Recent Progress and Future Outlook for Kinetic Simulations of Magnetic Reconnection

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Computing:
- DOE - ASC: Roadrunner
- NSF: Kraken
Wide range of applications to consider:

- Planetary magnetospheres → magnetopause, magnetotail

- Solar applications → chromosphere, transition region, corona - flares, prominences, coronal mass ejections

- Solar wind

- Laboratory fusion machines

- Astrophysical problems
  
  \[ \text{Hydrogen} \quad \{ \text{stellar flares, galactic magnetotails, accretion disks} \} \]
  
  \[ \text{Electron-Positron} \quad \{ \text{pulsar winds, gamma-ray bursts, jets from AGN} \} \]
When do we need a kinetic description?

\[
\log(\frac{L}{\rho_i}) \quad \log(\sqrt{\beta} \sqrt{\frac{m_i}{m_e}})
\]

**Collisionless Regime**

**Collisional - Sweet-Parker Regime**

**Collisional MHD with Plasmoids**

- Runaway fields for \( R \approx 0.05 \)

When do we need a kinetic description?
Kinetic Particle-in-cell Simulations
First-Principles Approach for all Regimes

\[
\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \sum_{s'} C_{ss'}
\]

+ Maxwell’s Equations


1. Relativistic, full Maxwell treatment
2. Monte-Carlo Coulomb collisions
3. Optimized to exploit newest computers
Difficult due to vast scale separation

Computational cost in 3D \( \propto \left( \frac{m_i}{m_e} \right)^{5/2} \left( \frac{L}{d_i} \right)^4 \)
Peta-scale machines offers new opportunities, but also new challenges

Roadrunner
Cell CBE Chip
~110,000 cores

Kraken - NSF
Cray XT5
~100,000 cores

Jaguar - DOE
Cray XT5
~220,000 cores
These efforts are permitting an exponential increase in problem size.

![Graph showing exponential increase in problem size from 2000 to 2020. Different years and computational resources are marked on the graph.]
What 3D problems are we looking at?

$m_i/m_e = 300$

$m_i/m_e = 64$

$m_i/m_e = 200 - 400$
Extended current sheets and secondary-islands are a common feature of large-scale 2D studies.

\[ \frac{m_i}{m_e} = 1 \rightarrow 1836 \]

Strong and weak guide fields

Collisional → Collisionless
Motivation for 3D Kinetic Treatment

- Thin sheets are the preferred sites for the onset of reconnection
- Reconnection leads to the formation of new current sheets
- Secondary magnetic islands may play an important role in the reconnection rate, energy partition & particle acceleration
Results confirmed for hydrogen mass ratio

\[ \frac{m_i}{m_e} = 1836 \]

Recent VPIC run on Kraken

Open boundary conditions
Fokker-Planck Treatment of Collisions

- Rigorous treatment of transition between fluid & kinetic regimes
- Benchmarks with Braginskii
- For $S \sim 1000$, transition from Sweet-Parker to kinetic observed
  \[ \delta_{sp} \approx d_i \]  critical resistivity
- Simple estimate fails completely in large systems due to plasmoids
- New electron layers also unstable to plasmoids

Daughton et al, PRL, 2009
Daughton et al, PoP, 2009
Reconnection Rate Modulated with Plasmoid Formation

$600d_i \times 600d_i$, $t \Omega_{ci} = 2.50$, Open BC

$N_e$

Measure Reconnection Inflow

$\frac{m_i}{m_e} = 1$

Daughton & Karimabadi, PoP, 2007
How do these results extend to real 3D systems?

- Secondary magnetic islands \(\rightarrow\) Flux ropes
- More freedom to form islands in 3D
- Can interact in complex ways not possible in 2D
- Stochastic magnetic fields?
- Influence of pre-existing turbulence upstream?
- Potential influence on nearly every aspect of the problem - basic cartoon, dissipation rate, etc
- Real need for theory - primary & secondary islands
Focus in detail on one problem:

Island Formation in Guide Field Reconnection

\[ \frac{m_i}{m_e} = 1 \rightarrow 1836 \]

\[ \frac{m_i}{m_e} = 1 \rightarrow 64 \]
Island formation is more complicated in 3D


Galeev et al, 1986

Drake et al, 2006
Harris Sheet Geometry with a Guide Field

Uniform background plasma
No initial temperature gradients

Consider:
1. Electron-positron plasma
2. Hydrogen plasma

\[ B_x = B_{xo} \tanh \left( \frac{z}{L} \right) \]
Tearing Modes are Localized about Resonant Surfaces

- Outer Region: $E_\parallel = 0$
- Singular Layer: $E_\parallel \neq 0$
- Outer Region: $E_\parallel = 0$

General Perturbation:

\[ \hat{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \hat{A}}{\partial t} \]

Electrostatic part "shorts out" response - except when

\[ k \cdot B = 0 \]

\[ \hat{A} = \tilde{A}(z) \exp \left[ -i\omega t + ik_x x + ik_y y \right] \]

\[ \hat{\phi} = \tilde{\phi}(z) \exp \left[ -i\omega t + ik_x x + ik_y y \right] \]

\[ \hat{E}_\parallel = \mathbf{b} \cdot \hat{E} = -ik_\parallel \hat{\phi} + i\frac{\omega}{c} \mathbf{b} \cdot \hat{A} \]

Resonant surface
Resonant Surfaces for Harris Sheet Geometry

\[ \mathbf{k} \cdot \mathbf{B} = 0 \]

\[ \frac{z_s}{L} = -\tanh^{-1} \left( \frac{k_y B_{yo}}{k_x B_{xo}} \right) \approx -\frac{k_y B_{yo}}{k_x B_{xo}} \]

\[ \frac{z_s}{L} = -\tanh^{-1} \left( \frac{k_y B_{yo}}{k_x B_{xo}} \right) \approx -\frac{k_y B_{yo}}{k_x B_{xo}} \]

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**Kinetic Theory is Tricky in Thin Layers** → \( L \leq \rho_i \)

Use formally exact technique → Daughton, PoP, 2003

**Method of Characteristics**

\[
\tilde{f}_s = -\frac{q_s f_{os}}{T_s} \left[ \tilde{\phi} - \frac{U_s}{c} \tilde{A}_y + i(\omega - k_y U_s) \tilde{S} \right]
\]

Orbit Integral

\[
\tilde{\rho} = \sum_s q_s \int \tilde{f}_s \, dv
\]

\[
\tilde{J} = \sum_s q_s \int v \tilde{f}_s \, dv
\]

Numerically solve integro-differential eigenvalue problem
We also developed new asymptotic theory

Serious issues with previous theories

1. Incorrect outer layer equation
2. Which means $\Delta'$ is not right
3. Oblique modes sensitive to matching

$$\Delta' \approx \frac{2}{L} \left( \frac{1}{kL} - kL \right) \left[ 1 + \frac{\tanh^2(z_s/L)}{2} \left( 1 + \frac{1}{1 - kL} \right) \right]$$

$$\frac{\gamma}{\Omega_{ci}} = \frac{1}{\sqrt{\pi} \mathcal{F}} \left( \frac{\rho_i}{L} \right)^3 \left( \frac{m_e T_e}{m_i T_i} \right)^{1/2} \left( 1 + \frac{T_e}{T_i} \right) \left[ 1 - (kL)^2 \right] \frac{B_{xo}}{B_{yo}} \mathcal{G}$$

$$\mathcal{G} = \left[ 1 - \sin^2(\theta) \left( 1 - \left( \frac{B_{yo}}{B_{xo}} \right)^2 \frac{1 - kL/2}{1 - kL} \right) \right] \frac{1}{1 + \hat{n}_b \cosh^2(z_s/L)}$$
Conditions to Drive Oblique Modes

Range of allowable angles is limited \[ \theta < \tan^{-1}(B_{xo}/B_{yo}) \]

Fastest growing modes are oblique when

\[
\frac{B_{yo}}{B_{xo}} > \left( \frac{1 - kL}{1 - kL/2} \right)^{1/2}
\]

For long wavelength \( kL \ll 1 \) this is simply \( B_{yo} > B_{xo} \)

How well does asymptotic theory compare with exact linear Vlasov approach?
Example comparisons for pair limit

\[ m_i = m_e \]
\[ B_{yo} = B_{xo} \]
\[ n_b/n_o = 0.3 \]
2D Simulations Only Permit Resonant Surface at $z=0$

\[ J_e \quad \text{(Conducting + reflecting)} \]

\[ m_i = m_e \]
\[ B_y = B_o \]

$120d_i \rightarrow 512$ cells

$240d_i \rightarrow 1024$ cells

Periodic

Periodic
$J_e$  


$J_e$  


2D

\[ \sim 10^6 \text{ cells} \]

3D - slice

\[ \sim 10^9 \text{ cells} \]

Oblique modes totally change solution
Somewhat later $t \Omega_{ci} = 150$

Oblique modes begin to dominate at later times.
3D Evolution on Roadrunner

0.5 billion cells ~200 billion particles
New Run on Kraken - Scaling Study

\[ \sim 3.3 \times 10^9 \text{ cells} \quad \sim 1.3 \times 10^{12} \text{ particles} \]
3D Structure from Kraken Run

$\sim 3.3 \times 10^9$ cells  $\sim 1.3 \times 10^{12}$ particles

Current filaments
“Secondary Islands”
3D Complexity slows energy dissipation!

Fractional decrease in magnetic energy

$m_i = m_e$

0.02
0.04
0.06
0.08
0.1

0
200
400
600
800
1000

linear tearing

$L_x = 240d_i$

factor \sim 2 slower

factor \sim 3 slower

$L_x = 240d_i$

$L_x = 480d_i$

“fast” onset
Coherent flow pattern from 2D is disrupted

2D
\( \sim 10^6 \) cells

3D cut
\( \sim 10^9 \) cells
Does this work the same way for hydrogen plasmas?
For hydrogen - location of matching between inner and outer regions is important

\[
\Delta' \equiv \lim_{\epsilon \to 0} \left[ \frac{1}{\tilde{A}_\parallel(z_s)} \left( \frac{d\tilde{A}_\parallel}{dz} \bigg|_{z_s+\epsilon} - \frac{d\tilde{A}_\parallel}{dz} \bigg|_{z_s-\epsilon} \right) \right] \quad \delta_j = \frac{\omega l_s}{k V_{th_j}}
\]

Asymptotic \(\varepsilon = 0\)

Exact Vlasov

Asymptotic \(\varepsilon = \delta_i\)

Asymptotic \(\varepsilon = 0\)

Exact Vlasov

Asymptotic \(\varepsilon = \delta_i\)
Early Structure at High Mass Ratio

\[ \sim 3.3 \times 10^9 \text{cells} \quad \sim 1.1 \times 10^{12} \quad \text{particles} \]

\[ m_i/m_e = 64 \]

Primary Islands
Dynamics does NOT result in this picture based on the initial tearing modes.

What about secondary instabilities?
Electron layers that form along separatrices are also unstable to secondary islands.

Secondary islands along separatrix needs finite $k_y$.

Can't occur in 2D.
Time Evolution of Current Structures \( \frac{m_i}{m_e} = 64 \)

Secondary magnetic islands form oblique flux ropes
Electron current layers along separatrices produce strong magnetic shear

Near peak current

\[ \theta_J = \tan^{-1}\left(\frac{J_y}{J_x}\right) \approx 66^\circ \]

\[ \theta_B = \tan^{-1}\left(\frac{B_y}{B_x}\right) \approx 60^\circ \]

\[ k \cdot B = 0 \]

\[ \theta_T = \frac{\pi}{2} - \theta_B \approx 30^\circ \]
Secondary magnetic islands along separatrices form oblique flux ropes in 3D.
Summary & Future Outlook

- Petascale computing is allowing kinetic studies \((100-1000)\times\) larger than previous state-of-the-art efforts.

- Real potential for breakthrough progress - but computing will never be a substitute for thinking - still desperately need theory, laboratory experiments, space observations, etc.

- We can move beyond simple cartoons.

- New asymptotic theory offers simple predictions of when to expect this complex evolution - need similar theory for secondary islands.
Summary & Future Outlook

For guide field regimes, reconnection be inherently 3D, which may have far reaching implications for:

- Dissipation rate
- Generation of stochastic magnetic fields
- Structure of exhaust
- Transport and acceleration of particles

Studies of reconnection in large 3D systems will be increasingly interconnected with turbulence

Influence of *pre-existing* upstream turbulence may be huge issue!

Finally - we can also now start to think about 2D global kinetic modeling of many more kinds of problems