Transverse transport in first-order orbit theory prior to the diffusion regime

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Abstract

We discuss an analytical derivation for the **temporal** dependence of the transverse transport coefficient of a charged particle in a magnetic turbulence for times **SMALLER** than the correlation time of the magnetic turbulence, where the quasi-linear theory (QLT) is not valid. The transverse transport is assumed to be dominated by the **guiding center** motion.

Transverse transport

The diffusion theory [2] relies on the approximation [3] that a characteristic time T exists much larger than the correlation time t_c of the magnetic field fluctuations (as seen by the particle) but also much smaller than the time-scale of variation of these fluctuations and of the average distribution function. Diffusion coefficients can then be related to turbulence power spectrum.

An analytical investigation of the transport prior to the diffusion phase is presented here.

We compute the instantaneous diffusion coefficient [4] along the transverse coordinate x through:

$$d_{xx}(t) \equiv \frac{1}{2}\frac{d}{dt}\langle(\Delta x)^2\rangle(t) = \int_0^t d\tau \langle v_x(\tau) v_x(0)\rangle \,, \qquad (1)$$

where v_x is the velocity along the x-direction.

Guiding Center drift

We consider a stochastic inhomogeneous static magnetic field $\mathbf{B}(\mathbf{x}) = \mathbf{B}_0 + \delta\mathbf{B}(\mathbf{x})$, with average component $\mathbf{B}_0 = B_0\mathbf{e}_z$ and $\delta\mathbf{B}(\mathbf{x})\cdot\mathbf{e}_z = 0$; thus $\langle\delta\mathbf{B}(\mathbf{x})\rangle = 0$ and $\delta B(\mathbf{x})/B_0 \ll 1$. We use first-order orbit theory [6]: particle gyroradius r_g is much smaller than length-scale of magnetic field variation. The most general form of the guiding center velocity transverse (gradient/curvature) to the local field $\mathbf{B}(\mathbf{x})$ is [6, 1]

$$\mathbf{V}^G_\perp(t) = \frac{vpc}{ZeB^2}\left[\frac{1+\mu^2}{2B}\mathbf{B}\times\nabla B + \mu^2(\nabla\times\mathbf{B})_\perp\right] \,. \qquad (2)$$

The finite-time average square transverse displacement of the particle from the direction of average B due to drift $d_D(t)$ can be written in this approximation:

$$d_{D_{ii}}(t) = \int_0^t d\xi \langle\mathbf{V}^G_{\perp,i}(t')\mathbf{V}^G_{\perp,i}(t'+\xi)\rangle \,, \qquad (3)$$

where t' is an arbitrary initial time, ξ the time lag and i stands for any transverse coordinate, X or Y.

References

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Transverse motion decomposed

The average displacement in Eq.(3) contains terms of type

$$\partial_l \delta B_j(\mathbf{x}) = \Re\int_{-\infty}^{\infty} d^3k\, \delta B_j(\mathbf{k})(ik_l)e^{i\mathbf{k}\cdot\mathbf{x}(t)} \,. \qquad (4)$$

The particle position in Eq. (4) along $\mathbf{B}(\mathbf{x})$ is computed as: $\mathbf{x}(t) = \mathbf{x}_0(t) + \mathbf{x}_{MFL}(z(t))$, where $\mathbf{x}_0(t)$ is the unperturbed particle orbit; $\mathbf{x}_{MFL}(z(t)) = (x_{MFL}, y_{MFL}, 0)$ is the offset in the plane orthogonal to B_0 due to the Magnetic Field Line Random Walk (MFLRW) at $z = z(t)$. Ballistic motion along B_0 is assumed, i.e., $z = v_\parallel t$.
To first order in $\delta B(\mathbf{x})/B_0$ the MFL contribution is negligible: $e^{i\mathbf{k}\cdot\mathbf{x}(t)} \simeq e^{i\mathbf{k}\cdot\mathbf{x}_0(t)}$ The turbulence power spectrum is standard uncorrelated at different wavenumber vectors:

$$\langle\delta B_r(\mathbf{k})\delta B^*_q(\mathbf{k}')\rangle = \delta(\mathbf{k}-\mathbf{k}')P_{rq}(\mathbf{k}) \,. \qquad (5)$$

Power spectrum dependence

The general term contributing to the first-order transverse drift $d_{D_{ii}}(t)$ of a particle in a static magnetic field with first-order perturbation is

$$\left(\frac{vpc}{ZeB_0^2}\right)^2 F(\mu^2)\int_{-\infty}^{\infty} d^3k P_{rq}(\mathbf{k})k_l k_p \frac{\sin[k_\parallel v_\parallel t]}{k_\parallel v_\parallel} \,,$$

where v, p and Ze are the particle's velocity, momentum and charge and $F(\mu^2)$ is a function of the pitch angle assumed to be isotropically distributed and constant.
If the correlation function of the magnetic fluctuation is homogeneous in space, the mean square displacement of the MFL as seen by a low rigidity particle is:

$$d_{MFL}(t) = \int_{-\infty}^{\infty} d^3k P_{rq}(\mathbf{k})\frac{\sin[k_\parallel v_\parallel t]}{k_\parallel v_\parallel} \,.$$

Application to slab turbulence

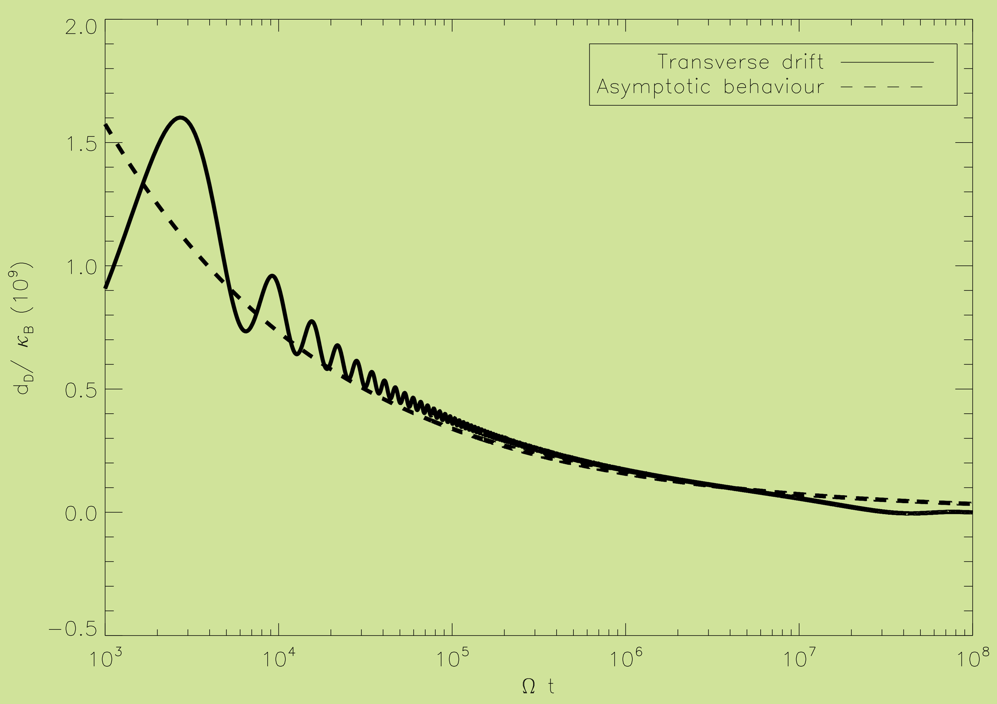
We consider the slab turbulence, i.e., static limit of transverse and parallel-propagating Alfven waves: $\delta\mathbf{B} = \delta\mathbf{B}(z)$ and $\delta\mathbf{B}(\mathbf{x})\cdot\mathbf{e}_z = 0$. The turbulence wave number is aligned to $\mathbf{B_0}$, thus we adopt the following form of the power spectrum: $P_{rq}(\mathbf{k}) = G(k_\parallel)(\delta(k_\perp)/k_\perp)(\delta_{rq} - k_r k_q/k^2)$ with $r, q = 1, 2$, and $P_{3i}(\mathbf{k}) = 0$ with $i = 1, 2, 3$. The 1D spectrum is assumed to be of Kolmogorov type. The power spectrum is assumed to be constant at scales larger than coherence length.
In units of the Bohm coefficient diffusion ($\kappa_B = (1/3)r_g v$), from Eqs. (4) and (5), we obtain

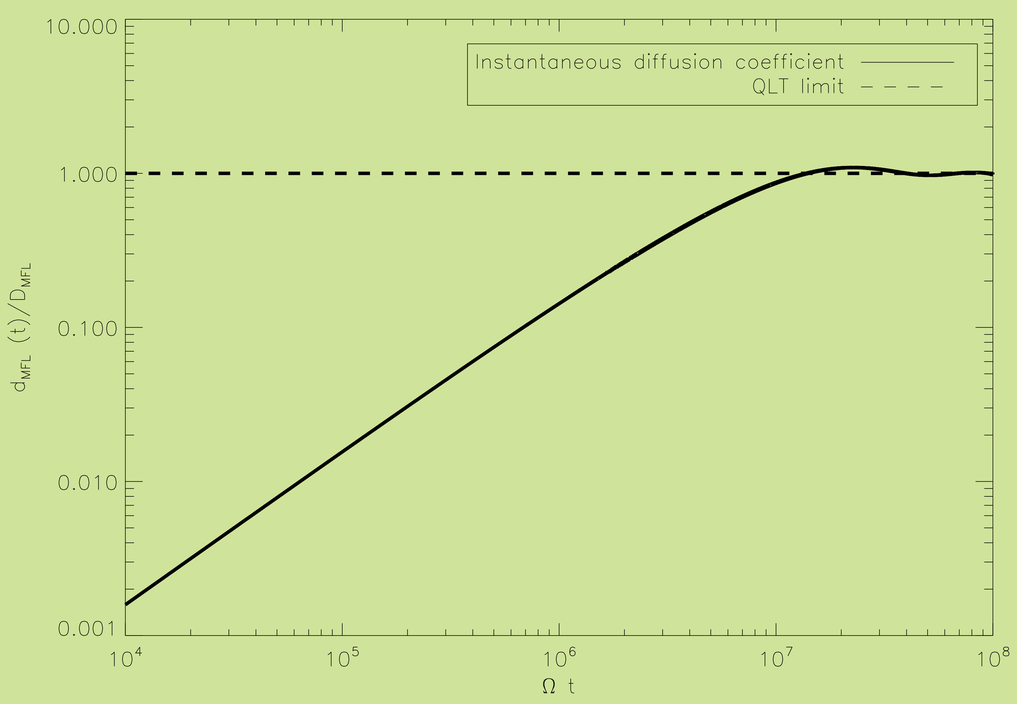
$$\frac{d_D(t)}{\kappa_B} = \frac{3}{20}\left(\frac{\delta B}{B_0}\right)^2 \frac{q-1}{q}\left[\left.\frac{\sin y - y\cos y}{k_\parallel^{min} r_g(\Omega t)^2}\right|_{y_{min}} + \frac{I(t; k_\parallel, 2-q)}{(k_\parallel^{min} r_g)^{1-q}(\Omega t)^{2-q}}\right] \qquad (6)$$

where $y_{min} = k_\parallel^{min} v_\parallel t \simeq k_\parallel^{min} r_g \Omega t$, Ω is the particle gyrofrequency and $I(t; k_\parallel, a)$ is a combination of incomplete gamma functions $\Gamma(a, z)$. $d_D(t)/\kappa_B$ is depicted in the Fig. on the left. The MFLRW, in units of coherence length $L_\parallel = (k_\parallel^{min})^{-1}$ (see also the comparison with the QLT-limit $D_{MFL}(t)$ depicted in the Fig. on the right), is given by

$$d_{MFL}(t)k_\parallel^{min} = \left(\frac{\delta B}{B_0}\right)^2 \frac{q-1}{2q}\left[\mathrm{Si}(y_{min}) + (k_\parallel^{min} r_g \Omega t)^q I(t; k_\parallel, -q)\right] \qquad (7)$$

where $\mathrm{Si}(x)$ is the Sine integral function.





$\Rightarrow$

1. Time evolution of individual charged particles drift and MFLRW across a static magnetic field with a first-order fluctuation is analytically described for a general turbulence
2. Motion perpendicular to the average magnetic field is assumed to be dominated by guiding center motion, i.e. MFL's meandering and drift
3. No prior diffusive regime is assumed both in perpendicular and parallel direction
4. For slab (Kolmogorov) turbulence drift transverse to the local field is suppressed as $t^{-1/3}$, slower than compound diffusion, which, transversally to the average field, is suppressed as $t^{-1/2}$. The MFL coefficient diffusion of QLT is recovered
5. 3D-isotropic turbulence, where numerical simulations disagree with theoretical scenarios, will be discussed in a paper in preparation

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