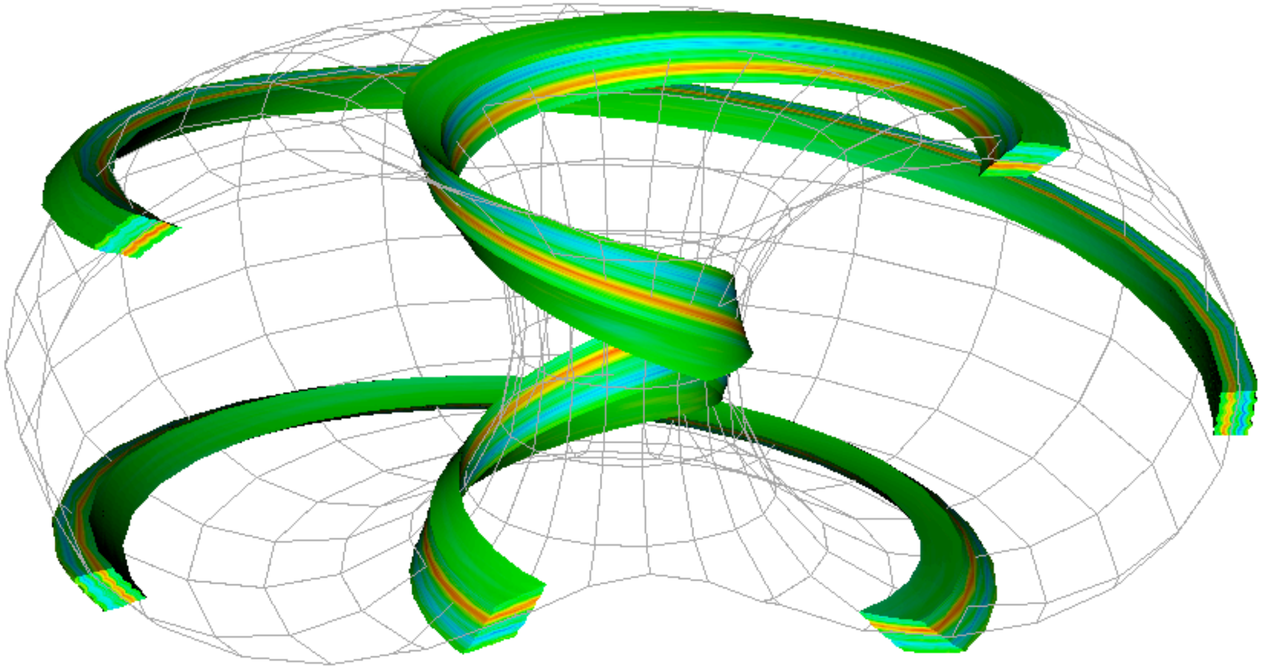
Multiscale Turbulence in Fusion and Gyrokinetics.

Steve Cowley -- Culham, Imperial

Gabe Plunk, Eric Wang, Ian Abel, Alex Schekochihin, Bill Dorland, Greg Hammett, Colin Roach, Michael Barnes, Felix Parra, Francis Casson

………..



CCFE CULHAM CENTRE FOR FUSION ENERGY

Casson and Hornsby

What am I going to say?.

- *The standard model of tokamak confinement and turbulence.*
- *Spatial scales -- time scales -- velocity space scales.*
- *Gyro-kinetic expansion.*
- *electron-ion separation.*
- *Possible problems*
  - *-- nonlocality, time/space*
  - *-- loss of scale separation*

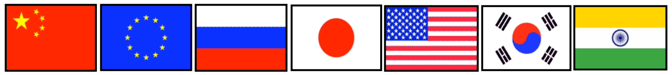
CCFE
CULHAM CENTRE FOR FUSION ENERGY

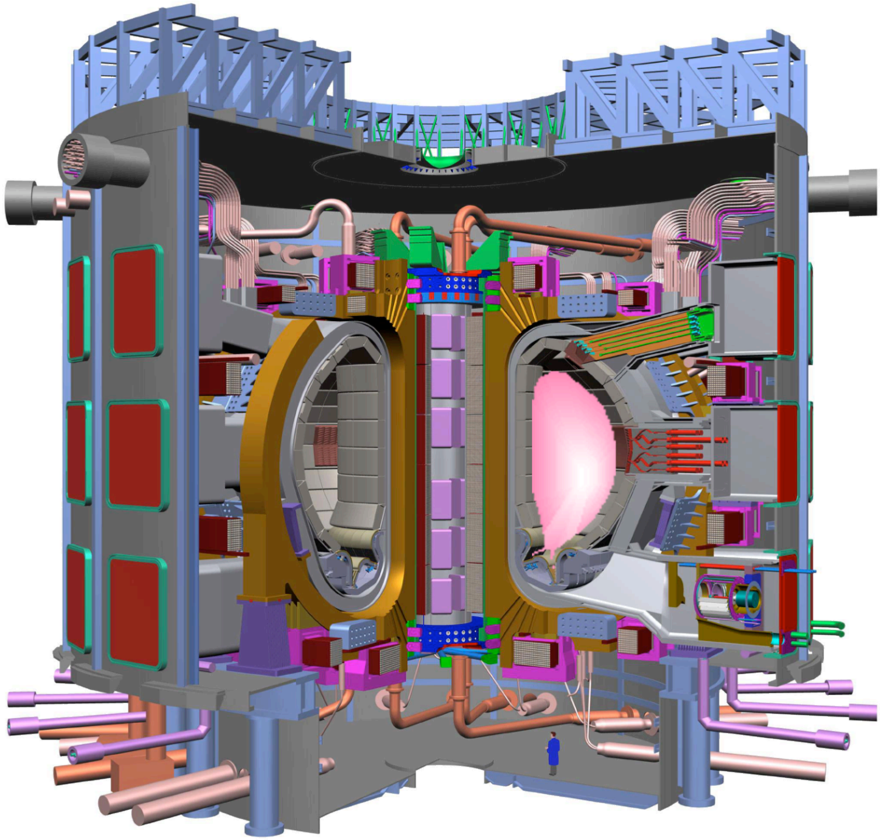
ITER

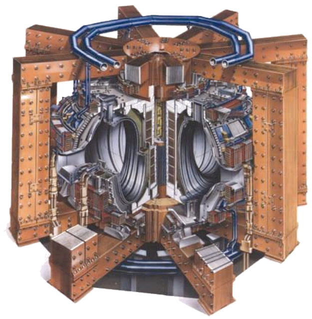
First Sustained Burning Plasma. Starts in 2019.

BASIC PARAMETERS.
Plasma Major Radius 6.2m
Plasma Minor Radius 2.0m
Plasma Current 15.0MA
Toroidal Field on Axis 5.3T
Fusion Power 500MW
Burn Flat Top >400s
Power Amplification Q>10

Cost is > 12 Billion Euro.





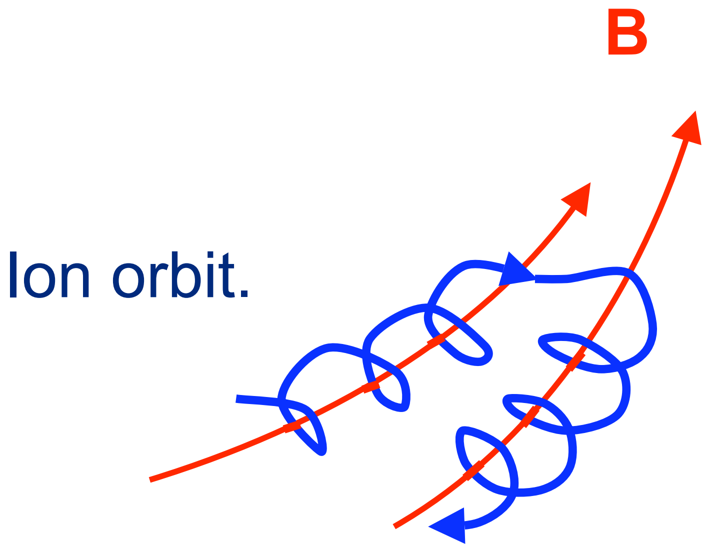


JET

CCFE
CULHAM CENTRE FOR FUSION ENERGY

Classical Transport.

Spitzer. 1951.



Random walk:
Step = ρ, larmor/cyclotron radius.
Decorrelation rate = ν = collision rate
Radius of plasma = a.

$$\tau_E \sim \frac{a^2}{\nu \rho^2}$$

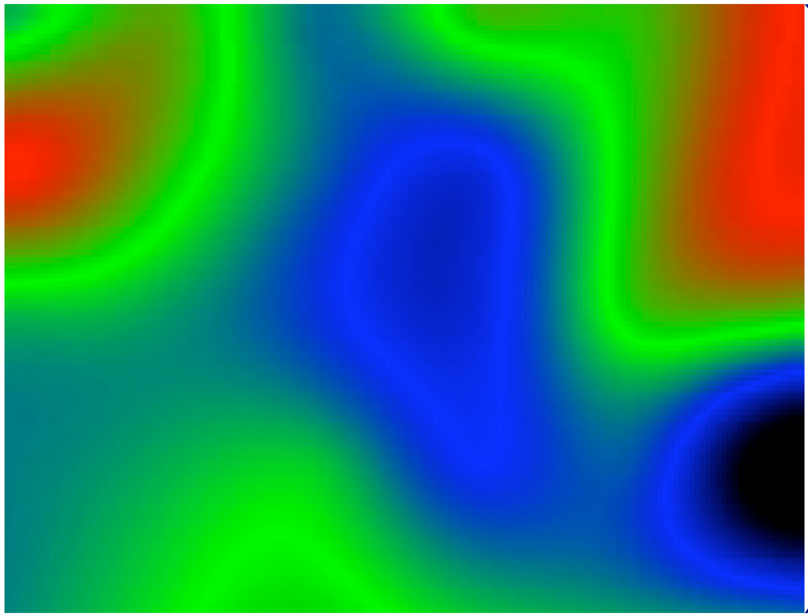
Collisions are rare and classical confinement can be very good.
Spitzer only needed a = 20cm, (τ_E>4s) for IGNITION.
Can't be right. Observed transport is much larger.

ITER neoclassical confinement time ~ 2000s

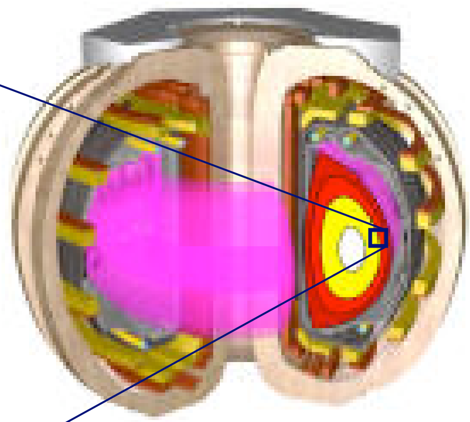
Turbulence Imaging, Beam Emission Spectroscopy

George McKee and Ray Fonck

~ 4cm



Density fluctuations



DIII-D in San-Diego

Plasma is 1m across

Eddies are small

compared to the device

Gyro-kinetic simulation.

DIII-D Shot 121717

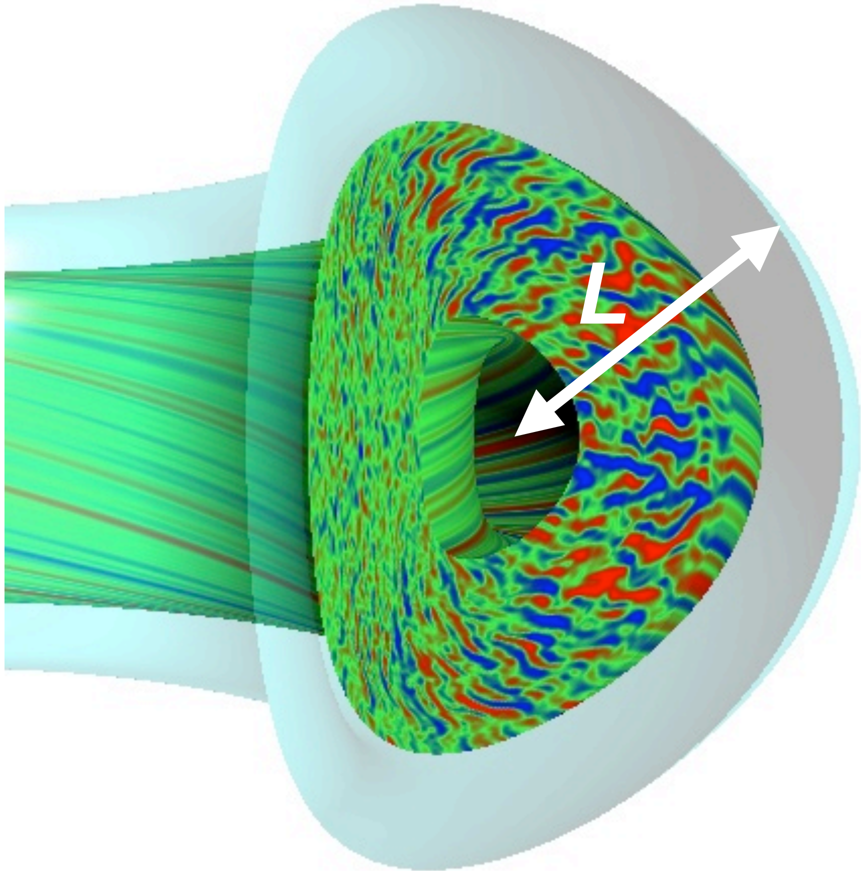
GYRO Simulation

Cray X1E, 256 MSPs

CCFE
CULHAM CENTRE FOR FUSION ENERGY

GYRO code simulations by Jeff Candy and Ron Waltz GA

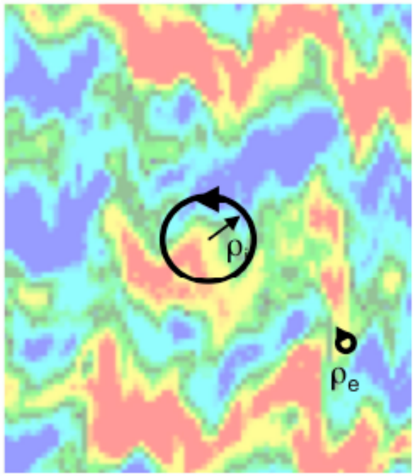
Spatial scales



L **=** *Equilibrium scale and parallel scale of turbulence*

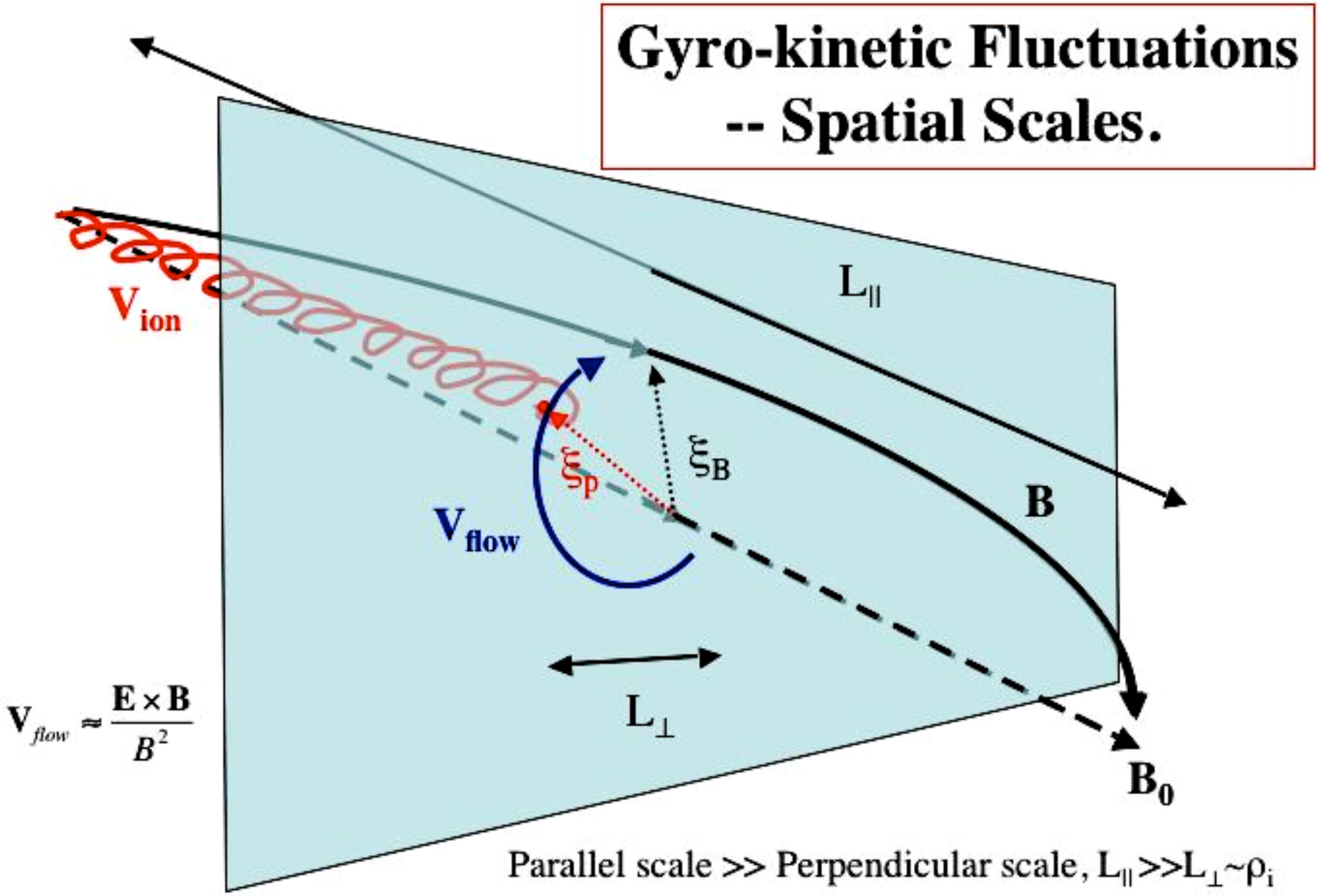
ρ = Ion larmor radius and perpendicular scale of turbulence

$\rho^* = \rho/L \sim 10^{-3}$ in ITER



CCFE
CULHAM CENTRE FOR FUSION ENERGY

Turbulent Scales

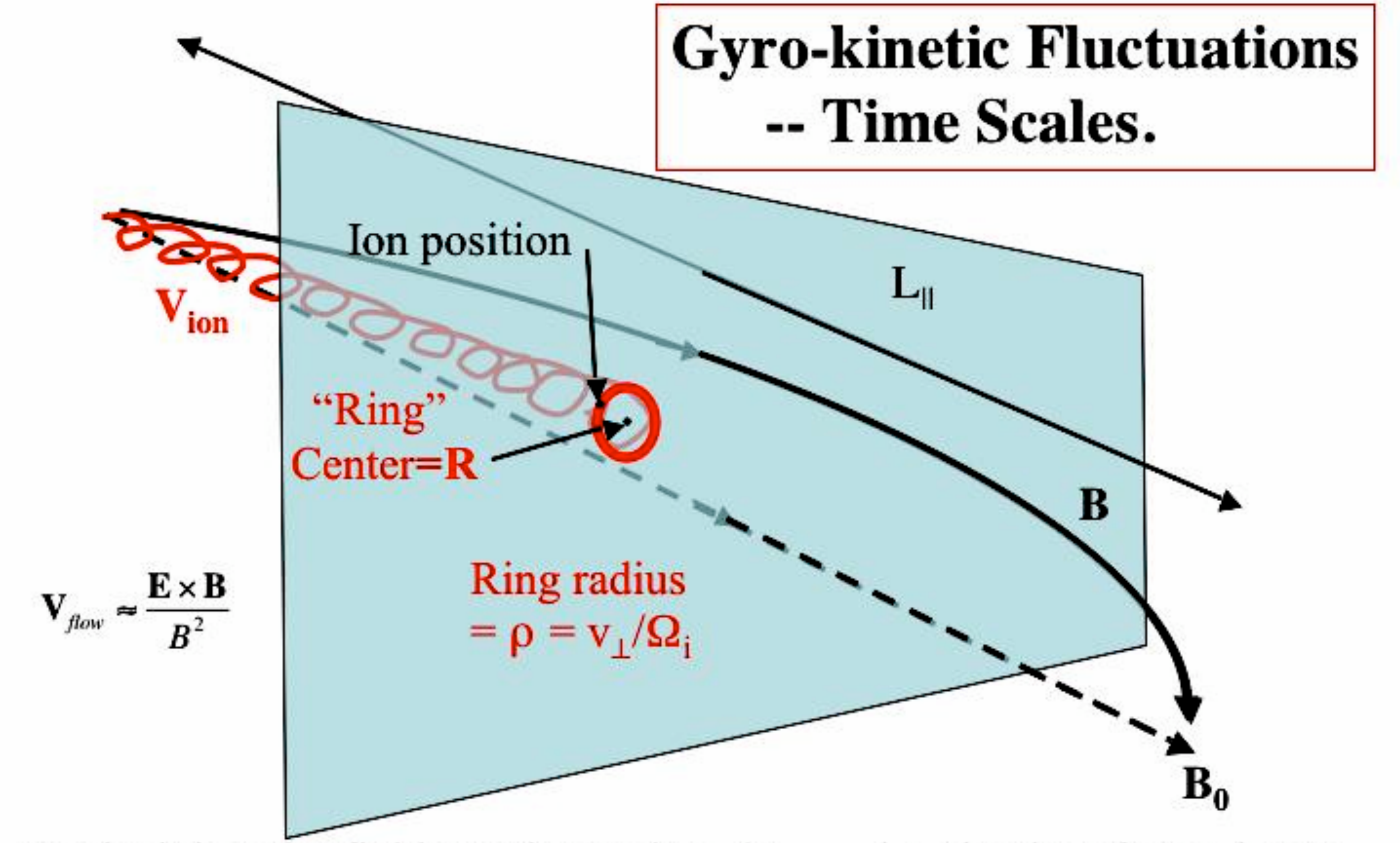


Parallel scale >> Perpendicular scale, $L_{||} >> L_{\perp} \sim \rho_i$

Particle displacement, $\xi_p \sim$ Of Order Field line displacement, $\xi_B \sim O(L_{\perp})$

CCFE CULHAM CENTRE FOR FUSION ENERGY

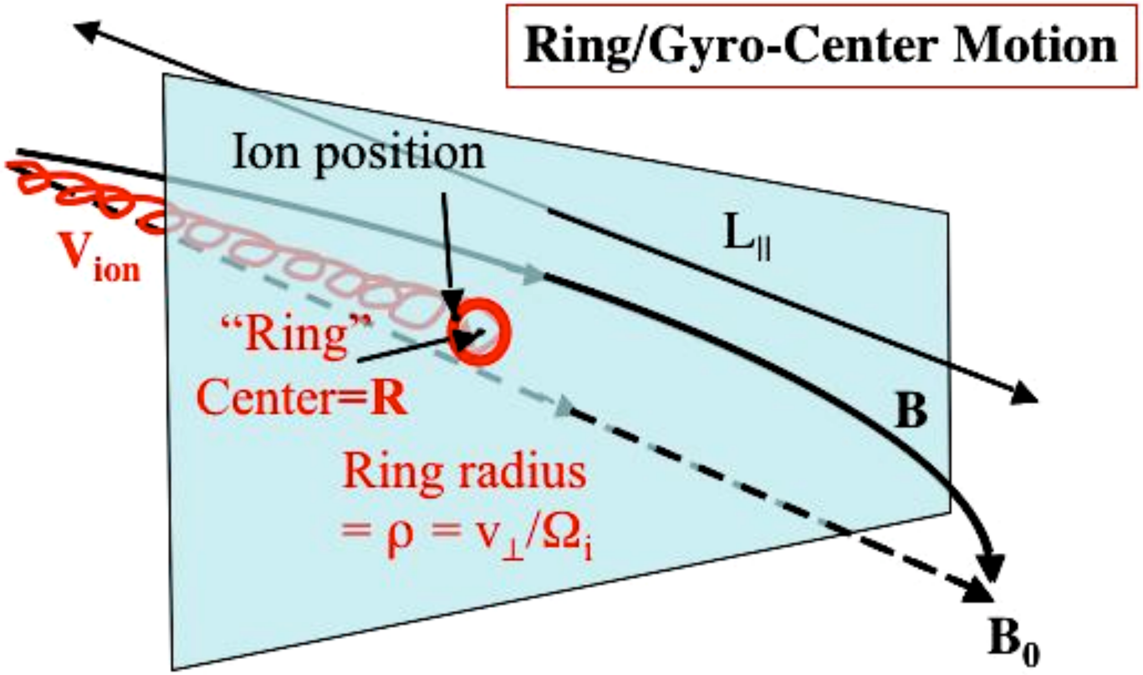
Turbulent Scales



Typical time for fields to change is $L_{||}/v_{ion}$ -- i.e. the time for an ion to go one parallel wavelength. Much longer than the gyration period. $\Rightarrow$ Ion senses the fields averaged over a **RING** of radius $\rho = v_\perp/\Omega_I$.

CCFE
CULHAM CENTRE FOR FUSION ENERGY

Guiding Centre Motion



$$< \frac{d\mathbf{R}}{dt} >= v_{\parallel}\mathbf{b}_0 \; - \; \frac{\partial < \chi >_{\mathbf{R}}}{\partial \mathbf{R}} \times (\frac{\mathbf{b}_0}{B_0}) + v_{\parallel}^2(\frac{\mathbf{b}_0}{\Omega_0}) \times \mathbf{b}_0 \cdot \nabla \mathbf{b}_0 + \frac{v_{\perp}^2}{2B_0}(\frac{\mathbf{b}_0}{\Omega_0}) \times \nabla B_0.$$

Parallel Motion along Equilibrium Field

Drifts in Perturbed Field

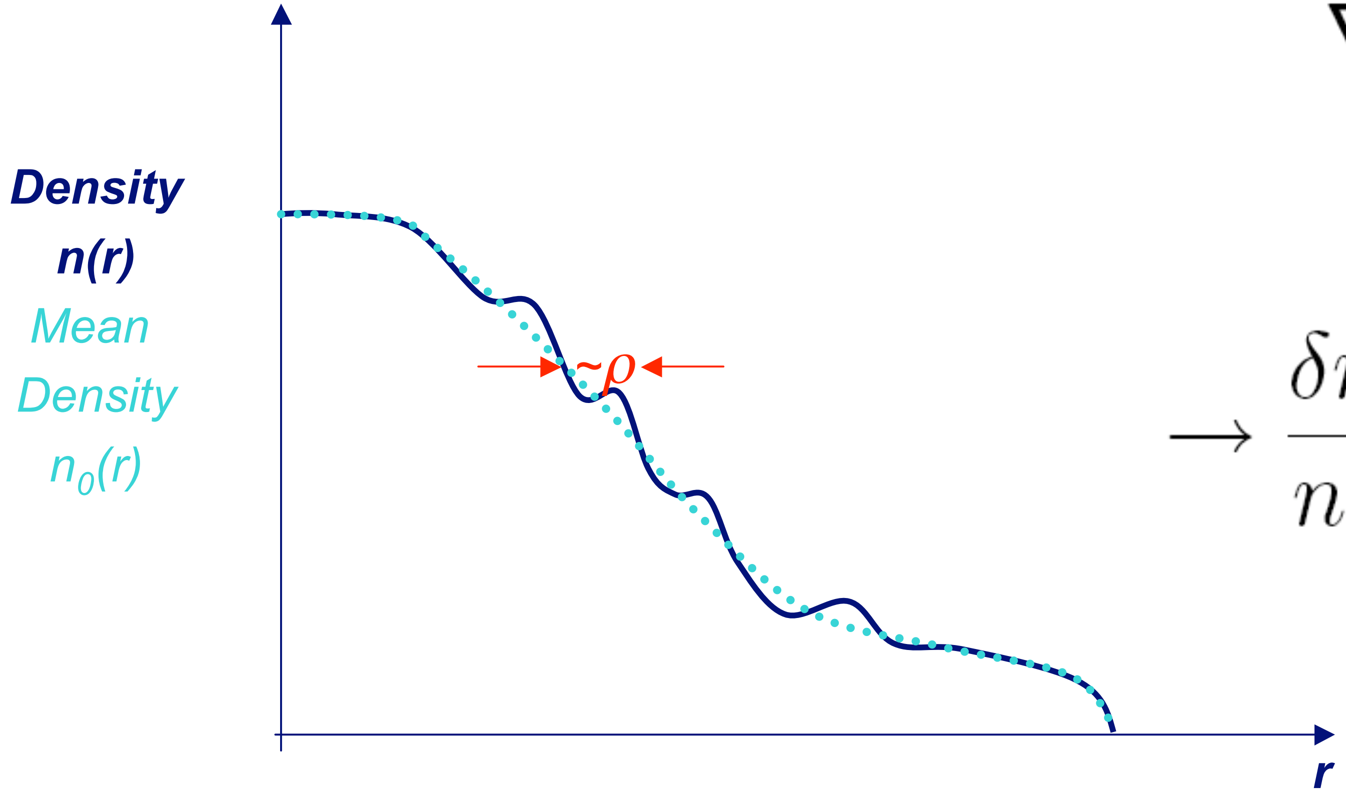
Curvature Drift in Equilibrium Field

∇B Drift in Equilibrium Field

$$\chi = \phi - \mathbf{v} \cdot \mathbf{A}$$

CCFE
CULHAM CENTRE FOR FUSION ENERGY

Ambient Gradient Argument



$$\nabla \delta n \sim \nabla n_0$$

$$\frac{\delta n}{\rho} \sim \frac{n_0}{L}$$

$$\rightarrow \frac{\delta n}{n_0} \sim \frac{\rho}{L} = \rho^*$$

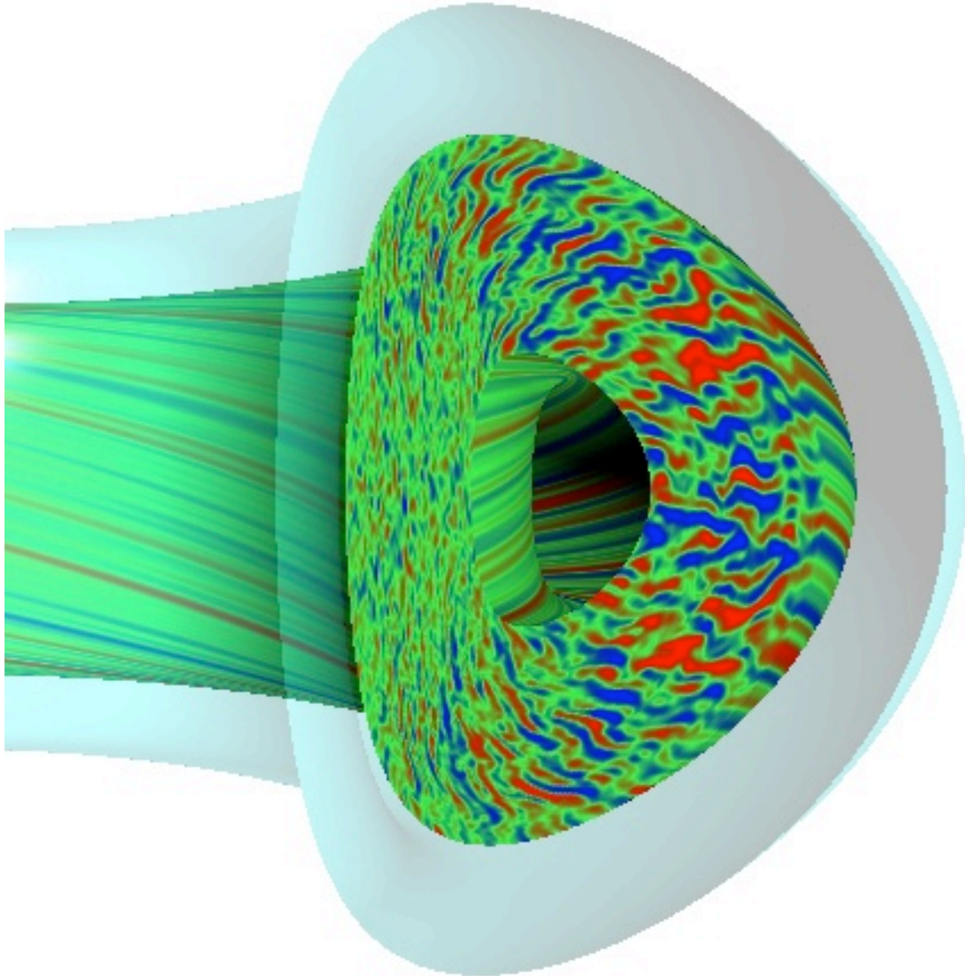
WARNING without collisions

$$\frac{df}{dt} = 0$$

Collisionless mixing produces Finite fluctuations -- collisions are essential.

CCFE
CULHAM CENTRE FOR FUSION ENERGY

Energy Confinement -- Random walk of heat/particles.



L = typical machine size

Δ= radial eddy size ∝ Ion larmor Radius ρ_i = random step.

N = number of steps to random walk out of plasma

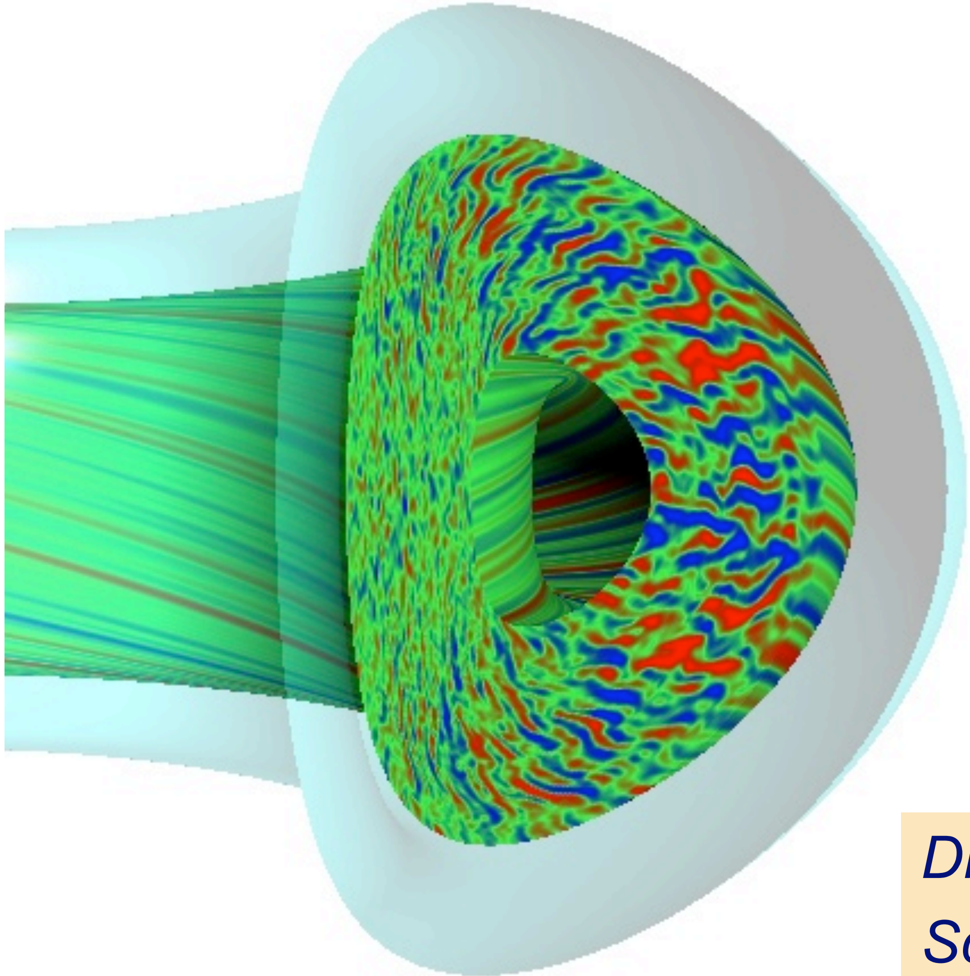
$$L \sim \sqrt{N}\rho_i$$

$$\rightarrow N = (\frac{L}{\rho_i}) = (\frac{1}{\rho^*})^2$$

For ITER N ~ 10^6.

CCFE CULHAM CENTRE FOR FUSION ENERGY

Energy Confinement -- Random walk of heat/particles.



Eddy turnover time =

$$\tau_{eddy} = \left(\frac{L}{v_{thi}}\right)$$

$$\tau_E \sim N\tau_{eddy} \sim \left(\frac{L^3}{\rho_i^2 v_{thi}}\right)$$

$$\propto L^3 B^2 T^{-1}$$

Dramatic scaling with size!
Scaling approximately agrees with data BUT geometry dependant.

CCFE
CULHAM CENTRE FOR FUSION ENERGY

Timescales -- ITER numbers -- ordering

Cyclotron -- $\sim 4 \times 10^{-9}$*s (ions) -- no activity* $\frac{1}{\Omega_i}$

Turbulence -- $\sim 10^{-5}$ *s -- fluctuating density, temperature, f,* ***E****.*

Collisions -- $\sim 10^{-2} - 10^{-3}$ -- *reestablish Maxwellian*

$$\left.\right\} (\frac{1}{\rho^*})\frac{1}{\Omega_i}$$

Transport -- ~ 4*s -- change mean density, temperatu* $(\frac{1}{\rho^*})^3\frac{1}{\Omega_i}$

Expand

$$f(\mathbf{r},\mathbf{v},t) = \underbrace{F_0(\mathbf{r},\mathbf{v},t)}_{1} + \underbrace{\frac{q\phi}{T}F_0 + h(\mathbf{R},\mathcal{E},\mu,\mathbf{t})}_{\rho^*} + \underbrace{\delta f_2(\mathbf{r},\mathbf{v},t)}_{(\rho^*)^2} + \quad ....$$

Varies on slow space and timescales

Varies on faster turbulence space and timescales

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0(\mathbf{r},t) + \delta\mathbf{B}(\mathbf{r},t), \qquad \mathbf{E}(\mathbf{r},t) = \delta\mathbf{E}(\mathbf{r},t)$$

At $\mathcal{O}(1)$ *collisions establish a local Maxwellian.*

$$F_0(\mathbf{r},\mathbf{v},t) = n(t,\psi)\left(\frac{m}{2\pi T(t,\psi)}\right)^{3/2}\exp\left[-\left(\frac{(1/2)mv^2}{T(t,\psi)}\right)\right]$$

$\boxed{\textit{Key issue is to find the slow evolution of } n(t,\psi) \textit{ and } T(t,\psi)}$

Solving for Fine Scale Turbulence

Evolution of turbulence comes from solving the gyro-kinetic equation:

e.g. the electrostatic version

$$\frac{\partial h}{\partial t}+v_{\parallel}\frac{\partial h}{\partial Z}+\mathbf{v_D}\cdot\frac{\partial h}{\partial \mathbf{R}}-\frac{\partial <\phi>_{\mathbf{R}}}{\partial \mathbf{R}}\times(\frac{\mathbf{b}_0}{B_0})\cdot\frac{\partial h}{\partial \mathbf{R}}-\left\langle \tilde{C}(h)\right\rangle_{\mathbf{R}}=q\frac{F_0}{T_0}\frac{\partial \left\langle \phi \right\rangle_{\mathbf{R}}}{\partial t}-\frac{\partial <\phi>_{\mathbf{R}}}{\partial \mathbf{R}}\times(\frac{\mathbf{b}_0}{B_0})\cdot\frac{\partial F_0}{\partial \mathbf{R}}$$

Comes from ring averaged kinetic equation $\mathcal{O}(\rho^*)$

Solved with quasi neutral approximation:

$$\cancel{\nabla^2\phi} = -\frac{1}{\epsilon_0}(qn_i - en_e)$$

In principle this equation should be solved everywhere

With F_0 held fixed in time letting h converge to statistical equilibrium.

th weak collisions h becomes very fine scale in velocity space due to mixir

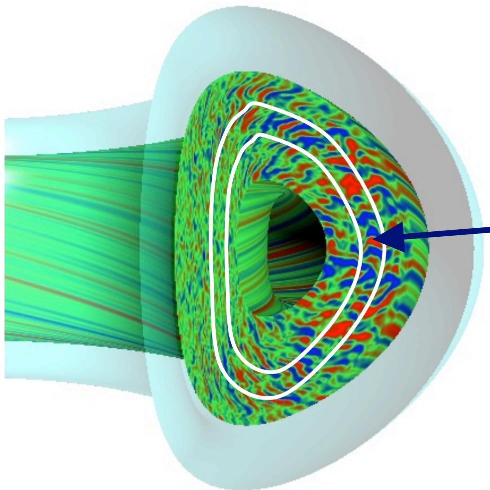
Solving for Slow Evolution

$$\mathcal{O}((\rho^*)^2) \qquad \frac{\partial F_0}{\partial t} + \frac{\partial \delta f_2}{\partial t} + \ldots\ldots$$

To solve for time evolution of F_0 (n & T) we have to annihilate the fast varying parts in space and time. It is easier to first take moments and use exact moment equations.

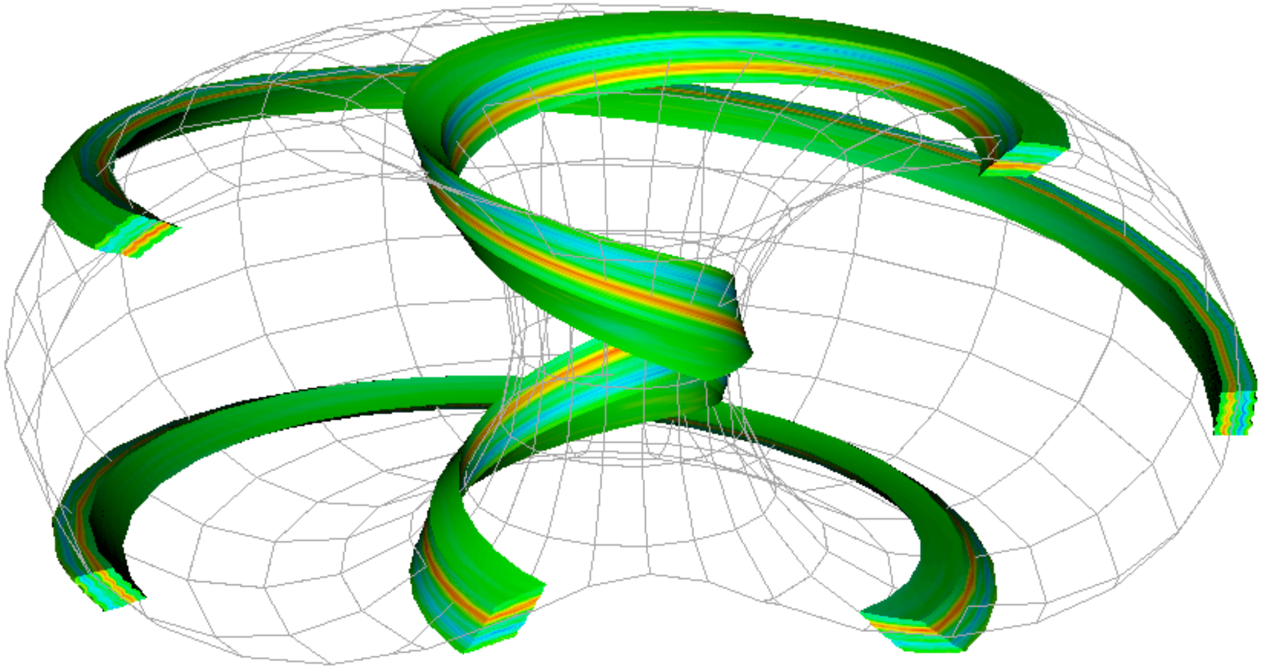
$$\frac{\partial n_i}{\partial t} + \nabla \cdot \mathbf{\Gamma_i} = 0. \qquad \mathbf{\Gamma_i} = \int \mathbf{v} f_i d^3\mathbf{v}.$$

Flux comes From turbulence.



Average flux over annulus (of thickness much greater than turbulence scale much smaller than radius) and time To get mean evolution of n and T.

Local simulation -- flux tubes



Beer 1993.

Is the turbulence in the flux tube determined by the conditions in the tube?

Does turbulence propagate -- correlated or not?

We should look at turbulent Greens function -- response to stirring.

What should we do?

- *Transport barriers -- can we use the "standard model".*

- *Detailed check of "standard model" needed.*

- *Full coupling -- (e.g. Trinity) needs finishing.*

- *The big prize is to find better configurations.*

CCFE
CULHAM CENTRE FOR FUSION ENERGY