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Full f Gyrokinetic Simulation of Tokamak Plasma Turbulence using ELMFIRE

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Outline

- The Sosenko PIC model

- ELMFIRE, the computational tool

- Neoclassical Benchmarking

- Momentum diagnostic

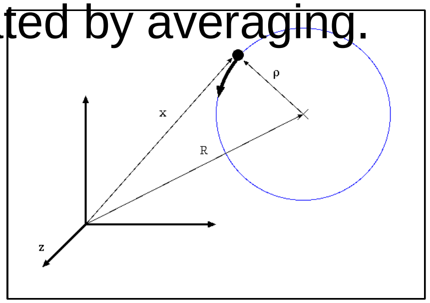
- Experimental Validation

- Conclusion & Future Prospects

The Sosenko PIC Model

Standard Gyrokinetic model

- Coordinates change to gyrocenter. Gyro angle eliminated by averaging.



$$(\vec{r},\vec{v}) \longrightarrow \left(\vec{R}, v_{\parallel}, v_{\perp}, \alpha,\right) \xrightarrow{\text{GK}} \left(\vec{R}, v_{\parallel}, v_{\perp}\right)$$

This introduces changes in equations that use real space coordinates (i.e. equations of motion & Poisson)

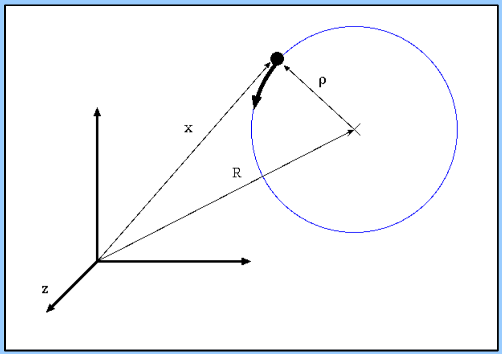
$$\frac{dR}{dt} = U\hat{b}_* + \frac{\mu}{e}\frac{\hat{b}\times\nabla B}{B_{\parallel}} + \frac{\hat{b}\times\nabla\Phi}{B_{\parallel}} \qquad \frac{dv_{\parallel}}{dt} = \frac{\hat{b}_*}{m}(\mu\nabla B + e\nabla\Phi)$$

$$\nabla^2\Phi + \frac{q^2}{mBe}\int \left| (\phi - <\phi>)\frac{\delta f}{\delta\mu} \right| dv = \frac{1}{\varepsilon_0}(en_e(\vec{r}) - qn_i(\vec{r}))$$

Sosenko's PIC model

- P. Sosenko's model uses an alternative definition of gyrocenter.

  P.P. Sosenko et al., Physica Scripta 64 (2001) 264



$$\vec{x} = \vec{R} - \vec{\rho} \quad \begin{cases} \vec{\rho} = \dfrac{\vec{b}\times\vec{v}}{\Omega} & \text{Lee} \\ \vec{\rho} = \dfrac{\vec{b}\times\left(\vec{v}-\vec{V}\right)}{\Omega} & \text{Sosenko} \end{cases}$$

- Polarization drift is included in the Eqs of motion.
- This introduces changes on Poisson's equation.

Sosenko's PIC model

Extra term in the equations of Motion:

$$\frac{dR}{dt} = U\hat{b}_* + \frac{\mu}{e}\frac{\hat{b}\times\nabla B}{B_\parallel} + \frac{\hat{b}\times\nabla\Phi}{B_\parallel} + \frac{1}{\Omega}\frac{d}{dt} - \nabla\Phi$$

$$\frac{dv_\parallel}{dt} = \frac{\hat{b}_*}{m}(\mu\nabla B + e\nabla\Phi)$$

And extra term in the Poisson equation
(for fluid model there is also a Jacobian term in the Poisson eq.):

$$\nabla^2\Phi + \frac{q^2}{mBe}\int |\,(\phi - <\phi>)\frac{\delta f}{\delta\mu} + \frac{m}{q} < f > \nabla_\perp^2 < \Phi > |\, dv = \frac{1}{\varepsilon_0}(en_e(\vec{r}) - qn_i(\vec{r}))$$

For $k_\perp\rho_i \to 0, \int |(\Phi - <\Phi>)\frac{\delta f}{\delta\mu} + \frac{m}{q} < f > \nabla_\perp^2 < \Phi > |dv \to 0$

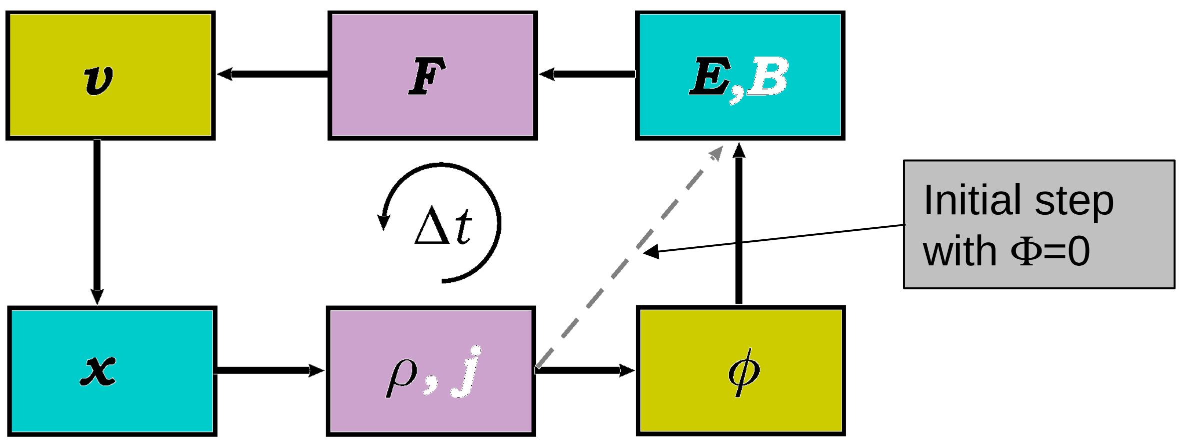
The ELMFIRE, Computational Tool

Computational Scheme

Acceleration and increment of velocity

Calculation of forces from fields and velocity

Computation of electric field. Magnetic is given.



Displacements and new positions. Boundary conditions.

Calculation of density. Current profile fixed.

Resolution of Poisson equation for the electrostatic potential.

Elmfire features

- Global full f nonlinear gyrokinetic particle-in-cell code
- Adiabatic and kinetic electrons
- Electrostatic
- Only core region, not magnetic axis
- Field alligned coordinates for particle following
- Concentric circle with low beta magnetic field background
- One impurity specie can be included
- Parallelized using MPI with very good scalability.
  - Based on free software: PETSc and GSL for math calc.

Collisions

Binairy collision model, Takizuka (1977)

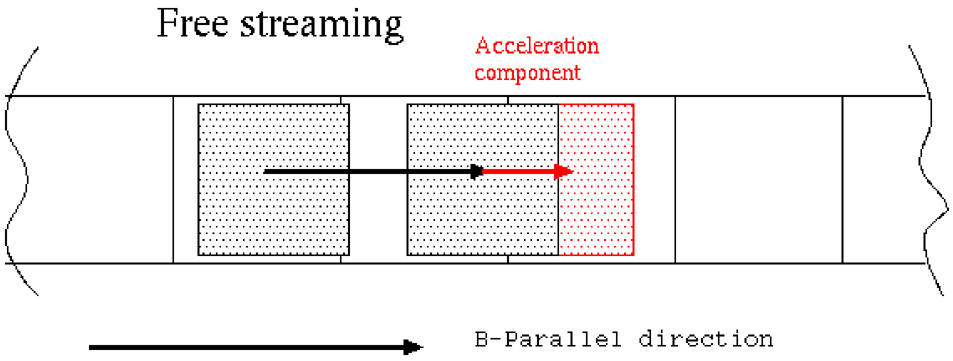
- Scattering between two test particles randomly selected from a collision grid.
- After that, the relative velocity vector of the two particles is rotated.
- The rotation angle is depending on the collision frequency and the timestep.
- The new velocity vectors for both particles are recalculated so that momentum and energy is conserved
- At every timestep all the particles collide atleast once.
- Benchmarked to the Landau collision model

Background collision model

- Monte Carlo collision opertor for energy and pitch angle

Implicit kinetic electron model

$$\Delta x_{fs} = v \cdot \Delta t \qquad \Delta x_a = \frac{1}{2} a \Delta t^2 = \frac{qE_{\parallel}}{2m} \cdot \Delta t^2$$



- E field in Δx_a is calculated at advanced time but at the position after free streaming
  - We demand that $|\Delta x_{fs}| \gg |\Delta x_a|$. It is a constraint in Δt

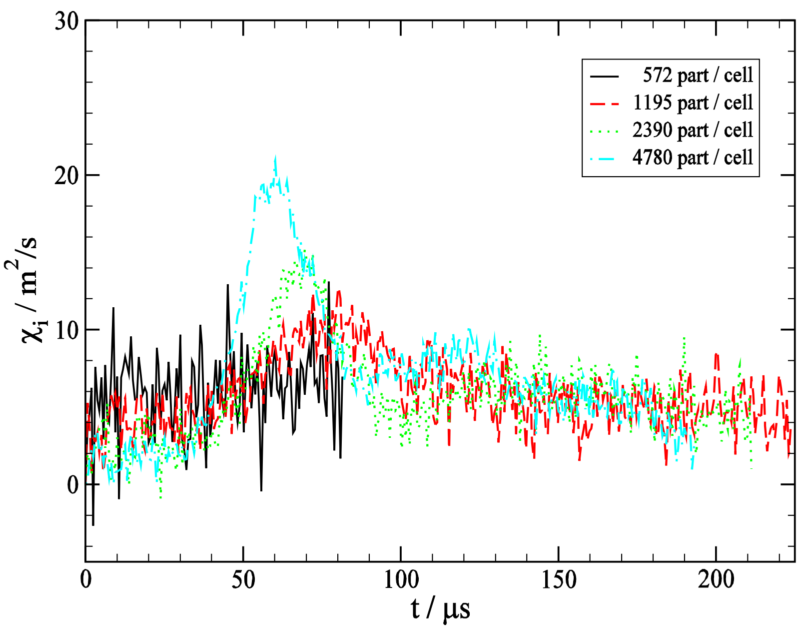
$$\Delta t \ll \frac{\sqrt{8 m_e k T}}{e \left| E_{\parallel} \right|} \qquad \begin{cases} kT = 100 \text{ eV} \\ E_{\parallel} = 50 \text{ V/m} \end{cases} \quad \Rightarrow \quad \Delta t \ll 1\, \mu\text{s}$$

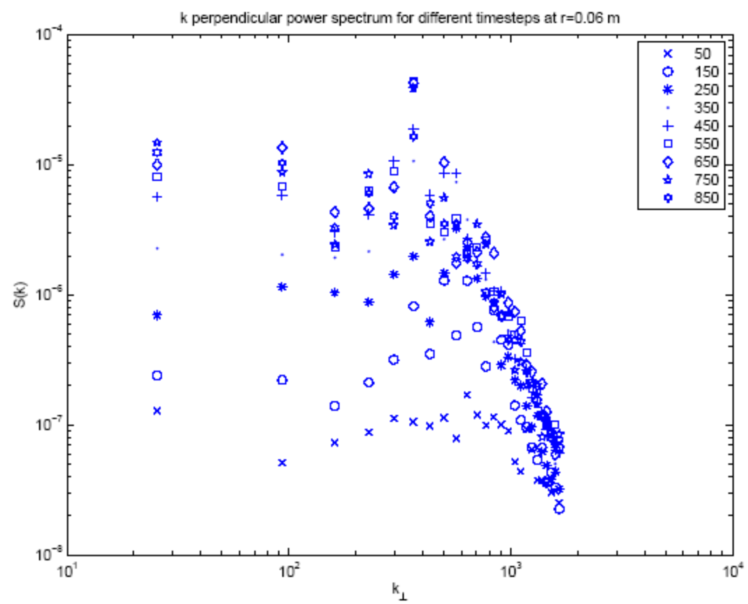
Neoclassical & Code Benchmarking

Convergence

- PIC codes are very noisy, error goes with $(N_p)^{-1/2}$
- Evolution of χi is studied with nonlinear runs

r of particles per cell.



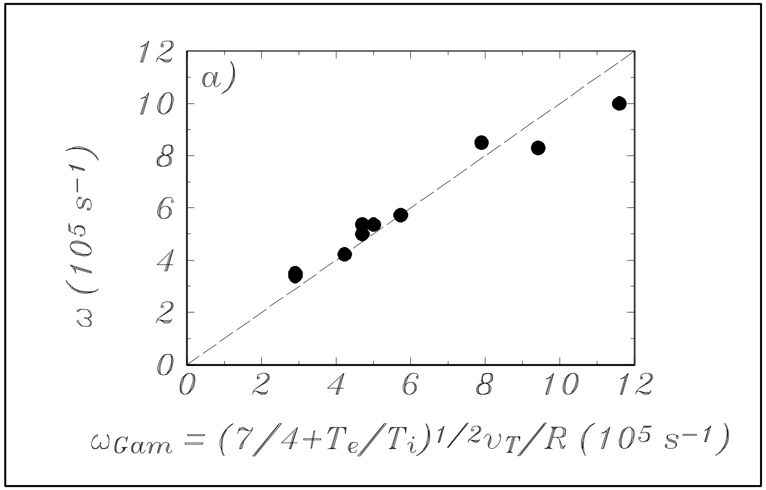


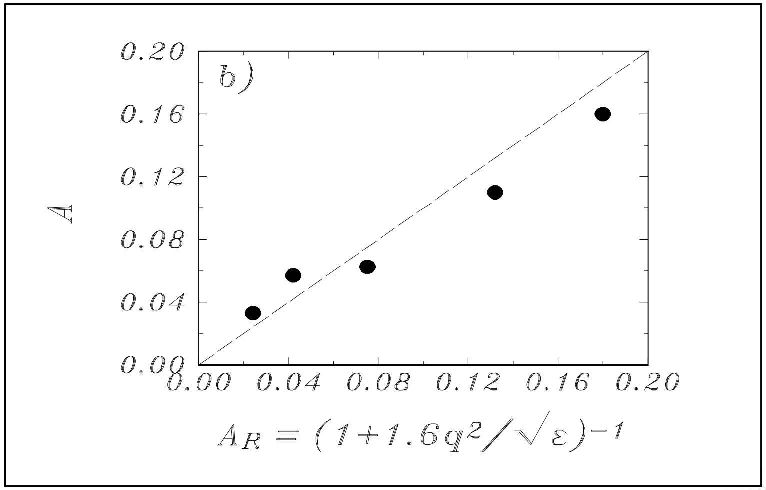
Geodesic Acoustic Modes

- Neoclassical theory predicts GAM frequency and Rosenbluth residual.

$$w_{Gam} = \frac{v_{Ti}}{R}\sqrt{7/4 + T_e/T_i} \qquad A_R = \left(1 + \frac{1.6q_s^2}{\sqrt{r/R}}\right)^{-1}$$

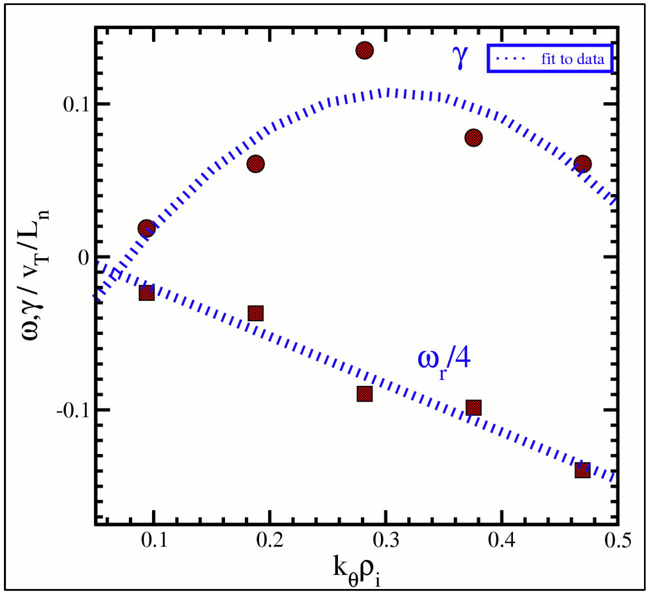
- Results show good wide agreement with theory.
- Simulations done on a plasma annulus.
  - R=0.3-0.9 m, a=0.08 m, B=0.6-2.45 T, q=1.28-2.91, Ti=90-360 eV, ni=5.1×10^{19}m^{-3} (r/a=0.75)

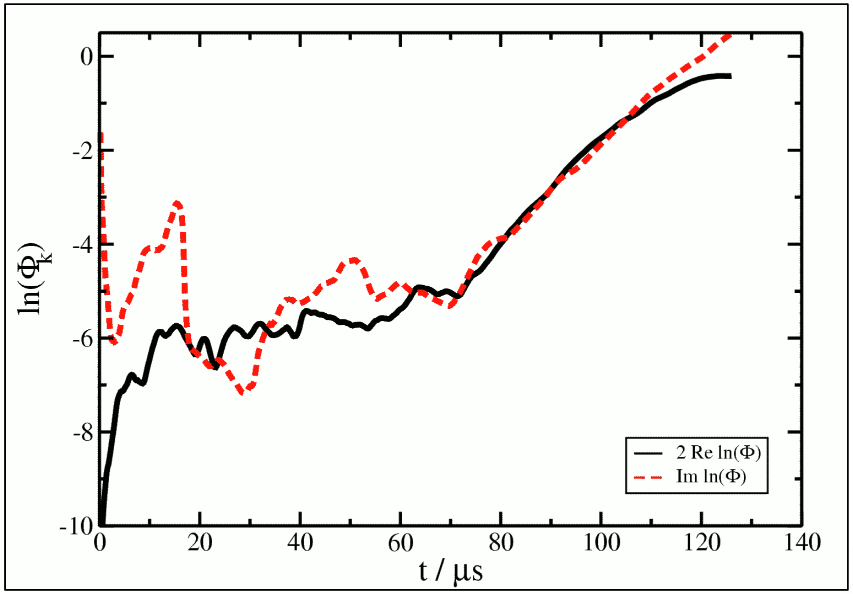




Linear growth of unstable modes

- Test based on Cyclone Base Case (Dimits PoP '00)
  - Red points from ELMFIRE, blue line: fit from article.
  - Figures show growth rates and typical time evolution for a mode with $k_\perp \rho_i = 0.3$





Momentum Diagnostic

Motivation

- Low order (2nd or 3rd) gyrokinetics produces an unphysical Lorenz force due to a radial current density $j = Ze \langle \int d^3v f_{i0} \partial_y \psi_0^{(3)} \rangle_r \sim \delta_i^3 p_i / L$ (F.I. Parra et al., 2009)

- This term is small, but so are the physical terms, in affecting toroidal angular momentum distribution and transport in tokamaks

- In gyrokinetic simulations, however, the accuracy of the numerical methods can sooner become an issue in diagnosing momentum.

Diagnostics of toroidal angular momentum

- Define a diagnostic region. Important to have overlap with the collision grid!

- Sample $p_\varphi = \sum mRv_\varphi - q\psi_p$ for all simulation particles (electrons and deuterons) that have the gyrocenter in the diagnostic area.

- v_φ has so far been defined by the toroidal projection of $v_{||}$. Projection of drifts have not yet been included.

- At every time step include the incoming p_φ sum and subtract the outflowing p_φ sum while taking all drifts into account.

Two different interpolation scheme's

In gyrokinetic particle simulation one needs the electric field evaluation at the particle position

IN PIC/CIC scheme one may interpolate in two ways

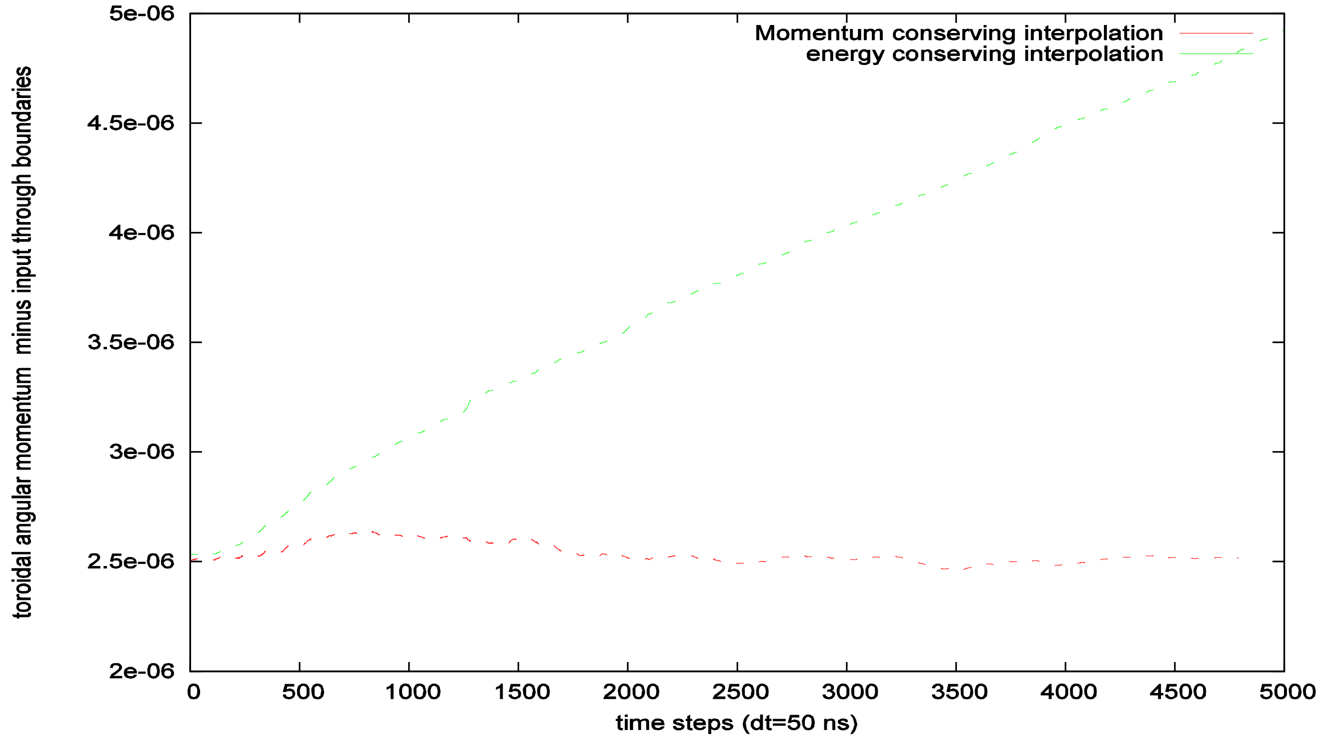
1) $\Phi(x)=\sum\Phi_k S_k(x)$; $E(x)=-\mathrm{grad}\Phi=-\sum\Phi_k \mathrm{grad} S_k(x)$

2) $E(x)=\sum E_k S_k(x)$

The scheme 1) conserves energy and is numerically stable against divergent ExB flow (J.A. Byers et al., 1994)

The scheme 2) conserves momentum, but is prone to numerical instability by divergent ExB flow

Scheme 1) and 2) comparison



Momentum drive:

$$\dot{P}_{\varphi} = -e\frac{\partial \phi}{\partial \varphi} - \left(\mu + \frac{e^2}{m}\rho_{par}^2 B\right)\frac{\partial B}{\partial \varphi}$$

Momentum conserving interpolation

quasie neutrality requires:

$$\rho = \Sigma q_i \delta(x - x_i) = 0$$

For a CIC code this gives:

$$\rho_k = \int S_k(x) \Sigma q_i \delta(x - x_i) dx = \Sigma q_i S_k(x_i)$$

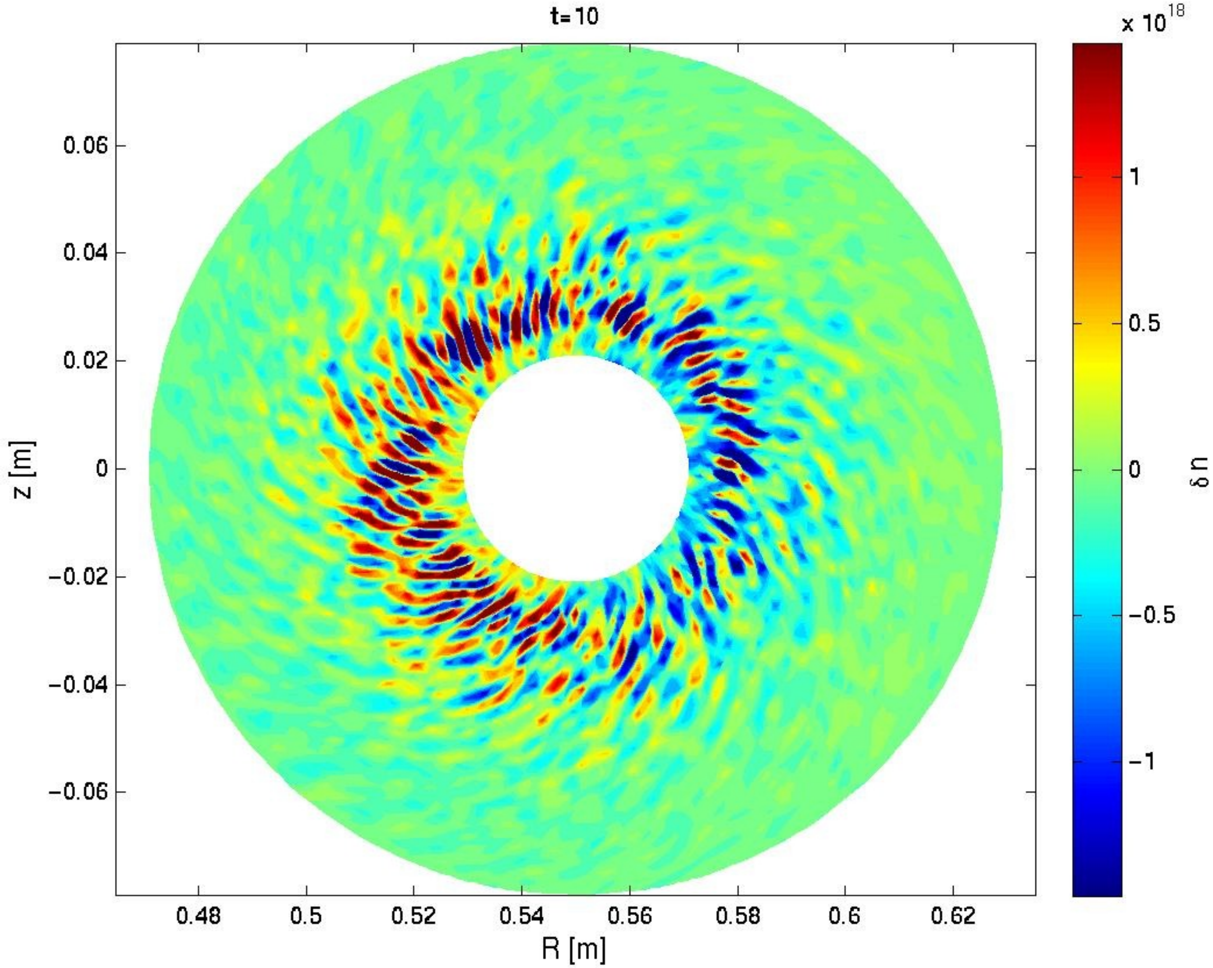
If we now use the same sampling for the calculation of the electric field for a particle:

$$E = \Sigma_k S_k(x) E_k$$

the momentum drive is forced to be zero:

$$\frac{dp_\varphi}{dt} = \int \rho E dx = \Sigma E_k \int S_k \Sigma q_i \delta(x - x_i) dx = 0$$

Scheme 2) was unstable in ELMFIRE



Advanced nonlinear scheme 2)

To preserve conservation of toroidal angular momentum but to ensure divergenceless ExB, scheme 2) was modified by using the radial electric field interpolation:

$$Er(r,\theta,\varphi)= (1\text{-}w)[\boldsymbol{\Phi}_{i,j,k}-(h_i-h_i^2/2)(\Phi_{i,j-1,k}-\Phi_{i,j+1,k)})/2 - (h_i^2/2)(\Phi_{i,j,k}-\Phi_{i,j+2,k)})/2 - \{i+1 \rightarrow i\}]/dr + w[k+1 \rightarrow k] /dr,$$

where $w=(\varphi-\varphi_k)/d\varphi$, $h_i=(\theta-\theta_{i,j})/d\theta_i$,and i,j,k refer to the grid points in radius r, poloidal angle θ, and toroidal angle φ, and $\boldsymbol{\Phi}_{i,j,k}=(\Phi_{i,j-1,k}+\Phi_{i,j,k}+\Phi_{i,j+1,k})/4$

Major change is in the radial current balance

Scheme 1) produces nonambipolar ExB radial current of electrons and ions due to gyrokinetic correction of <E> for ions (T. Stringer, 1991) → return ion current and flattening of the radial profile of flux surface averaged Er

→ Er Neoclassic is only reached for stongly collisional cases

Scheme 2) keeps the ExB radial current (by turbulence) as ambipolar (F. Parra & P. Catto, 2009) → Er approaches the neoclassical value

Results while using momentum conserving interpolation scheme

- There still is a increase of momentum, but much much smaller than before
- To check the contributions of the boundaries we have set dPhi/dzeta outside the diagnostic domain to zero and sum charge*dPhi/dzeta*tstep for all the particles inside and outside the domain
- Mometum still slowly grows but the growth is monotonically and strongly suppressed when the quasineutrality is improved
- Suppresion can be done by either simply by increasing the number of particles or by iteration of the implicit polarization matrix solution.
- The drive can be made as small as wished which proves that the code can be forced to conserve angular momentum with an accuracy wished.

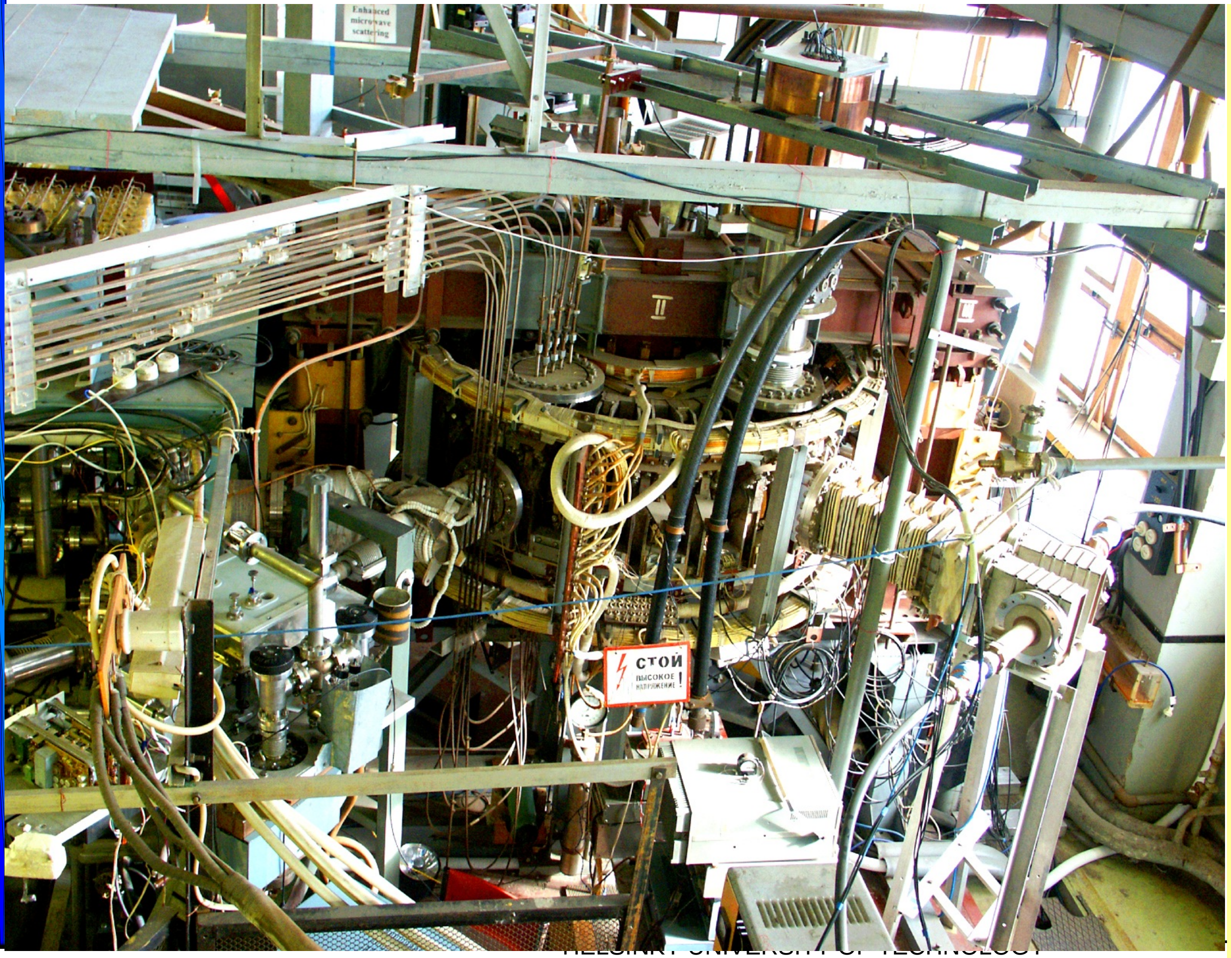
Summary momentum diagnostic

- The effect of scheme 1) and (advanced) 2) on p_φ is drastically different; only scheme 2) can conserve p_φ
- Even with scheme 1) the nonconservation of p_φ within an energy confinement time is not very large in strong collisional cases.
- It has however not been proven that there is no unphysical force in gyrokinetics due to too low ordering of gyrokinetics since drifts are not included in the calculation
- Estimation of this ordering error however is probably prohibited in simulations lacking translation and reflection symmetry in radius

Experimental Validation

Experimental Benchmarking: FT-2 tokamak

R_0 = 55 cm, limiter radius: a = 7.9 cm



$B_t < 2.7$ T

$I_p < 50$ kA

$n_e < 7\times 10^{19}$ m^{-3}

$T_e < 600$ eV

$T_i < 300$ eV

- ✓ 5-chanel CX analyser
- ✓ Multi-pulse TS
- ✓ 2 mm-interferometer
- ✓ 5-electrodes L. probes

+ RF-system:
  2-waveguide grill for LHW launching

P_{RF} = 30-180 kW

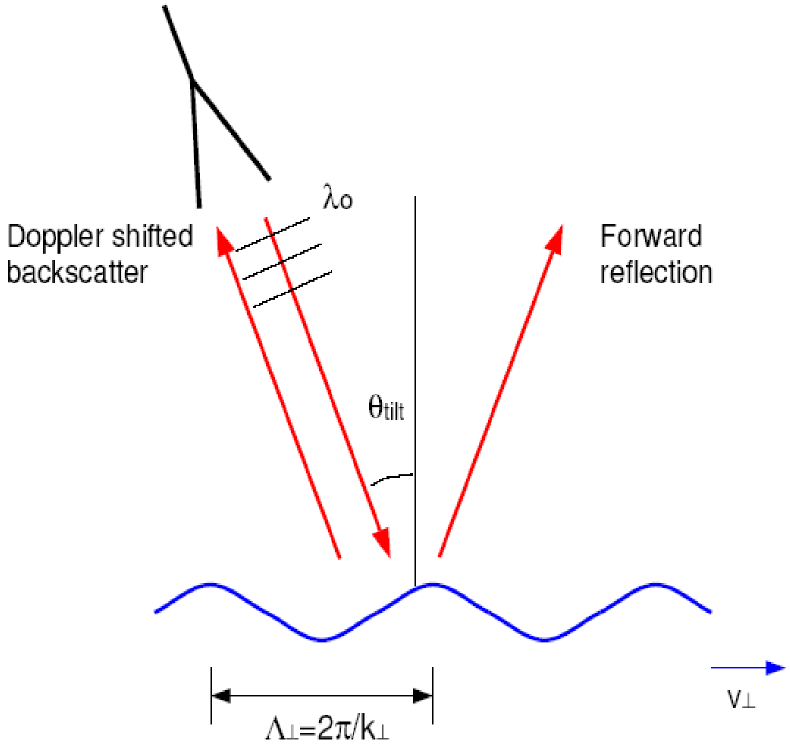
f_{LH} = 920 MHz

Doppler reflectometry

Doppler reflectometry measures the poloidal velocity by evaluation of the rotational velocity from the Doppler frequency shift of microwave back scattered radiation

The output signal of a quadrature detector is given by

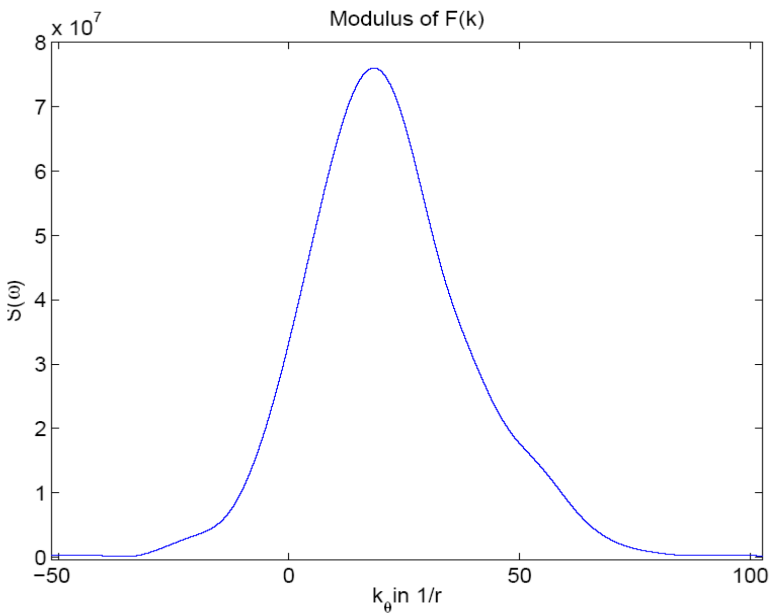
$$I(t)=\int w(r,\theta)\,\delta n(r,\theta,t)\,r\,dr\,d\theta$$



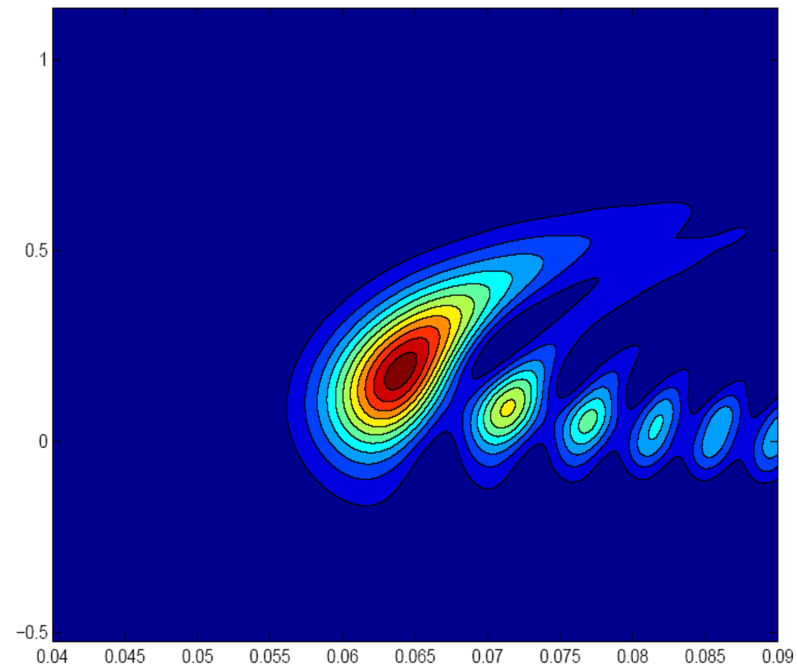
- W(r, θ) = Weightingfunction calculated by beam tracing code
- δn(r,θ,t) = Density fluctuations simulated by ELMFIRE code

Weightingfunction

W(r,θ)=wcos+i*wsin



Amplitude of W



$$F(k_\theta) = \int W(r,\theta)\, e^{i(k_\theta \theta)}\, r\,dr\,d\theta$$

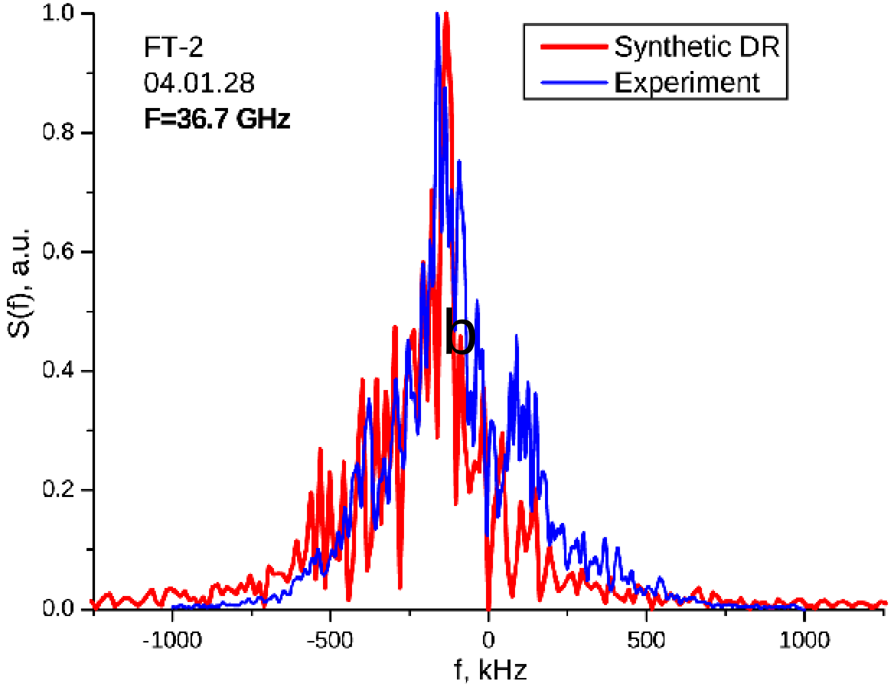
Simulation parameters

- Simulation Domain r=0.02-0.08 cm → ρ=0.25-1
- Ti=90-10 EV
- Te=300-40 eV
- Ne=3.5-0.2 10^{19}/m3
- Q=2-7

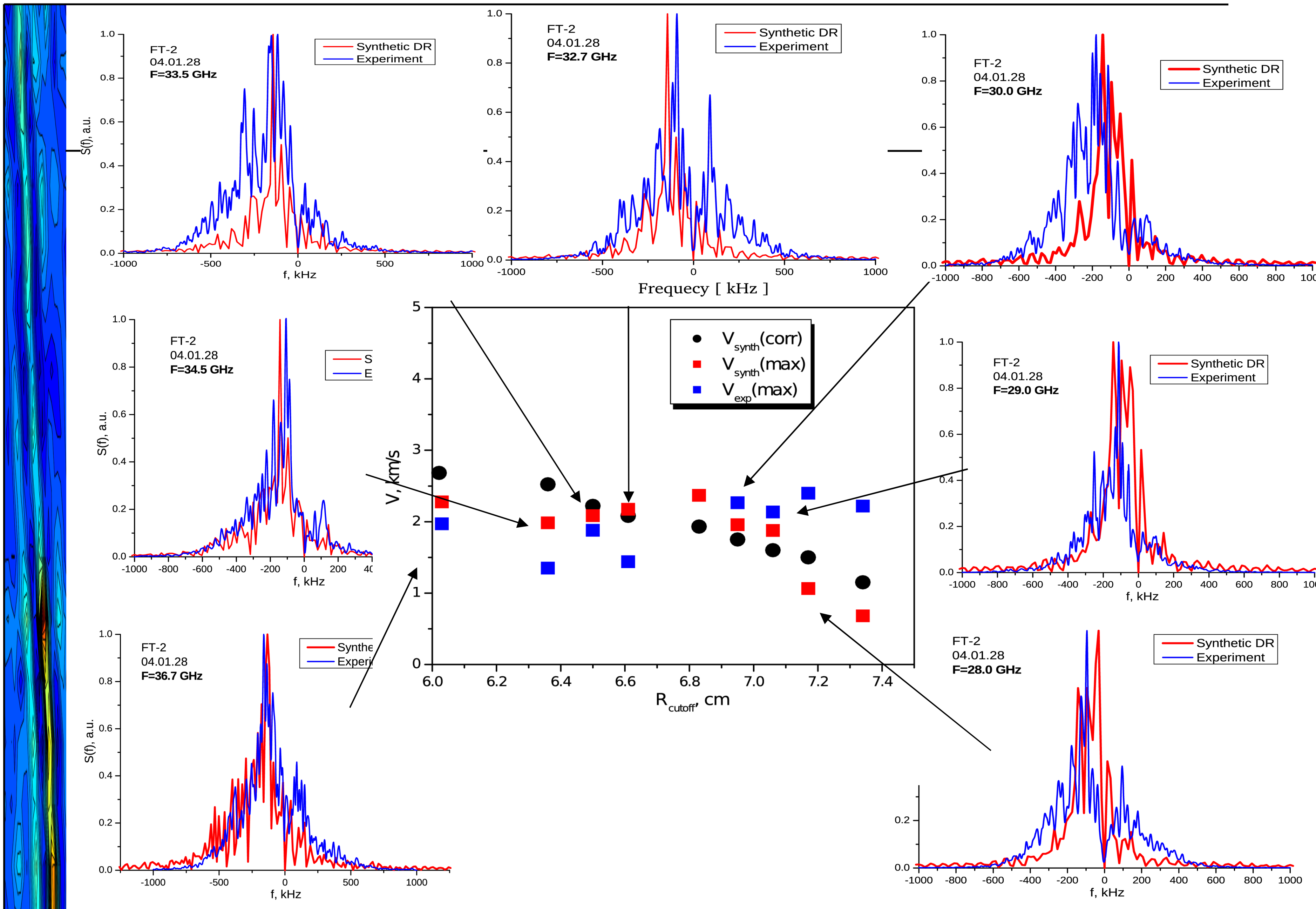
- Δt= 30 ns, Ion orbit time $t_{io}=v_t/(qR_0)$=8-84 μs = 260-2800 timesteps
- Simulation time=10 000 ts = 40-4 tio
- Grid 120rad x 300pol x 8tor @ 7000 part/gridcell
- 245 760 CPU hours @ 2.7 GHZ cray XT system

Doppler Spectra

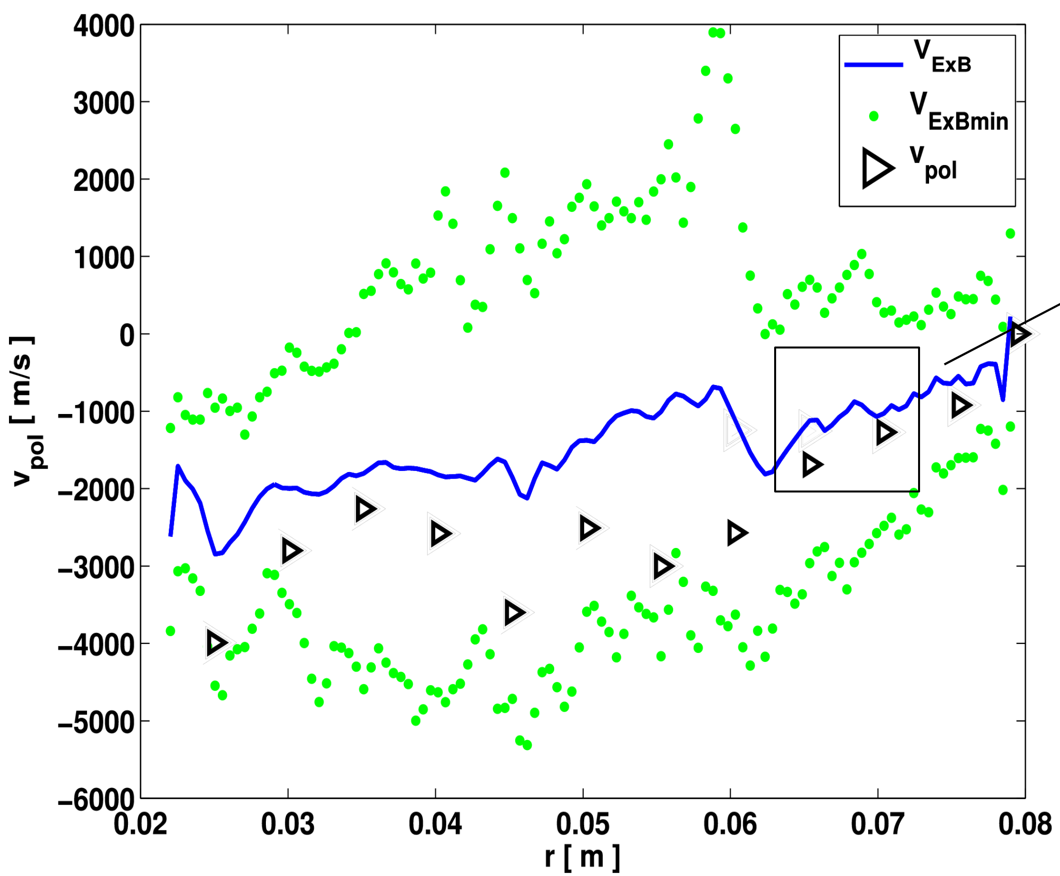
$I(t)=\int w(r,\theta)\, \delta n(r,\theta,t)\, r\, dr\, d\theta$



Averaging for spectra 128 µs
$\Delta f = \int f S(f)df / \int S(f)df = 132$ kHz
$v_\theta = 2\pi\Delta f / k_\theta = 1927$ m/s
$k_\theta = 2k_0 \sin\alpha$


FT-2
04.01.28
F=33.5 GHz
Synthetic DR
Experiment
S(f), a.u.
f, kHz
FT-2
04.01.28
F=32.7 GHz
Synthetic DR
Experiment
Frequecy [kHz]
FT-2
04.01.28
F=30.0 GHz
Synthetic DR
Experiment
FT-2
04.01.28
F=34.5 GHz
S(f), a.u.
f, kHz
V_{synth}(corr)
V_{synth}(max)
V_{exp}(max)
V, km/s
R_{cutoff}, cm
FT-2
04.01.28
F=29.0 GHz
Synthetic DR
Experiment
f, kHz
FT-2
04.01.28
F=36.7 GHz
S(f), a.u.
f, kHz
FT-2
04.01.28
F=28.0 GHz
Synthetic DR
Experiment
f, kHz

V_{ExB} & v_{phase} versus V_{pol}

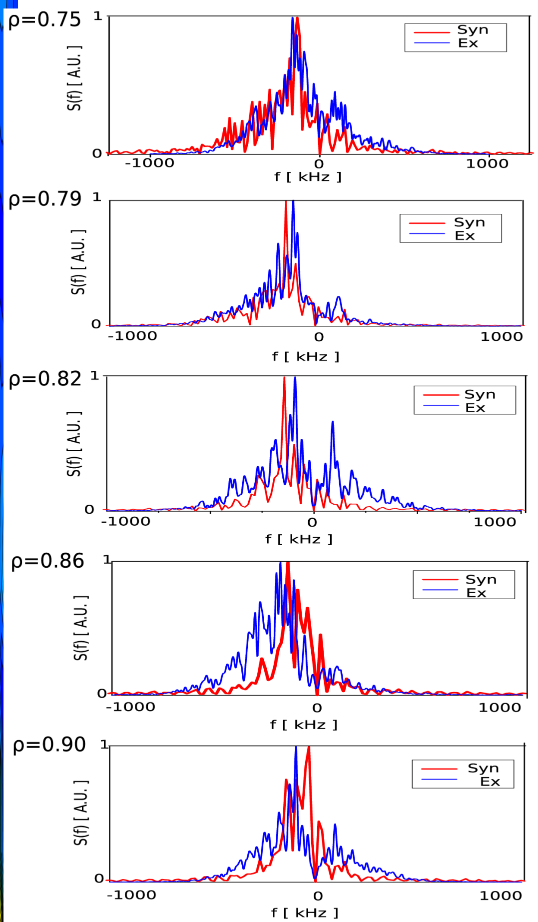


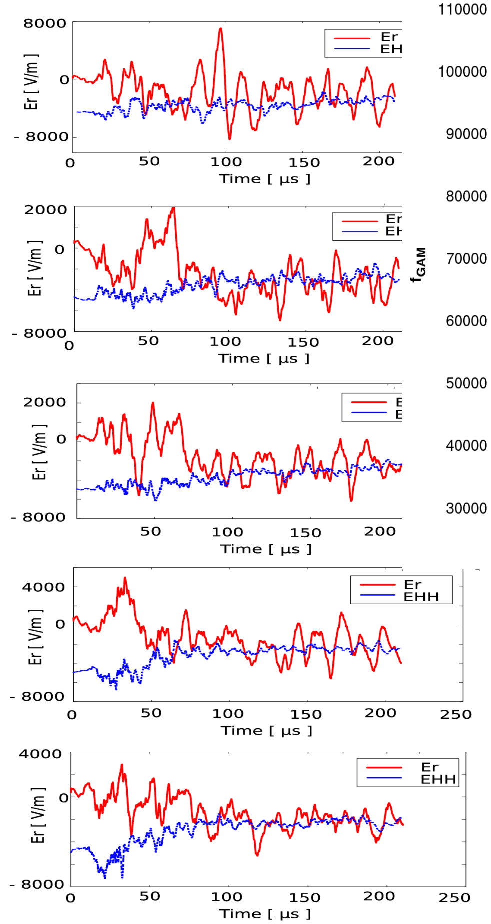
$v_{pol}=v_{ExB}+v_{phase}$

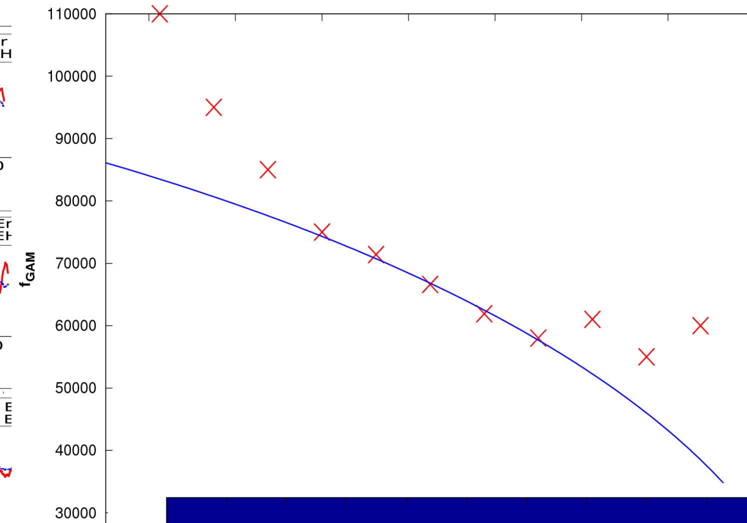
For the area in which the Doppler reflectometry benchmarking is done

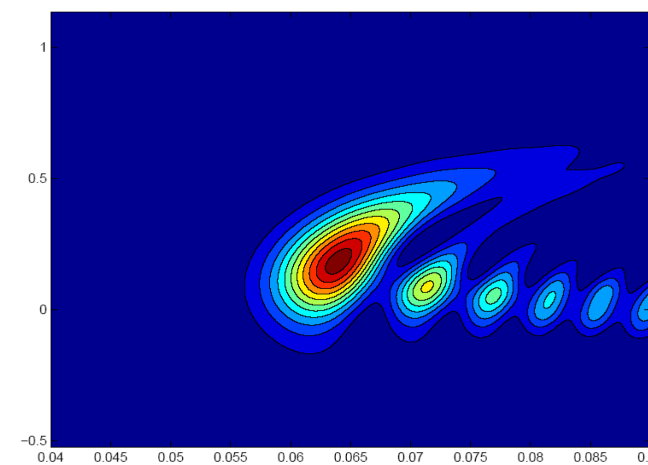
$v_{pol} \sim v_{ExB}$

Er Time traces : GAMS









Conclusion & Future Prospects

Conclusion & Future Prospects

- Successful gyrokinetic full *f* PIC simulations of tokamak electrostatic turbulence and transport for core plasma are performed and benchmarking of the codes is performed in appropriate limits for the turbulence saturation and neoclassical characteristics.
- Mometum diagnostic has been added to the code, and some results are obtained....
- Experimental validation has been performed in cooperation with the FT-2 tokamak. A reasonable good agreement has been found for comparison with Doppler Reflectometry spectra so far. GAM's are expected to play a role in the broadening of the DR spectrum.
- A Sol model including the two poloidal limiters of the ft-2 tokamak is implemented and tested while speaking. This will hopefully increase the particle outflow at the edge.