SOME THINGS (WE THINK) WE HAVE LEARNED ABOUT PEDESTALS

Peter J. Catto, Plasma Science and Fusion Center, MIT
Matt Landreman (MIT PSFC), Grigory Kagan (LANL)
& Istvan Pusztai (Chalmers)

Cambridge, 27 July 2010
Motivation: pedestal = transport barrier

- Higher energy content
- Larger energy confinement time

Existence of the pedestal associated with decreased transport and turbulence

Density pedestal results in strong radial electric field and electrostatically confined banana regime ions

Pedestal width $w \sim \rho_{\text{pol}}$
Particle orbits in pedestal

Strong radial electric field: 
\[(v_\parallel \tilde{n} + \tilde{v}_E) \cdot \nabla \theta \approx (v_\parallel + c I \Phi' / B) \tilde{n} \cdot \nabla \theta\]

\[\text{ExB drift } \sim v_i \rho / w \sim v_i \rho / \rho_{pol} \ll v_\parallel \sim v_i, \text{ but geometry makes it comparable to poloidal projection of } v_\parallel\]
Overview of Topics

- Version of gyrokinetics useful in pedestal: $w \sim \rho_{\text{pol}}$
- Ion temperature profile: DIII-D
- Subsonic flow implications
- Ion and impurity flow with finite ExB
- C-Mod comparisons
- Bootstrap current enhanced since ion flow modified
- Neoclassical ion heat flux
- Intrinsically ambipolar but diffusivities depend on E
- Modification to the Rosenbluth-Hinton zonal flow
Gyrokinetic variables

\[ \tilde{B} = I(\psi) \nabla \zeta + \nabla \zeta \times \nabla \psi = B \vec{n} \]

Canonical angular momentum

\[ \psi_* = \psi - (M_c / Z_e) R^2 \nabla \zeta \cdot \vec{v} \]
\[ = \psi + \Omega^{-1} \vec{v} \times \vec{n} \cdot \nabla \psi - I \nu_n / \Omega \]

gyration drift

\[ (\rho / a) \psi \quad (\rho_{pol} / a) \psi \]

Toroidal angle \( \zeta_* \)

Poloidal angle \( \theta_* \)

Total energy \( E \)

Magnetic moment \( \mu \)

Gyrophase \( \varphi \)
Axisymmetric gyrokinetic equation

Axisymmetric ($\partial/\partial \zeta = 0$) gyrokinetic equation

$$\frac{\partial \langle f \rangle}{\partial t} + \left\langle \frac{d \theta_*}{dt} \right\rangle \frac{\partial \langle f \rangle}{\partial \theta_*} = \langle C \{ f \} \rangle - \frac{Ze}{M} \frac{\partial \langle \Phi \rangle}{\partial t} \frac{\partial \langle f \rangle}{\partial E}$$

Steady state ($\partial/\partial t = 0$) to leading order in $\rho_{pol}$:
- transit averaging in banana regime

$$\langle C \{ f_* \} \rangle = 0$$

where \( \bar{Q} = \int d\tau Q / \int d\tau \) with \( d\tau = d\theta_* / \langle \theta_* \rangle \)

Are there non-Maxwellian solutions in pedestal?

Entropy production analysis: no!

*Kagan & Catto 2008 PPCF 50 085010*
Pedestal ion temperature variation

In the banana regime $\partial f_*/\partial \theta_*=0$ so $f_*(\psi_*,E,\mu)$

The only Maxwellian possible is

$$f_* = \eta \left(\frac{M}{2\pi T}\right)^{3/2} \exp\left(-\frac{Ze\Phi}{T} + \frac{M\omega^2R^2}{2T} - \frac{Ze\omega\psi}{cT}\right) \exp\left[-\frac{M(\bar{v} - \omega R\bar{\xi})^2}{2T}\right]$$

where $\eta$, $\omega$, and $T$ are constants to lowest order, $n$ is Maxwell-Boltzmann, and

$$\omega = -c[\partial \Phi/\partial \psi + (1/Zen)\partial p/\partial \psi]$$

Non-isothermal modifications can only enter to next order in the $B_p/B$ expansion

$T$, $\eta$, $\omega$ must vary slowly compared to $\rho_{pol}$
Physical interpretation

Core gradients are weak so ion departures from a flux surface are unimportant - any flux surface is a closed system.

Pedestal gradients are as large as $1/\rho_{pol}$ so drift departures affect the equilibration of neighboring flux surfaces - the entire pedestal region is a closed system (rather than individual flux surfaces).
The $T_i$ gradient for the bulk ion in a DIII-D ECH H-mode pedestal is small relative to gradients in electron temperature and density.

DIII-D: Edge $T_i$ for bulk ion He$^{++}$

diii-d 120239.01800 ne(bk) 4ti(r) 4te(bl) vs R midplane

- The thermal ion full banana width is computed to be $2\rho_0 = 10$ mm for He$^{++}$ at the top of the density pedestal.

- The smooth spline fits to the data (solid lines) end at the LCFS as computed by EFIT. Note the clear break in slope for $T_i$ beyond the LCFS.

- In a nominally identical companion discharge we measured $T_i$ for the minor C$^6+$ impurity constituent. The $T_i$ profile for C$^6+$ has a very similar slope to that for He$^{++}$, but is $\sim 150$ eV greater in this region, probably because this discharge had an increase in $\beta_N$ of $\sim 10\%$ compared with the one shown here.

Pedestal pressure balance

Radial ion pressure balance using \( \vec{V}_i = \omega_i R \vec{\zeta} + u_i \vec{B} \) gives

\[ \omega_i \approx -c \left[ \frac{\partial \Phi}{\partial \psi} + \left( \frac{1}{en} \right) \frac{\partial p_i}{\partial \psi} \right] \]

subsonic pedestal (\( w \sim \rho_{pol} \))

\[ \frac{\omega_i}{\left[ (T_i/en) \frac{\partial n}{\partial \psi} \right]} \sim \frac{\omega_i R}{v_i} \ll 1 \]

\( \frac{\partial \Phi}{\partial \psi} \approx - \frac{1}{en_i} \frac{\partial p_i}{\partial \psi} > 0 \)

pedestal electric field inward for subsonic ion flow

Radial electron pressure balance: \( \vec{V}_e = \omega_e R \vec{\zeta} + u_e \vec{B} \)

\[ \omega_e = -c \left[ \frac{\partial \Phi}{\partial \psi} - \left( en_e \right)^{-1} \frac{\partial p_e}{\partial \psi} \right] \]

Additive, making \( \omega_e R \sim v_i \) so that \( J_{ped} \sim env_i \) & co-current

Thus, the electric field balancing the \( 1/\rho_{pol} \) density gradient requires a stationary ion Maxwellian & large electron flow
Pedestal orderings & ExB drift effects

Drift departure $\rho_{pol}$ is of order pedestal width $w$

Finite drift orbits effects enter in leading order

Estimating $Ze\nabla \Phi \sim T/\rho_{pol}$ gives

$\vec{v}_E \cdot \nabla \theta \sim v_{||} \vec{n} \cdot \nabla \theta$

where $\vec{v}_E$ is the ExB drift velocity

ExB $\sim$ poloidal streaming

Orbit localization from $\varepsilon = a/R \ll 1$

Decouple neoclassical & classical by assuming $\rho_{pol} \gg \rho$
Trapped particles: $\Phi(\psi) \neq \Phi(\psi_*)$

ExB drift:

i) Increases effective potential well depth: $\mu = 0$ trapped by $\Phi$

ii) Shifts the axis of symmetry of the trapped particle region - fewer trapped!

Trapped fraction decays exponentially if $u = c\Phi'/B > v_i$

Neoclassical and polarization phenomena strongly modified

Recall $u \approx (\rho_{pol}/\rho)v_E >> v_E$ so particle dynamics qualitatively changed by a finite subsonic ExB drift
Neoclassical ion heat flux & parallel flow

\[ \nabla T \text{ drive only} \]

\[ C_1\{g + f_M \frac{Iv_\parallel M v^2}{2\Omega T^2} \frac{\partial T}{\partial \psi}\} = 0 \]

Need a model for the collision operator - must keep energy scatter as well as pitch angle scatter

\[ \text{Solve for } g \]

\[ \text{Calculate flow & transport (take appropriate moments of the distribution function)} \]
Ion motion for $\varepsilon = a/R \ll 1$

Assume a quadratic potential well and expand about $\psi_*$

$$\Phi = \Phi_* + \frac{Iv_{||}}{\Omega} \Phi'_* + \frac{I^2 v_{||}^2}{2\Omega^2} \Phi''_*$$

$$\psi_* \approx \psi - Iv_{||}/\Omega$$

Using $E - Ze\Phi_*/M$, $\mu$ and $\psi_*$ invariance:

$$\frac{(v_{||} + u)^2}{2S} + \mu B + \frac{u_*^2}{2S} = E - \frac{Ze\Phi_*}{M}$$

$$v_{||} + u = Sv_{||} + u_*$$

Squeezing orbit

Magnetic dipole energy

ExB energy

$S > 0$ ($S < 0$) trapped particles outboard (inboard)

For $\varepsilon \ll 1$ can find the useful form

$$\frac{1}{2}(v_{||} + u)^2 = W(1 - \lambda/h)$$

$$h = 1 - 2\varepsilon \sin^2(\vartheta/2) = B_0/B$$

with $\lambda = 1/(1+2\varepsilon)$ at trapped-passing boundary
Collisions in the pedestal

Pitch-angle scattering is not sufficient to retain transitions across the trapped-passing boundary!

Kagan & Catto 2010 PPCF 52 055004 and 079801

Convenient variables are $\lambda$ and $W$ ($S = 1$):

$$\lambda = \frac{\mu B_0 + u^2/h^2}{W}$$

$$W(1 - \lambda/h) = \frac{1}{2} (v_{\parallel} + u)^2$$
Neoclassical parallel ion flow

Localized portion of $g$ higher order in $\varepsilon$

\[ V_{\parallel i} = -\frac{cI}{B} \left( \frac{\partial \Phi}{\partial \psi} + \frac{1}{\text{Zen}} \frac{\partial p_i}{\partial \psi} \right) - \frac{7cI}{6ZeB_0} \frac{\partial T_i}{\partial \psi} J(u/v_i) \]

(Kagan & Catto PPCF 2010 + errata)

No orbit squeezing effect on ion flow

$J$ changes to Pfirsch-Schluter sign at $u/v_i \sim 1.2$

Seems to explain C-Mod flow measurements in pedestal

More pedestal bootstrap current

$J(0) = 1$
Pedestal impurity flow

Change in poloidal ion flow alters impurity flow

For Pfirsch-Schluter impurities & banana ions:

\[ V_{Z}^{\text{pol}} = V_{i}^{\text{pol}} - \frac{cIB_{\text{pol}}}{eB^{2}} \left( \frac{1}{n_{i}} \frac{\partial p_{i}}{\partial \psi} - \frac{1}{Zn_{Z}} \frac{\partial p_{Z}}{\partial \psi} \right) \]

&

\[ V_{i}^{\text{pol}} \approx \frac{7cIB_{\text{pol}}}{6eB_{0}^{2}} \frac{\partial T_{i}}{\partial \psi} J \left( \frac{u}{v_{i}} \right) \]

C-Mod pedestal flow (Marr et al 2010):

Pfirsch-Schluter: \( \sim \) agree

Banana: problem - need \( E_{r} \)
Poloidal ion flow: C-Mod vs theory

**Pfirsch-Schluter plateau**

*Marr et al*

**banana**

*(shaded)*
Pedestal bootstrap

$J(u)$ alters electron friction with ions thereby modifying the bootstrap current ($p = \text{total pressure}$)

\[ Z \gg 1: \quad J_{\|}^{bs} = -1.46\sqrt{\varepsilon} \frac{cIB}{\langle B^2 \rangle} \left[ \frac{dp}{d\psi} - 1.17J\left(\frac{u}{v_i}\right)\frac{n_e}{Z} \frac{\partial T_i}{\partial \psi} \right] \]

Arbitrary $Z$:

\[ J_{\|}^{bs} = -1.46\sqrt{\varepsilon} \frac{cIB}{\langle B^2 \rangle} \left[ \frac{Z^2 + 2.21Z + 0.75}{Z(Z + 1.414)} \right] \]

\[ \left[ \frac{dp}{d\psi} - \frac{(2.07Z + 0.88)n_e}{(Z^2 + 2.21Z + 0.75)} \frac{\partial T_e}{\partial \psi} - 1.17J\left(\frac{u}{v_i}\right)\frac{n_e}{Z} \frac{\partial T_i}{\partial \psi} \right] \]

$J(u/v_i)$ changes sign at $u/v_i \sim 1.2$ to enhance bootstrap current in pedestal \cite{(Kagan & Catto PRL 2010)}
Pedestal ion heat flux

Modified ion heat flow:

\[
\langle \vec{q} \cdot \nabla \psi \rangle = -\frac{McIT}{Ze} \left( \int d^3v \left( \frac{Mv^2}{2T} - \frac{5}{2} \right) \frac{v_\parallel}{B} C_1 \{g_{local} \} \right)
\]

Evaluating:

\[
\langle \vec{q} \cdot \nabla \psi \rangle = -1.35 \sqrt{\varepsilon n v_i} \frac{I^2 T_i}{\Omega^2} \frac{\partial T_i}{\partial \psi} \frac{G(u)}{\sqrt{S}}
\]

Radial ion heat flux & trapped population exponentially small for \( u/v_i > 1 \)

Ion heat flux more sensitive to \( \Phi' \) than \( \Phi'' \)

conventional result

\[ S = 1 + c l^2 \Phi''/B\Omega \]
Neoclassical polarization in the pedestal

Ignore collisions, but retain strong radial electric field:

\[ \frac{\partial g}{\partial t} = f_m \frac{Ze}{T} \frac{\partial \Phi^*}{\partial t} J_0 \left( \frac{k \cdot v}{\Omega} \right) e^{iQ} \]

Solving keeping distinction between \( \psi \) and \( \psi^* \) gives

\[ \frac{\varepsilon_{nc}^{pol}}{\varepsilon_{nc}^{RH}} = \left( \frac{T(U)}{\sqrt{S}} + i \frac{\Xi(U,S)}{k \rho_{pol}} \right) \]

Exponential reduction due to decrease in trapped making

\[ \frac{\Phi_1(t \to \infty)}{\Phi_1(t = 0)} \to 1 \]
Summary

• Pedestal ions nearly isothermal \( (\rho_{pol} \nabla T_i \ll 1) \): subsonic ions electrostatically confined + magnetically confined electrons

• Banana regime ion heat flux reduced & poloidal ion flow can change sign in the pedestal due to \( \Phi' \) as in C-Mod

• Pedestal bootstrap current enhanced!

• Pedestal zonal flow turbulence regulation stronger due to \( \Phi' \) (see Kagan PoP and Landreman PPCF)

• Plateau regime ion heat flux increases before decreasing, no sign change for ion flow (remains PS sign) or bootstrap current, no orbit squeezing effects (Pusztai & Catto)

• QSS almost the same as a tokamak! (Landreman & Catto)