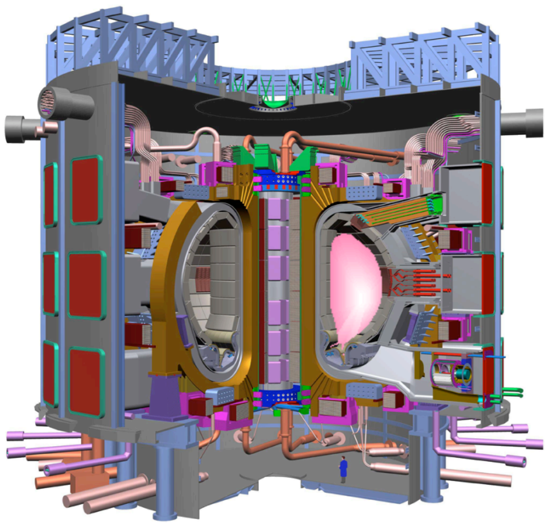
Shape Matters -- Building a Better Bottle?

Steve Cowley *Culham, Imperial*

Bill Dorland, K^3, Omar Hurricane and Pierre Gourdain,

.



United Kingdom Atomic Energy Authority

Causes Without Results

"I am afraid I rather give myself away when I explain. Results without causes are much more impressive."

Sherlock Holmes in <u>The Stock-Broker's Clerk</u>, Arthur Conan Doyle.

United Kingdom Atomic Energy Authority

Better Bottle?

We have a magnetic configuration that will take us to burning plasmas in ITER. This will probably be the configuration of the first generation of fusion reactors.

Can we improve this? What would that mean?

Better confinement? .... τ_E
Higher pressure -- lower B? .... β
More reliable? .... stability.
Less heat flux? .... Divertor.

Has every configuration been tried? In 2D?

United Kingdom Atomic Energy Authority

Requirements For Fusion.

$$Fusion \quad Power \propto n_D n_T T^2 \propto \beta^2 B^4$$

$$10\,keV < T < 20\,keV\,.$$

Rough criterion for ignition. $\boxed{nT\tau_E > 3 \times 10^{15} cm^{-3} keV\ s}$

Physics limits the achievable values of these quantities.

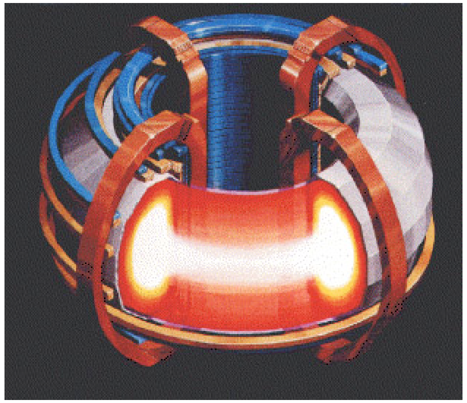
n :Density “Greenwald” limit.

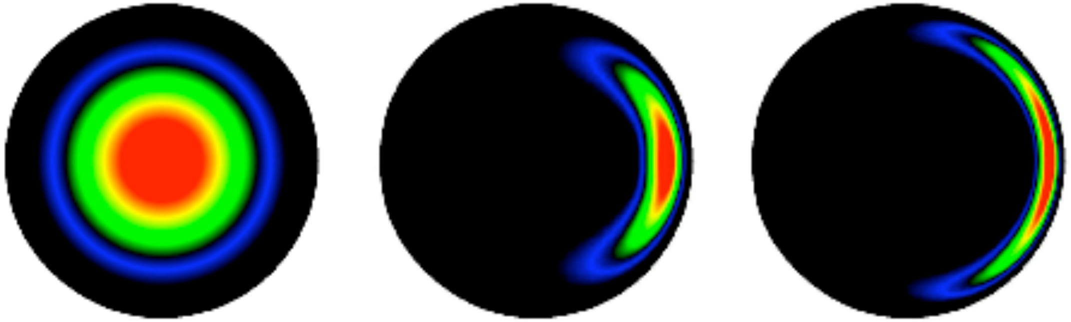
$nT = \beta \dfrac{B^2}{8\pi}$: Beta Limit. $\beta = \beta_N \dfrac{I}{aB}$

τ_E : Turbulence.

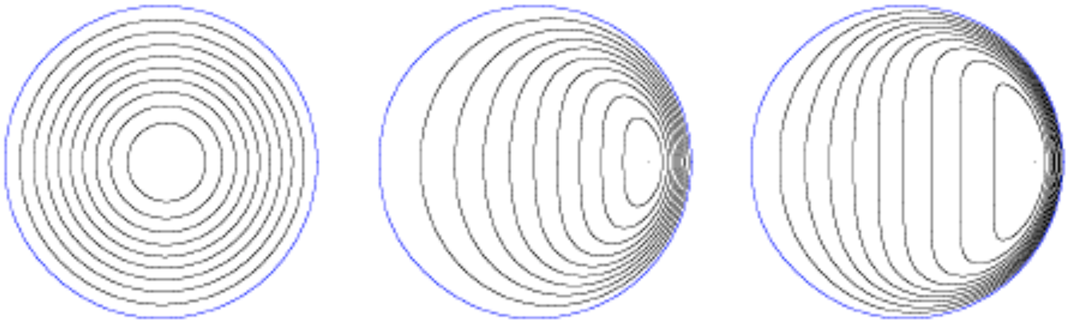
United Kingdom Atomic Energy Authority

Raising Beta





$$\mathbf{J} \times \mathbf{B} = \nabla p$$



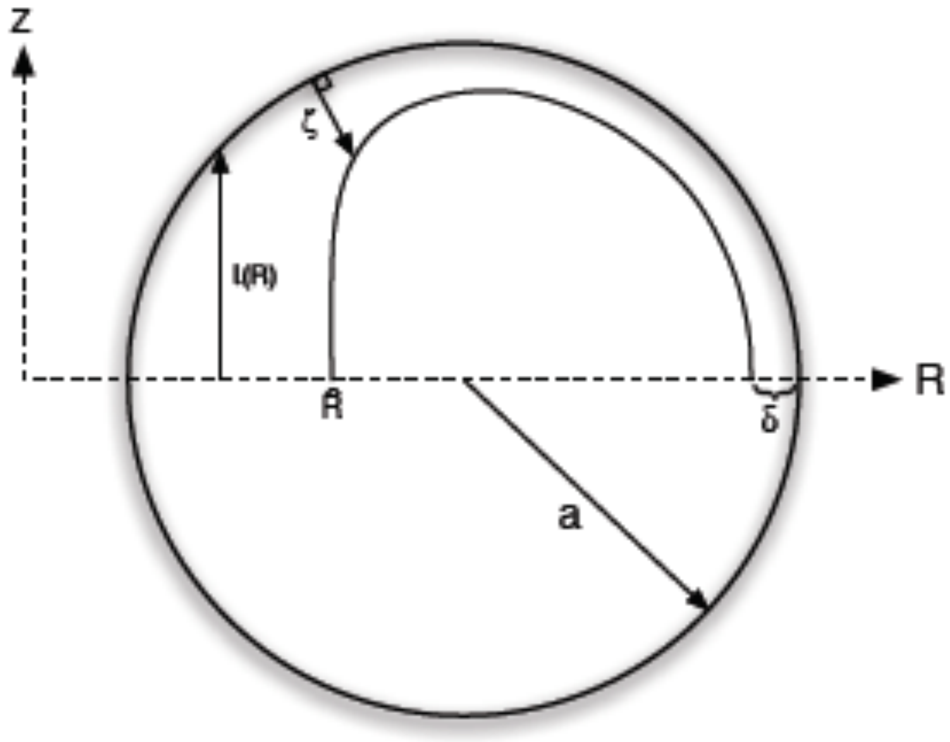
Radial Force Balance $\nabla\psi \cdot [\mathbf{J} \times \mathbf{B} = \nabla p]$

$$\left[R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial Z^2}\right]\psi = -\mu_0 R^2 \frac{dp}{d\psi} - F\frac{dF}{d\psi}$$

Grad-Shafranov Equation.

where

$$\mathbf{B} = \frac{\nabla\psi \times \mathbf{e}_T}{R} + \frac{F(\psi)}{R}\mathbf{e}_T$$



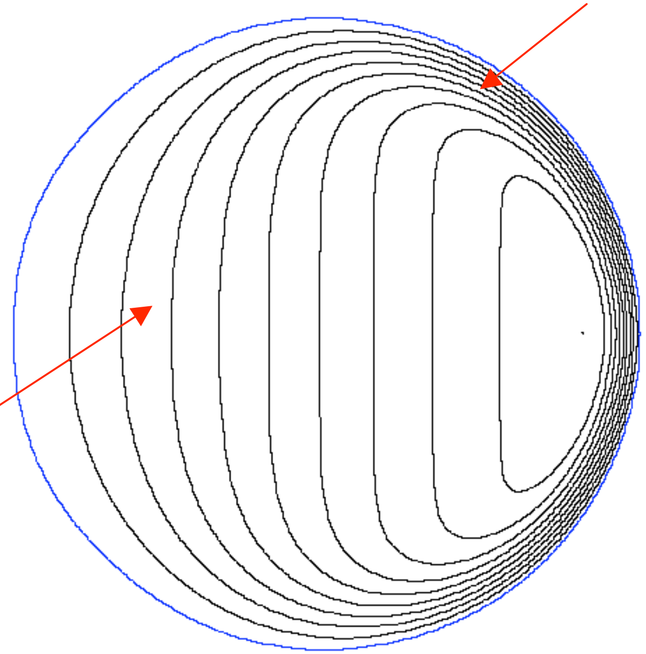
United Kingdom Atomic Energy Authority

The Small Parameter

$$\frac{\epsilon}{q^2} = \frac{a}{q^2 R} \ll 1$$

$$\mathbf{B}_p \ll \mathbf{B}_T = F/R$$

Surfaces shift and
Squash against edge.



$$\underbrace{\nabla p}_{\mathcal{O}(\beta)} + \underbrace{\frac{\nabla B_{\mathrm{T}}^2}{2}}_{\mathcal{O}(1)} + \underbrace{\frac{\nabla B_{\mathrm{P}}^2}{2}}_{\mathcal{O}(\epsilon^2/q^2)} + \underbrace{\frac{B_{\mathrm{T}}^2 \nabla R}{R}}_{\mathcal{O}(\epsilon)} + \underbrace{B_{\mathrm{P}} \cdot \nabla B_{\mathrm{P}}}_{\mathcal{O}(\epsilon^2/q^2)} = 0$$

CORE

$$\boxed{\left[R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right)+\frac{\partial^2}{\partial Z^2}\right]\psi} = -\mu_0 R^2\frac{dp}{d\psi} - F\frac{dF}{d\psi}$$

small $\mathcal{O}(\epsilon)$

$$\mu_0 R^2\frac{dp}{d\psi} = -F\frac{dF}{d\psi}$$

Toroidal Field Confinement

$$\boxed{R = R(\psi) \quad or \quad \psi = \psi(R)}$$

Straight vertical flux surfaces.

United Kingdom Atomic Energy Authority

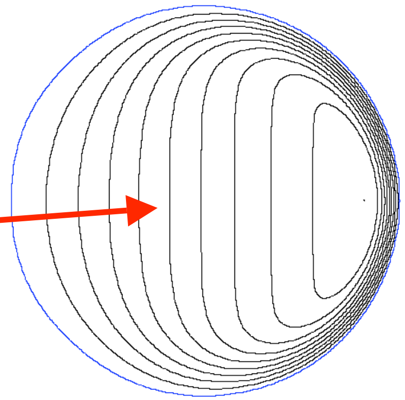
CORE

$$F(\hat{R}) = \sqrt{2\left(C - \mu_0 \int_{R_{\min}}^{\hat{R}} \hat{R}'^2 \frac{dp}{d\hat{R}'} d\hat{R}'\right)}$$

C = constant & p increases and F decreases towards the axis

$$\boxed{R = R(\psi) \;\; or \;\; \psi = \psi(R)}$$

Straight vertical flux surfaces in core



United Kingdom Atomic Energy Authority

Boundary Layer -- BL

Gradients are large perpendicular to wall ξ = distance to wall.

$$\frac{\partial^2\psi}{\partial\xi^2} = -\mu_0(R^2 - \hat{R}^2(\psi))\frac{dp}{d\psi}$$

Width of Boundary Layer is small and Poloidal Field is strong

$$\left(\frac{\partial\psi}{\partial\xi}\right)^2 = -2\mu_0\int_R^{\hat{R}}(R^2 - \hat{R}''^2)\frac{dp}{d\psi}\frac{\partial\psi}{\partial\xi}d\xi$$

Poloidal field pressure forces balance the residual force from Lack of cancellation of pressure and toroidal field forces.

$$|\mathbf{B}_p| \sim \sqrt{\epsilon p} \ll |\mathbf{B}_T|$$

United Kingdom Atomic Energy Authority

Boundary Layer --- BL

$$\xi(R,\hat{R}) = \int_{R_{\min}}^{\hat{R}} \frac{d\hat{R}'\,\frac{\partial\psi}{\partial\hat{R}'}}{\sqrt{-2\mu_0\int_R^{\hat{R}'} d\hat{R}''\,\frac{dp}{d\hat{R}''}(R^2-\hat{R}''^2)}}$$

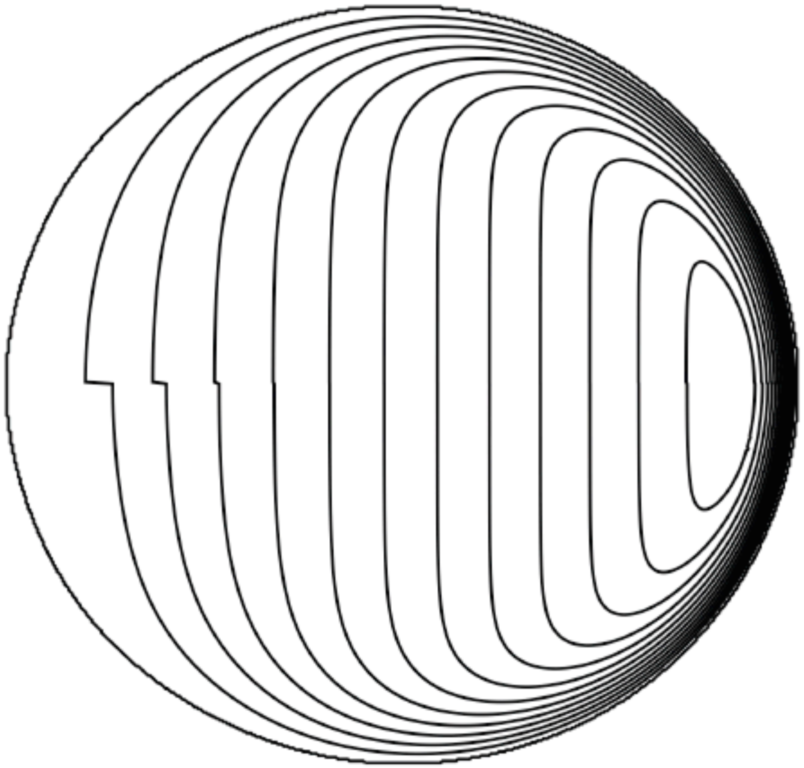
$$\delta = a\sqrt{\frac{\epsilon}{q^2\beta}}$$

Boundary layer width.
Expansion works if $\delta < a$

Poloidal field increases outwards in Boundary Layer.

United Kingdom Atomic Energy Authority

Comparison



Agreement gets better as we increase beta

FIG. 3: Comparison of a equilibrium soluton computed in CUBE (top) and the same solution calculated using the analytic theory (bottom).

United Kingdom Atomic Energy Authority

Good properties.
1. Good Average Curvature

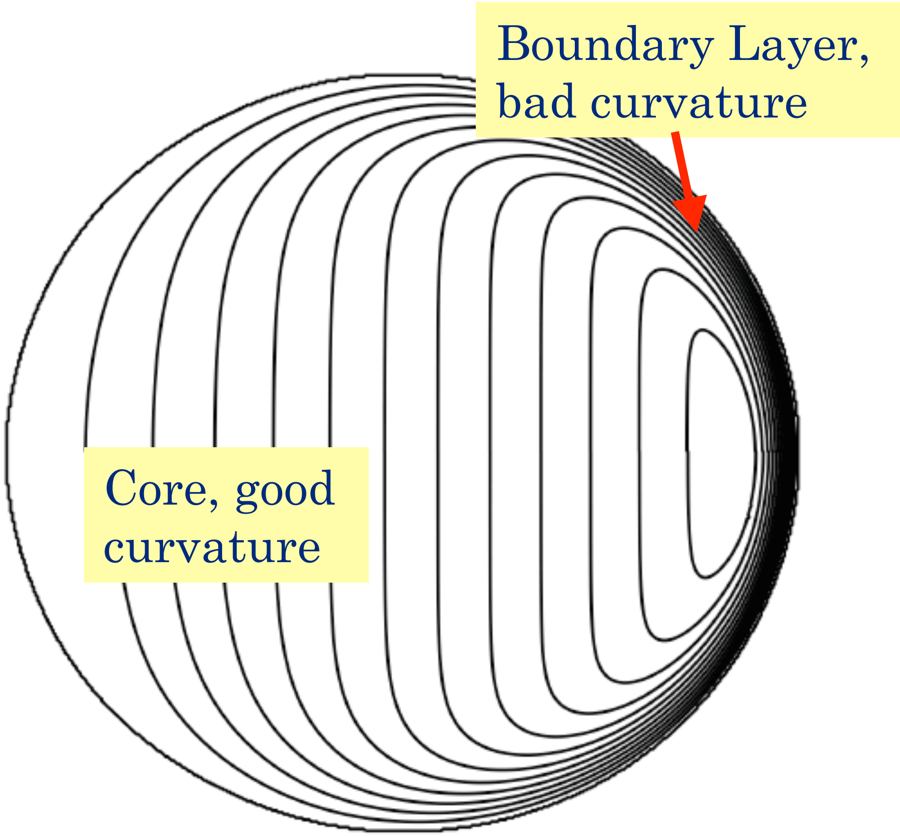


FIG. 3: Comparison of a equilibrium soluton computed in CUBE (top) and the same solution calculated using the analytic theory (bottom).

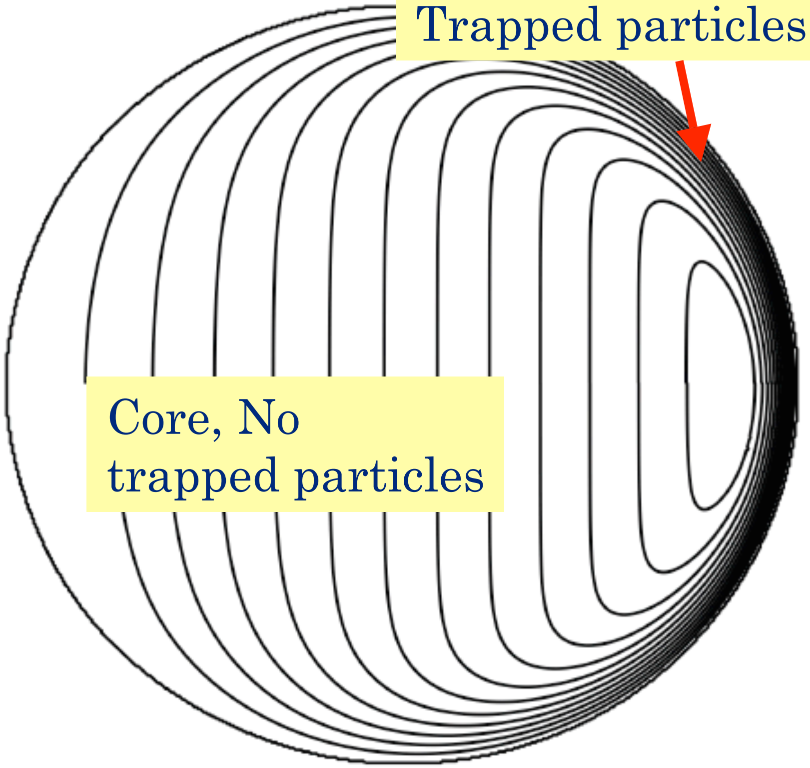
Bad field line curvature in the boundary layer only. Core dominates average

$$<\nabla p\cdot(\mathbf{b}\cdot\nabla\mathbf{b})>\sim -\frac{p}{aR}$$

Mercier stable and tearing mode stable.
Cowley Phys. Fluids B 1991.

United Kingdom Atomic Energy Authority

Good properties.
2. Small trapped particle fraction



|B| constant on flux surface in core $\Rightarrow$ no bounce points in core. Trapped particle fraction…..

$$f_T \sim \left(1 - \frac{B_{min}}{B_{max}}\right)^{1/2} \frac{\Delta V}{V}$$

As beta increases both factors decrease. **|B|** constant on flux surface in BL too **(omnidigeneity)**. The volume fraction in BL is

$$\frac{\Delta V}{V} \sim \sqrt{\frac{\epsilon}{q^2 \beta}} \ll 1$$

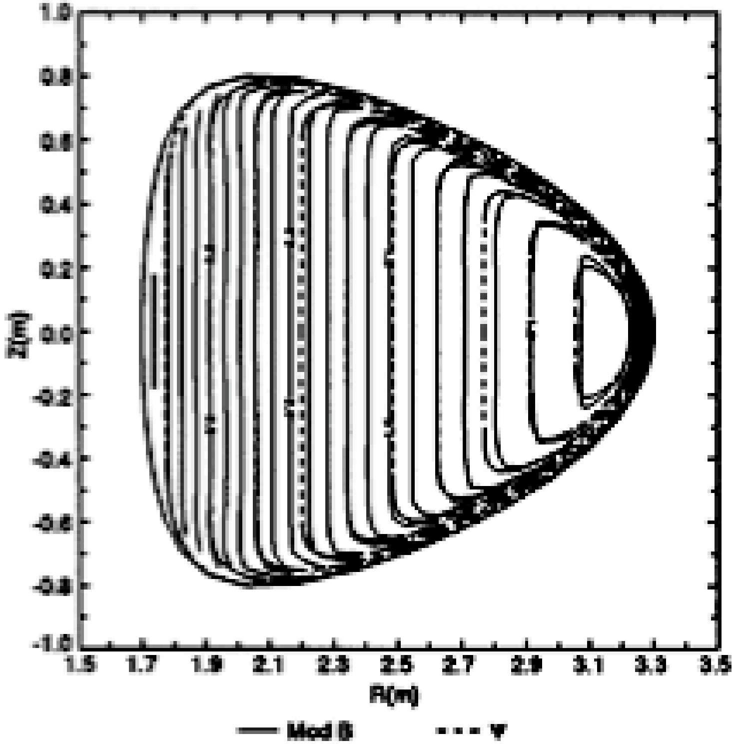
Banana width is squeezed by strong B_p. Neoclassical transport reduced by more than $\sim \sqrt{\frac{\epsilon}{q^2 \beta}}$ *SCC PPPL report.*

United Kingdom Atomic Energy Authority

Good properties.
3. Magnetic well

$$\mathbf{B}^2 = \frac{F^2}{R^2} + \frac{|\nabla\psi|^2}{R^2}$$

$$p + \frac{\mathbf{B}^2}{2} = constant$$



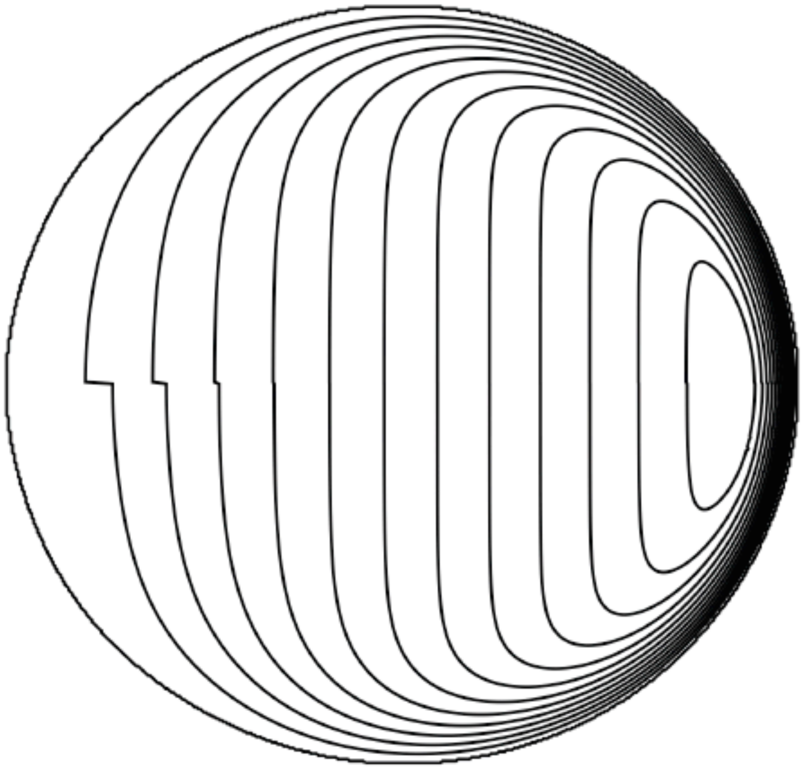
|B| is small in the center of
The plasma.

$$\mu = \frac{v_\perp^2}{B} = constant$$

Got to give particles energy
To get them out. Helps
Stability, *Taylor 1963*

United Kingdom Atomic Energy Authority

Good properties.
4. Short Connection length

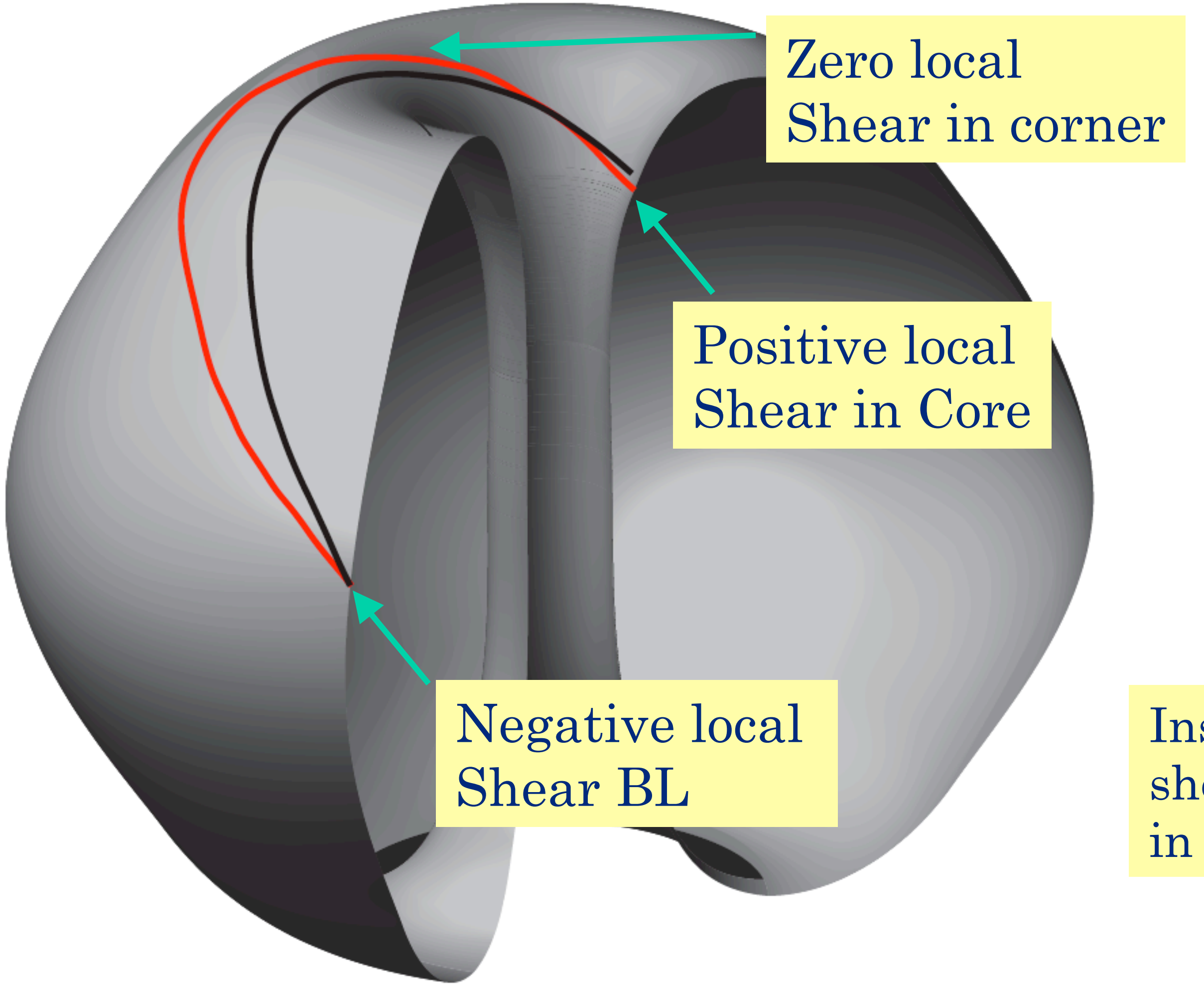


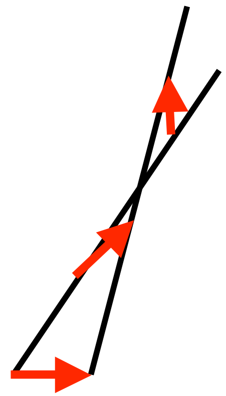
B_p is large in BL so distance along field from bad to Good curvature is

$$L_C \sim qR\sqrt{\frac{\epsilon}{q^2\beta}}$$

Stabilizing.

Good properties.
5. Strong negative local shear in BL





Shear length in boundary layer =

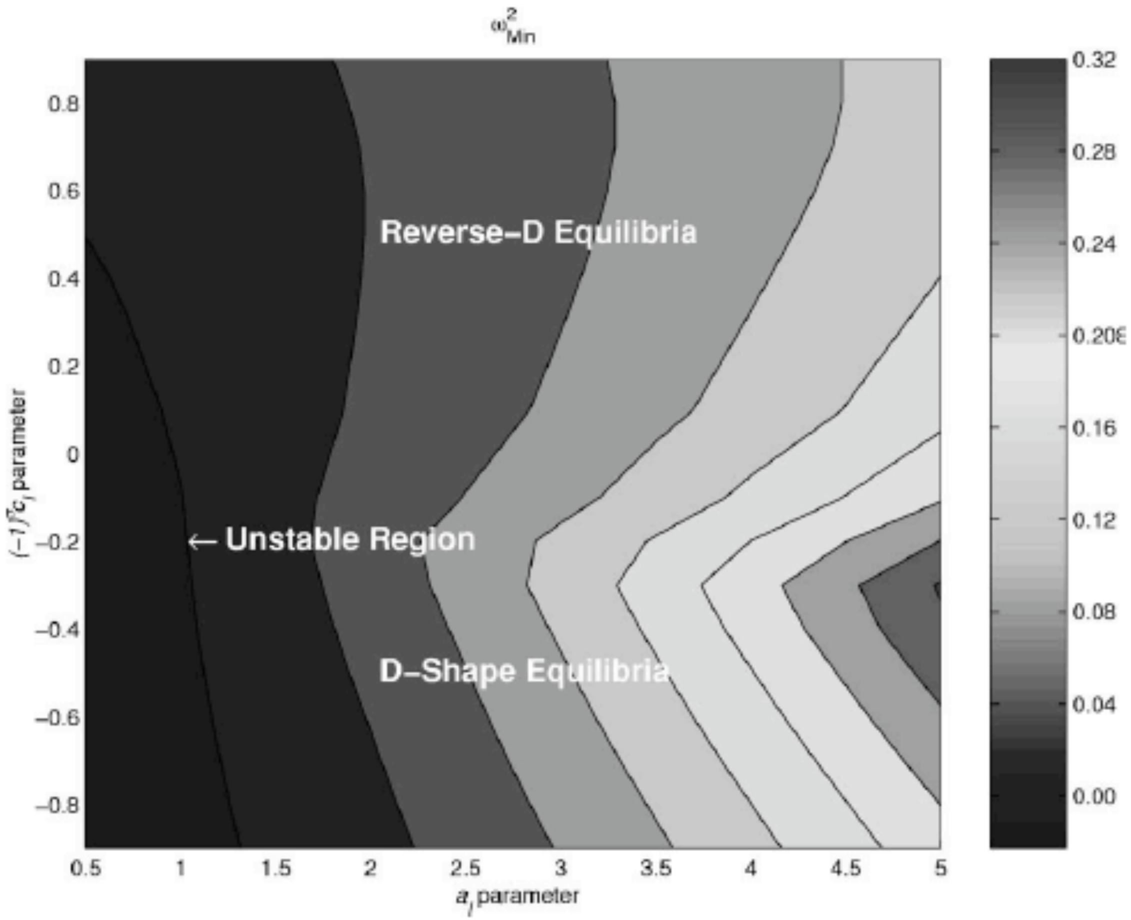
$$L_s \sim \mathcal{O}(a/q\beta) \ll L_c$$

Instabilities are heavily sheared by Magnetic shear in BL Stabilizes ballooning.

United Kingdom Atomic Energy Authority

Internal Kink Stability

Analytic result just depends on shape -- not on **β or q.**



Triangularity

Elongation

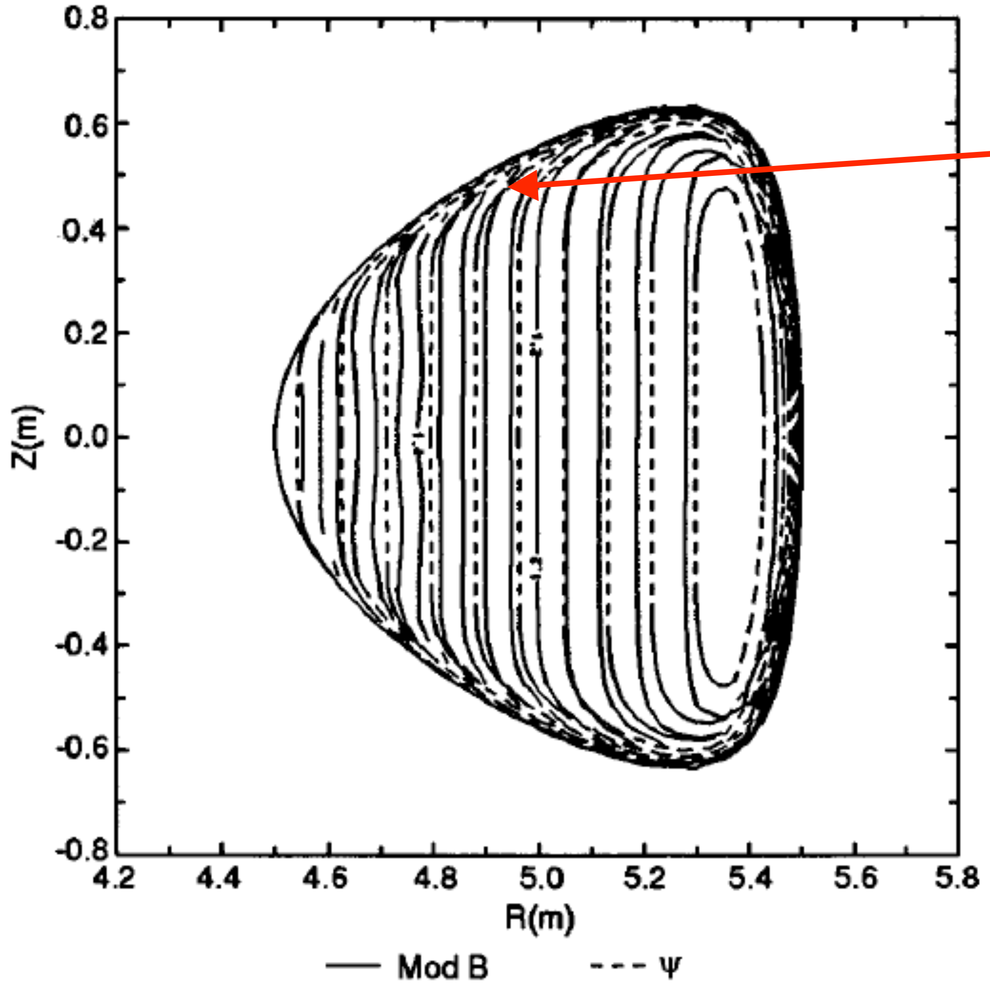
Hurricane et. al.
Phys. Plas. 2000

What about
Rotation?

External kinks
are the problem.

United Kingdom Atomic Energy Authority

Negative Triangularity - Reverse D.



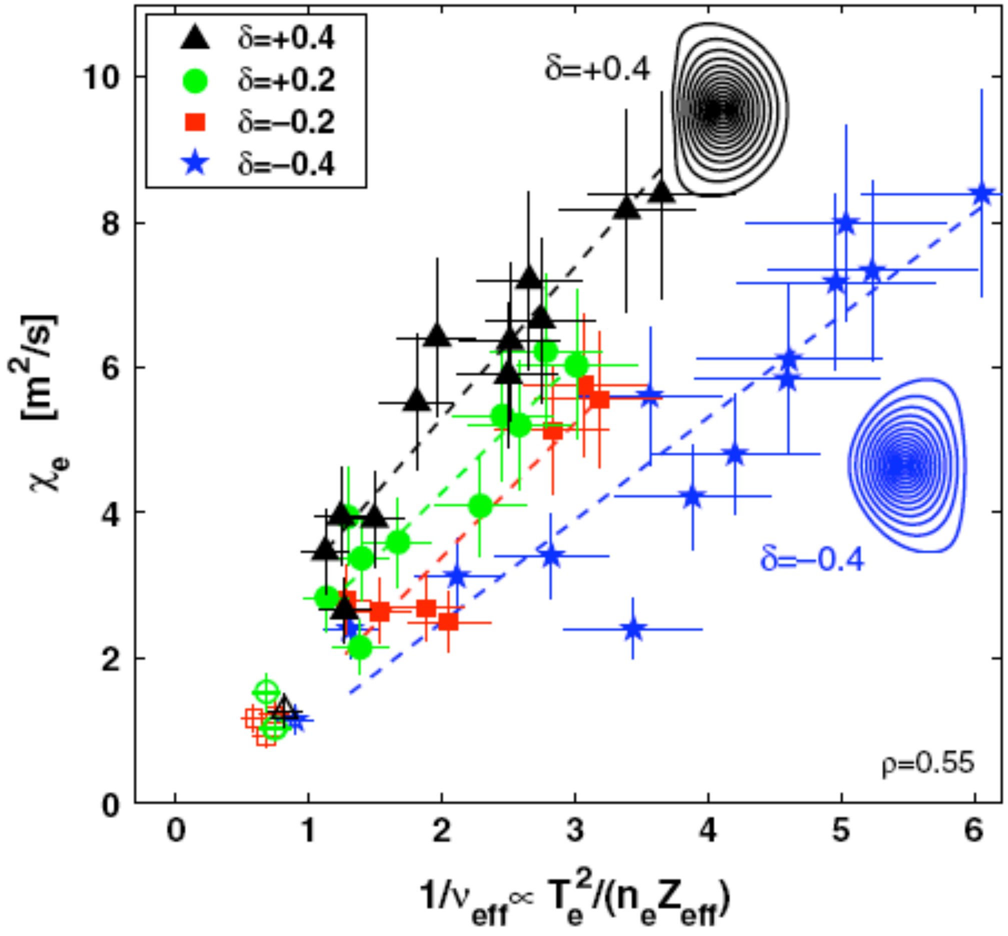
Zero local Shear in Corner firmly in good Curvature region.

Particles drift reversed

Bill Dorland will show Interesting results on Similar configuration.

United Kingdom Atomic Energy Authority

Negative Triangularity - Reverse D. TCV



Less transport
In reverse D

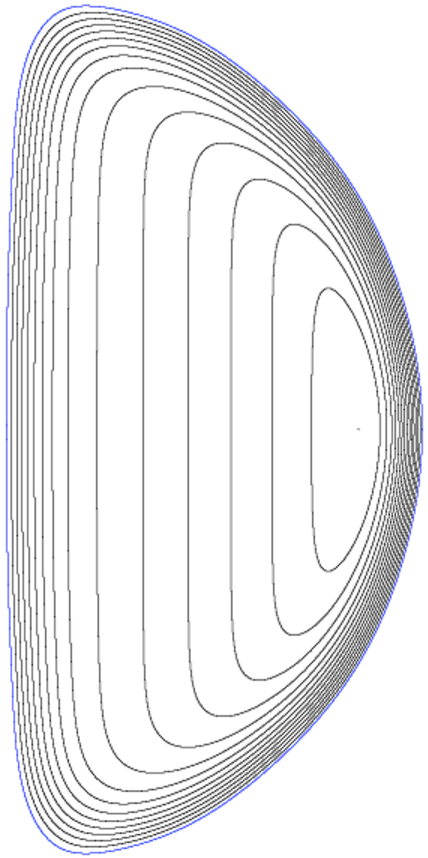
Camenen et. al. Nucl Fus. 2007

United Kingdom Atomic Energy Authority

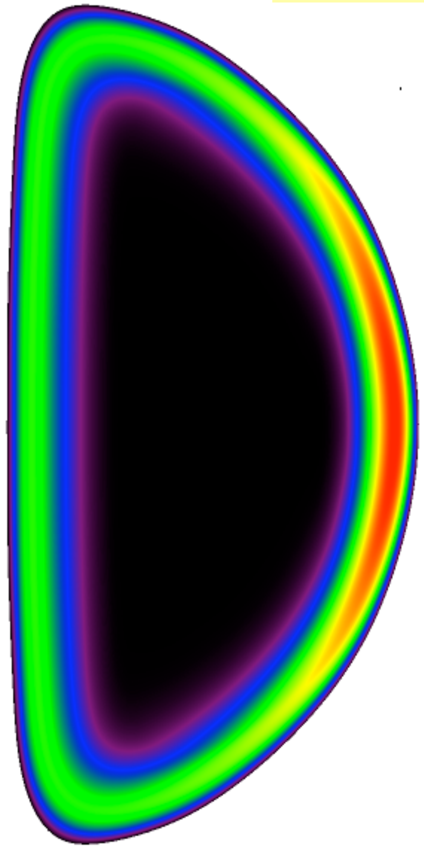
Unity beta current hole equilibrium

This equilibrium is stable to all ideal MHD criteria including internal and external modes for n =1, 2 and 3… ***Note that the β_N is "small" despite the large value of beta.***

Pierre Gourdain's work



Flux Surfaces

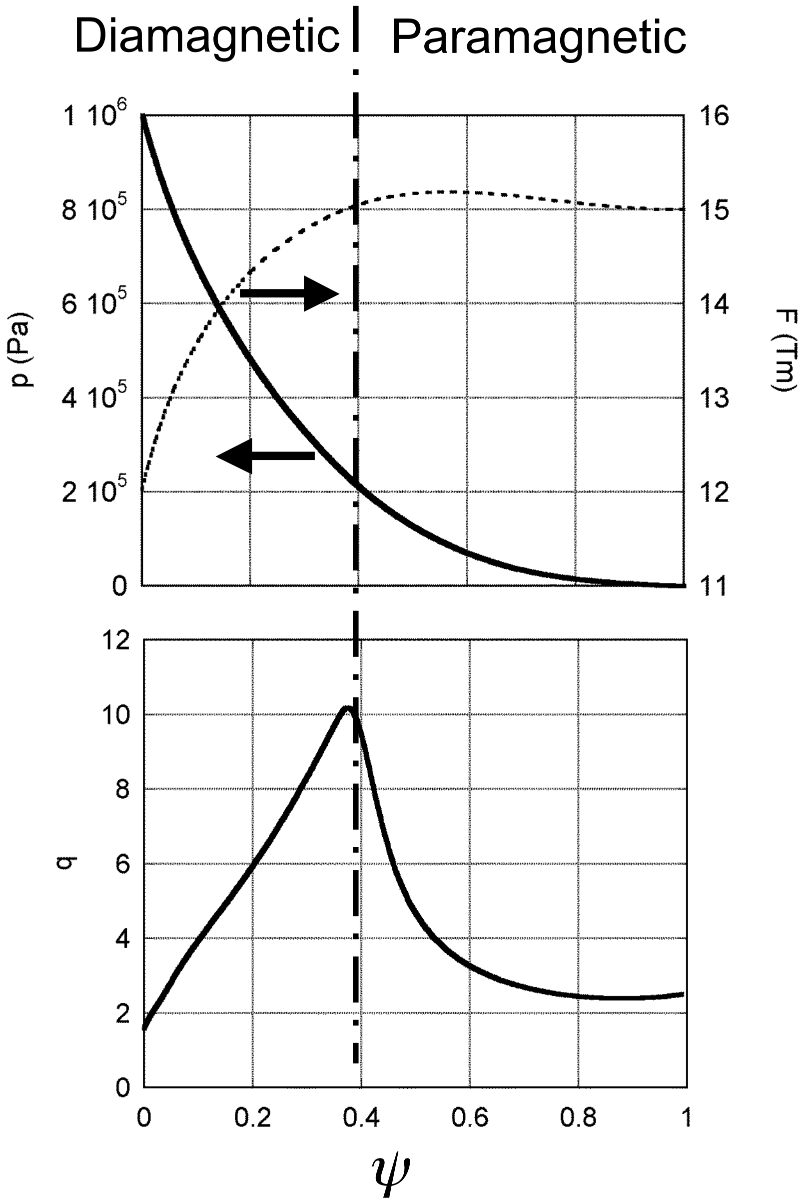


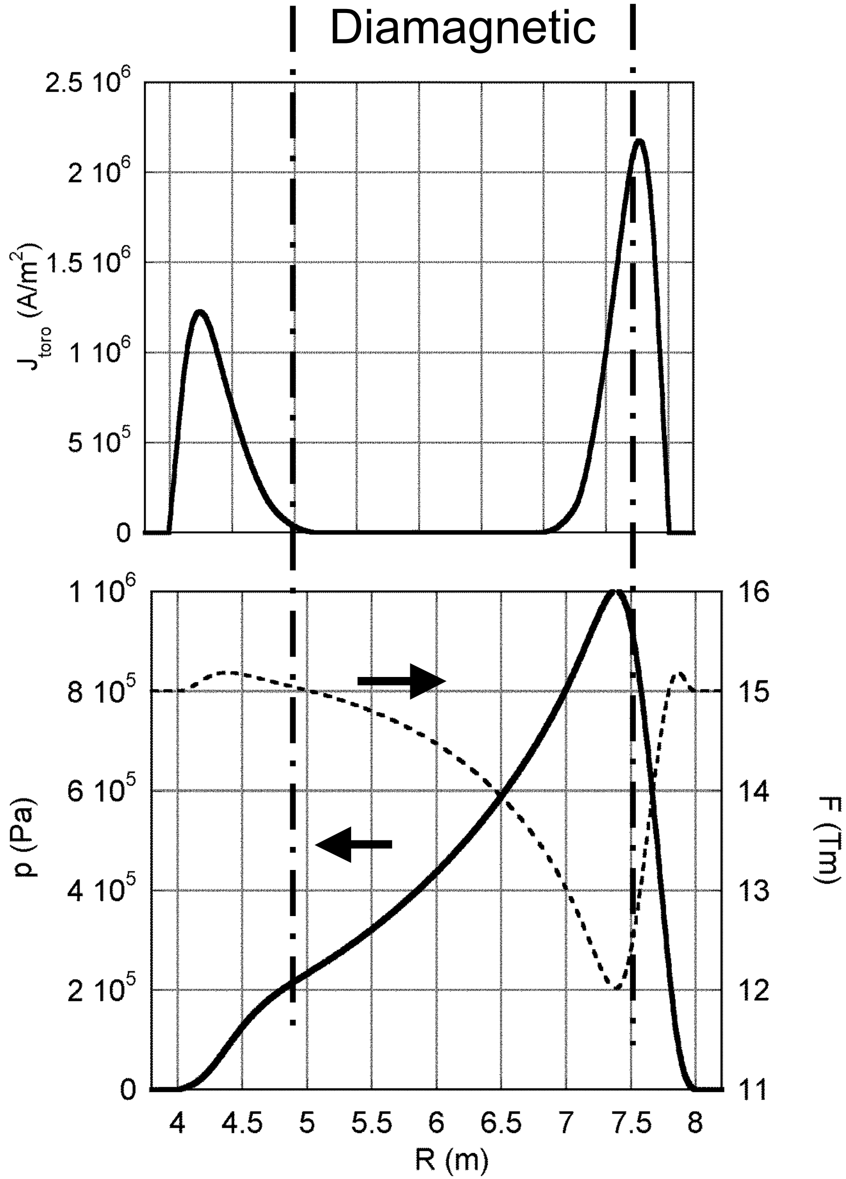
Toroidal Current Distribution

R	6 m
a	2 m
B_T	2.5 T
β	100%
<β>	12%
β_N	4.6
q_{min}	1.5

United Kingdom Atomic Energy Authority

Unity beta current hole profiles



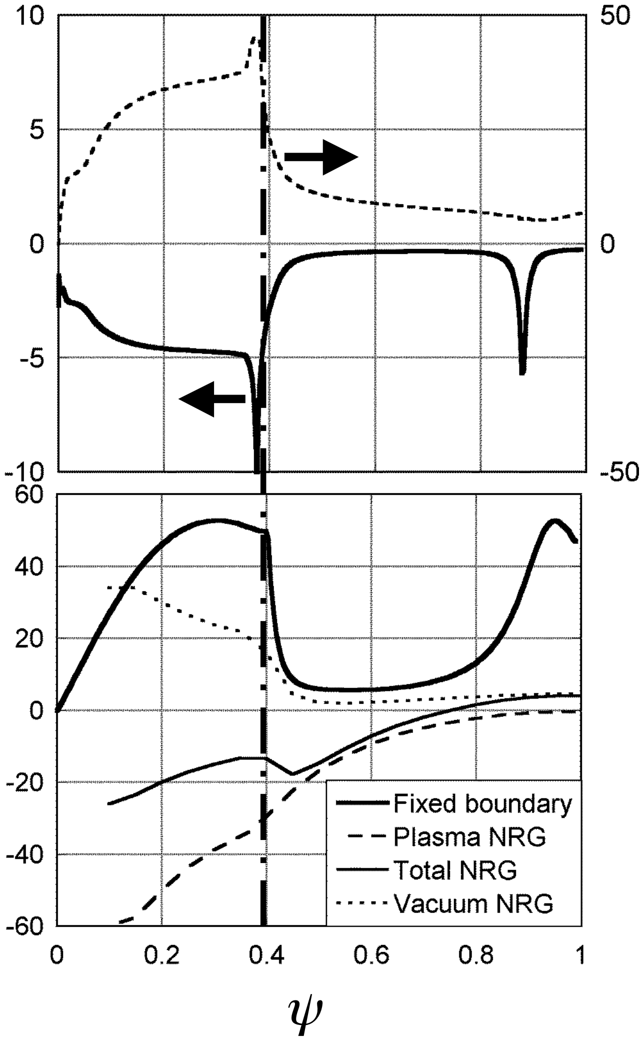


Internal and external kink stability

Mercier
Stable when negative

High-n ballooning
Stable when positive

Fixed and free boundary mode n=1
Stable when positive



DCON finds stability for Mercier, high-n ballooning as well as fixed boundary kink modes (n=1).

The free boundary mode n=1 is also stable (stability criteria obtained for $\psi = 1$).

Stability for n=2 and n=3 was also demonstrated.

United Kingdom Atomic Energy Authority

Better Bottle -- better shape?

- It certainly isn't clear that we can find a better bottle.   But we should use our best tools to look hard.

- Start with transport considerations -- gyro-kinetics?

United
Kingdom
Atomic
Energy
Authority