Transverse transport in first-order orbit theory prior to the diffusion regime

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**Abstract**
We discuss an analytical derivation for the temporal dependence of the transverse transport coefficient of a charged particle in a magnetic turbulence for times SMALLER than the correlation time of the magnetic turbulence, where the quasi-linear theory (QLT) is not valid. The transverse transport is assumed to be dominated by the guiding center motion.

**Transverse transport**
The diffusion theory \[2\] relies on the approximation \[3\] that a characteristic time \(t_c\) of the magnetic field fluctuations (as seen by the particle) but also much smaller than the time-scale of variation of these fluctuations and of the average distribution function. Diffusion coefficients can then be related to turbulence power spectrum.

An analytical investigation of the transport prior to the diffusion phase is presented here. We compute the instantaneous diffusion coefficient \[4\] along the transverse coordinate \(x\) through:

\[
d_x(t) = \frac{1}{2} \left[ (\Delta x)^2 \right]_d(t) = \int_0^t d\tau (v_x(\tau)v_x(0)).
\]

(1)

where \(v_x\) is the velocity along the \(x\)-direction.

**Guiding Center drift**
We consider a stochastic inhomogeneous static magnetic field \(B(x) = B_0 + \varepsilon B(x)\), with average component \(\overline{B_0} = B_0\varepsilon\) and \(\overline{B(x)} \cdot \varepsilon = 0\); thus \(\langle B(x) \rangle = 0\) and \(\overline{B(x)}\). We use first-order orbit theory \[6\]; particle gyroradius \(r_g\) is much larger than the scale-length of magnetic field variation. The best general form of the guiding center velocity transverse (gradient/curvature) to the local field \(B(x)\) is \(\overline{V_{\perp}^2} = \frac{v_{pc}}{22^2} \left\{ 1 + \frac{\rho^2}{2B} + \frac{\rho^2}{(\nabla \times B)} \right\} \)

(2)

The finite-time average square transverse displacement of the particle from the direction of average \(B\) due to drift \(d_{DR}(t)\) can be written in this approximation:

\[
d_{DR}(t) = \int_0^t d\tau (V_{\perp}^2(t) V_{\perp}^2(t + \tau + \xi)),
\]

(3)

where \(\tau\) is an arbitrary initial time, \(\xi\) the time lag and \(\xi\) stands for any transverse coordinate, \(X\) or \(Y\).

**References**


**Transverse motion decomposed**
The average displacement in Eq.(3) contains terms of type \(\delta \delta B(t) \equiv \int \delta B(t) \delta B(t) dt = (x(t) - x_0(t)) \propto x_0(t) = x_{MFLR}(t)\), where \(x_0(t)\) is the unperturbed particle orbit; \(x_{MFLR}(t) = (x_{orb}, y_{MFLR}, 0)\) is the offset in the plane orthogonal to \(B_0\) due to the Magnetic Field Line Random Walk (MFLRW) at \(z = 1/2\). Ballistic motion along \(B_0\) is assumed, i.e., \(z = v_t B_0\).

To first order in \(\delta B(t)/B_0\) the MFL contribution is negligible: \(e^{i k \cdot x(t)} \approx e^{i k \cdot x_0(t)}\). The turbulence power spectrum is standard unchanged at different wavenumber vectors:

\[
\langle \delta B(t) \delta B(t') \rangle = \delta(k - k') P_{\delta B}(k).
\]

(5)

**Power spectrum dependence**
The general term contributing to the first-order motion \(d_{DR}(t)\) of a particle in a static magnetic field with first-order perturbation is

\[
\left( \frac{v_{pc}}{\varepsilon B_0} \right)^2 \int_{-\infty}^{\infty} d^2k \left( \frac{1}{k^2} \right) P_{\delta B}(k) \sin[kq] e^{i\tau q} e^{i\tau q}.
\]

(6)

where \(v_p\) and \(Ze\) are the particle’s velocity, momentum and charge and \(P(t^2)\) is a function of the pitch angle assumed to be isotropically distributed and constant.

If the correlation function of the magnetic fluctuation is homogeneous in space, the mean square displacement of the MFL as seen by a low rigidity particle is:

\[
d_{MFL}(t) = \int_{-\infty}^{\infty} d^2k P_{\delta B}(k) \sin[kq] e^{i\tau q} e^{i\tau q}.
\]

(7)

**Application to slab turbulence**
We consider the slab turbulence, i.e., static limit of transverse and parallel-propagating Alfvén waves: \(B = B_0(z) + \delta B(x) \cdot e_0 = 0\). The turbulence wave number is aligned to \(B_0\); thus we adopt the following form of the power spectrum:

\[
P_{\delta B}(k) = G(k) \delta(k) \delta(k_{\perp}) \delta(k_{\parallel} - q_k k_{\parallel} / k^2)^2 \)

(8)

with \(r_q, q, k_{\parallel}\) is the parallel wavenumber, \(k_{\perp}\) is the perpendicular wavenumber. The turbulence power spectrum is assumed to be constant at scales larger than coherence length. In units of the Bohm coefficient diffusion \((D_B = (1/3)) v_p v_0^2\), from Eqs. (4) and (5), we obtain

\[
d_{DR}(t) = \frac{3}{20} B_0 \mu^2 q^2 \left( \int_{-\infty}^{\infty} d^2k P_{\delta B}(k) \sin[kq] e^{i\tau q} e^{i\tau q} \right)
\]

(9)

1. Time evolution of individual charged particles drift and MFLRW across a static magnetic field with a first-order fluctuation is analytically described for a general turbulence
2. Motion perpendicular to the average magnetic field is assumed to be dominated by guiding center motion, i.e., MFL’s meandering and drift
3. No prior diffusive regime is assumed both in perpendicular and parallel direction
4. For slab (Kolmogorov) turbulence drift transverse to the local field is suppressed as \(t^{-1/3}\), slower than compand diffusion, which, transversally to the average field, is suppressed as \(t^{-1/2}\). The MFL coefficient diffusion of QLT is recovered
5. 3D-isotropic turbulence, where numerical simulations disagree with theoretical scenarios, will be discussed in a paper in preparation

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