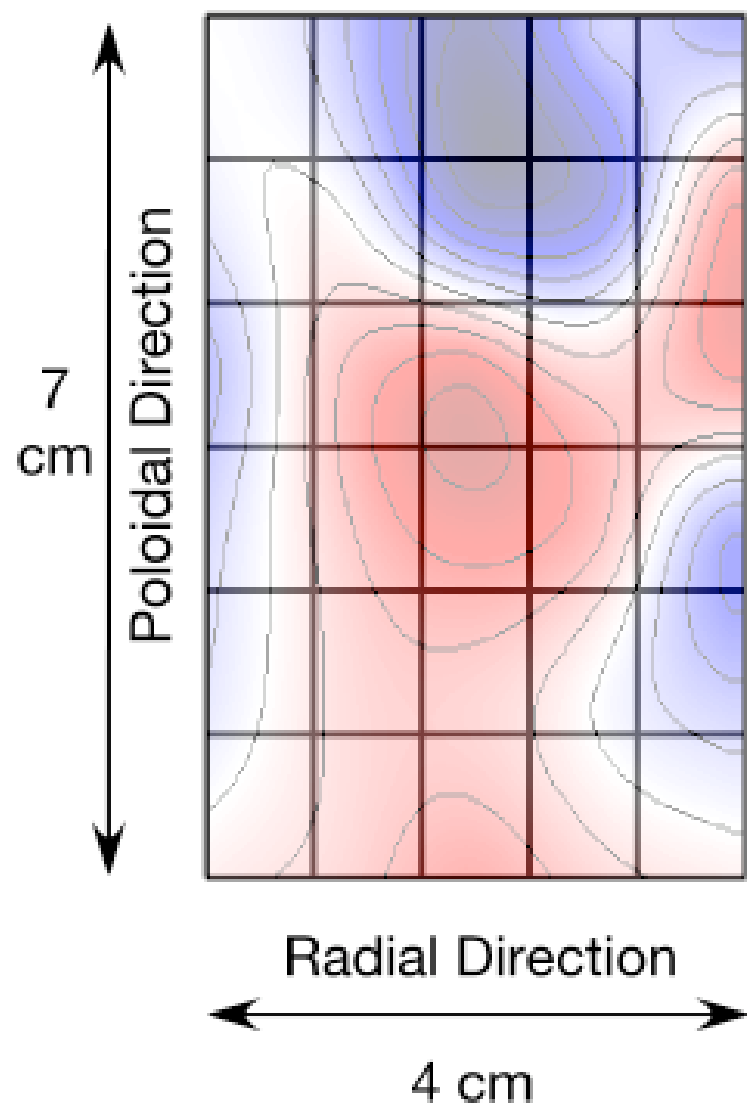
- Regulate turbulence via fluctuating  $E_r \times B_T$  ( $v_\theta$ ) flows
  - Observed in turbulence simulations
- Radially-localized,  $n=0$ ,  $m=0$ , electrostatic potential
  - 1) Zero-Mean-Frequency zonal flow (ZMF-ZF,  $\Delta f \sim \nu_{ii} < 10$  kHz)
  - 2) Geodesic Acoustic Mode (GAM, coherent, 10-20 kHz)



*P. Diamond, S. Itoh, K. Itoh, T.S. Hahm, PPCF 47, R35 (2005).*

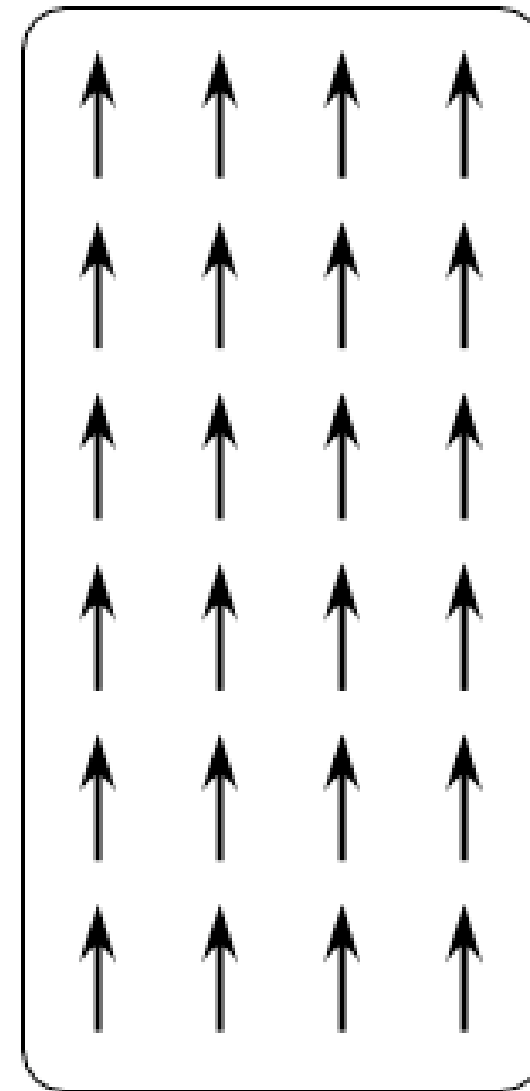
# Time-Varying Turbulence Flows Measured Via 2D $\tilde{n}$ with BES to Discern Zonal Flow Characteristics

Turbulence imaged with discrete channels deployed on 2D grid



**Time Delay Estimation between poloidally-adjacent channels:**

- 1) Wavelet-based cross-phase ( $\Delta\theta(t)$ )
- 2) Time-resolved cross-correlation ( $\tau_{\rho, \max}(t)$ )
- 3) Dynamic Programming (vector matching method)

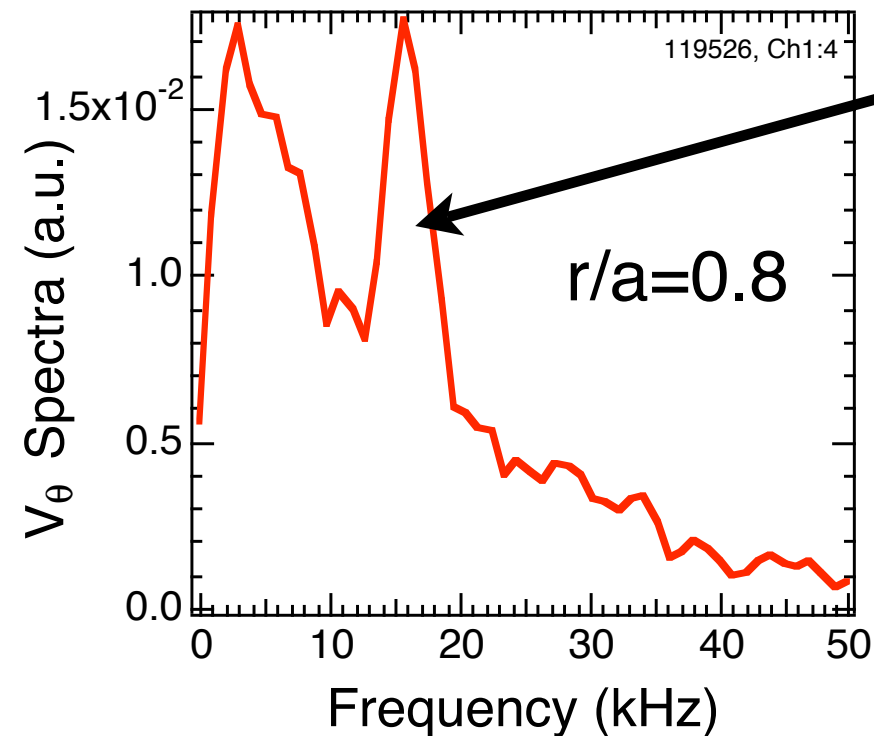


$V_{\theta}(R, Z, t)$   
on relevant time scale  
( $0 < f < 200$  kHz)

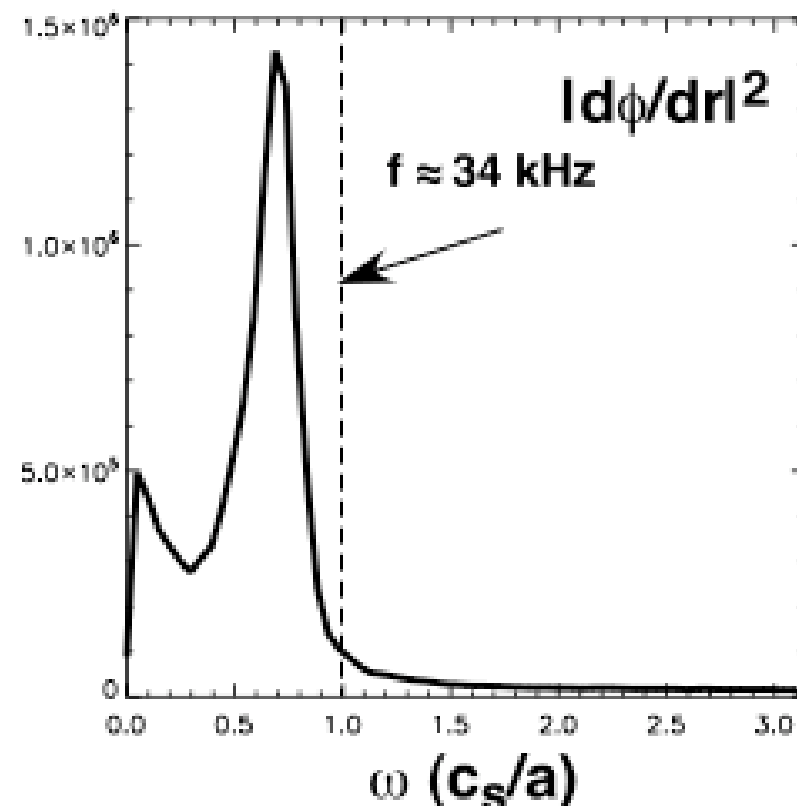
Spectral and spatial analysis of  $v_{\theta}(R, Z, t)$  to search for flow features

# Zonal Flow Features Observed in the $V_\theta$ Spectrum

- Spectrum shows broad, low-frequency structure:
  - Peaks near zero frequency
  - Width,  $\Delta f \sim 20$  kHz
- GAM also clearly observed near  $f = 15$  kHz
  - Observed on DIII-D and other experiments (JFT-2M, ASDEX, HL-2A, JIPP-TIIU, CHS)
- GYRO simulation of zonal flow spectrum exhibits qualitative similarity to measured spectrum



**GAM**  
Poloidal velocity spectrum measured with BES

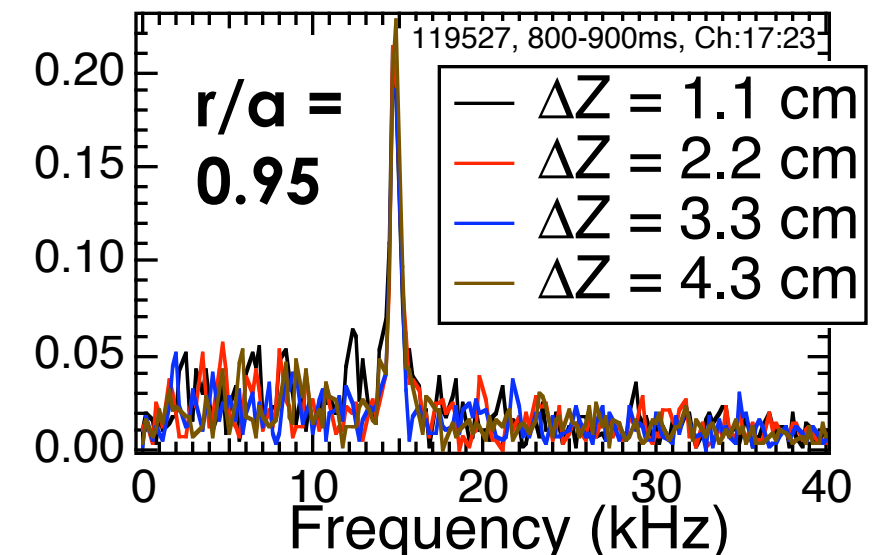
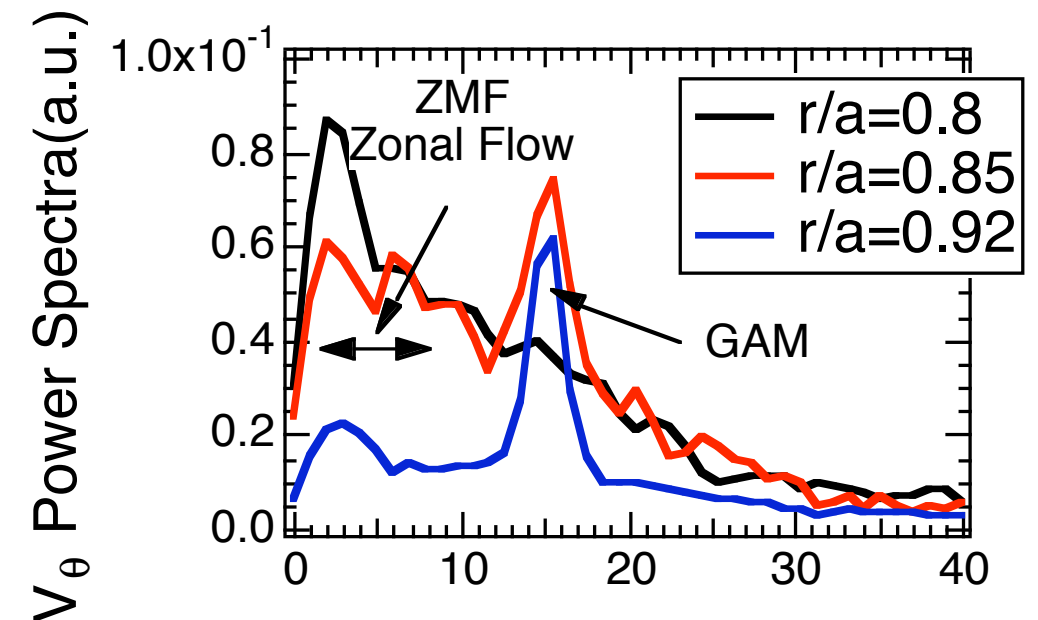
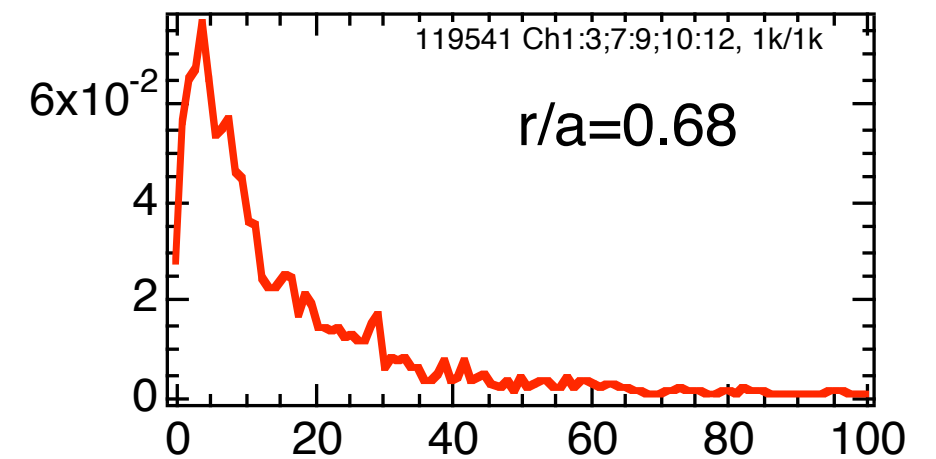


**GYRO**  
( $q=3$ , flux tube, kinetic electrons)

# Transition from a low-frequency Zonal Flow in core to a GAM-dominated flow in edge region

- **Velocity spectra exhibit broad Zero-Mean-Frequency Zonal Flow spectrum for  $r/a < \sim 0.8$**
- **Broad ZMF-ZF spectrum and GAM superimposed near  $r/a=0.85$**
- **Geodesic Acoustic Mode dominates spectrum for  $r/a > 0.9$** 
  - $f_{\text{GAM}} = c_s/2\pi R$
- **Theory and simulation predict ZMF-ZF to dominate at lower  $q$  (core) while GAM dominates at higher  $q$  (edge)**
- **High coherence,  $f/\Delta f > 20$ , indicates GAM lifetime ( $\tau_{\text{GAM}} > 1$  ms), two orders of magnitude longer than turbulence decorrelation time:**

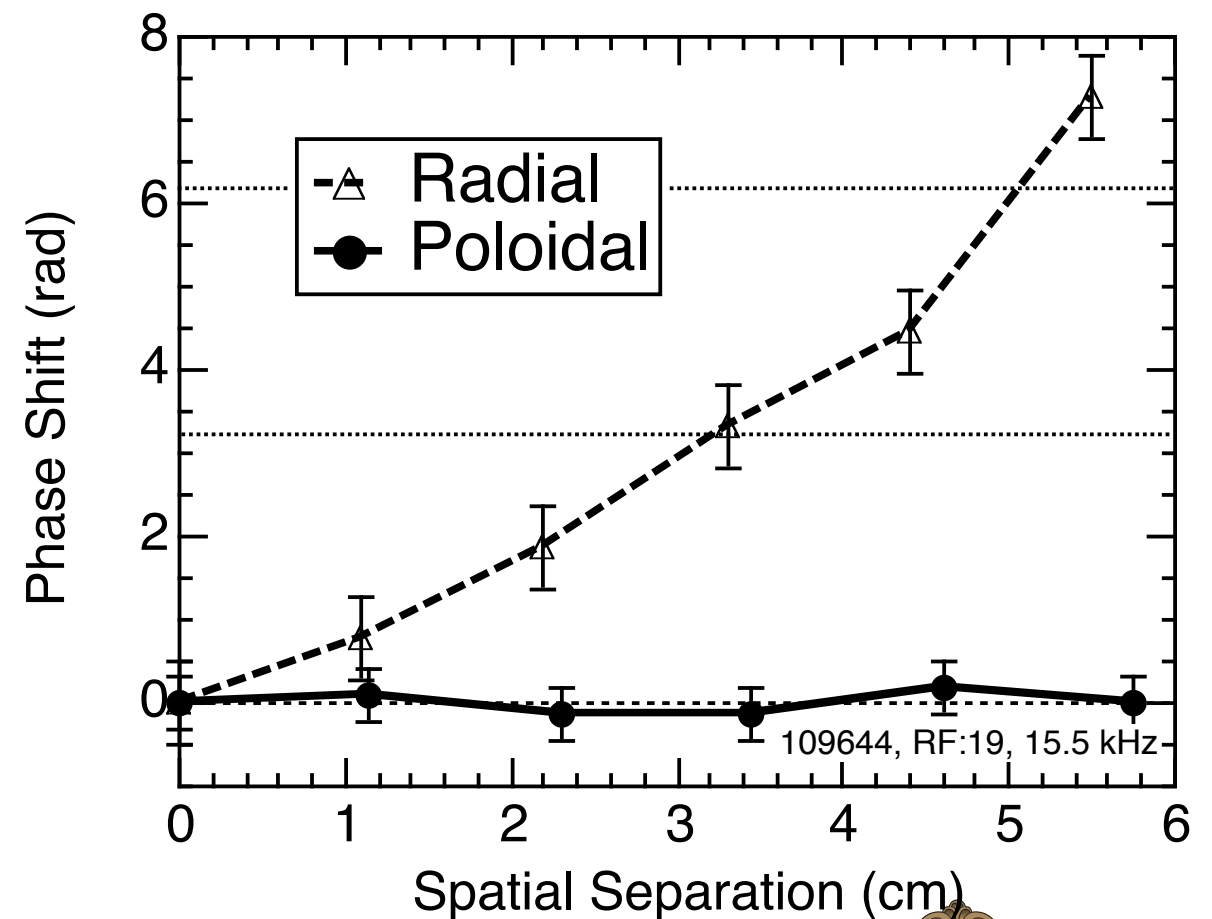
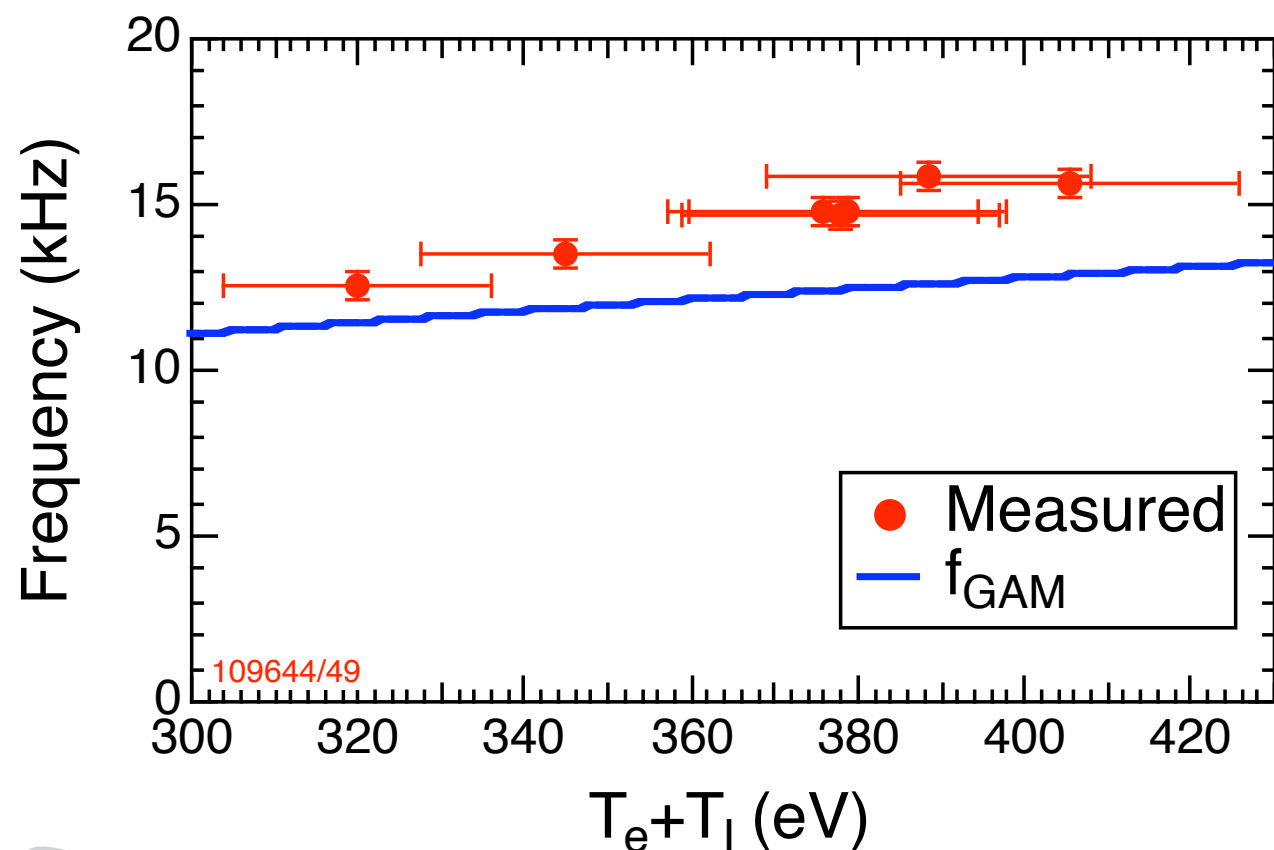
$$\tau_{\text{GAM}} \gg \tau_{\text{Turbulence}} (\sim 10 \mu\text{s})$$



# GAM Exhibits Theoretically-Expected Temperature and Spatial Dependence

- Frequency scales closely with sound speed
- Velocity oscillation exhibits little or not poloidal phase shift
  - $|m| < 3$ , consistent with expected  $m=0$
- Radial phase shift, finite radial wavenumber:  $k_r \sim 1 \text{ cm}^{-1}$ 
  - Finite shearing rate sufficient to affect turbulence,  $\omega_{s,GAM} \approx 0.3\tau_c^{-1}$

$$f_{GAM} \cong c_s / 2\pi R = \sqrt{(T_e + T_i) / M_i} / 2\pi R$$



# Nonlinear Transfer of Energy Can be Measured Experimentally

- Consider a model of density evolution:

$$\begin{aligned} \frac{\partial \tilde{n}}{\partial t} &\approx -V_x \frac{dn_0}{dx} - V_x \frac{\partial \tilde{n}}{\partial x} - V_y \frac{\partial \tilde{n}}{\partial y} + D \nabla_{\perp}^2 \tilde{n} \\ x &\rightarrow r \\ y &\rightarrow r\theta \end{aligned}$$

$$\rightarrow \frac{1}{2} \frac{\partial \langle |\tilde{n}|^2 \rangle}{\partial t} = -\langle \Gamma_x \rangle \frac{dn_0}{dx} - \text{Re} \left\langle \tilde{n}^* V_x \frac{\partial \tilde{n}}{\partial x} \right\rangle - \text{Re} \left\langle \tilde{n}^* V_y \frac{\partial \tilde{n}}{\partial y} \right\rangle + D \langle |\nabla_{\perp} \tilde{n}|^2 \rangle$$

$$= -\langle \Gamma_x(f) \rangle \frac{dn_0}{dx} + \sum_{f'} T_n^X(f, f') + \sum_{f'} T_n^Y(f, f') + D \langle |\nabla_{\perp} \tilde{n}(f)|^2 \rangle$$

Coupling of flux to background density gradient (source)

Collisional dissipation of fluctuation energy (sink)

Nonlinear “three-wave” interactions which exchange energy between different space/timescales

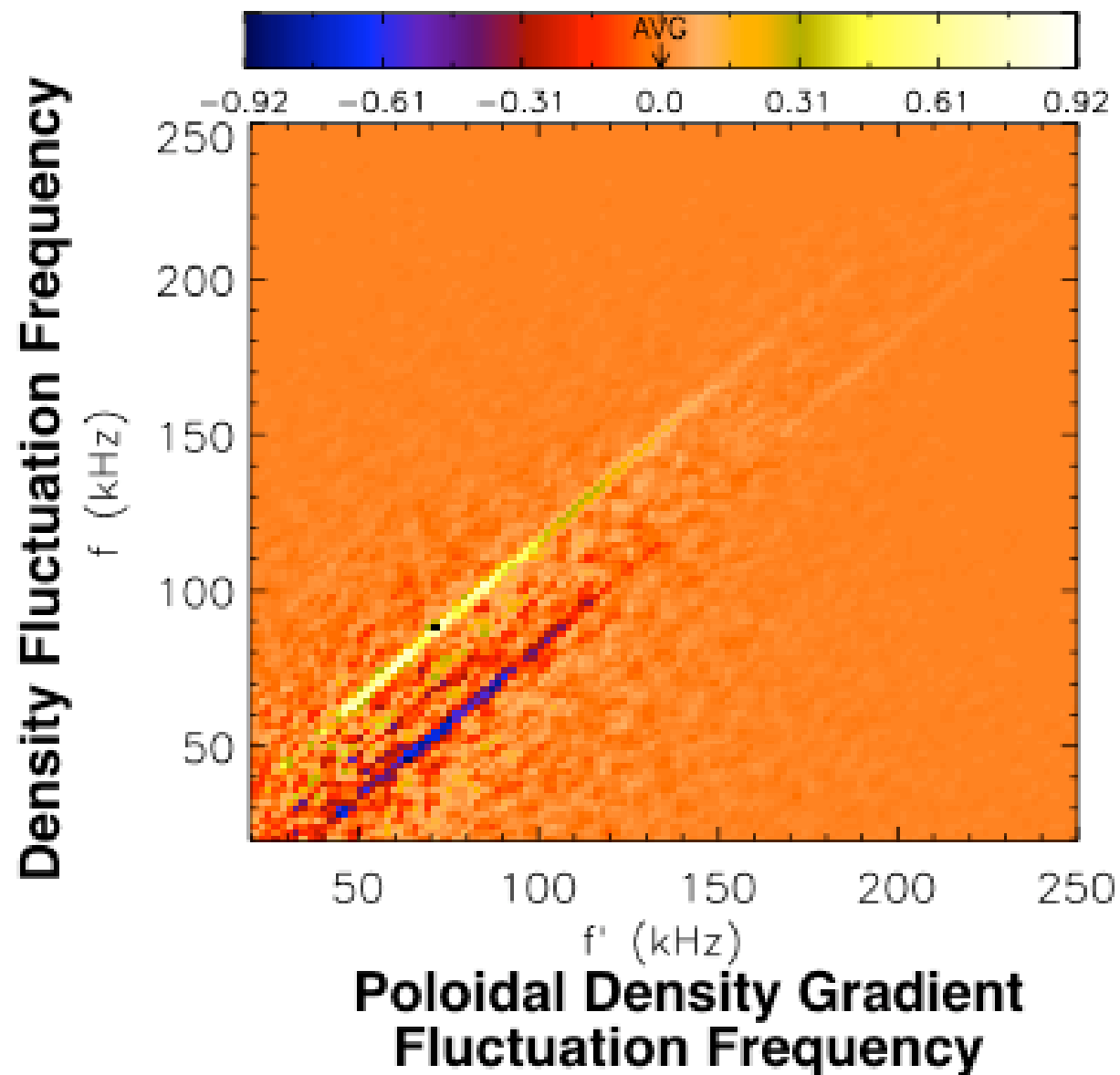
$$T_n^X(f, f') = -\text{Re} \left\langle \tilde{n}^*(f) V_x(f - f') \frac{\partial \tilde{n}}{\partial x}(f') \right\rangle$$

$$T_n^Y(f, f') = -\text{Re} \left\langle \tilde{n}^*(f) V_y(f - f') \frac{\partial \tilde{n}}{\partial y}(f') \right\rangle$$

# GAM Interacts Nonlinearly with Ambient Turbulence: Drives Forward Cascade of Energy to Higher Frequency

$$T_n^Y(f', f) = -\text{Re} \left\langle n^*(f) V_y(f - f') \frac{\partial n}{\partial y}(f') \right\rangle$$

Bispectrum measures 3-wave interactions

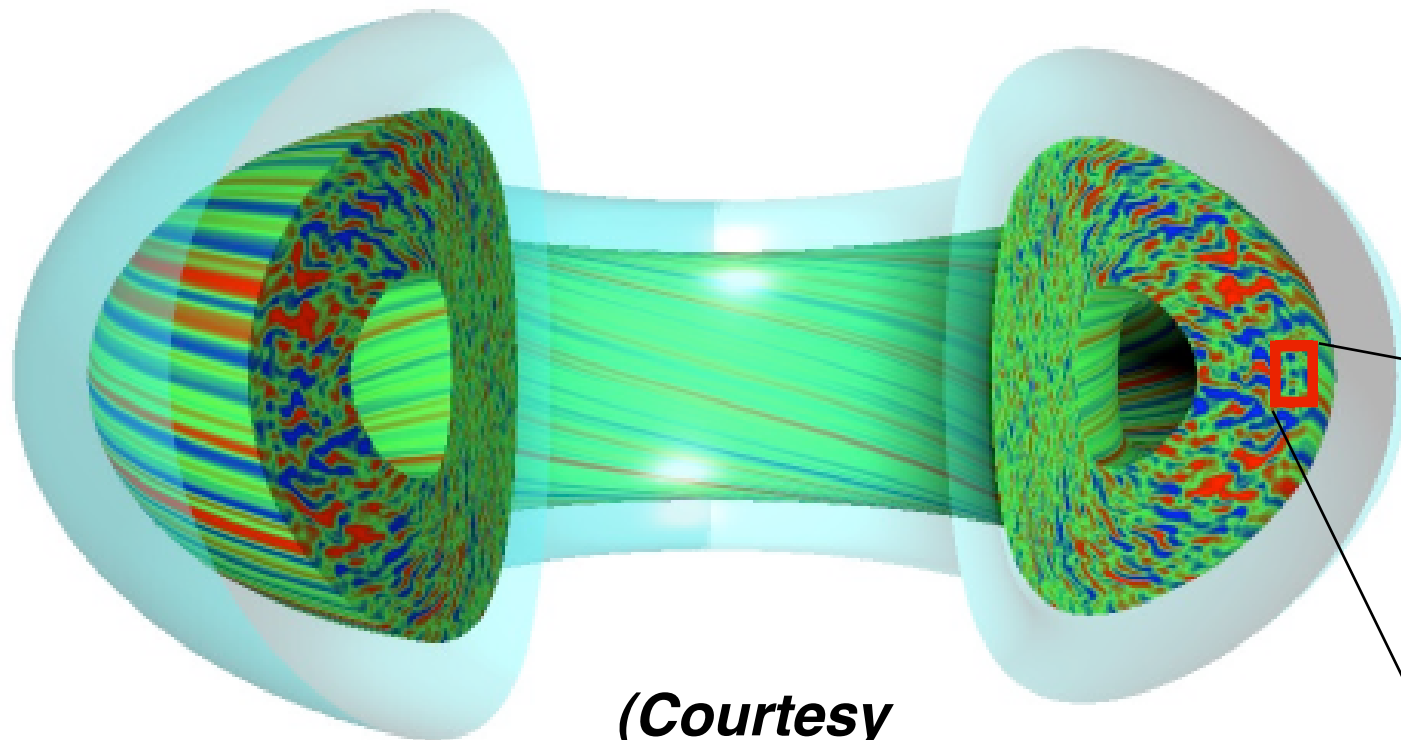


- All quantities are experimentally measured with 2D BES  $\tilde{n}$  data
- Strong interaction at  $|f - f'| = f_{\text{GAM}}$
- Density fluctuations at  $f$  gain energy from poloidal density gradient fluctuations at  $f' = f - f_{\text{GAM}}$ , and lose energy to those at  $f' = f + f_{\text{GAM}}$
- Energy moves between  $n$ ,  $dn/dy$  to higher  $f$  in steps of  $f_{\text{GAM}}$
- Convection of density fluctuations by the GAM leads to a cascade of energy to higher  $f$

*GAM plays an active role in mediating turbulence spectrum*

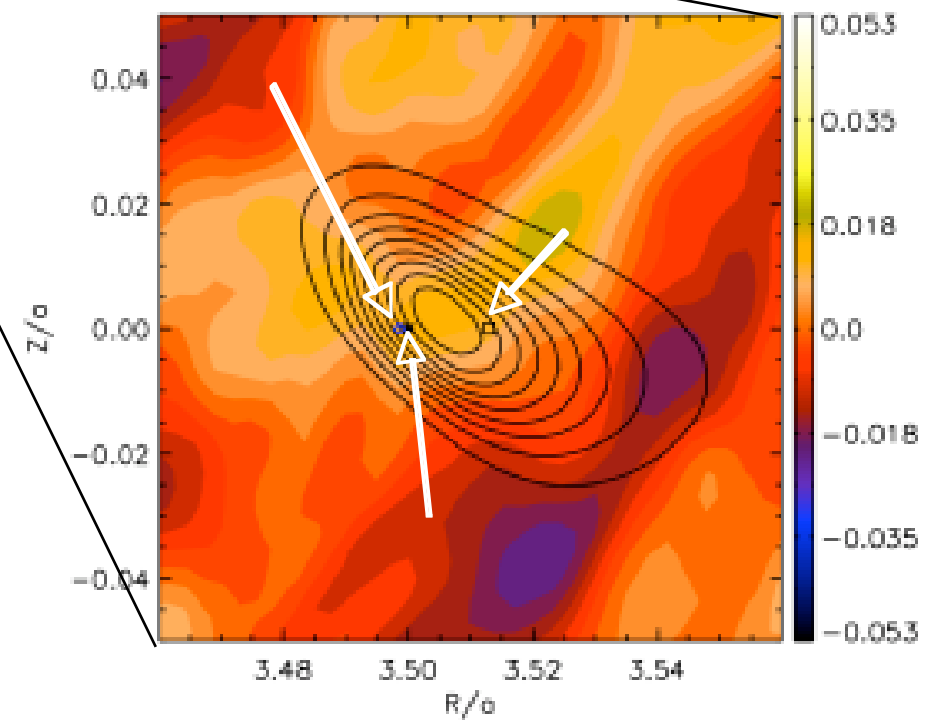


# Experimental Fluctuation Measurements are Quantitatively Compared with Nonlinear Simulations



(Courtesy  
J. Candy, R. Waltz, GA)

**GYRO, a Eulerian continuum code, simulates plasma turbulence and transport with full physics and geometry, experimental profiles**

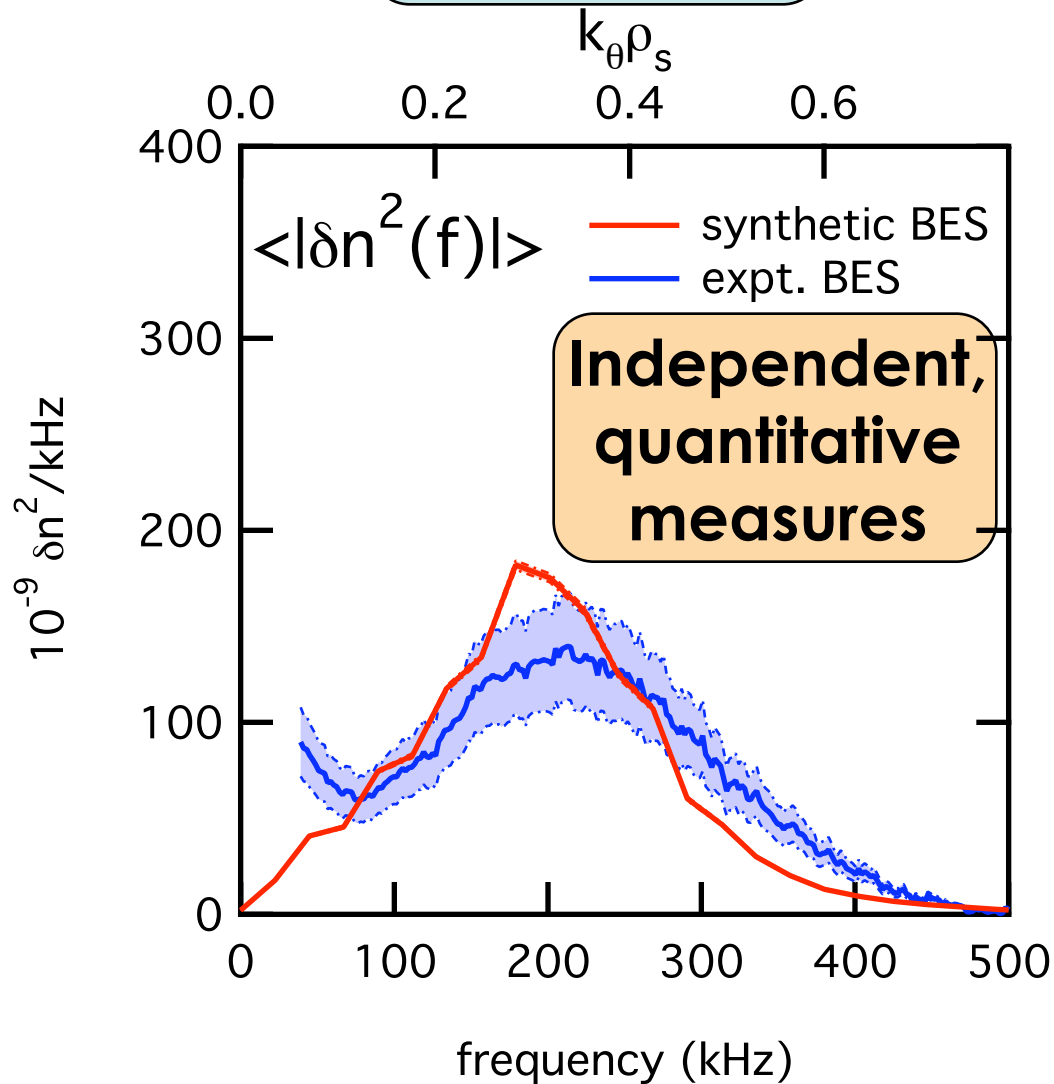


**“Synthetic Diagnostics” model measurement physics and diagnostic parameters to allow quantitative comparisons between simulation and measurements**

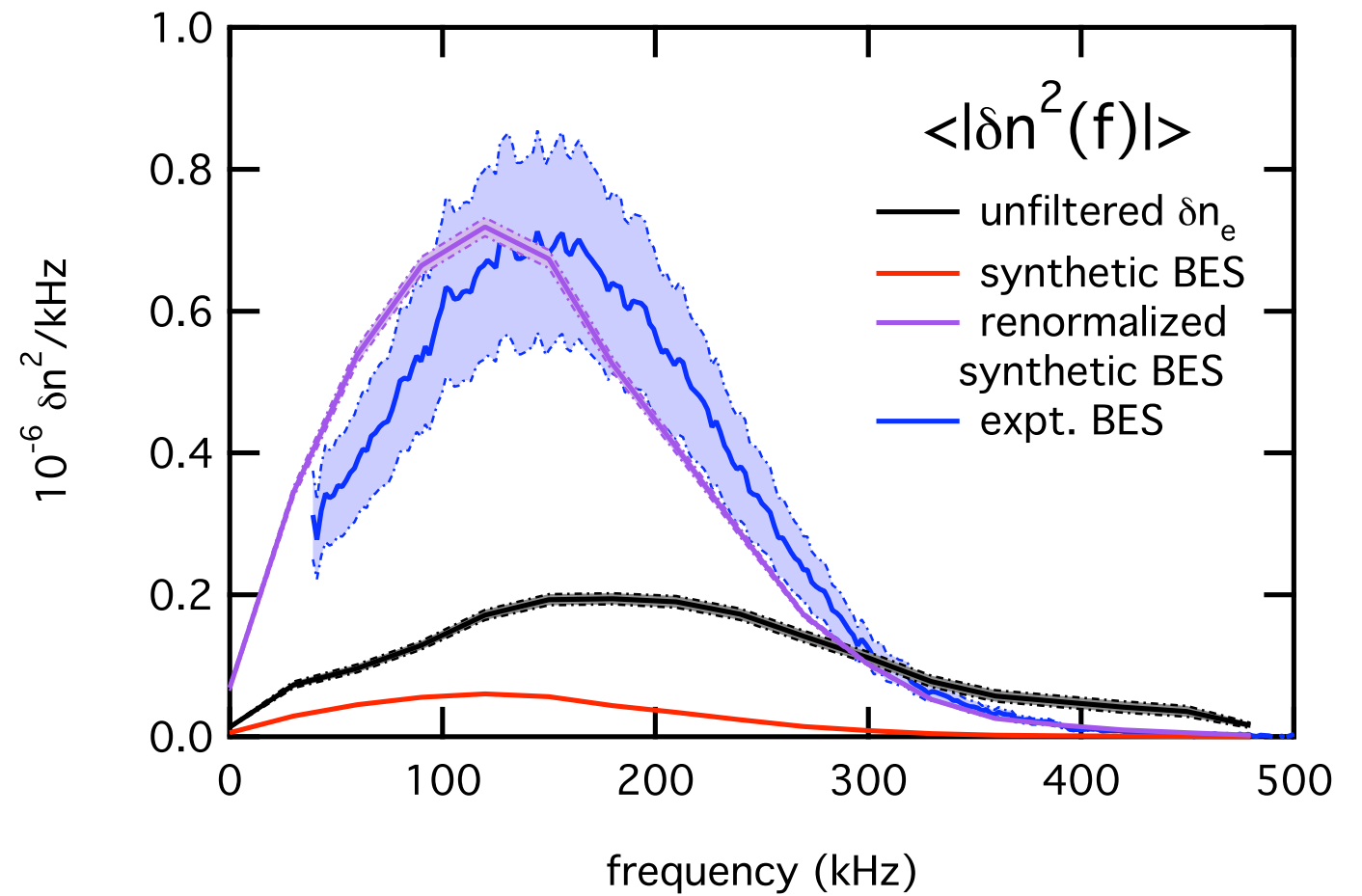
$$\delta n_{\text{synthetic}}(x, y, t) = \frac{\int d^2x' \psi_{\text{PSF}}(x - x', y - y') \delta n_e^{\text{GYRO}}(x', y', t)}{\int d^2x' \psi_{\text{PSF}}(x - x', y - y')}$$

# Density Fluctuation Spectra from BES and GYRO are Compared at 2 Radii

$r/a = 0.5$



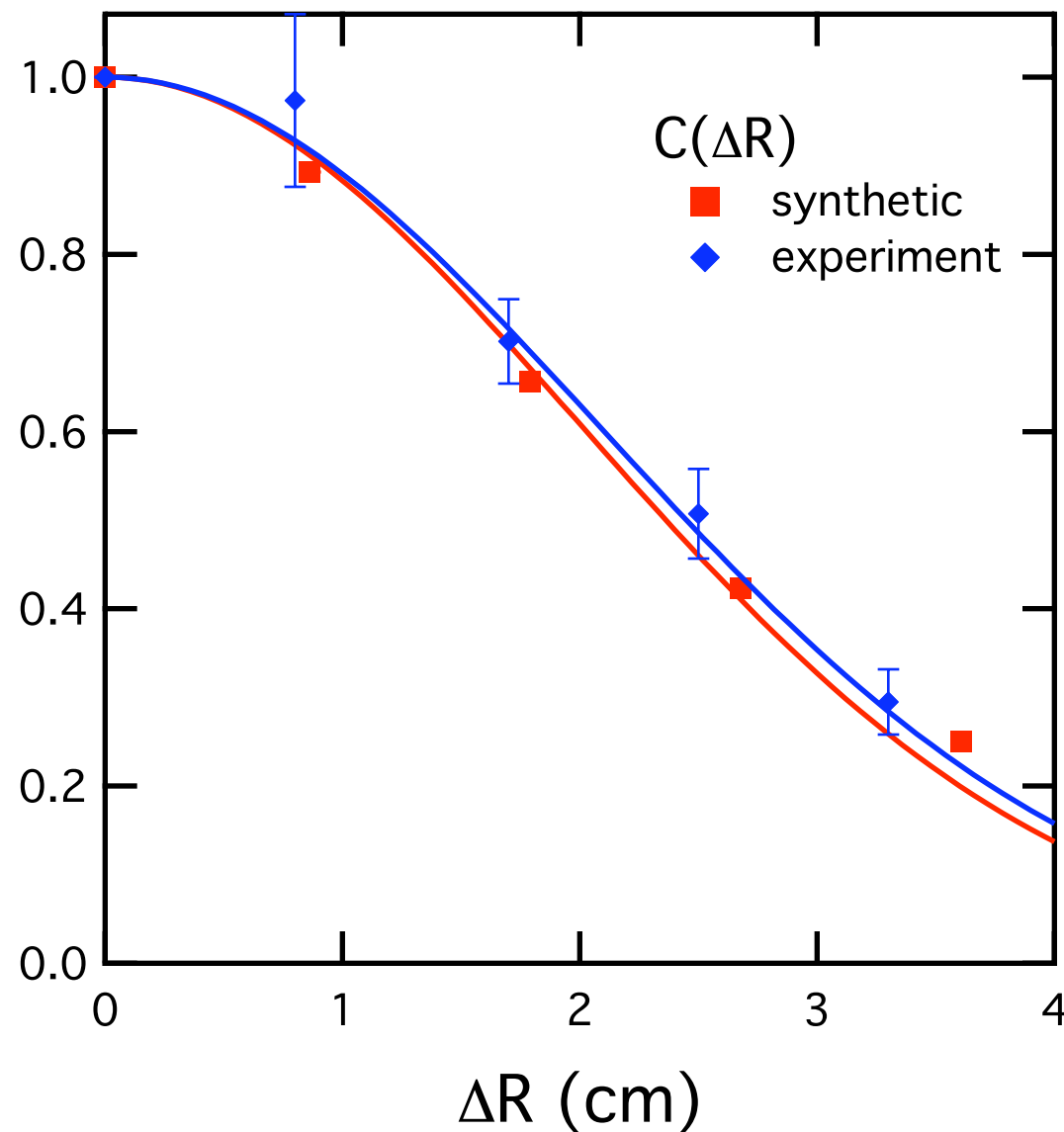
$r/a = 0.75$



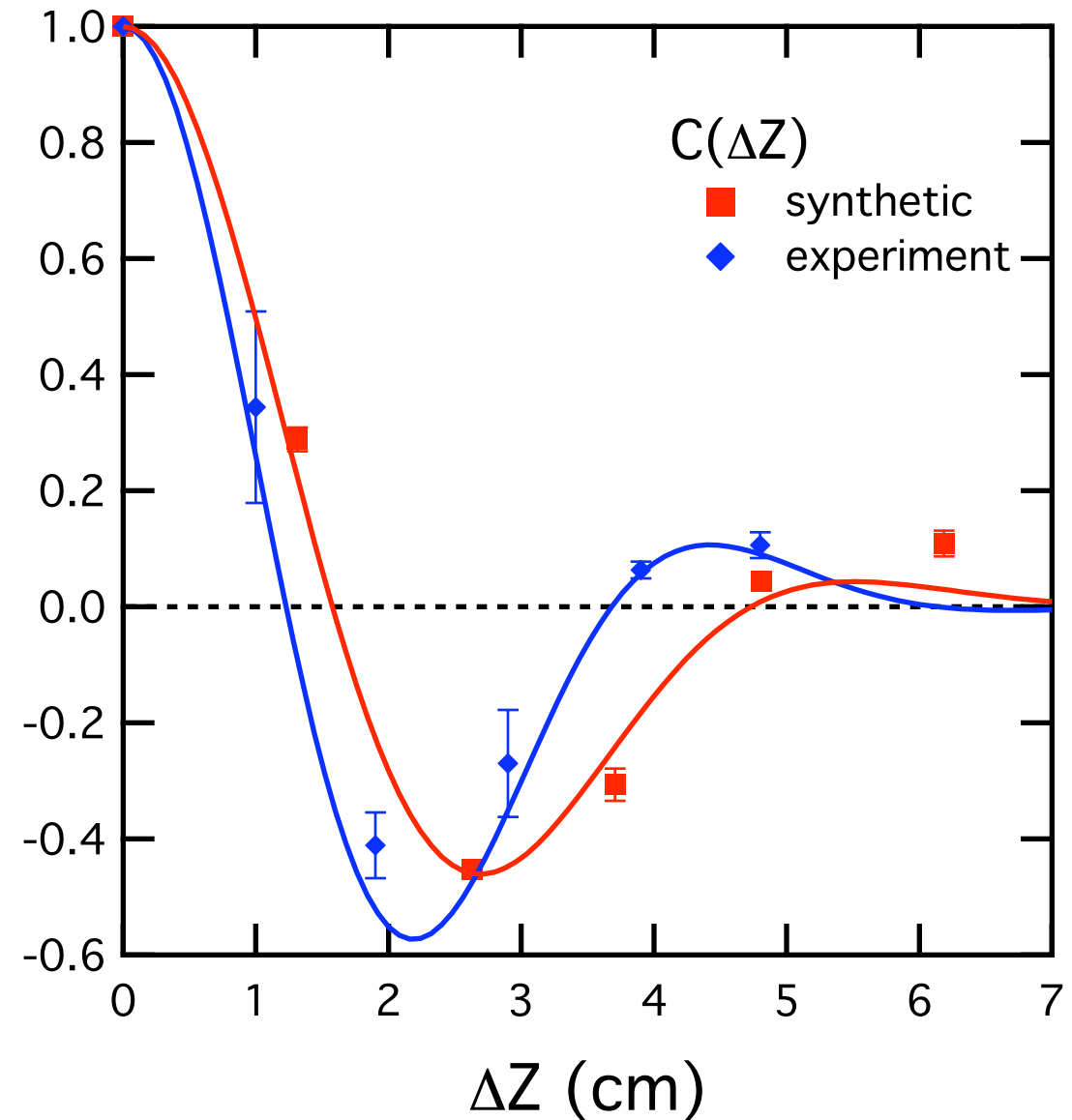
- At mid-radial location (0.5), excellent agreement is found between measurement and simulation
- At outer location (0.75), GYRO underestimates measured fluctuation spectrum
  - Turbulent diffusivities also underestimated by similar factor

# Spatial Correlations Exhibit Good Quantitative Comparison

## Radial Correlation



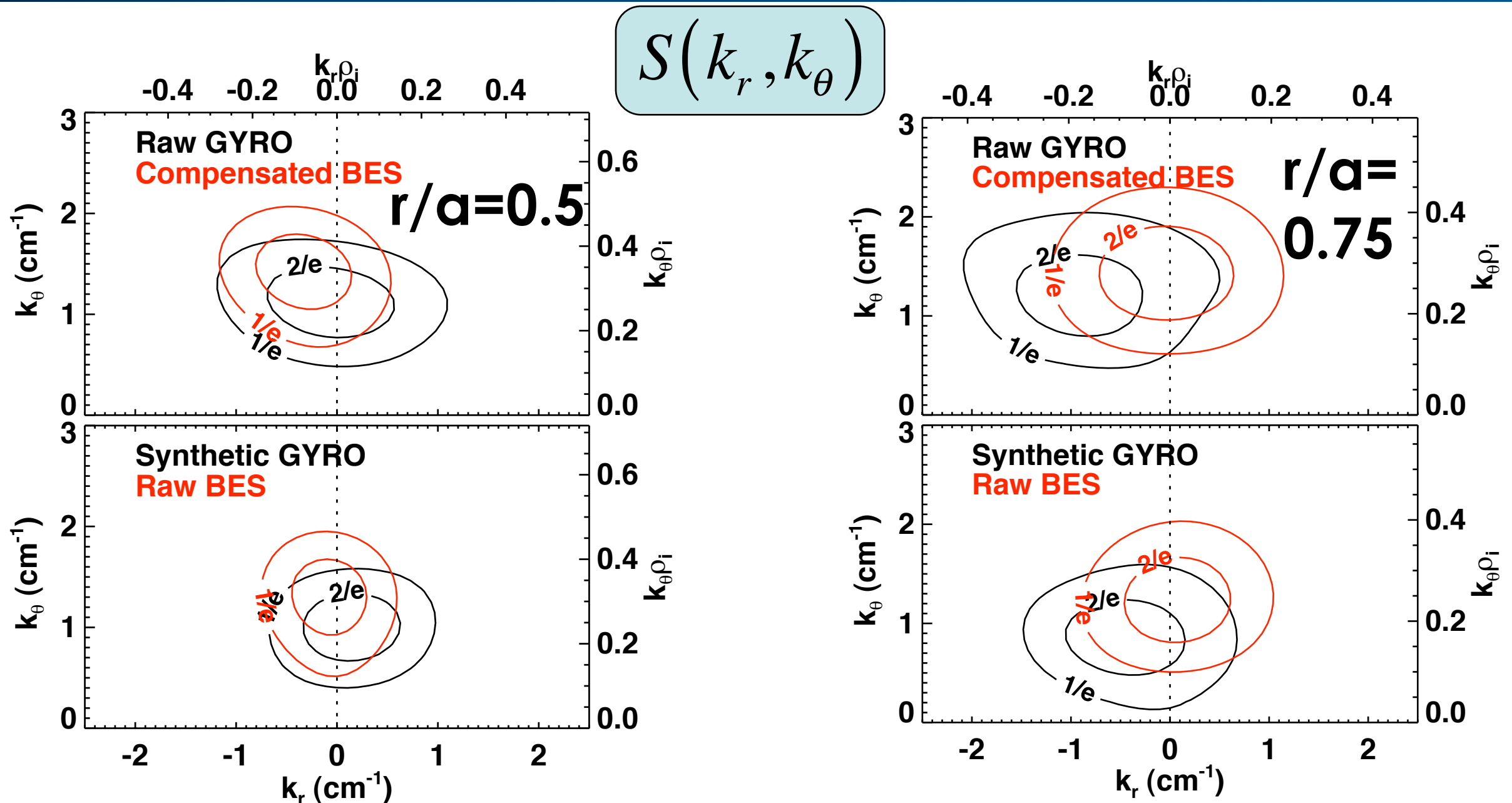
## Poloidal Correlation



- **Radial and poloidal correlation functions in good overall agreement**
  - Poloidal correlations differ modestly
- **Demonstrates the GYRO is reasonably correctly predicting scales**

# Comparison of BES-GYRO 2D $S(k_r, k_\theta)$ Wavenumber Spectra

## Reveal Notable Differences



- 2D wavenumber spectra show that GYRO predicts a finite  $k_r$  that is not observed experimentally
  - Might reflect an exaggeration of ExB shear effect on turbulent structure, which is also consistent with underprediction of turbulent-driven energy fluxes

# Conclusion

- **2D Measurements providing key insights into the characteristics and dynamics of long-wavelength density turbulence**
- **Visualizations of turbulence demonstrate:**
  - Turbulence flow patterns
  - Eddy interactions
  - Shear effects
- **Spectra typically peak near  $k_r \approx 0$ ,  $k_\theta \approx 1 \text{ cm}^{-1}$ ,  $k_\theta \rho_i \approx 0.3$** 
  - Largely consistent with expectations from gyrokinetics
  - Correlation lengths scale with  $\rho_i$
- **Advanced analysis methods applied to 2D measurements reveal:**
  - Zonal flow/GAM features in the turbulence flow-field
  - Nonlinear transfer of internal energy
- **Quantitative comparisons with fully nonlinear simulations are providing critical information towards validating models of turbulent transport**
  - Find cases of good agreement, and those where improvements are required