Dispersion of fast ions in models of anisotropic plasma turbulence

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and the TORPEX team

Generalized dispersion of particles

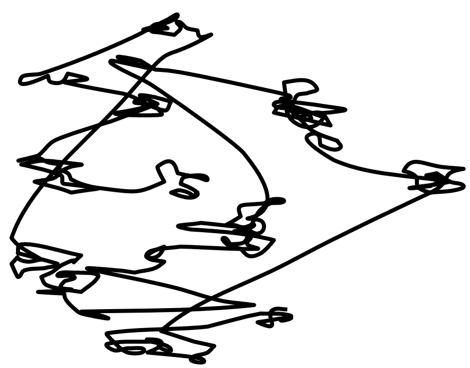
i.e. Metzler/Klafter Phys Rep 339 (2000) 1-77; Balescu book and articles

$$\delta x = x(t) - x(0)$$

$$\sigma^2 = \overline{(\delta x - \overline{\delta x})^2}$$

$$\sigma^2(t) \sim t^\gamma$$

- $\gamma < 1$ subdiffusive
- $\gamma = 1$ diffusive
- $\gamma > 1$ superdiffusive
- $\gamma = 2$ ballistic

$\gamma = 1$  $\gamma > 1$  $\gamma < 1$ 

Δt_1 = Δt_2 = Δt_3

Brownian walk $\subset$ continuous time random walks

Random walks in magnetized plasmas

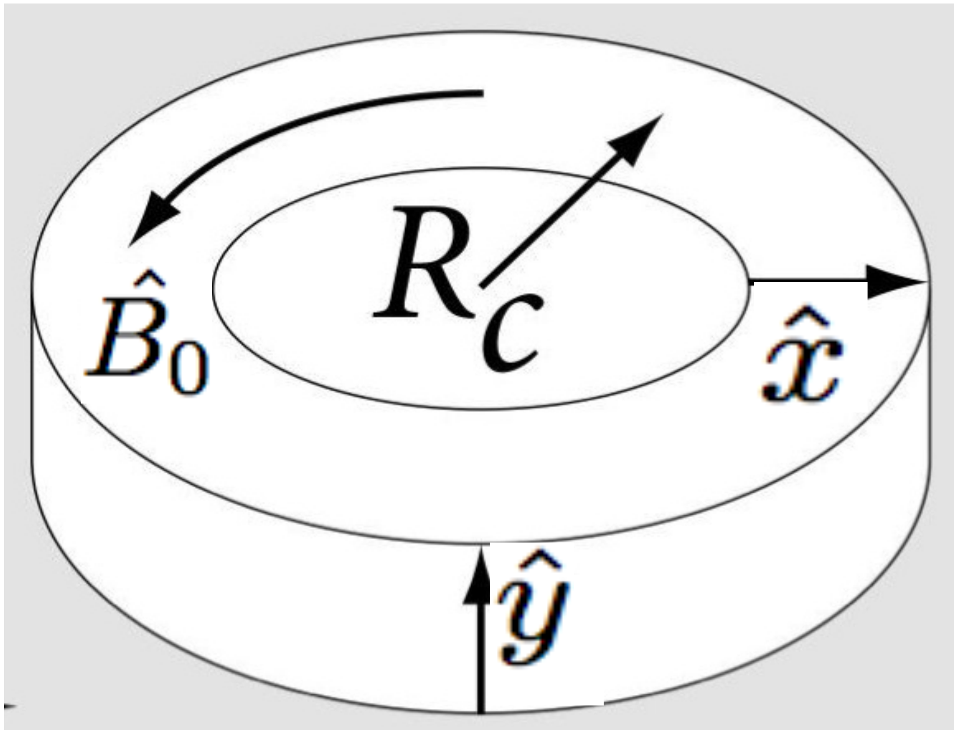
1) **Microscale**:
Gyrokinetic Z-pinch turbulence with zonal flows in a δf PIC code

2) **Macroscale**:
Interchange turbulence with blobs in TORPEX

What is the result of the interplay between plasma turbulence and ions with Larmor radii comparable to the scale of the turbulence?

Is the dispersion of these ions described by a diffusion equation or a generalized non-diffusive dispersion?

Z pinch geometry



$$\Omega_s = \frac{q_s B}{m_s c} \qquad \mathbf{v}_B = \frac{v_\parallel^2 + \frac{1}{2}v_\perp^2}{\Omega_s R_c}\hat{y}$$

- Magnetic field only toroidal, B ~ 1/r to edge.
- Gyroaveraged ExB drift produces radial (x) and axial (y) turbulent dispersion.
- Axial B drift $\mathbf{v}_B$ is constant for each particle.

Pressure-gradient-driven instability in a gyrokinetic Z-pinch

Brömstrup PhD, U Maryland 2008; Ricci *et al* PRL 97 245001 (2006)

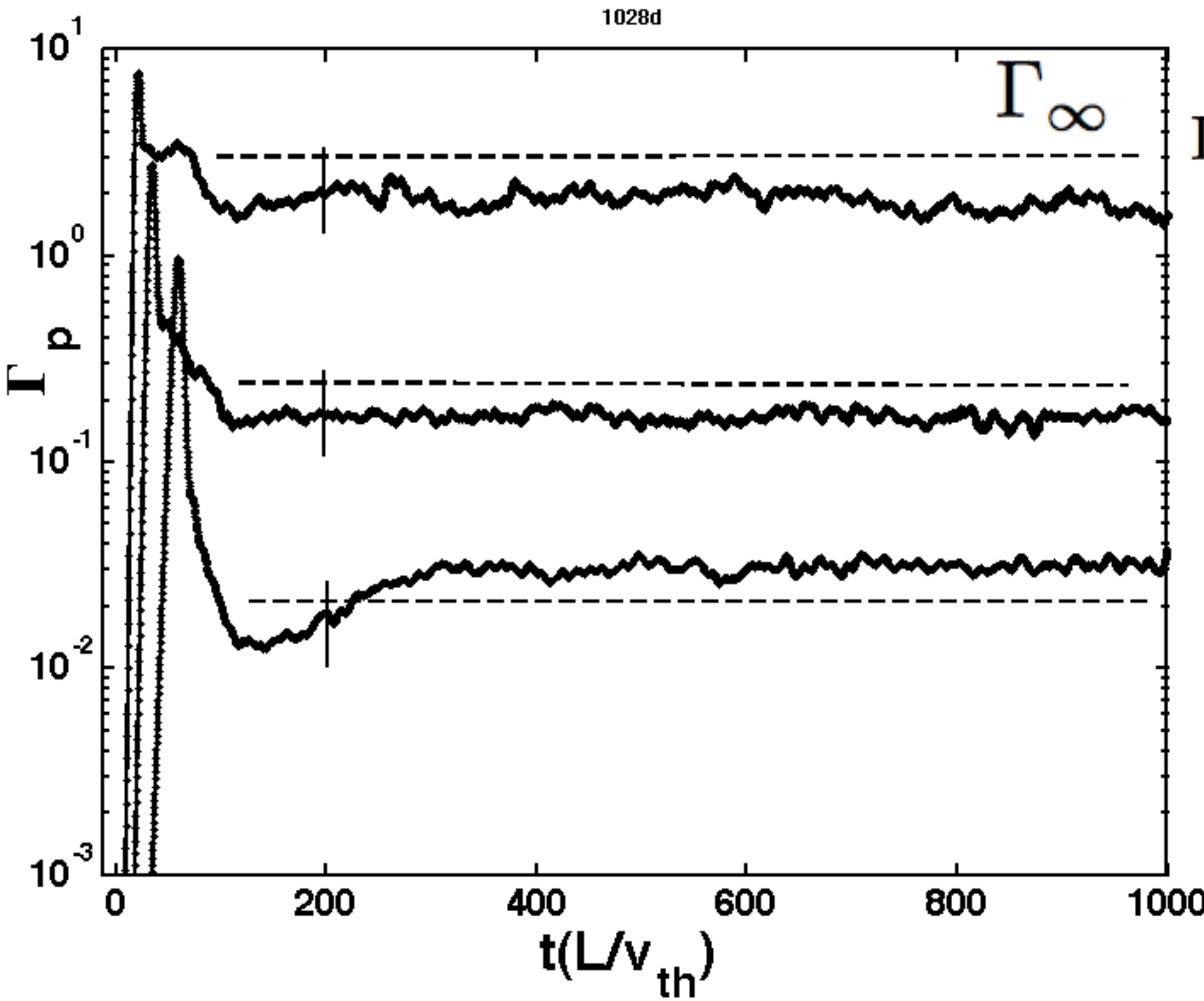
- Our new gyrokinetic, local ($1/L_n = -\nabla n/n$), δf, particle-in-cell (PIC) code (GSP), uses electrons and ions in the Z-pinch geometry.
- We extract a subset of the marker particles and study their dispersion in the nonlinear phase of the turbulence.
- This turbulence is driven by a density gradient ($1/L_T = 0$) and curvature, and is unstable for $2/7 < L_n/R < \pi/2$

Results from nonlinear gyrokinetic simulations

- Benchmarked fluxes with GS2 – within 50% agreement
- Axially: superdiffusive to ballistic
- Radially: diffusive for several density gradients
- Comparing diffusion coefficients from test-particles and flux/gradient relation
- Dependence of test-particle diffusion coefficient on perpendicular velocity

Particle flux compared with GS2

Ricci *et al* PRL 97 245001 (2006) for GS2 data



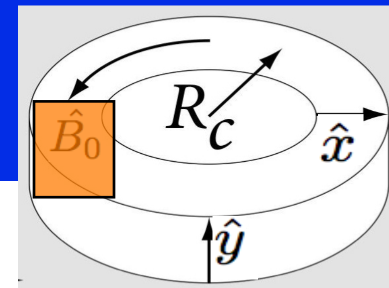
$$\Gamma_p = \sum_i^{N_{part}} \delta f_1 \langle v_x \rangle_R$$

Top to bottom: strong to weak gradient;

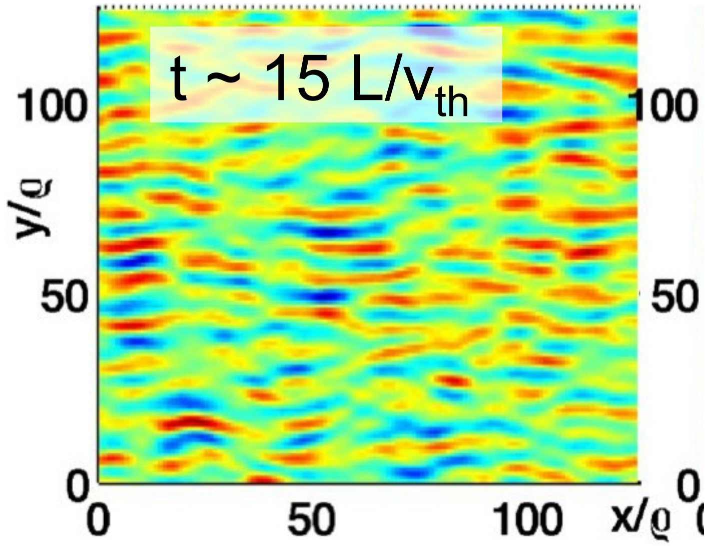
Dashed lines:
published GS2 values

Units for Γ_p :
$(\rho_i/R)^2 n_0 v_{th_i}$

Gyrokinetic turbulence

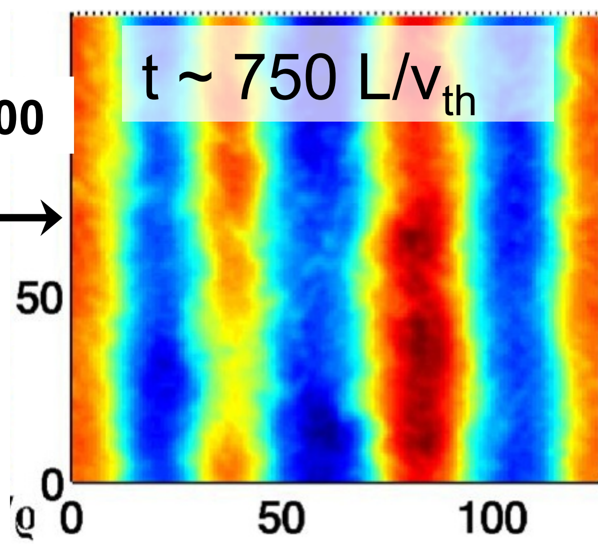


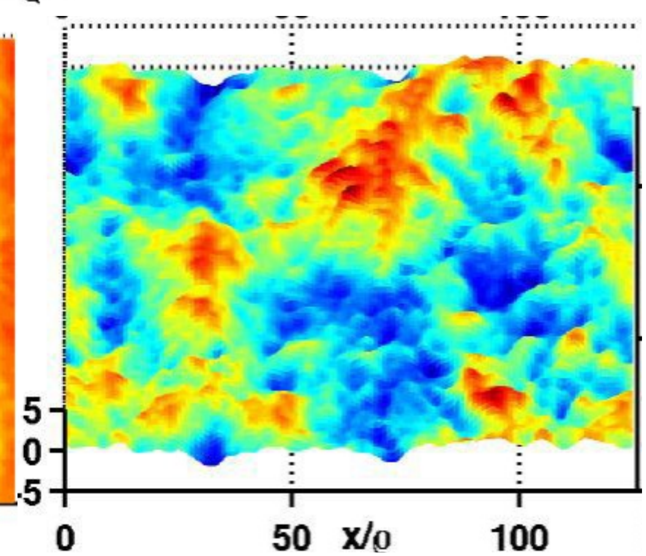
Linear phase - growing k_y modes



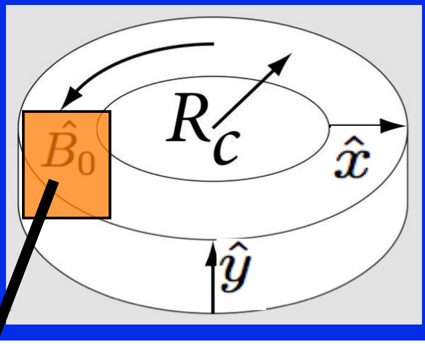
Surface plots of electrostatic potential, $\Phi(x,y)$

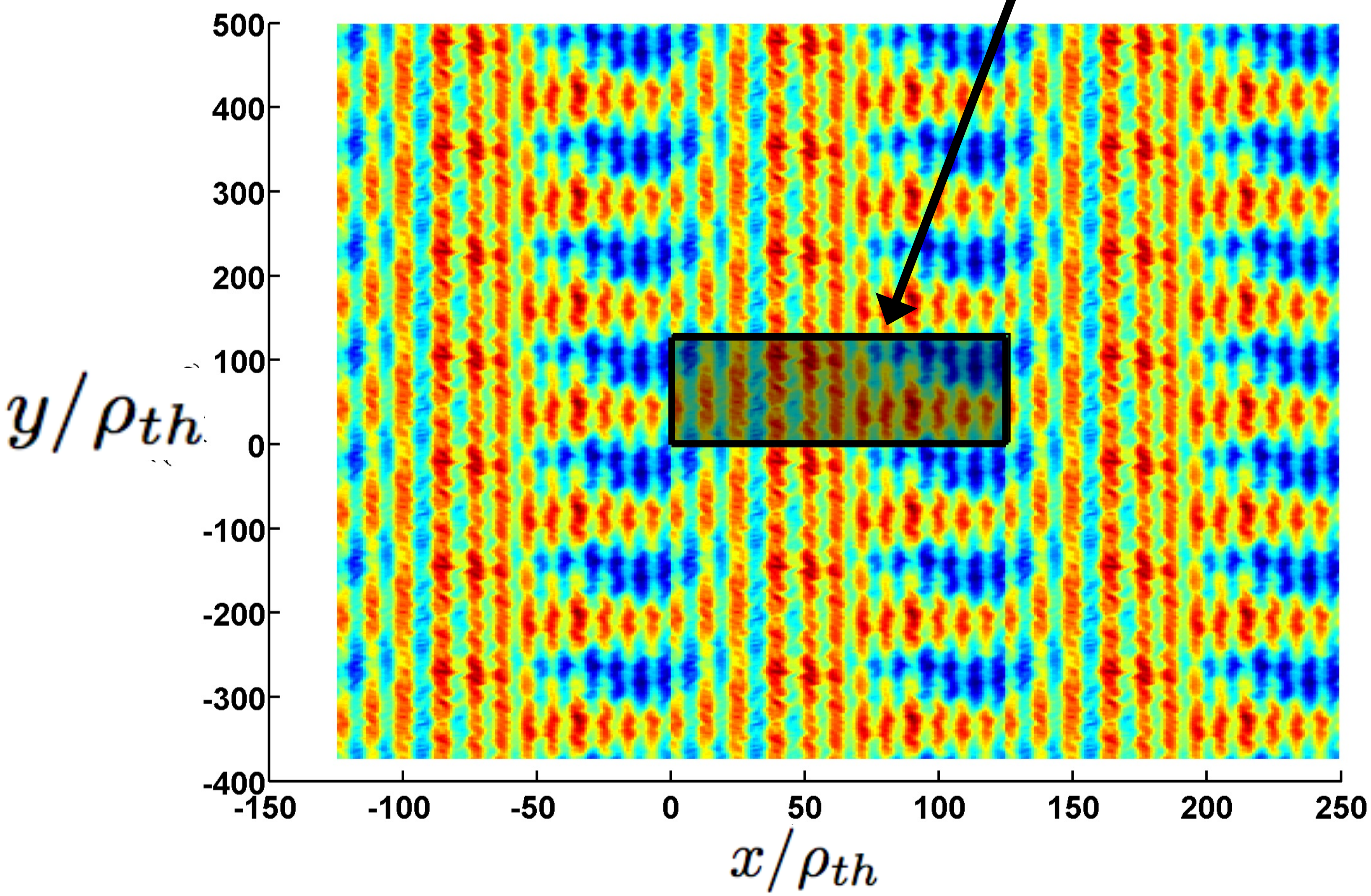
Nonlinear phase - steady zonal flow with background turbulence





Periodic copies for tracers to follow





Kubo number

$$K = v\tau_c/\lambda_c$$

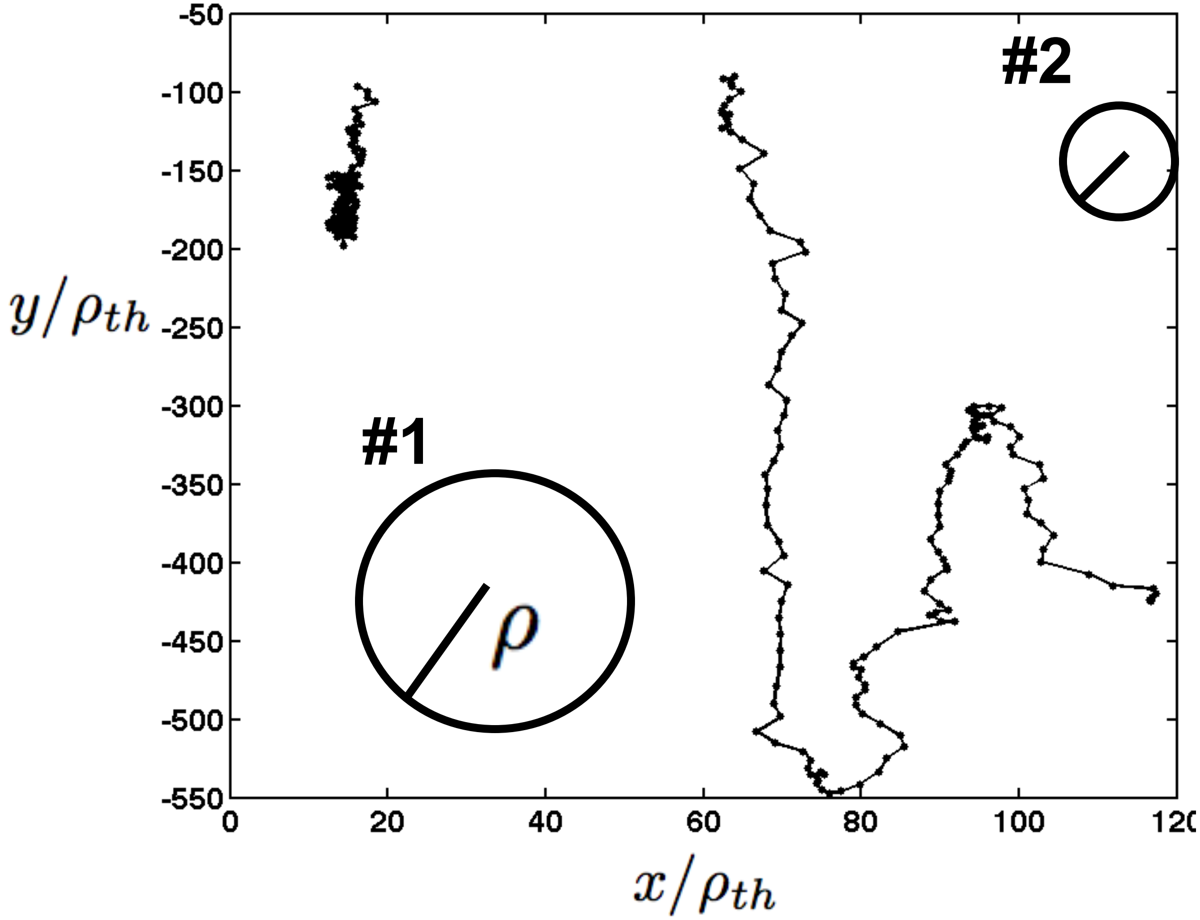
- Particle velocity
- Correlation time
- Correlation length

Hauff and Jenko (PoP 15, 2008) point out:
K ≥ 1 implies constant transport for gyroradii less than the correlation length
K ≤ 1 implies steady decrease in transport for increasing gyroradius

Sample gyrocenter trajectories

K ~ 10, depending on strength of the gradient

Marker particles distributed on $v_\perp$ grid



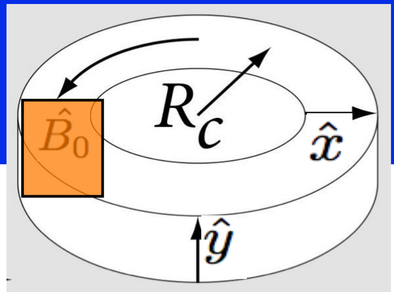
$\Delta x_1 \sim 5\ \rho_{th}$

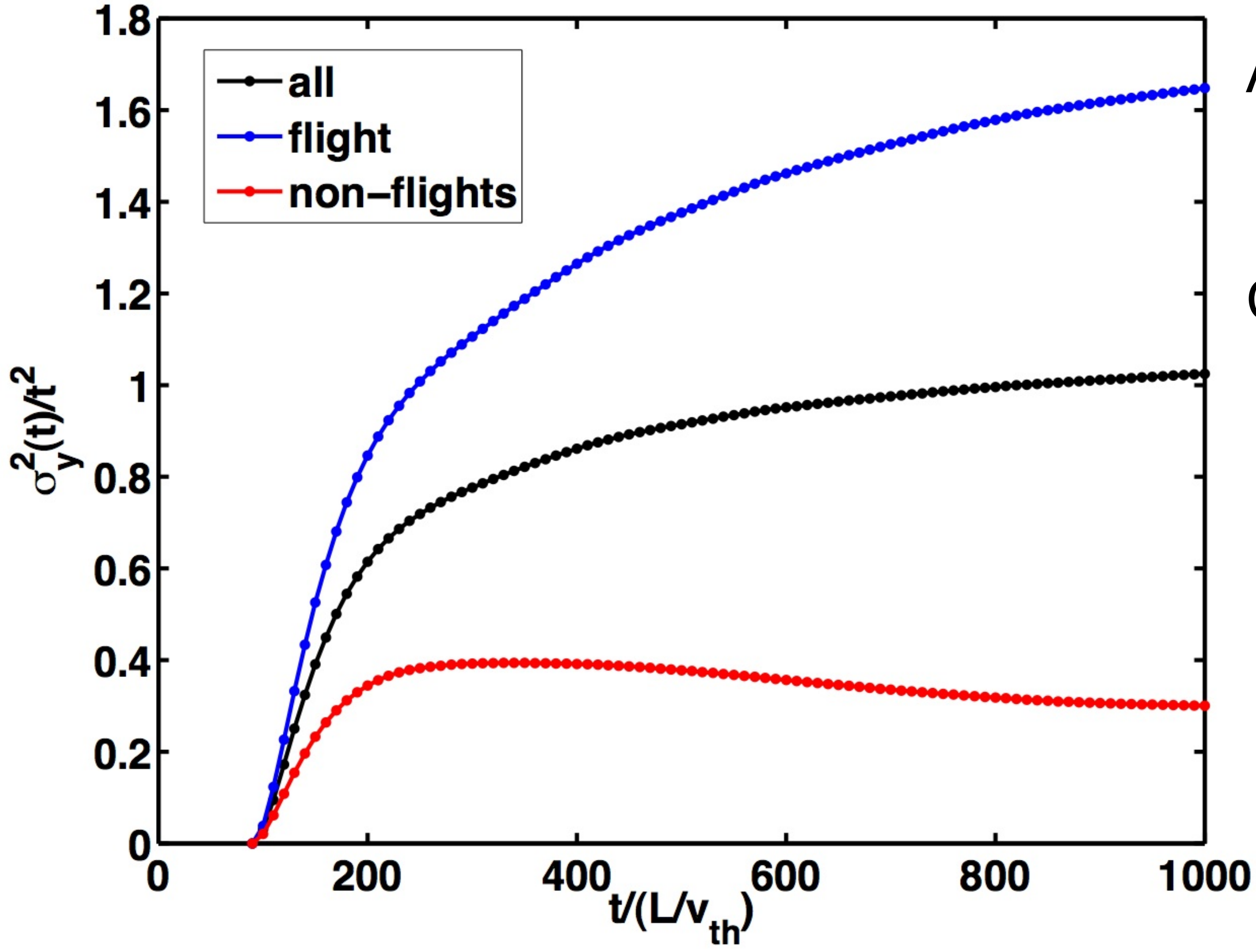
$\Delta x_2 \sim 60\ \rho_{th}$

Larger $v_\perp$ implies more trapping in E x B vortices (for a given turbulence scale)

Axial displacements

compare to Gustafson, del Castillo Negrete, Dorland, PoP 102309 (2008)

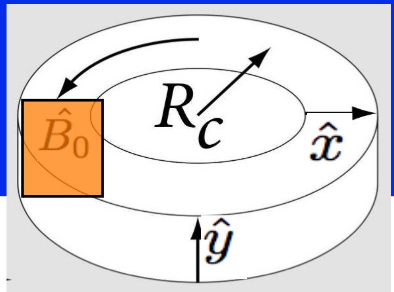




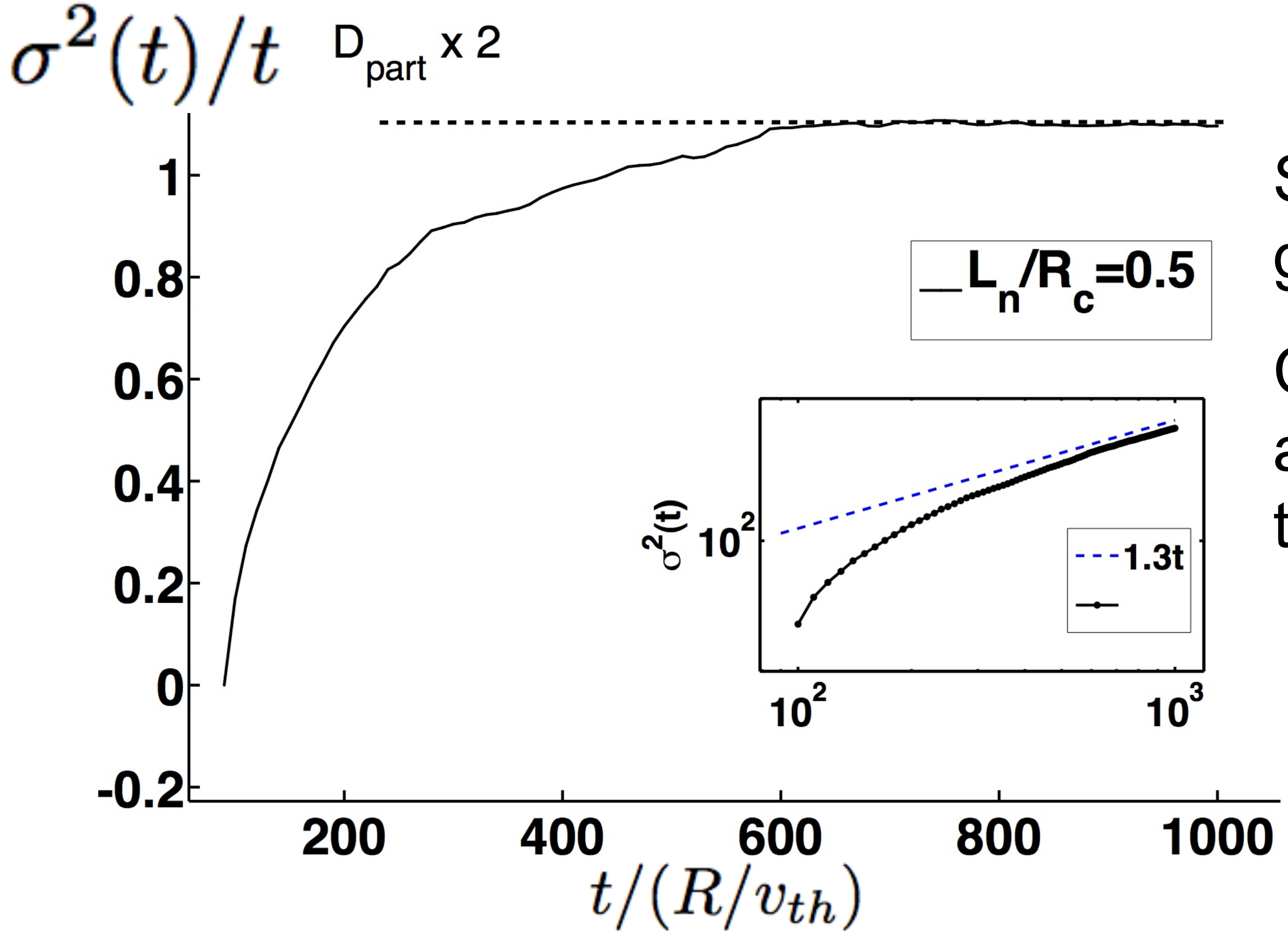
Approaching ballistic (γ~2)

Consistent with our study of a prescribed vortex chain in shear flow (see ref.)

Radial displacements



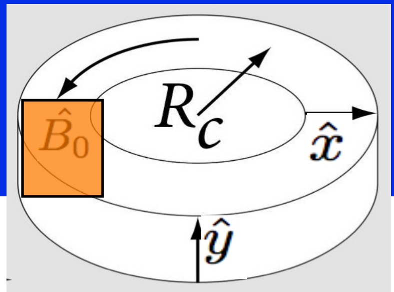
c.f. Manfredi & Dendy PRL 1996; Zhang *et al* PRL 2008; Sanchez et al PRL 2008; Hauff *et al* PRL 2009

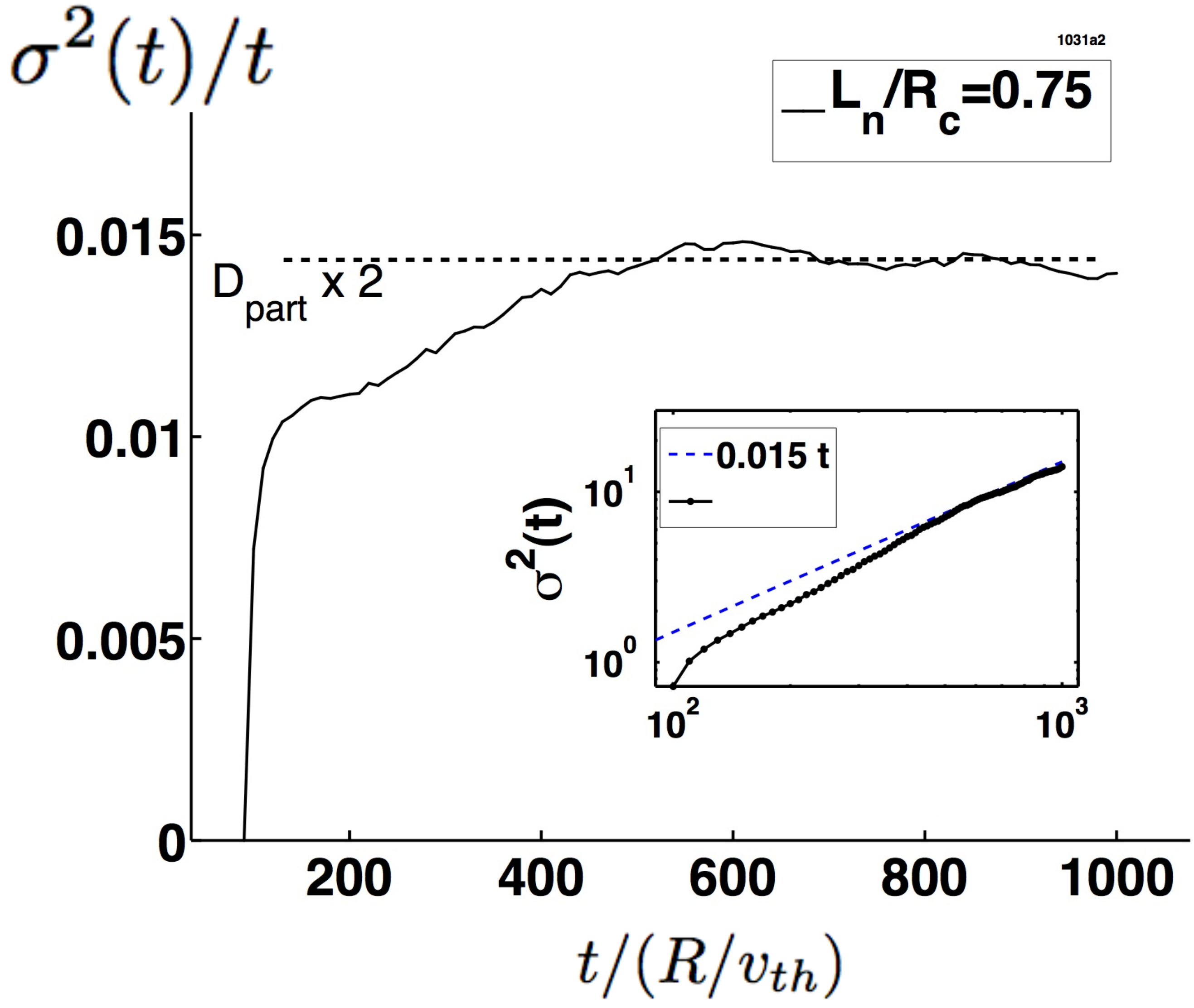


Strongest gradient tested

Clearly diffusive after $t = 600\ R/v_{th}$.

Radial displacements





Medium gradient (L_n)

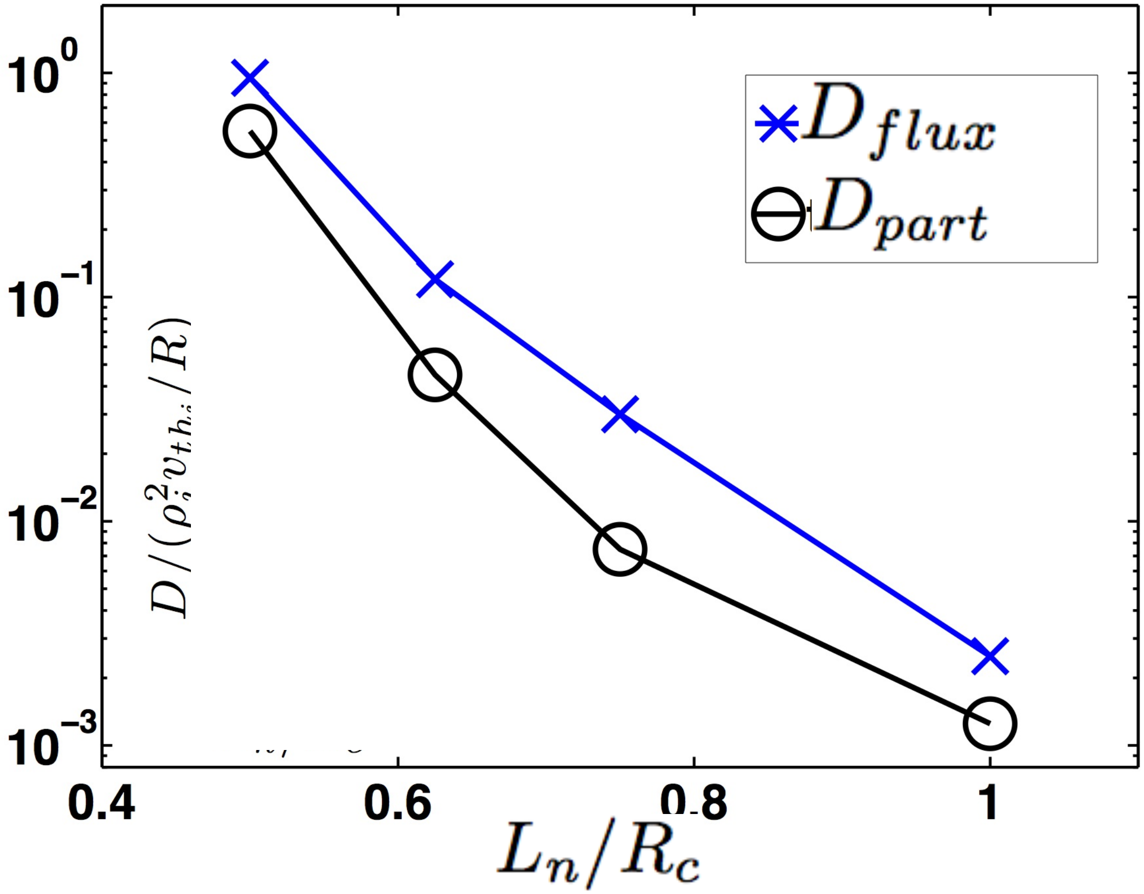
Clearly diffusive after t = 600 R/v_{th}.

Smaller diffusivity consistent with observed change in flux

Diffusivity two ways

compare with Basu *et al*
Phys Plasmas, 10 2003

$$D_{part} = \frac{\sigma_x^2(t)}{2t} \quad D_{flux} = \Gamma_\infty L_n$$



Conservation of potential vorticity,

$$\Pi = \nabla^2\phi - n + x \ ,$$

as in the inviscid local L_n Hasegawa-Wakatani limit, implies these should be equal.

Disagreement : artificial dissipation and Krook collision operator, or new version of potential vorticity is needed.

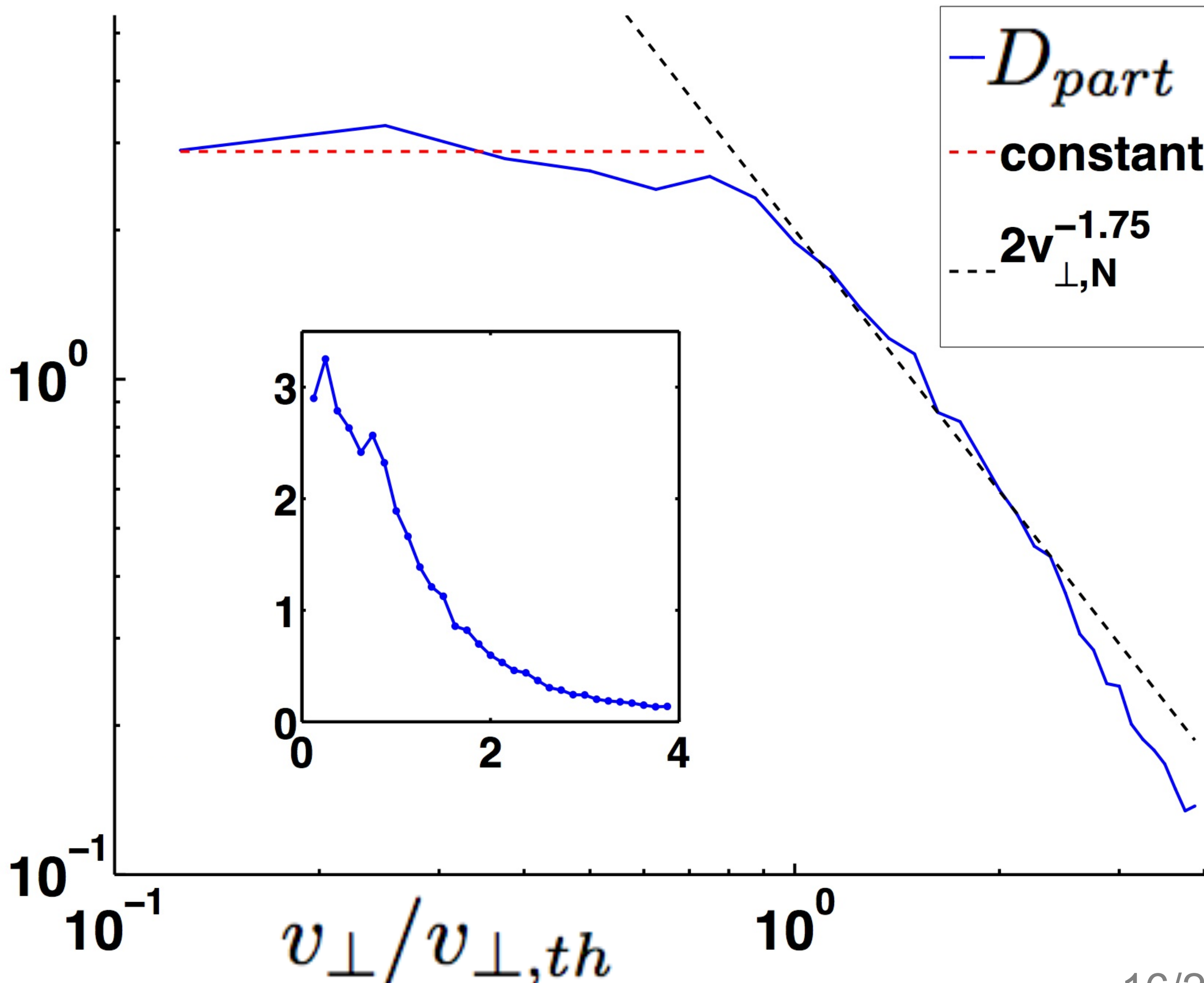
Ion diffusivity depends on v perp

$L_n/R_c = 0.5$

Strong gradient

Decay rate of diffusivity increases near $v_{\perp,th}$

Combination of gyroaveraging and drift averaging

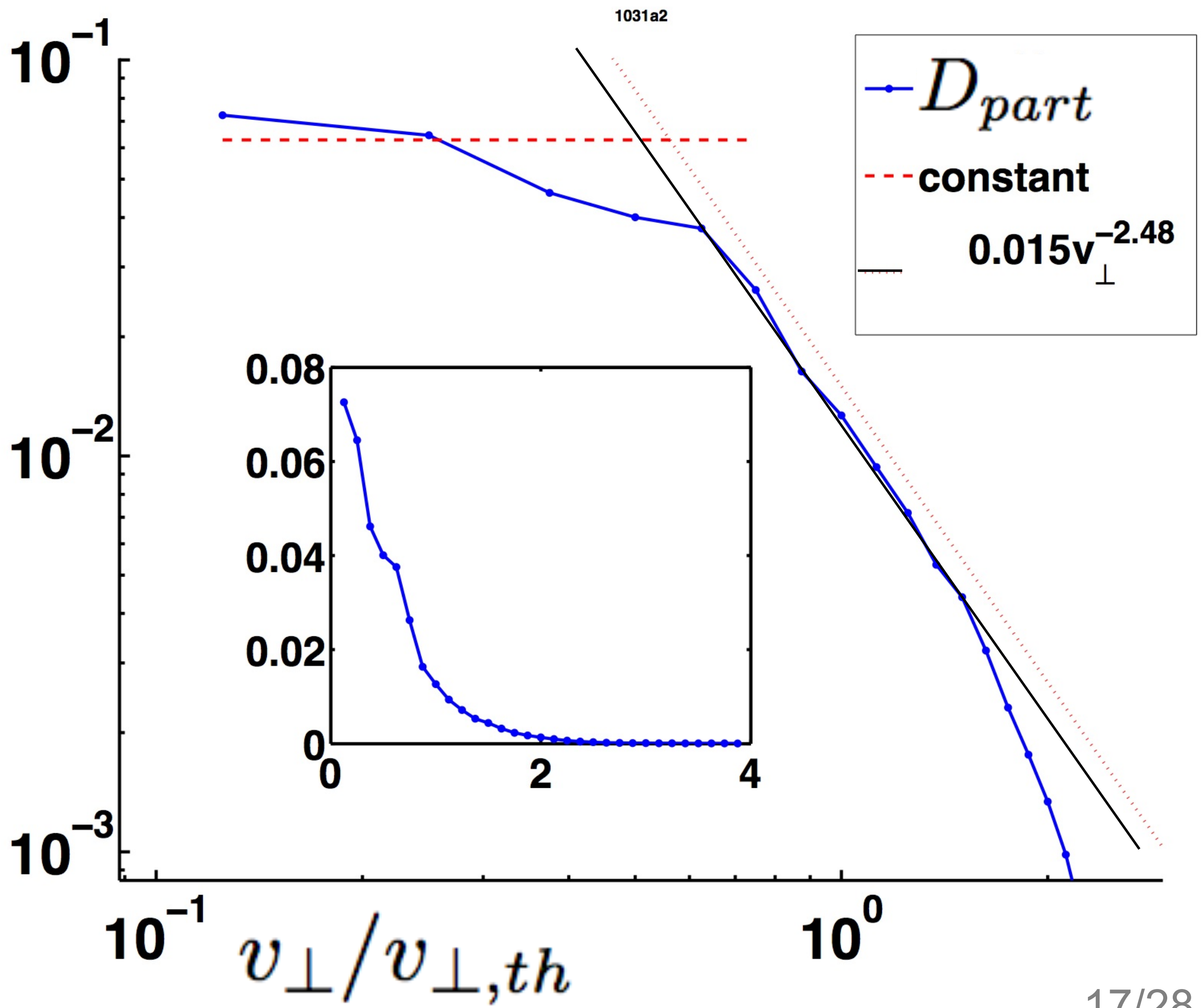


Ion diffusivity depends on v perp

$L_n/R_c = 0.75$

Weaker gradient

Sharp decrease in particle diffusivity arrives at a smaller scale



Summary of gyrokinetic results

- New δf PIC code, valid at large $k_\perp\rho$, is benchmarked with GS2 and convenient for studying particle dispersion in Z-pinch geometry

- Zonal flows in a local gyrokinetic simulation give diffusive radial test-particle transport at the ion Larmor radius scale, across several wavelengths of the flow

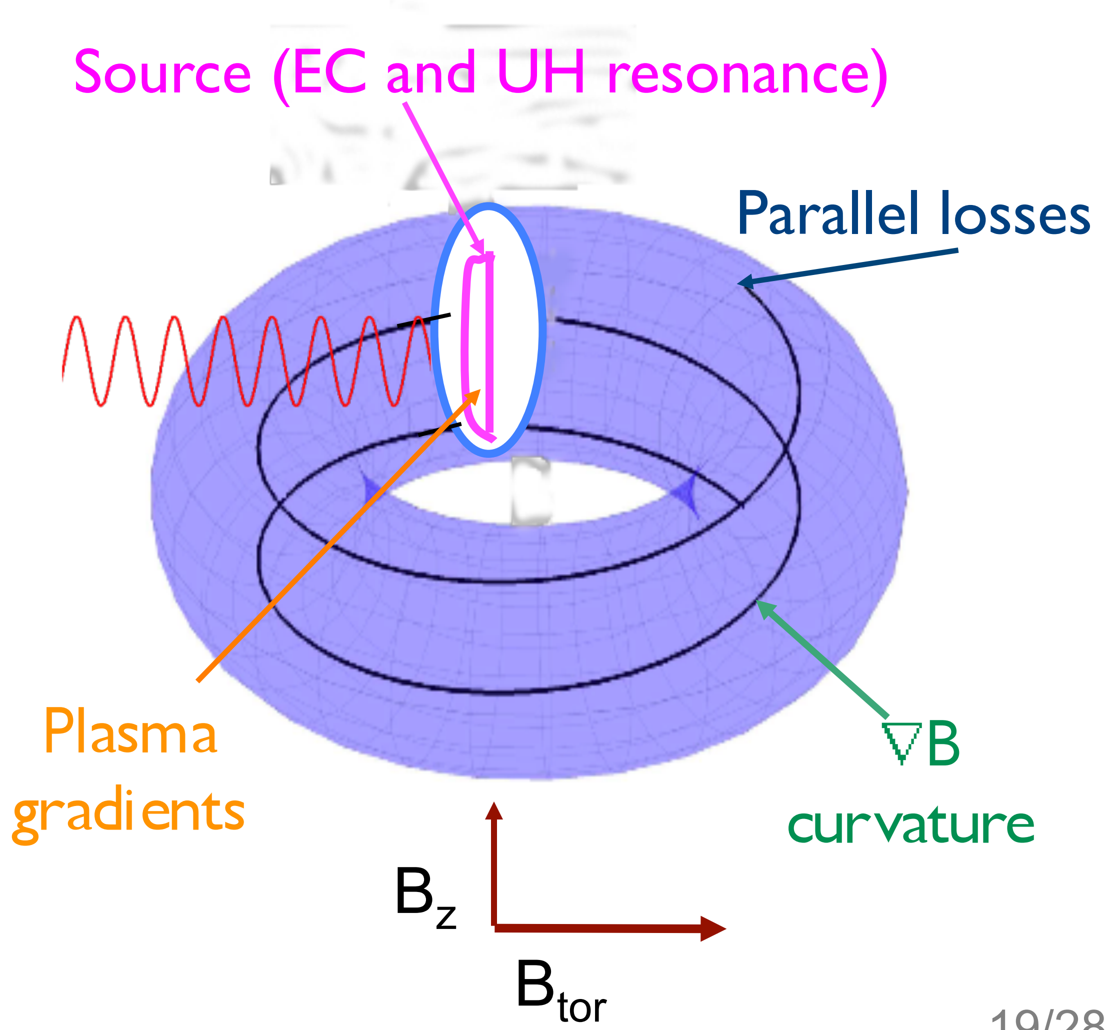
- Test-particle diffusivity, D_{part}, is correlated with Fick's law diffusivity, D_{flux}.

- Energy dependence of diffusivity shows influence of gyroaveraging for several values of the density gradient

New project: fast ions in TORPEX

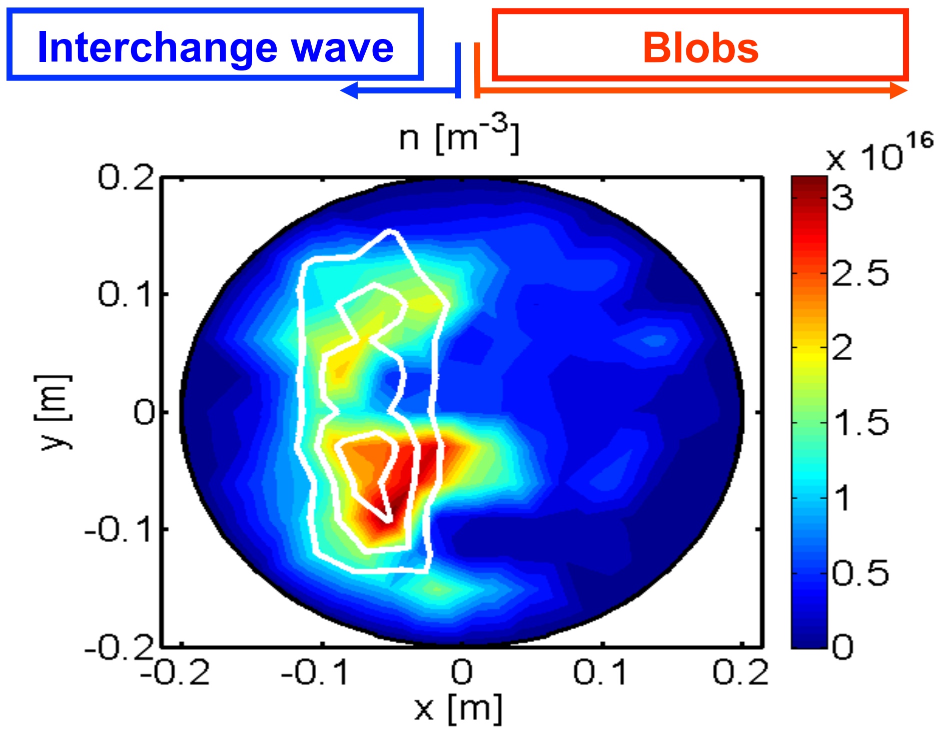
A. Fasoli, *et al*, Phys. Plasmas 13, 055902 2006

- Simple Magnetized Torus: experimental model of scrape-off-layer
- Interchange turbulence and blobs drive nondiffusive transport



Simulated fast ions in turbulent E-field: direct comparison with experiment

Periodic boundary conditions



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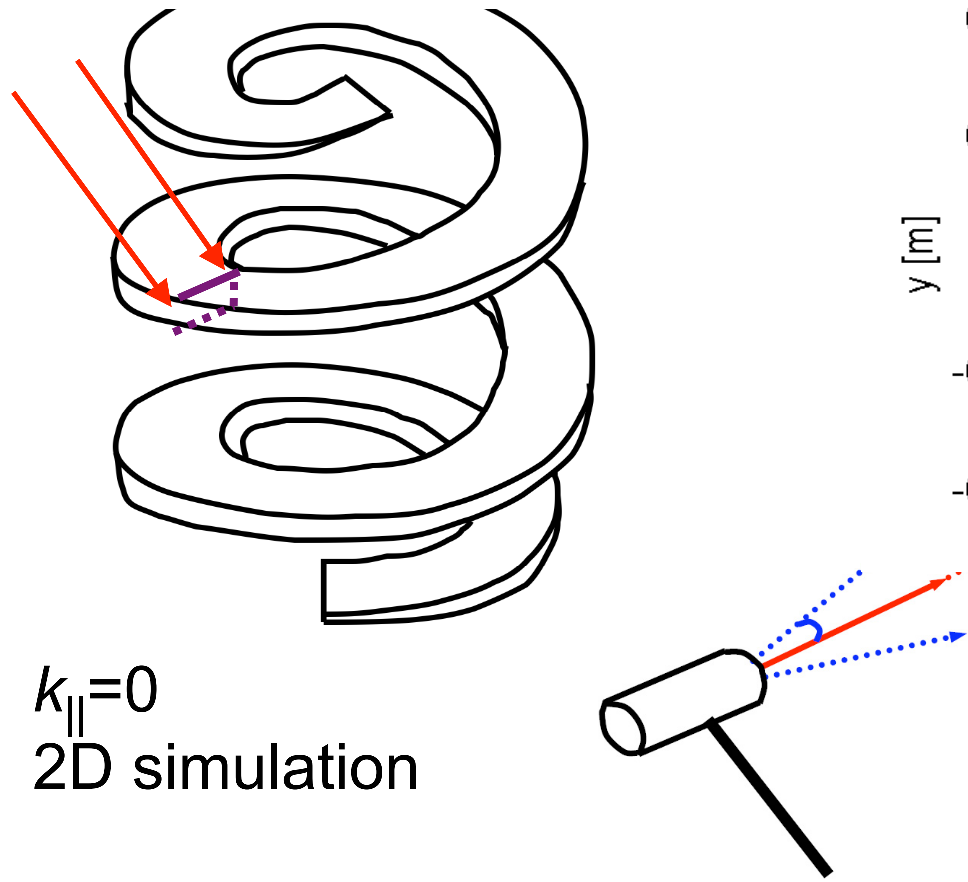
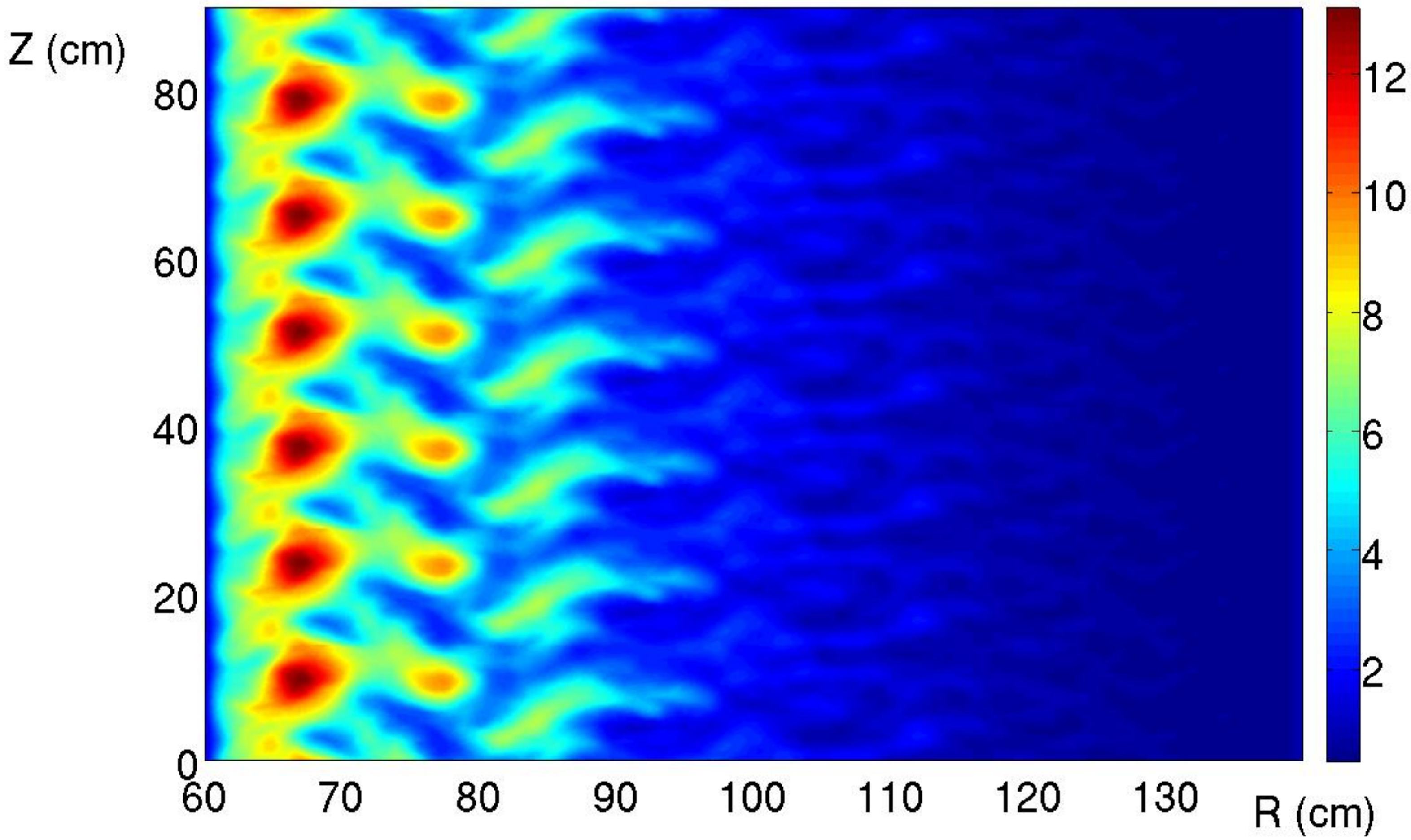


$k_{||}$=0
2D simulation

Tracer particle source with realistic spread in energies (10%) and in angular distribution (0.2rad)

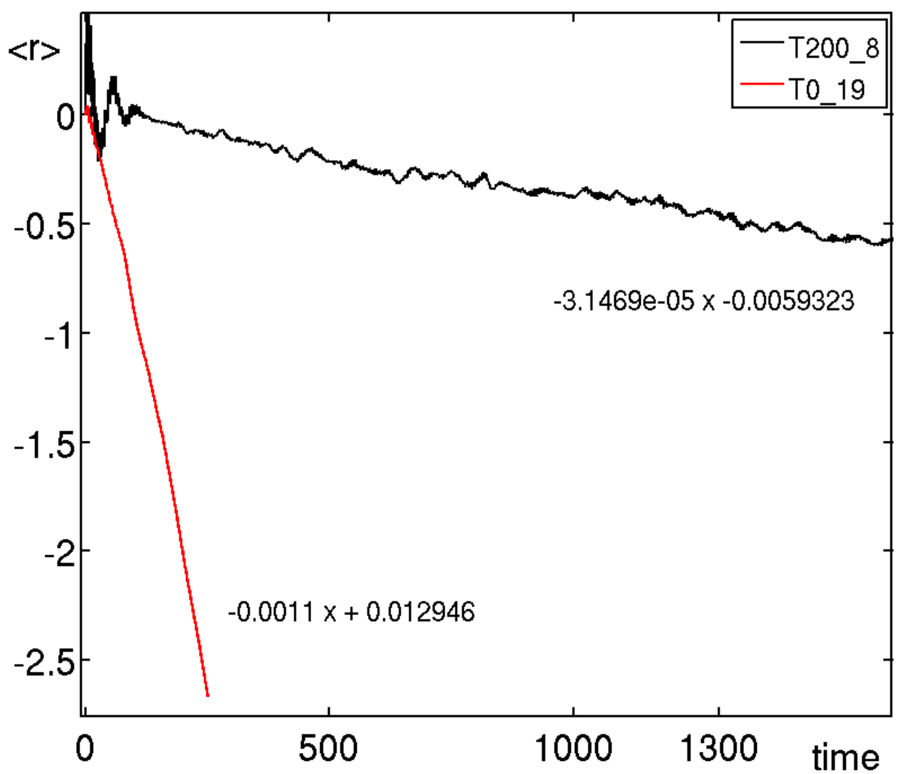
Simulated e.s. potential for $k_{||}$

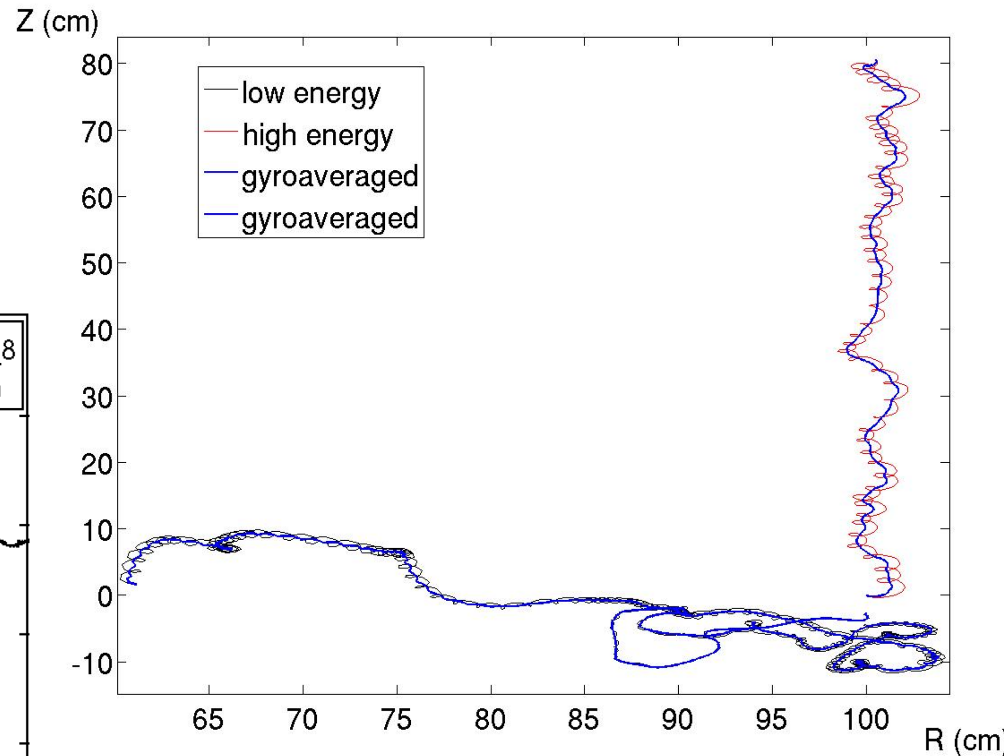


Kubo number: 4 < K < 200 depending on the input energy of the fast ions.

Trajectories with a boundary

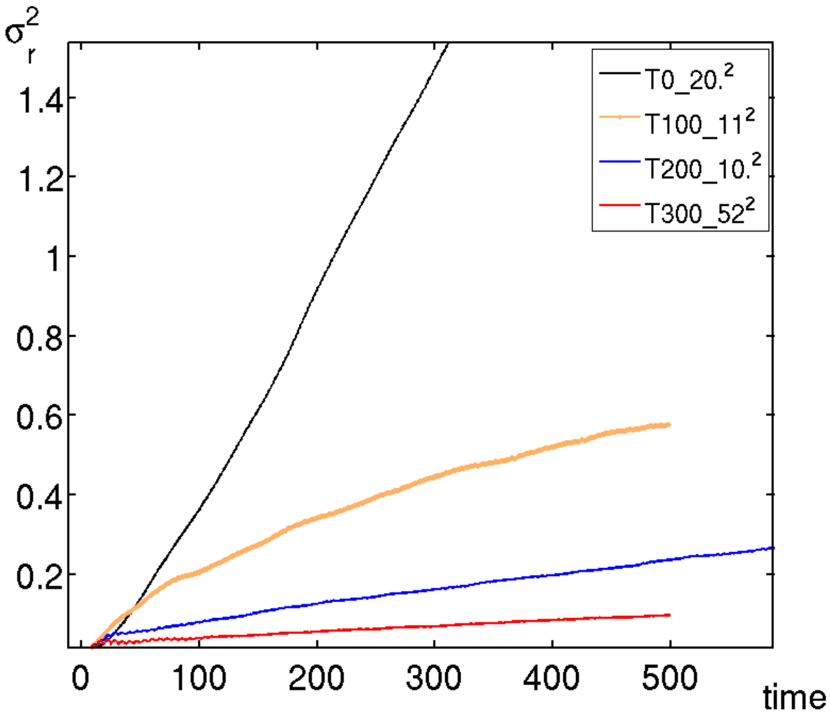
Full trajectory – rather than gyrocenters

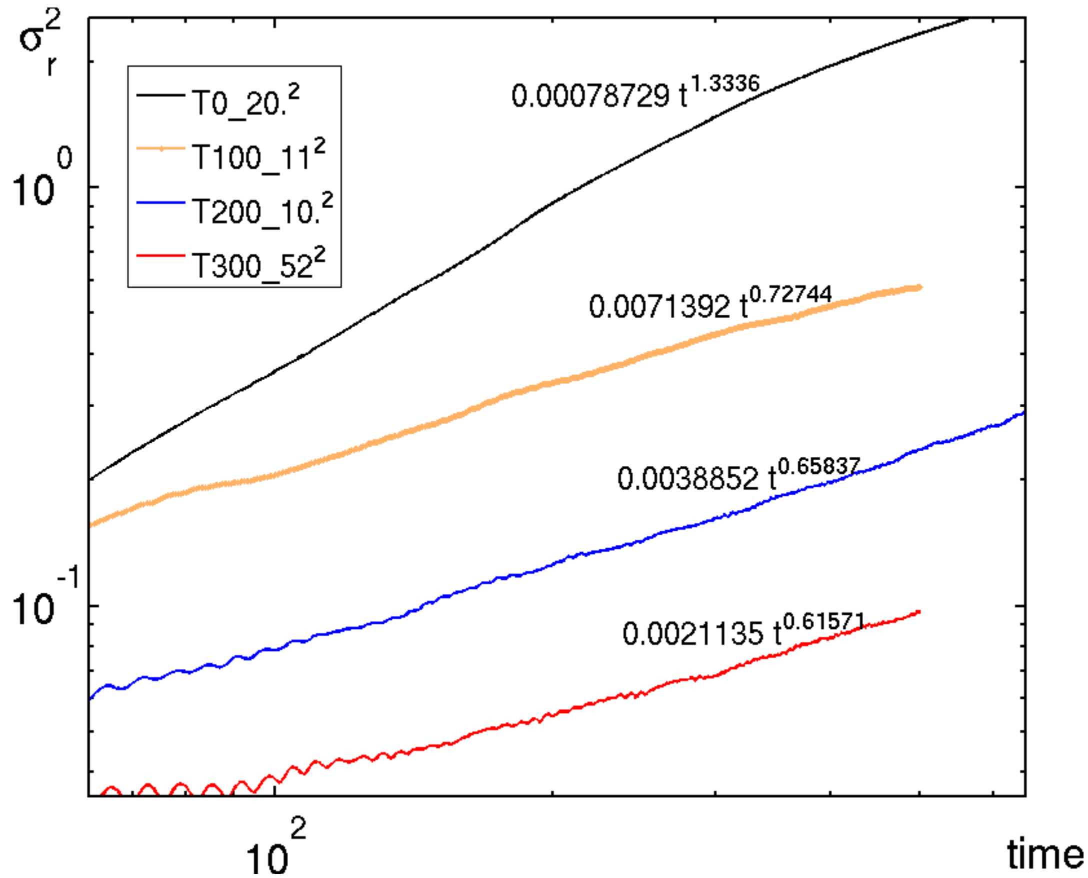




Asymmetry of the displacements for low energy

Superdiffusion to subdiffusion





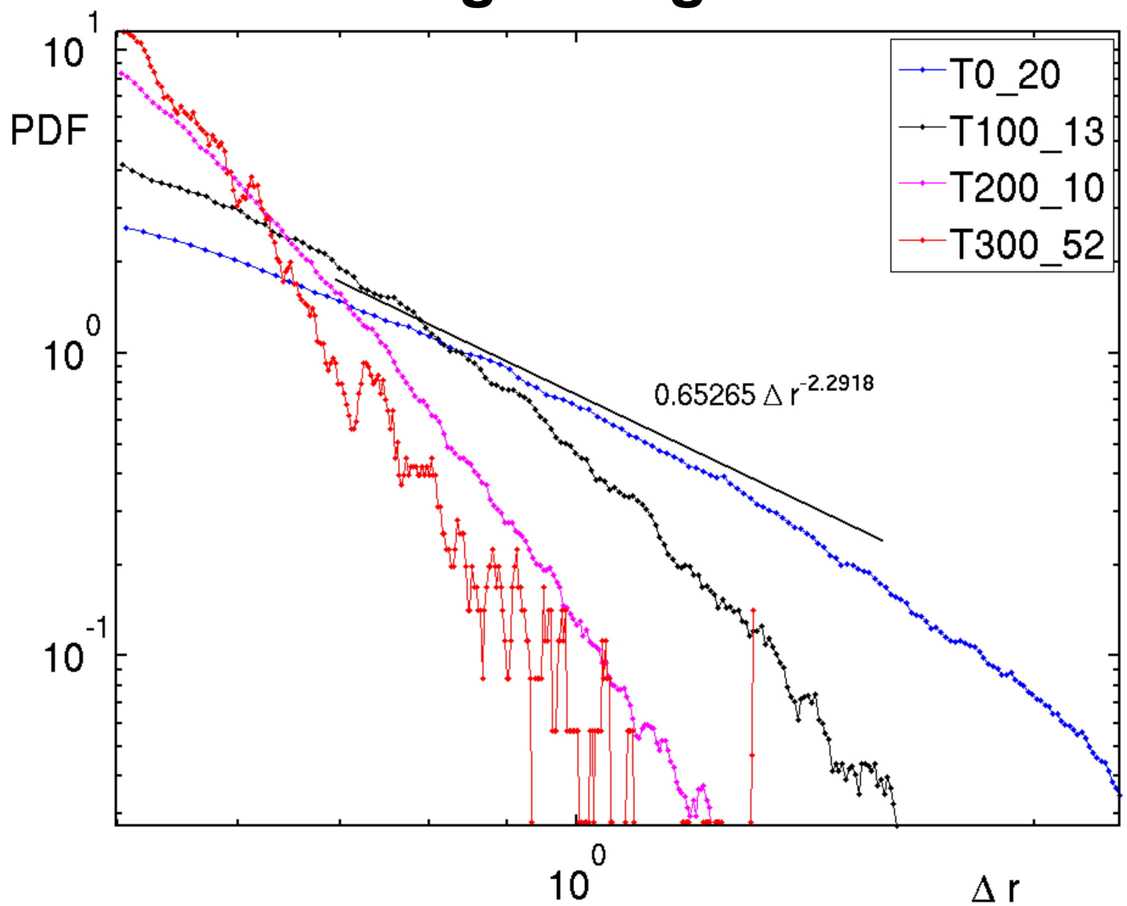
$$\sigma_r^2(t) = D_{eff} * t^{\gamma}$$

Higher energy (fast ion velocity) leads to less radial transport – even slower than diffusive. At the highest energies tested (1000 eV), most ions remain radially stuck.

CTRW and Lévy walks

The superdiffusive, low energy, flight distribution is consistent with a $\gamma = 4 - \mu$ law as expected from a Lévy walk, a restriction of the Lévy flight to a constant velocity approximation (see Phys. Rev. A 35-7, 1987).

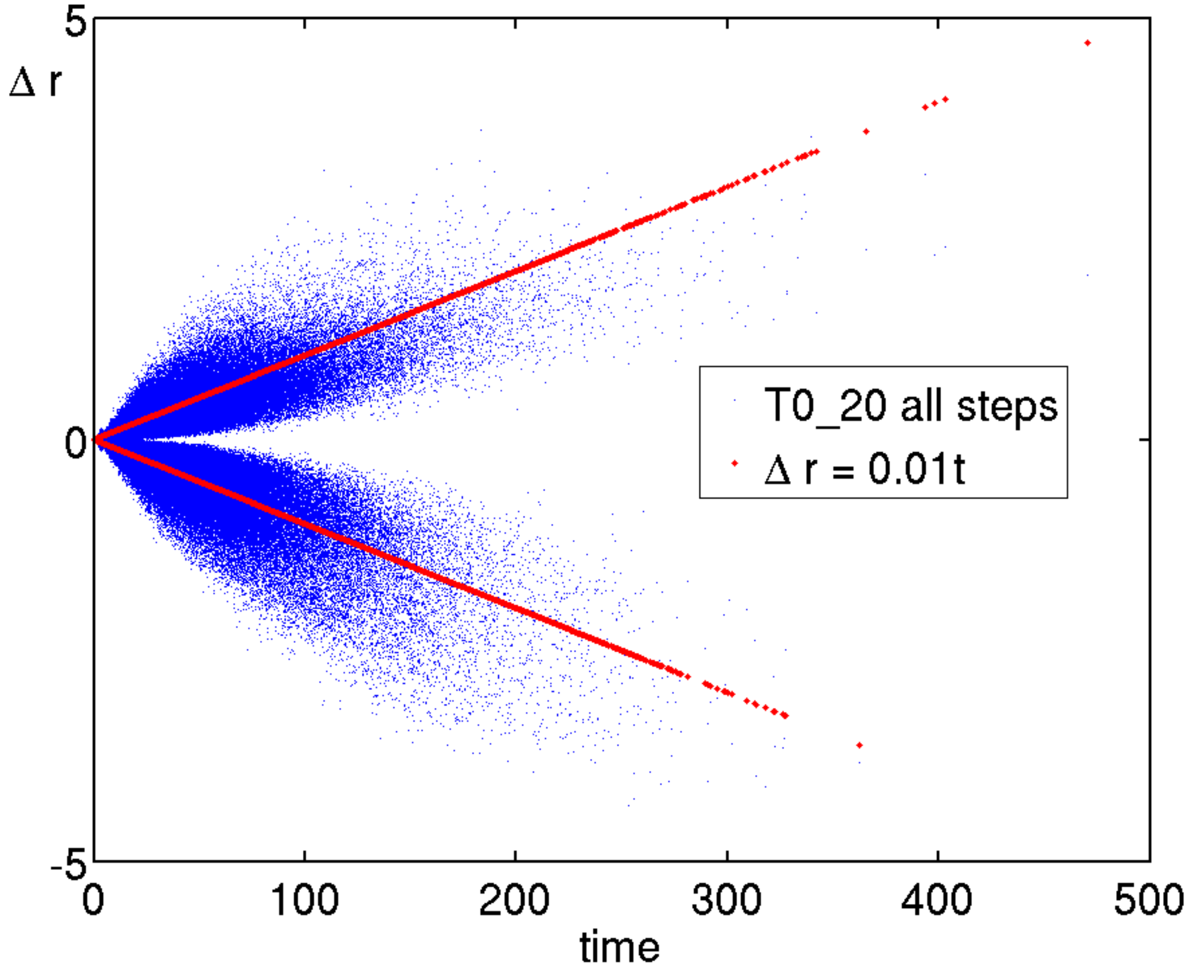
Flight lengths



Lévy walk CTRW kernel: $\Psi(\Delta r, t) = \Delta r^{-\mu}\delta(r - t^{\nu})$

$$\psi(r) = \Delta r^{-\mu}$$

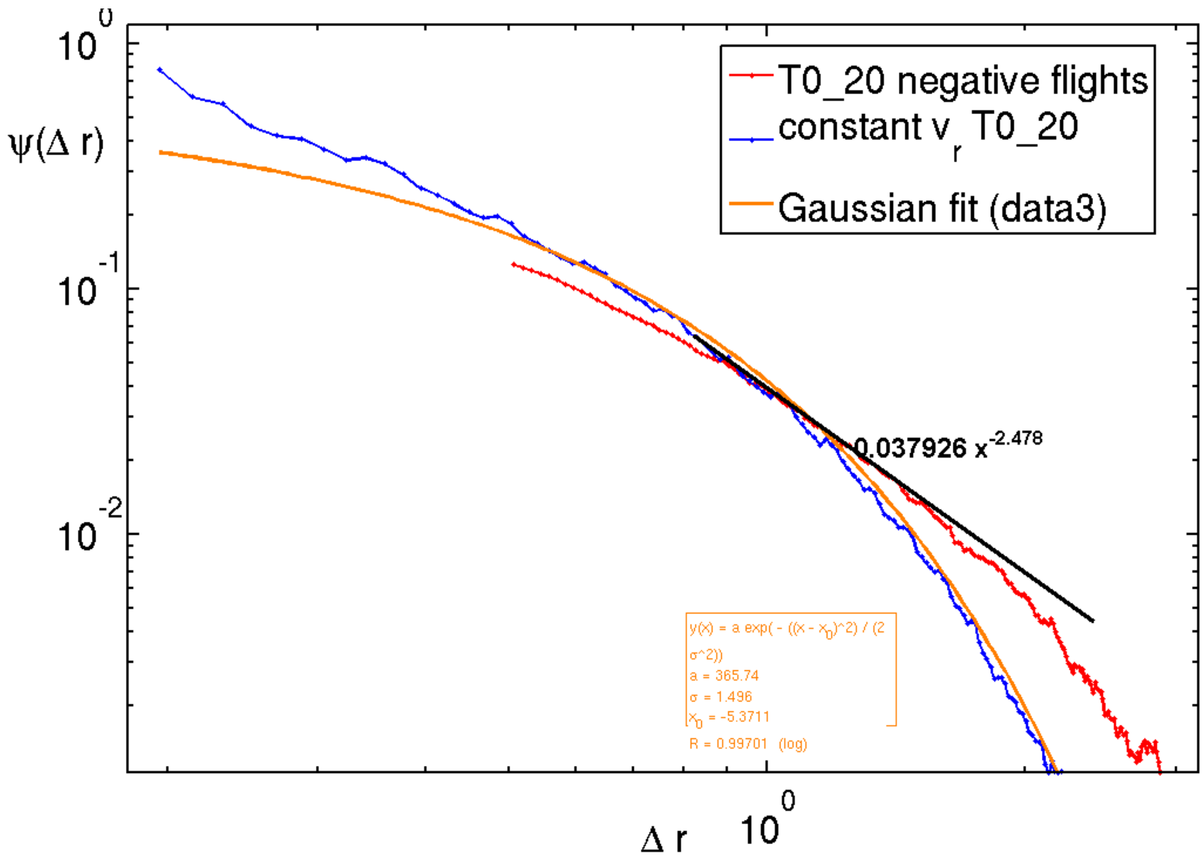
Lévy walk approximation



Deviation from constant velocity shown in blue, and collapsing the cone onto the line of constant velocity in red.

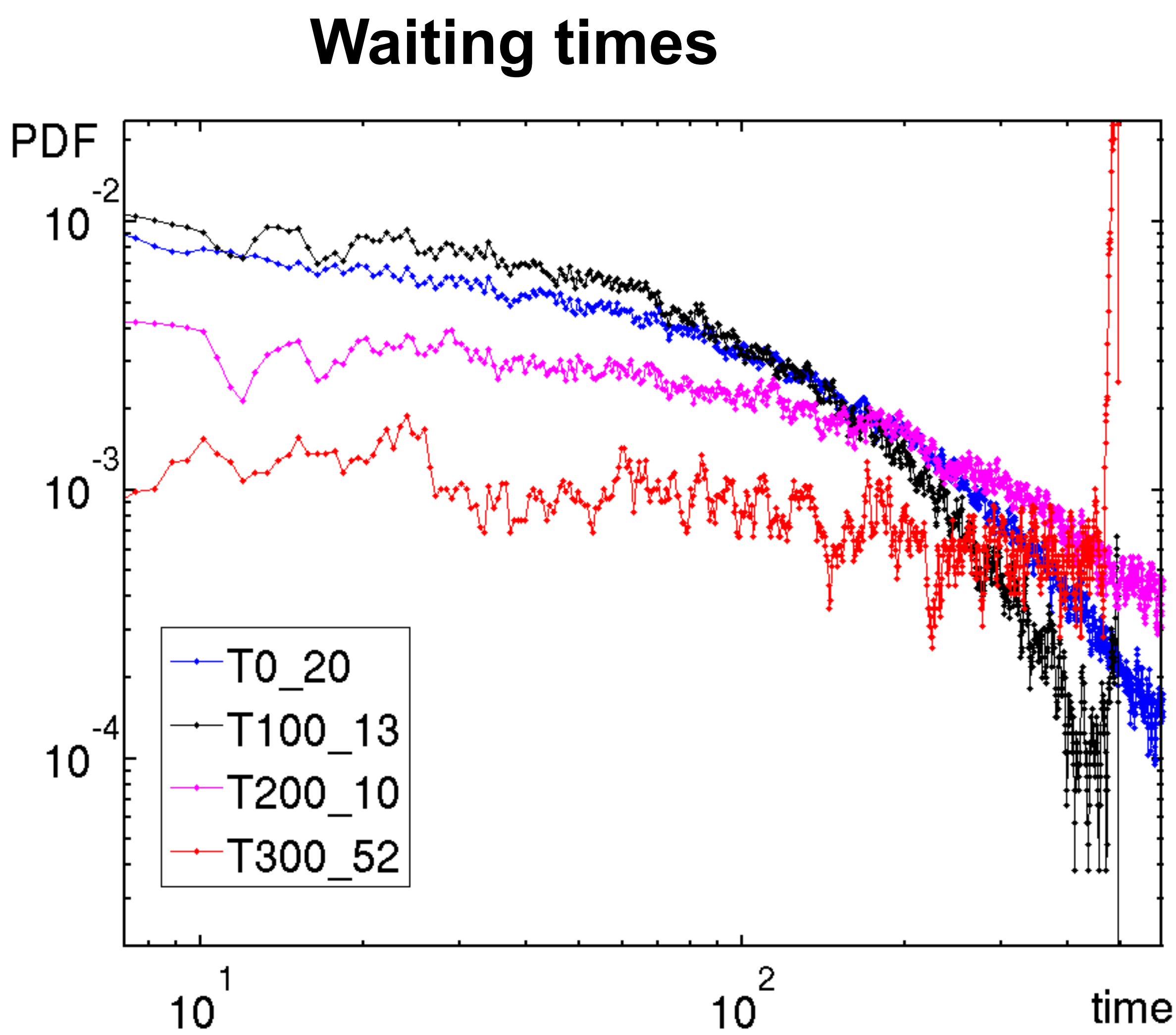
Compare to the assumptions of the canonical Lévy walk.

Lévy walk approximation



Artificially correcting the steps to have the same velocity (see previous slide) removes the flat part of the curve with favorable slope.

CTRW and Lévy walks



The subdiffusive, higher energy, fast ions show a high probability of waiting times at the length of the simulation.

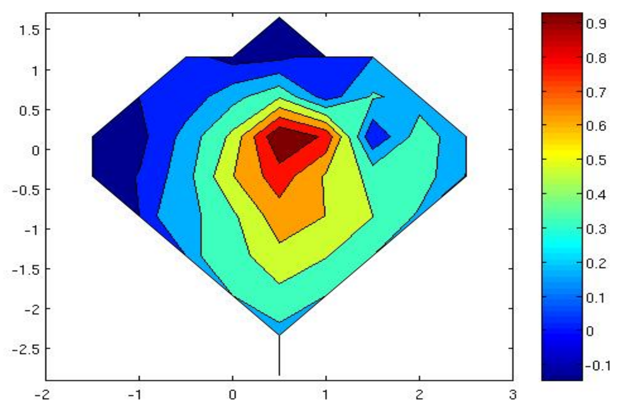
This spike of long waiting times shrinks if the length of the simulation is increased, but the long sticky events still cause subdiffusion.

Experimental comparison

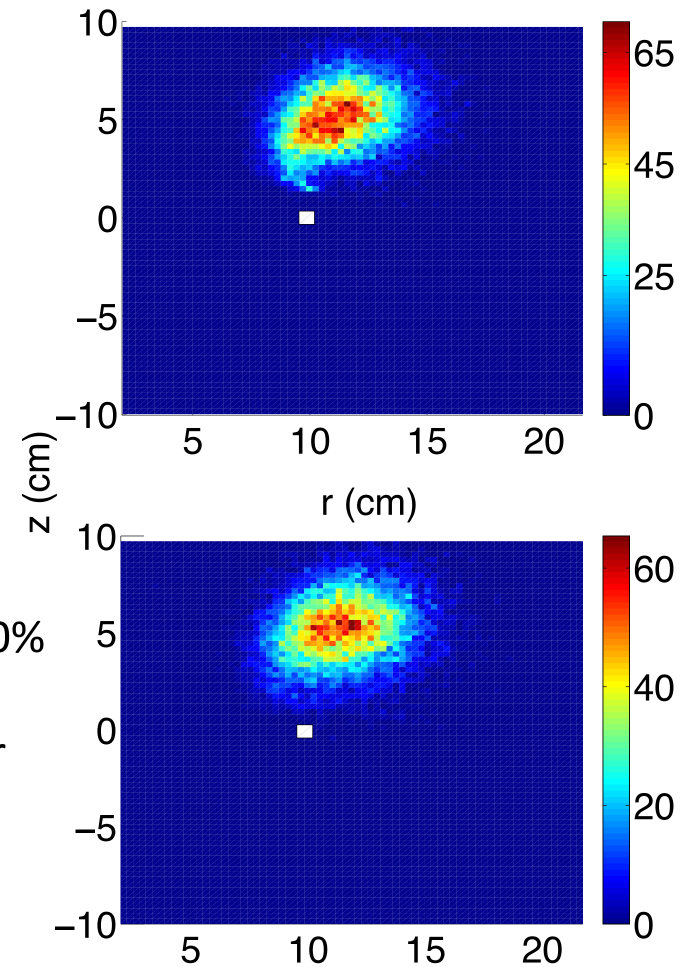
Data

Synthetic diagnostic

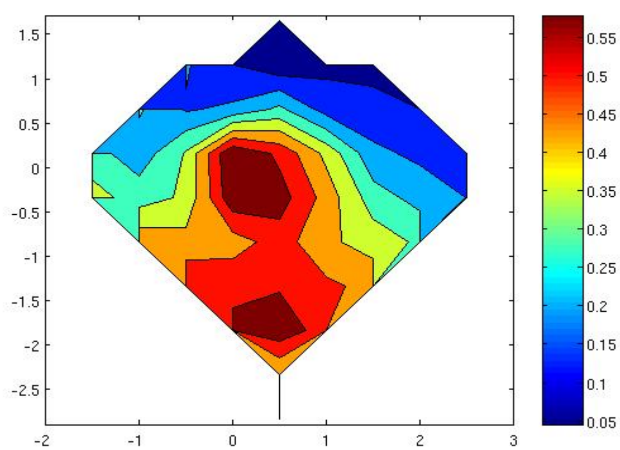
Without plasma



E_{fast}= 300eV



With plasma



ΔR/R increases by 10%

More data points for the spreading as a function of time are planned…

Plyushchev, PhD EPFL 2010