Gyrokinetic Modeling for Basic Turbulence Experiments on the LAPD

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The Center for Multi-scale Plasma Dynamics

Outline

- Basic Turbulence Experiments on the LAPD
    - Theoretical Background
    - Experimental Setup

- Challenges for Gyrokinetic Modeling of LAPD Experiments

- Validation of Gyrokinetics and AstroGK
    - Validity of Gyrokinetics at LAPD frequencies
    - AstroGK results for Linear Kinetic Alfven Waves in LAPD

- Next Steps

- Conclusions

MHD Turbulence Theory

The Incompressible MHD Equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla (p + \frac{B^2}{8\pi}) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi\rho} \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times B) \qquad \nabla \cdot \mathbf{B} = 0$$

Elsasser Variables $\mathbf{z}^{\pm} = \mathbf{u}_{\perp} \pm \delta \mathbf{B}_{\perp} / \sqrt{4\pi\rho}$

$\mathbf{z}^{+}$ travels down $\mathbf{B}_0$ field

$\mathbf{z}^{-}$ travels up $\mathbf{B}_0$ field

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp v_A \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} = -\nabla p$$

Linear Term

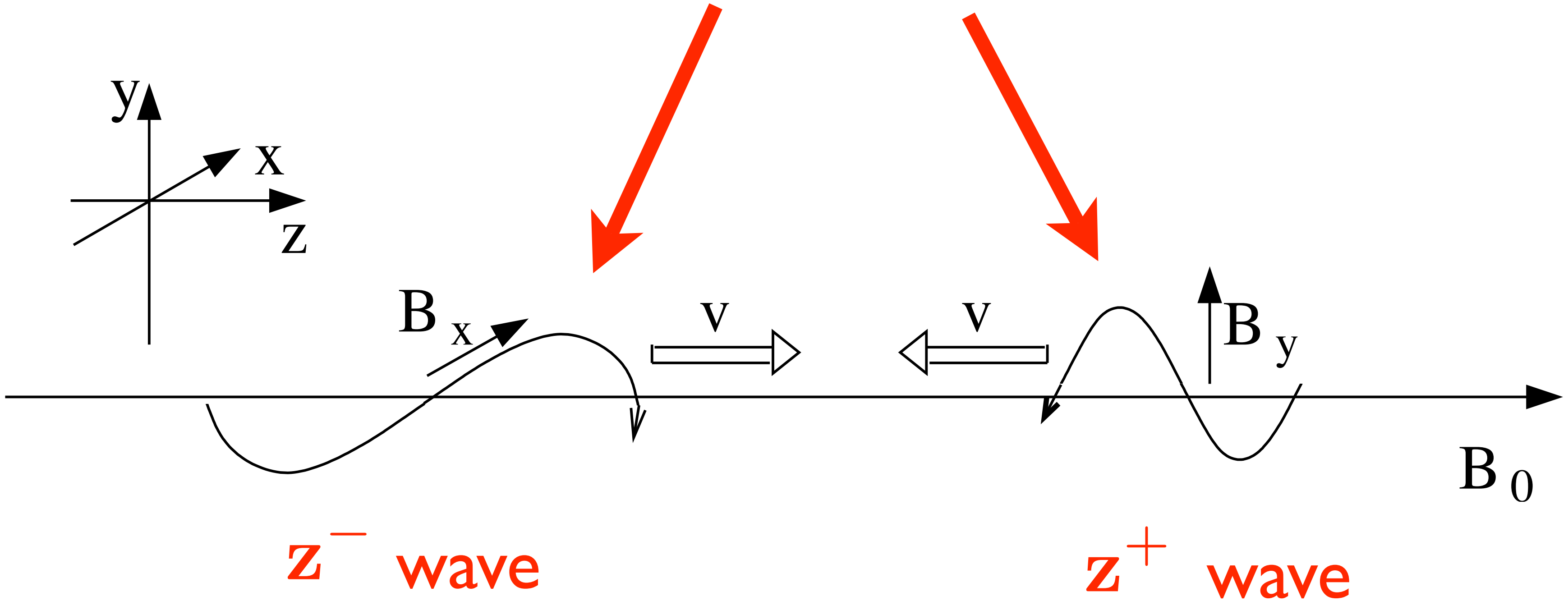
Nonlinear Term → Responsible for "collisions" between oppositely propagating Alfven waves

Alfven Wave "Collisions"

The Incompressible MHD Equations:

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp v_A \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} = -\nabla p$$

Counterpropagating
Alfven waves "collide"

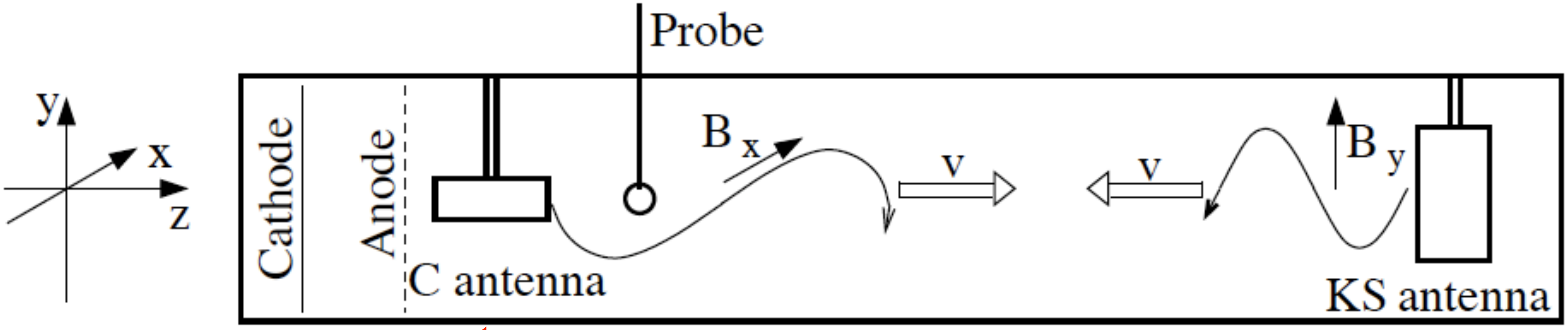


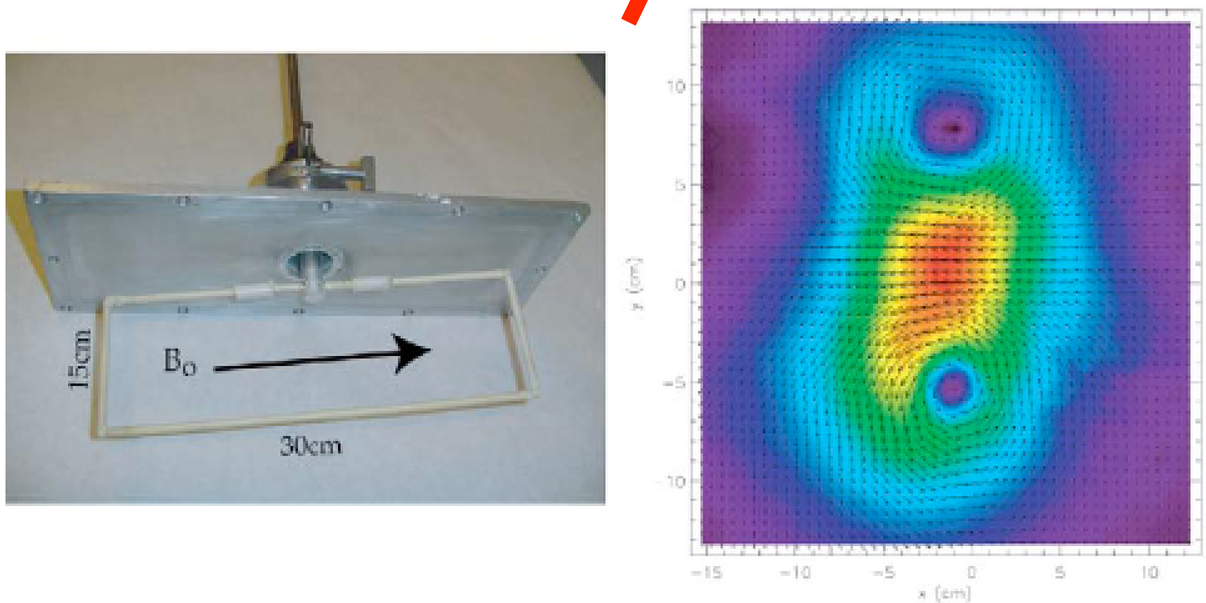
Nonlinear term leads to transfer
of energy to higher wavenumber

Often described as
scattering of the waves

Experimental Setup

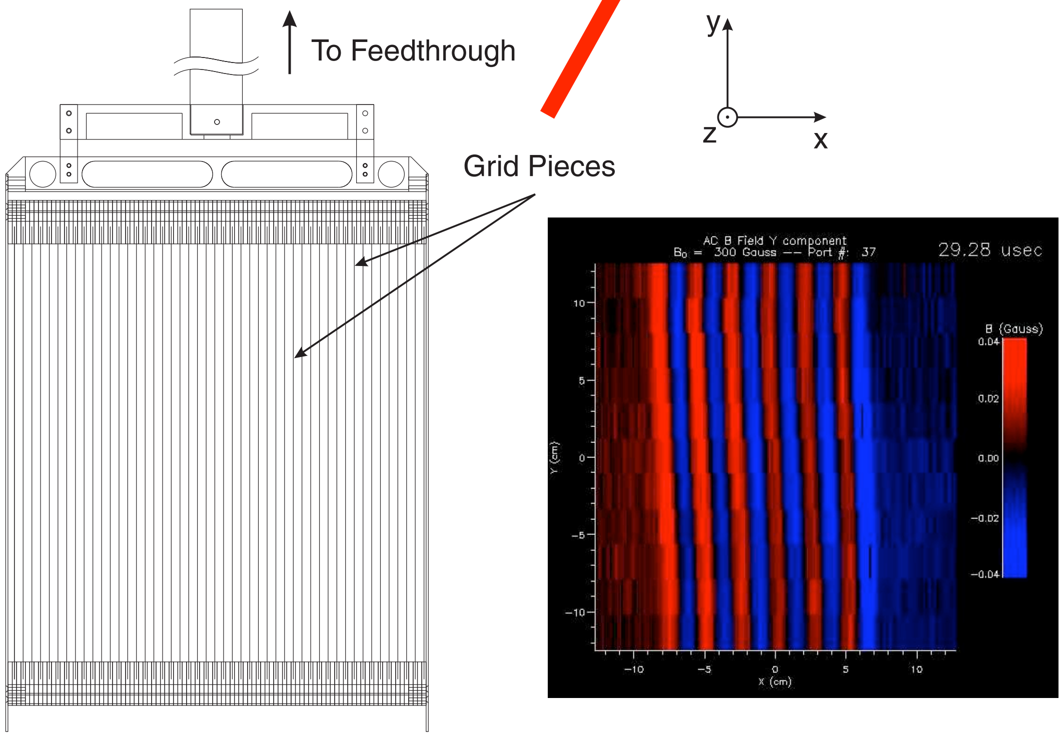
Basic Experiments of Alfven Wave Collisions on the LAPD





C Loop Antenna

(Carter, UCLA)



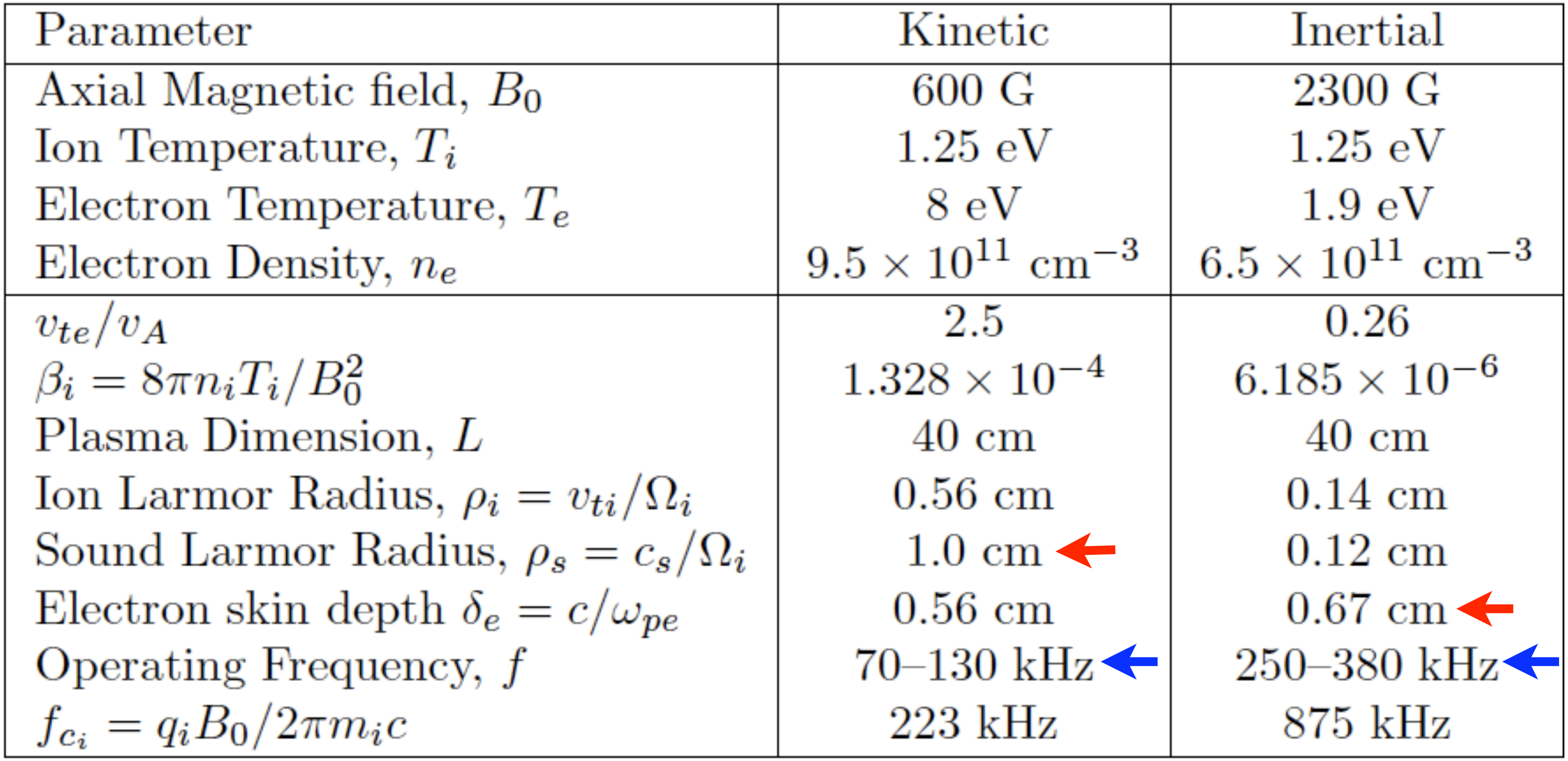
KS Antenna

(Kletzing & Skiff, Univ. Iowa)

Experimental Parameters

LAPD Operating Parameters for

- Kinetic Alfven Wave Regime
- Inertial Alfven Wave Regime



Parameter	Kinetic	Inertial
Axial Magnetic field, B_0	600 G	2300 G
Ion Temperature, T_i	1.25 eV	1.25 eV
Electron Temperature, T_e	8 eV	1.9 eV
Electron Density, n_e	9.5×10^{11} cm^{-3}	6.5×10^{11} cm^{-3}
v_{te}/v_A	2.5	0.26
$\beta_i = 8\pi n_i T_i / B_0^2$	1.328×10^{-4}	6.185×10^{-6}
Plasma Dimension, L	40 cm	40 cm
Ion Larmor Radius, $\rho_i = v_{ti}/\Omega_i$	0.56 cm	0.14 cm
Sound Larmor Radius, $\rho_s = c_s/\Omega_i$	1.0 cm ←	0.12 cm
Electron skin depth $\delta_e = c/\omega_{pe}$	0.56 cm	0.67 cm ←
Operating Frequency, f	70–130 kHz ←	250–380 kHz ←
$f_{c_i} = q_i B_0 / 2\pi m_i c$	223 kHz	875 kHz

Complications for Experimental Program

- LAPD plasma is not well described by Incompressible MHD
  - Moderate Collisionality $\nu \sim \omega$ Requires Kinetic Description
  - Finite Larmor Radius effects $(v_{te} > v_A)$ $k_\perp \rho_s \gtrsim 1$ Kinetic regime
  - Finite Electron Skin Depth $(v_{te} < v_A)$ $k_\perp \delta_e \gtrsim 1$ Inertial regime
  - Finite Frequency $\omega \lesssim \Omega_i$
- We want to study nonlinear effects, but the properties above lead also to linear non-ideal MHD effects
  - We need to separate non-ideal from nonlinear effects to interpret experimental results

AstroGK can model both non-ideal and nonlinear effects!
(in the gyrokinetic limit)

Challenges for Gyrokinetic Modeling

- Finite Larmor radius and finite electron skin depth
  - Correctly modeled in gyrokinetics

- Moderate Collisionality
  - Requires use of advanced, fully conservative collision operator

(Abel, Barnes, Cowley, Dorland, & Schekochihin, 2008)
(Barnes, Abel, Dorland, Ernst, Hammett, Ricci, Rogers, Schekochihin, & Tatsuno, 2009)

- Finite Frequency
  - GK excludes cyclotron frequency effects
  - For each case, must determine magnitude of cyclotron effects

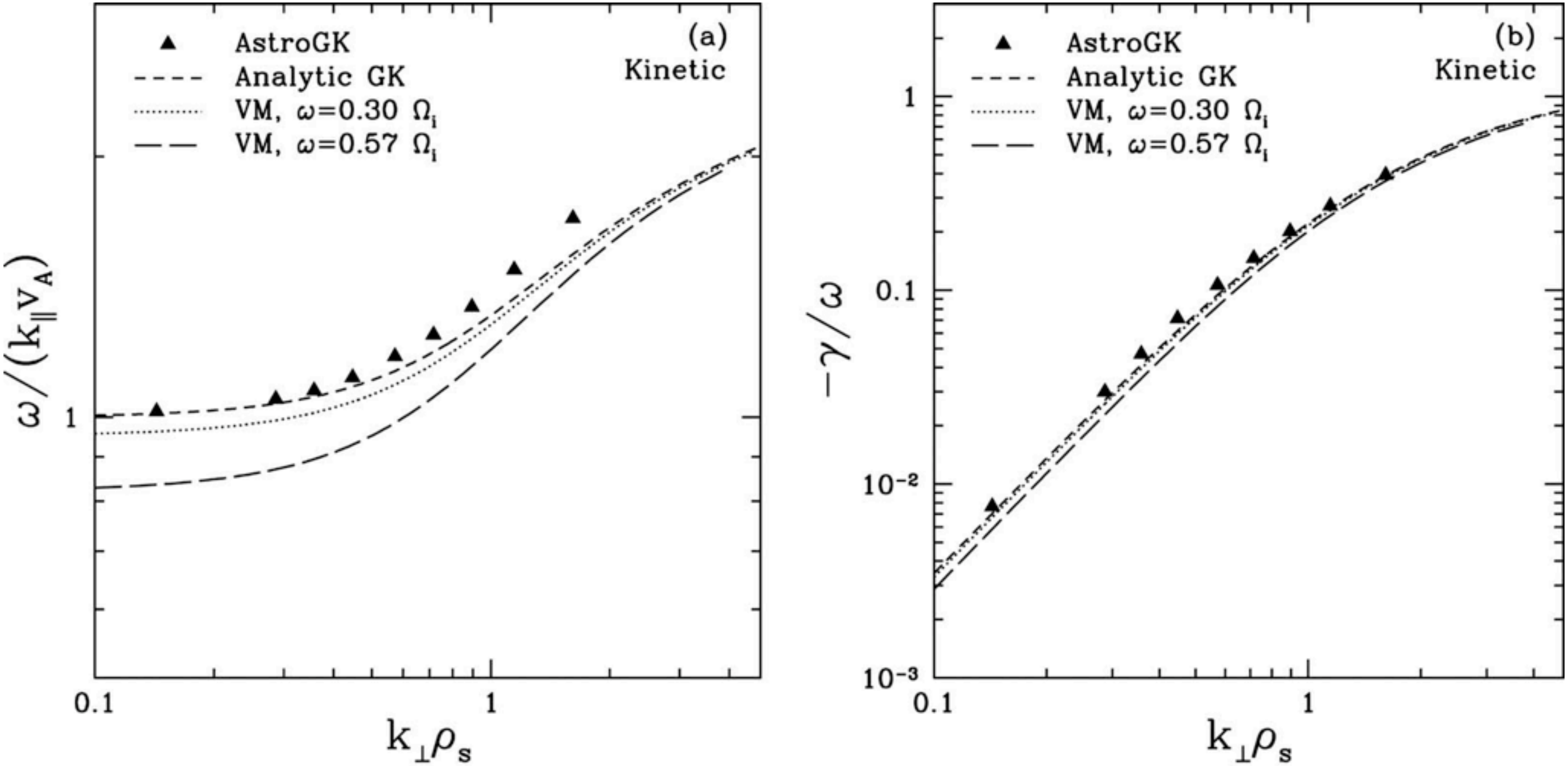
- Geometry and Boundary Effects
  - For direct comparison between simulation and experiment, plasma geometry and boundary effects may be important

Validation of Gyrokinetics in LAPD Plasma

Finite frequency effects are excluded from GK description

- Must verify that cyclotron effects are not too significant

Kinetic Alfven Wave Regime



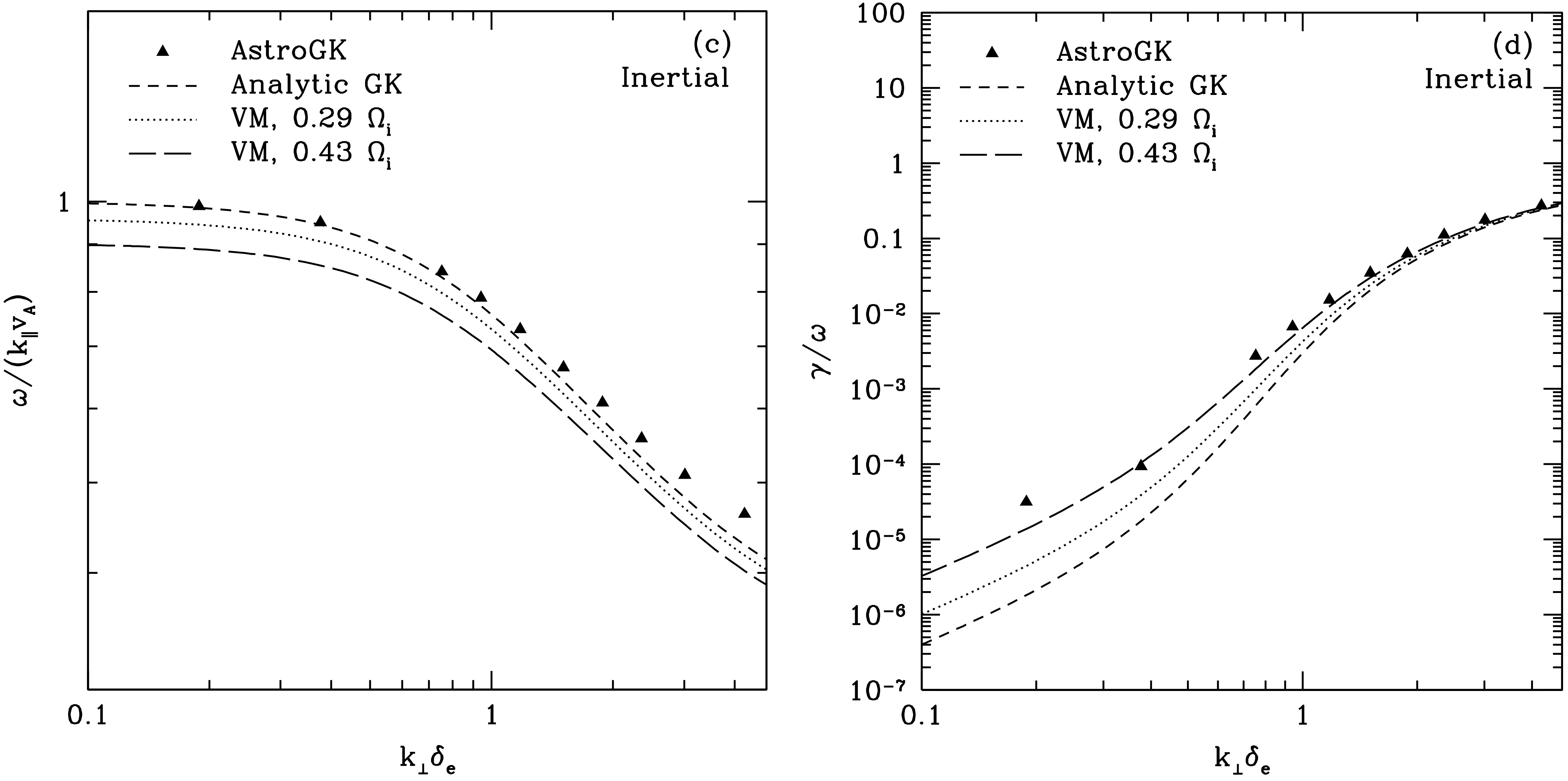
Frequency off by less than 5-20% at large $k_\perp \rho_s$

Validation of Gyrokinetics in LAPD Plasma

Finite frequency effects are excluded from GK description

- Must verify that cyclotron effects are not too significant

Inertial Alfven Wave Regime

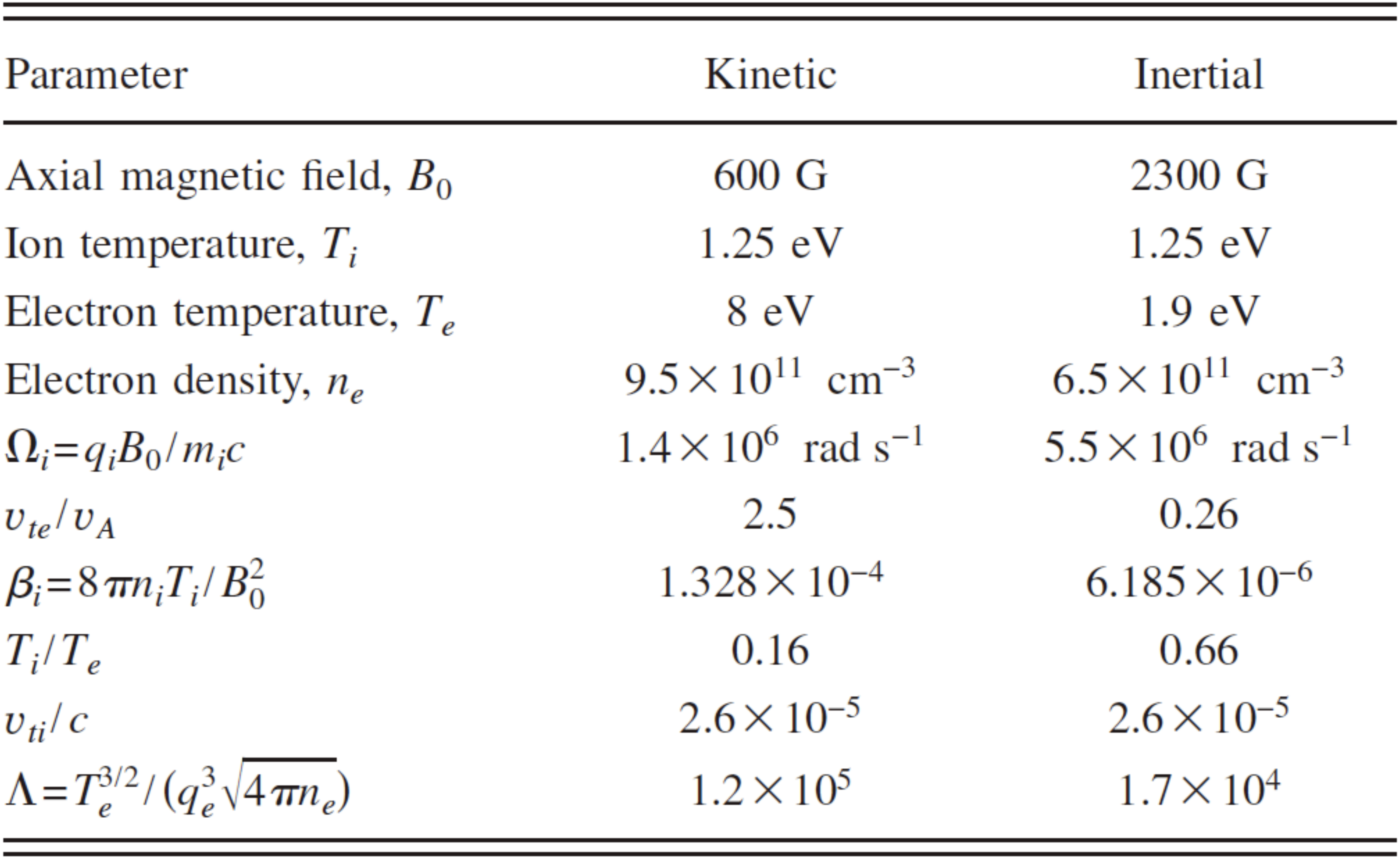


Frequency off by less than 10%

AstroGK Results for KAWs in LAPD

THE UNIVERSITY OF IOWA

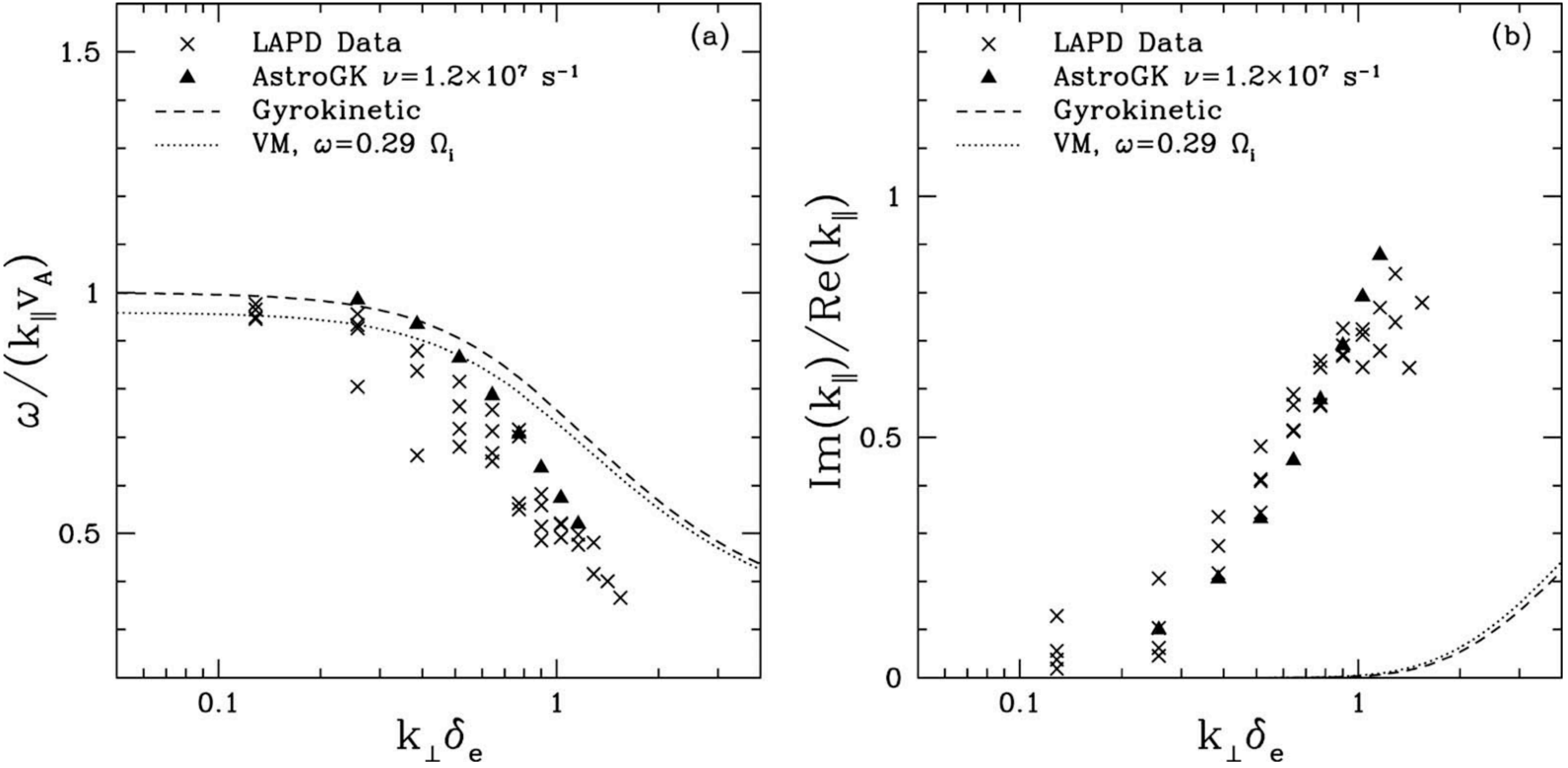
Experimental Parameters



Parameter	Kinetic	Inertial
Axial magnetic field, B_0	600 G	2300 G
Ion temperature, T_i	1.25 eV	1.25 eV
Electron temperature, T_e	8 eV	1.9 eV
Electron density, n_e	9.5×10^{11} cm^{-3}	6.5×10^{11} cm^{-3}
$\Omega_i = q_i B_0 / m_i c$	1.4×10^{6} rad s^{-1}	5.5×10^{6} rad s^{-1}
v_{te}/v_A	2.5	0.26
$\beta_i = 8\pi n_i T_i / B_0^2$	1.328×10^{-4}	6.185×10^{-6}
T_i/T_e	0.16	0.66
v_{ti}/c	2.6×10^{-5}	2.6×10^{-5}
$\Lambda = T_e^{3/2}/(q_e^3\sqrt{4\pi n_e})$	1.2×10^{5}	1.7×10^{4}

AstroGK Results for KAWs in LAPD

Inertial Alfven Wave Regime $\omega = 0.29\ \Omega_i$

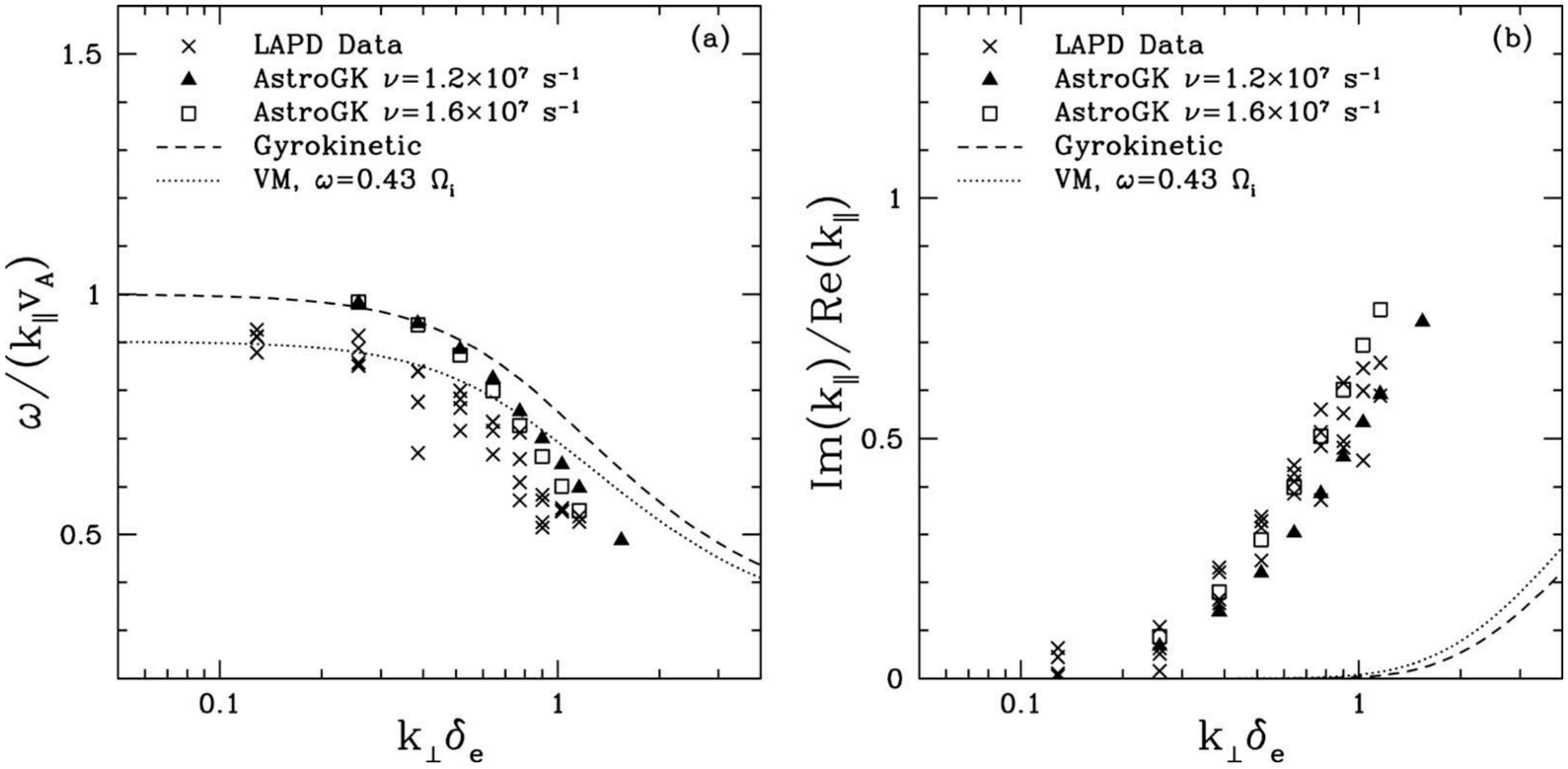


Braginskii $$\nu_e = \frac{16\sqrt{\pi}e^4 Z n_0 \ln\,\Lambda}{3m_e^2 v_{te}^2} = 7 \times 10^6 s^{-1}$$

AstroGK $$\nu_e = 1.2 \times 10^7 s^{-1}$$

AstroGK Results for KAWs in LAPD

Inertial Alfven Wave Regime $\omega = 0.43\ \Omega_i$



Braginskii

$$\nu_e = \frac{16\sqrt{\pi}e^4 Z n_0 \ln\Lambda}{3m_e^2 v_{te}^2} = 7\times10^6 s^{-1}$$

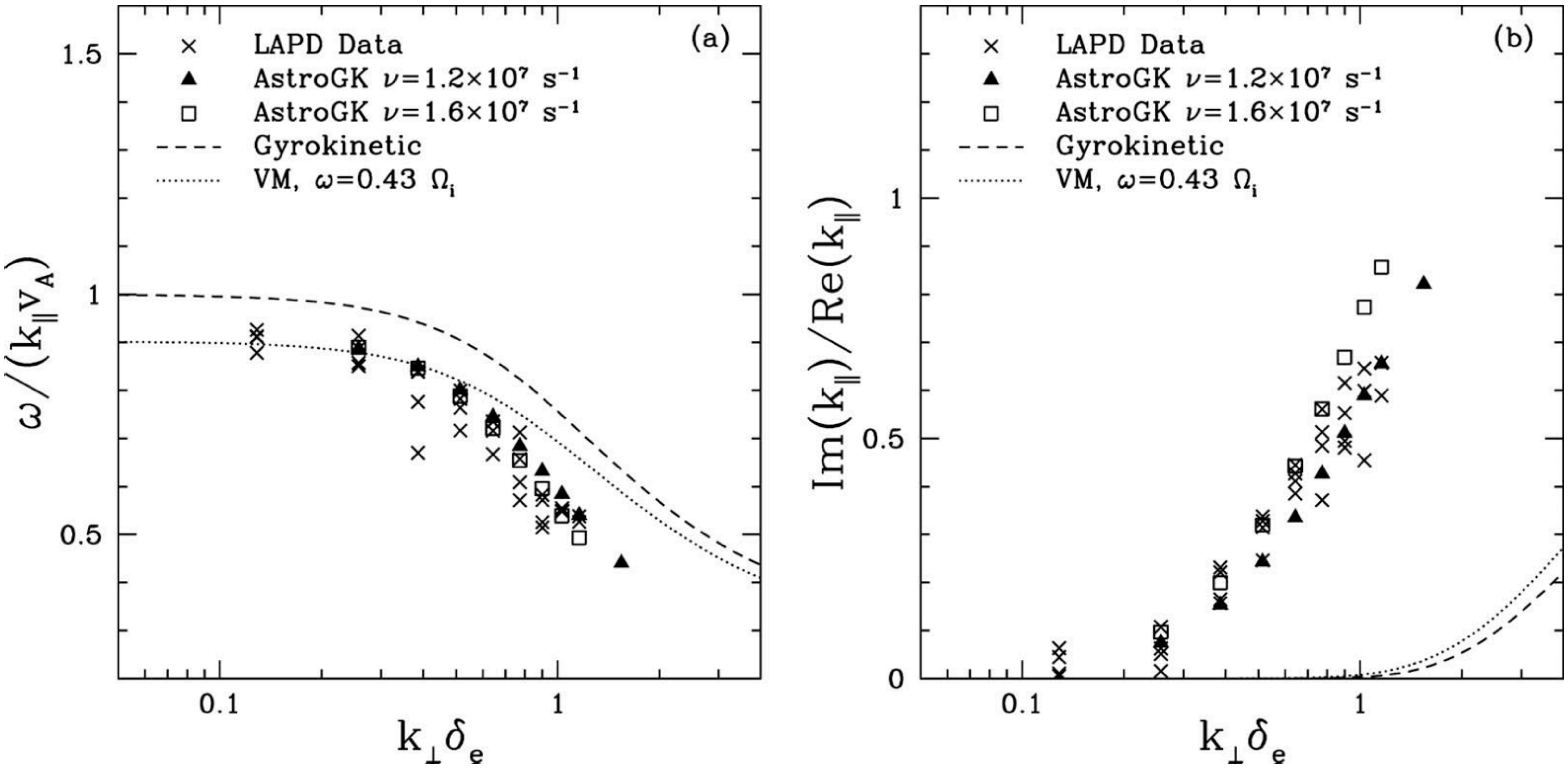
AstroGK

$$\nu_e = 1.2\times10^7 s^{-1}$$
$$\nu_e = 1.6\times10^7 s^{-1}$$

"Correcting" for Cyclotron Effects

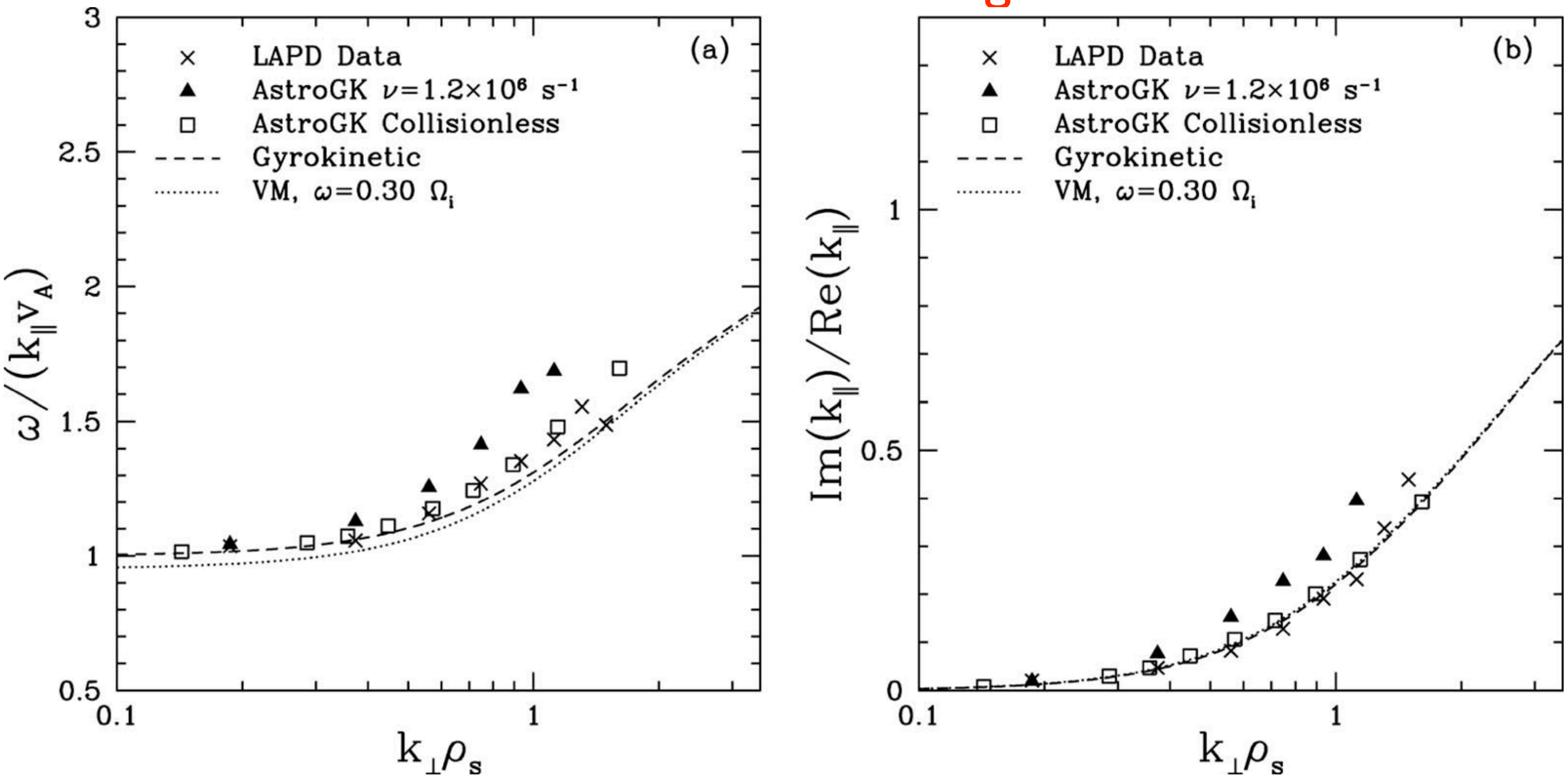
We can subtract the difference between the collisionless gyrokinetic and Vlasov-Maxwell eigenfrequencies to "correct" LAPD results for finite frequency effects

Corrected



AstroGK Results for KAWs in LAPD

Kinetic Alfven Wave Regime $\omega = 0.30\ \Omega_i$



Braginskii

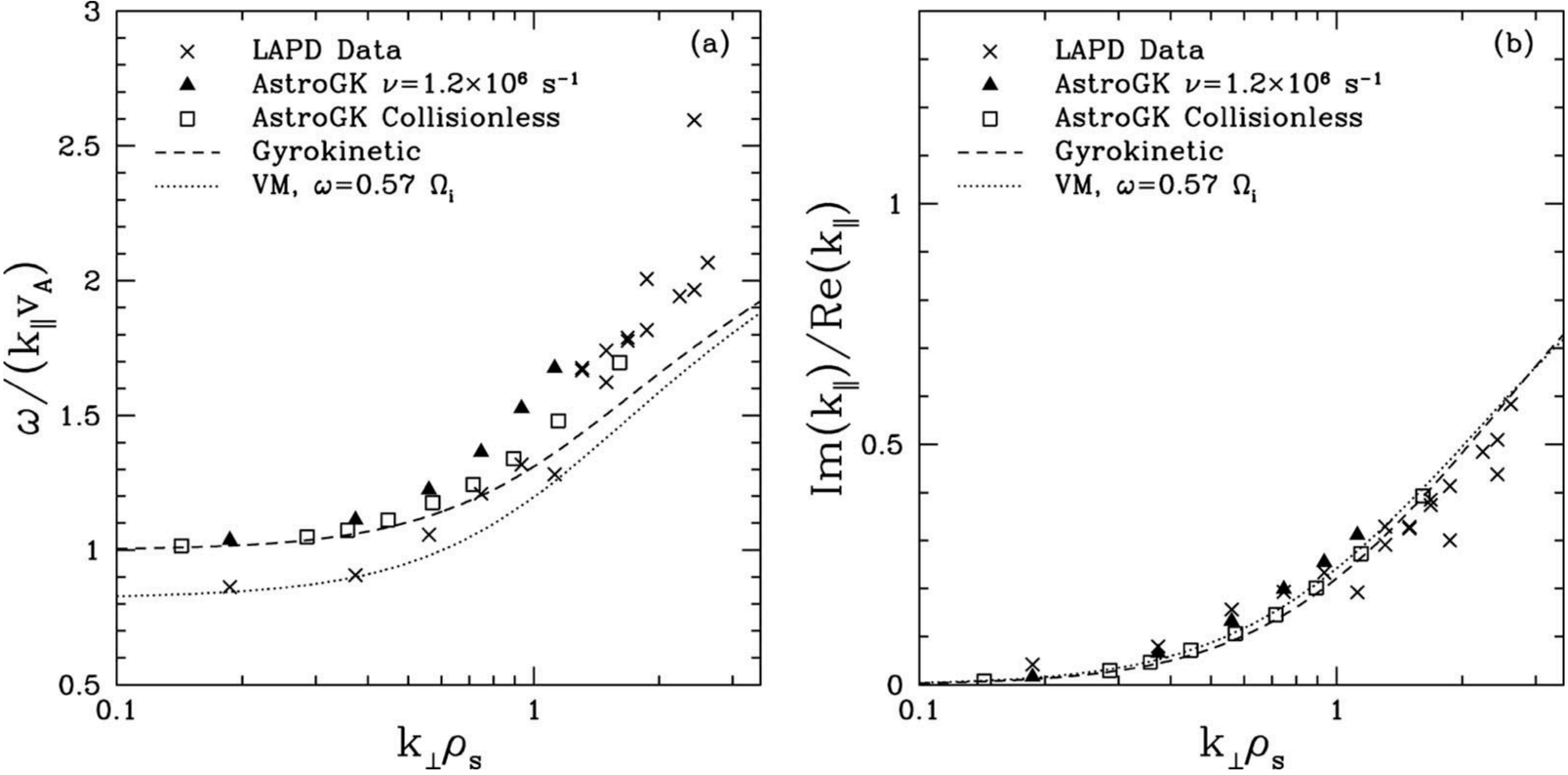
$$\nu_e = \frac{16\sqrt{\pi}e^4 Z n_0 \ln\Lambda}{3m_e^2 v_{te}^2} = 1.4\times 10^6 s^{-1}$$

AstroGK

$$\nu_e = 1.2\times 10^6 s^{-1}$$

AstroGK Results for KAWs in LAPD

Kinetic Alfven Wave Regime $\omega = 0.57\ \Omega_i$



Braginskii $$\nu_e = \frac{16\sqrt{\pi}e^4 Z n_0 \ln\,\Lambda}{3m_e^2 v_{te}^2} = 1.4 \times 10^6 s^{-1}$$

AstroGK $$\nu_e = 1.2 \times 10^6 s^{-1}$$

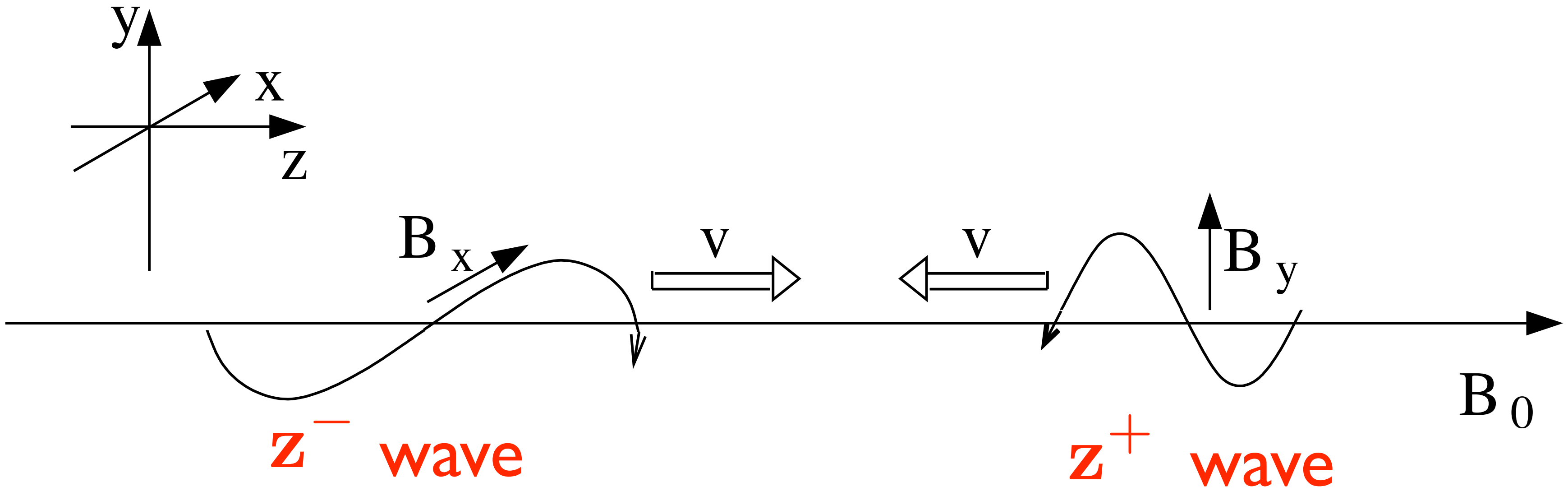
Current and Future Gyrokinetic Modeling

- Nonlinear Energy Transfer Rate dependence on amplitude
  -Determine amplitudes necessary in experiment

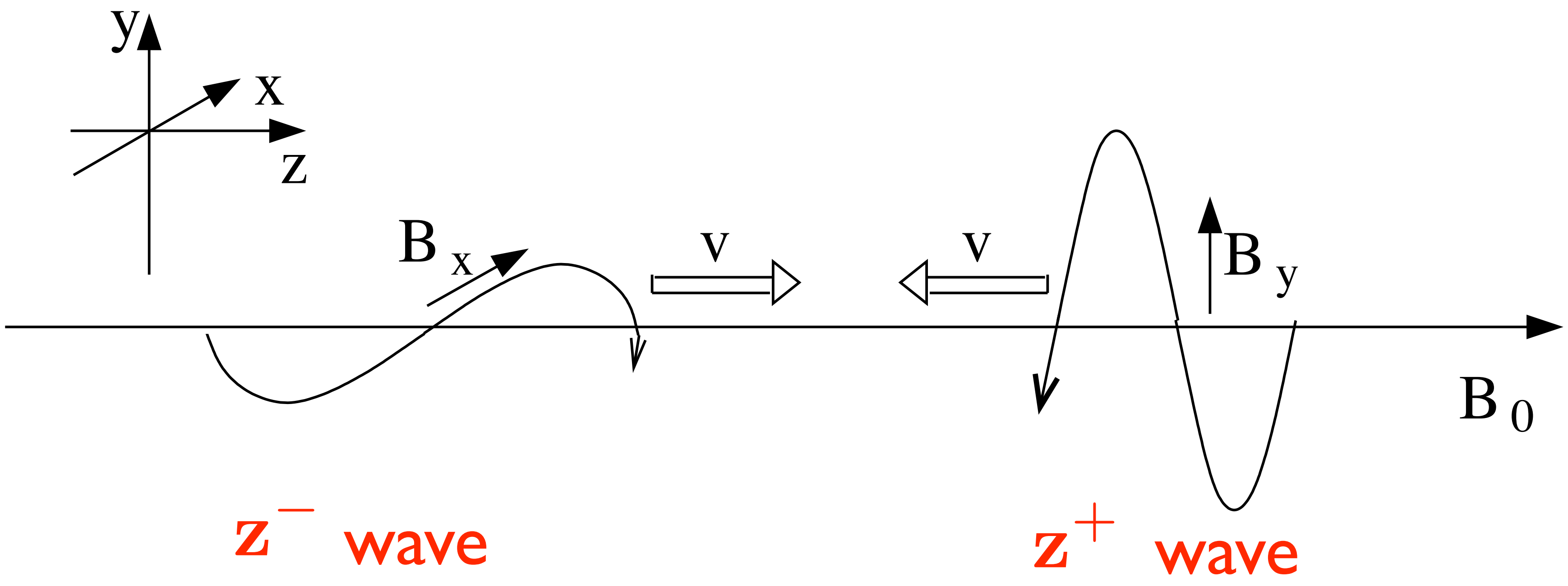
- Predict Nonlinear Product for comparison to experiment

- Develop improved diagnostics: The Elsasser Probe

Amplitude Dependence of Nonlinear Energy Transfer

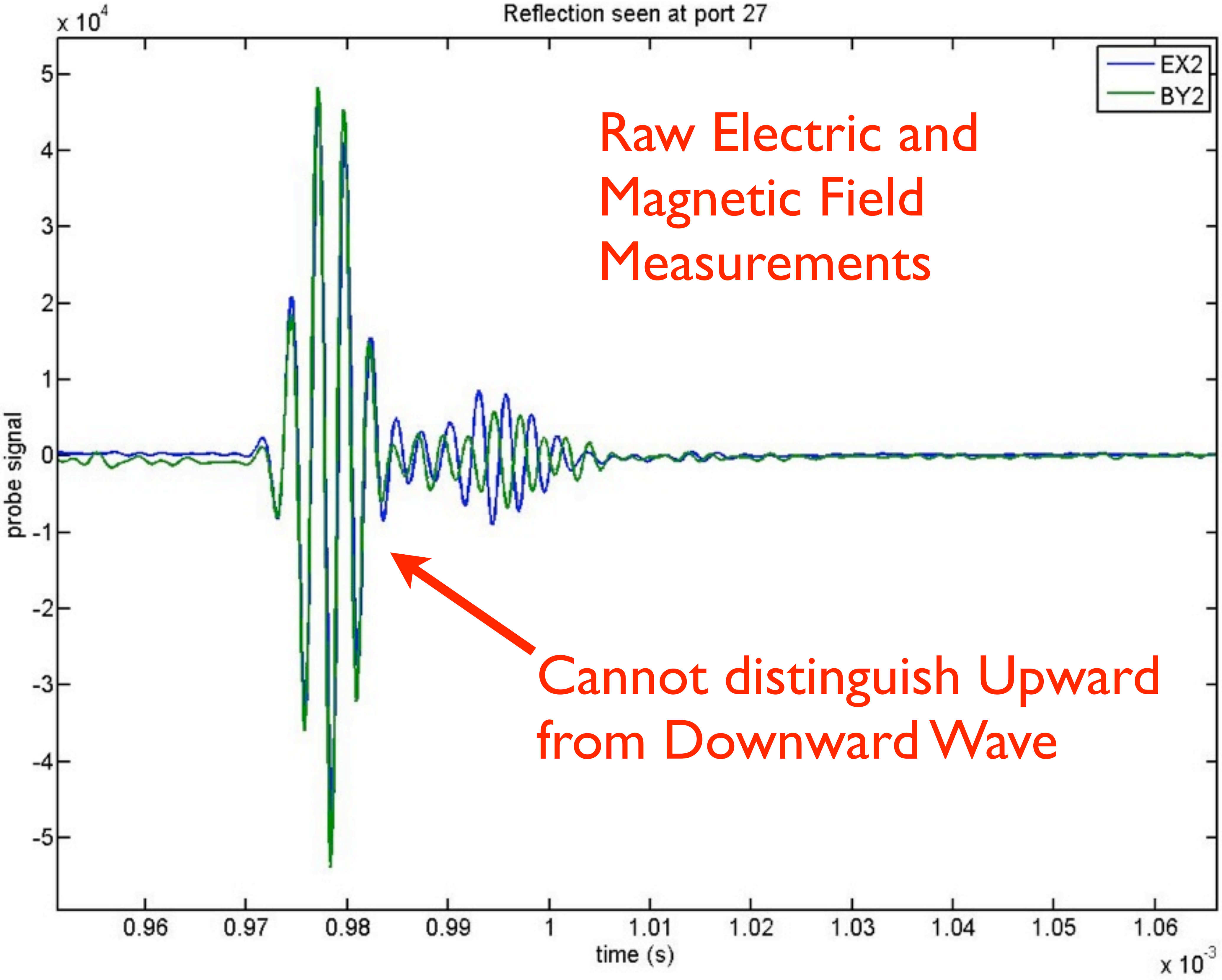


- Consider evolution of $\mathbf{z}^-$ wave: $\dfrac{\partial \mathbf{z}^-}{\partial t} + v_A \cdot \nabla \mathbf{z}^- + \mathbf{z}^+ \cdot \nabla \mathbf{z}^- = -\nabla p$
- Nonlinear term depends on amplitude of $\mathbf{z}^+$ wave: $(\mathbf{z}^+ \cdot \nabla)\mathbf{z}^-$



Sufficiently fast nonlinear transfer is needed to observe nonlinearity in experiment!

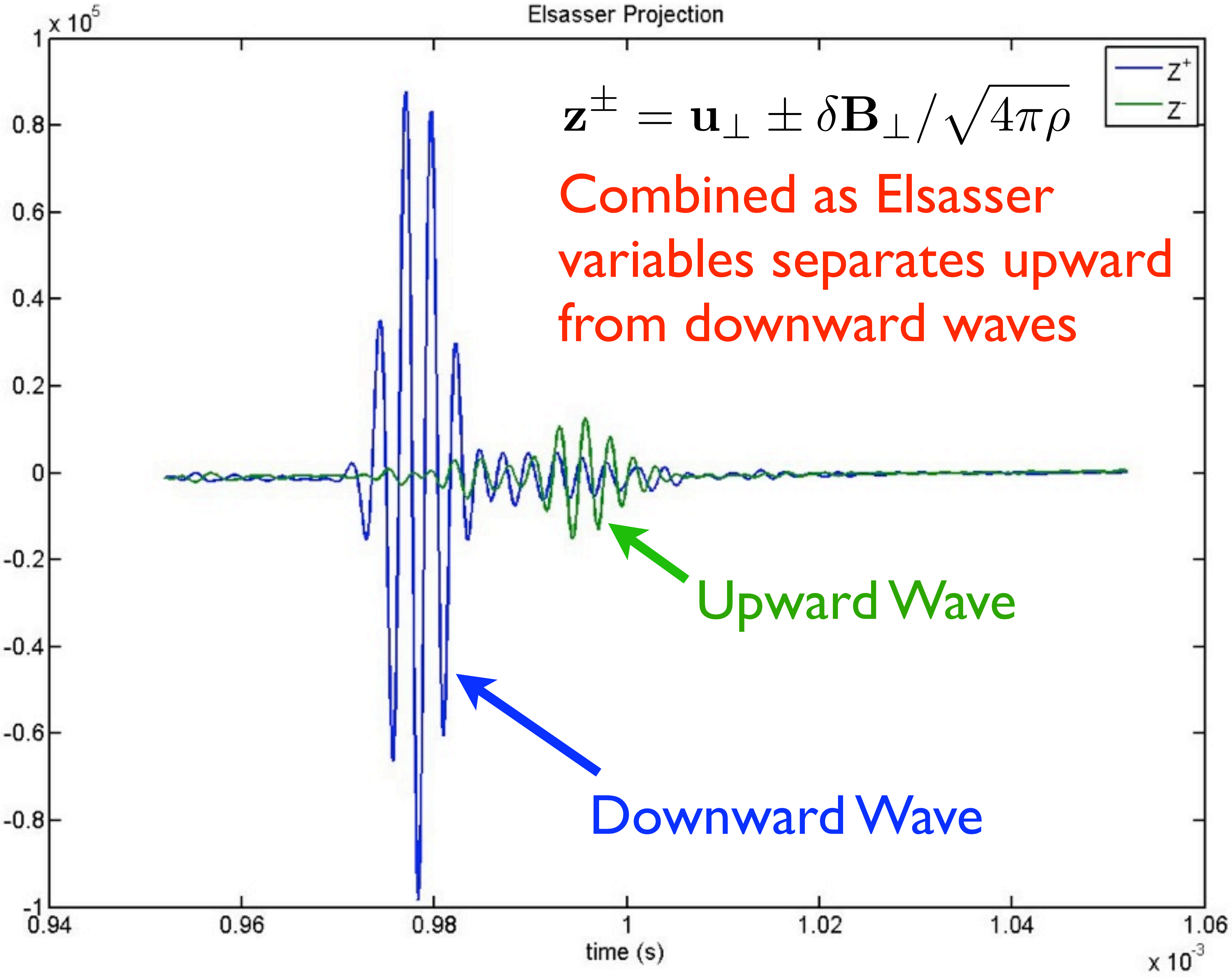
Elsasser Probe



Raw Electric and Magnetic Field Measurements

Cannot distinguish Upward from Downward Wave

Elsasser Probe



$$\mathbf{z}^{\pm} = \mathbf{u}_{\perp} \pm \delta\mathbf{B}_{\perp}/\sqrt{4\pi\rho}$$

Combined as Elsasser variables separates upward from downward waves

Upward Wave

Downward Wave

Conclusions

- LAPD experiments in simple geometries can be used to test fundamental concepts in plasma turbulence

- Basic concepts are based in MHD theory, but lab plasmas are poorly described by MHD

- Gyrokinetics can be applied to model LAPD experiments
  - Can model both linear non-ideal and nonlinear effects
  - Moderate collisionality is a big challenge
  - Finite-frequency effects must be closely monitored

- Linear gyrokinetic simulations have successfully modeled both kinetic and inertial Alfven waves in the LAPD

- Next we will perform nonlinear gyrokinetic simulations to:
  - Guide the design of experiments
  - Interpret the results of experiments

THE END