Gyrokinetic simulations of a strongly rotating tokamak plasma

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Inclusion of the centrifugal force

- When rotation velocities are of order of the Mach number, the **centrifugal** force is kept in the **rotating frame of reference**.
  - Mach numbers up to 0.9 are observed in spherical tokamaks
  - Mach numbers up to 0.6 are observed in conventional tokamaks
  - Heavy impurities have large Mach number even at low bulk rotation
- The centrifugal force has been implemented in the **local** flux tube code GKW [1] using the formulation of Brizard [2]:

Guiding centre velocity

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = v_\parallel \mathbf{b} + \frac{\mathbf{b}}{eB^*_\parallel} \times (e\nabla\phi + \mu\nabla B + m\mathbf{u}^*_0 \cdot \nabla \mathbf{u}^*_0)$$

Inertial term

Parallel velocity

$$\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = -\frac{\mathbf{B}^*}{mB^*_\parallel} \cdot (e\nabla\phi + \mu\nabla B + m\mathbf{u}^*_0 \cdot \nabla \mathbf{u}^*_0).$$

Inertial term

$$\mathbf{u}^*_0 = \mathbf{u}_0 + v_\parallel \mathbf{b} \qquad \mathbf{u}_0 = \mathbf{\Omega} \times \mathbf{X} = R^2 \Omega \nabla\varphi$$

Toroidal background rotation — Angular frequency

- A rigid body rotation is assumed. The rotation of the frame is chosen to be the rotation of the plasma on the **local** flux surface.
  - In a local model the co-moving system yields compact equation similar in form to the non rotating system.
  - The large ExB velocity of strong toroidal rotation is transformed away
  - Not suited for a global description since a gradient in the rotation would lead to a time dependent metric
- The inertial terms have three effects [3]:
  - Coriolis drift (gives a momentum pinch) [4]
  - Centrifugal drift
  - Enhanced trapped (from the parallel component of the centrifugal force)
- The enhanced trapping is kept in the equilibrium:

Normal trapping. Any distribution (like the Maxwellian) which is isotropic is an equilibrium distribution

$$v_\parallel \mathbf{b}\cdot\nabla F - \frac{1}{m}\mathbf{b}\cdot[Ze\nabla\langle\phi_0\rangle + \mu\nabla B - m\Omega^2 R\nabla R]\frac{\partial F}{\partial v_\parallel} = 0$$

Mass dependent centrifugal force different for electrons and ions

Must retain a background electro-static potential which is a function of the poloidal angle in order to satisfy quasi-neutrality

$$n(\theta) = n_{R_0}\exp(-\mathcal{E}(\theta)/T)$$

$$\left.\frac{R}{L_n}\right|_{\mathrm{eff}} = -\frac{R}{n}\frac{\partial n}{\partial r} = \frac{1}{T}\frac{\partial \mathcal{E}}{\partial r} + \frac{\mathcal{E}}{T}\frac{R}{L_T} + \left.\frac{R}{L_n}\right|_{R_0}$$

Density and rotation are not independent parameters

$$\mathcal{E}_\Omega = Ze\Phi - \frac{1}{2}m\Omega^2(R^2 - R_0^2)$$

Integration constant - A choice for density definition location

$$e\Phi = \frac{1}{4}m_i\Omega^2(R^2 - R_0^2).$$

"Centrifugal potential" found by solving for quasineutrality – here for a 2 species plasma

New centrifugal terms kept in equations solved

$$\frac{\partial f}{\partial t} + (v_\parallel\mathbf{b} + \mathbf{v}_D + \mathbf{v}_E)\cdot\nabla f - \frac{\mathbf{b}}{m}\cdot(\mu\nabla B + \nabla\mathcal{E}_\Omega)\frac{\partial f}{\partial v_\parallel} = S.$$

Enhanced trapping

$$\sum_s Z_s n_{R_0,s}\left[2\pi B\int \mathrm{d}v_\parallel \mathrm{d}\mu J_0(k_\perp\rho_s)\hat{f}_s + \frac{Z_s}{T_s}[\Gamma(b_s) - 1]\exp(-\mathcal{E}_s/T_s)\hat{\phi}\right] = 0$$

Modified polarization

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = v_\parallel\mathbf{b} + \frac{1}{Ze}\left[\frac{mv_\parallel^2}{B} + \mu\right]\frac{\mathbf{B}\times\nabla B}{B^2} + \frac{\mathbf{b}\times\nabla\langle\phi + \Phi\rangle}{B} + \frac{2mv_\parallel}{ZeB}\mathbf{\Omega}_\perp - \frac{m\Omega^2 R}{ZeB}\mathbf{b}\times\nabla R$$

New drifts

$$F_M = \frac{n_{R_0}}{\pi^{3/2}v_{\mathrm{th}}^3}\exp\left[-\frac{(v_\parallel - (RB_t/B)\omega_\phi)^2 + 2\mu B/m}{v_{\mathrm{th}}^2} - \mathcal{E}_\Omega/T\right]$$

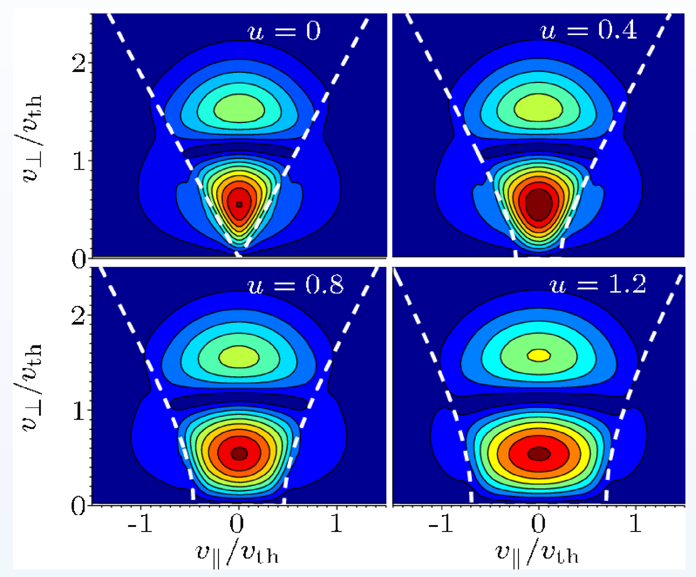
Modified equilibrium

$$S = -(\mathbf{v}_E + \mathbf{v}_D)\cdot\left[\frac{\nabla n_{R_0}}{n_{R_0}} + m\Omega^2 R_0\frac{\partial R_0}{\partial\psi}\nabla\psi + \left(\frac{v_\parallel^2}{v_{\mathrm{th}}^2} + \frac{(\mu B + \mathcal{E}_\Omega)}{T} - \frac{3}{2}\right)\frac{\nabla T}{T} + \frac{mv_\parallel}{T}\frac{RB_t}{B}\nabla\omega_\phi\right]F_M - \frac{Ze}{T}[v_\parallel\mathbf{b} + \mathbf{v}_D]\cdot\nabla\langle\phi\rangle F_M.$$

Modified Source

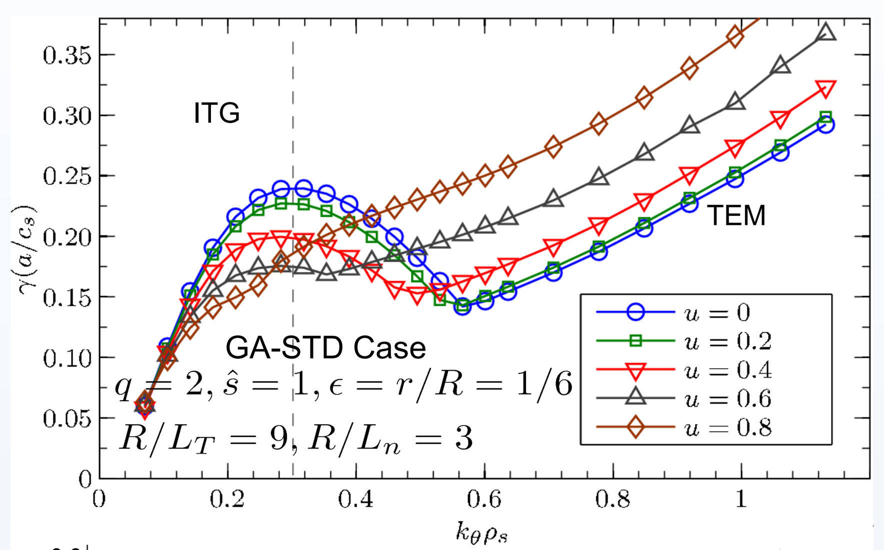
- In order that the density gradient has the intuitive meaning of being in the radial direction, the radial derivative of R_0 (at constant θ) must be kept.
- The other parts of the derivative of the density exponential cancel with the $(dv_\parallel/dt)\partial F_M/\partial v_\parallel$ term of the Maxwellian
- The "centrifugal potential" traps electrons, but detraps ions, resulting in the same trapping condition for both species.

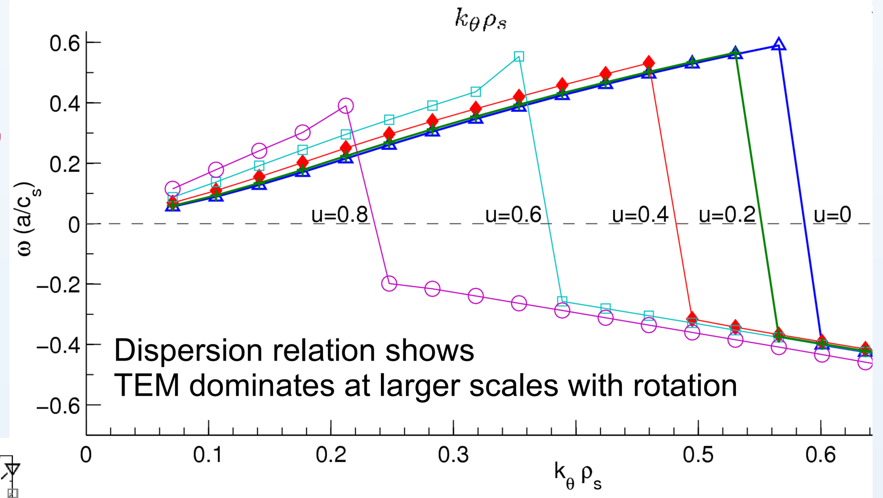
Linear results

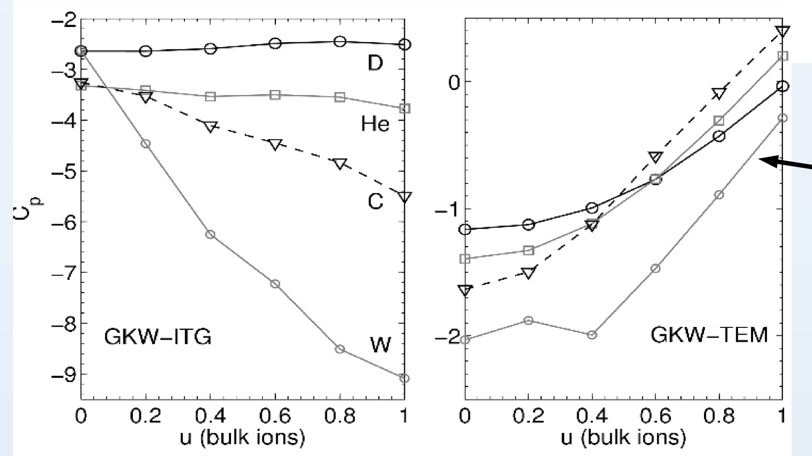


Trapped region enlarges with rotation, so TEM is enhanced

$$v_\perp^2 > \left[\frac{v_\parallel^2}{2\epsilon} + \Omega^2 R^2\right](1+\epsilon)$$





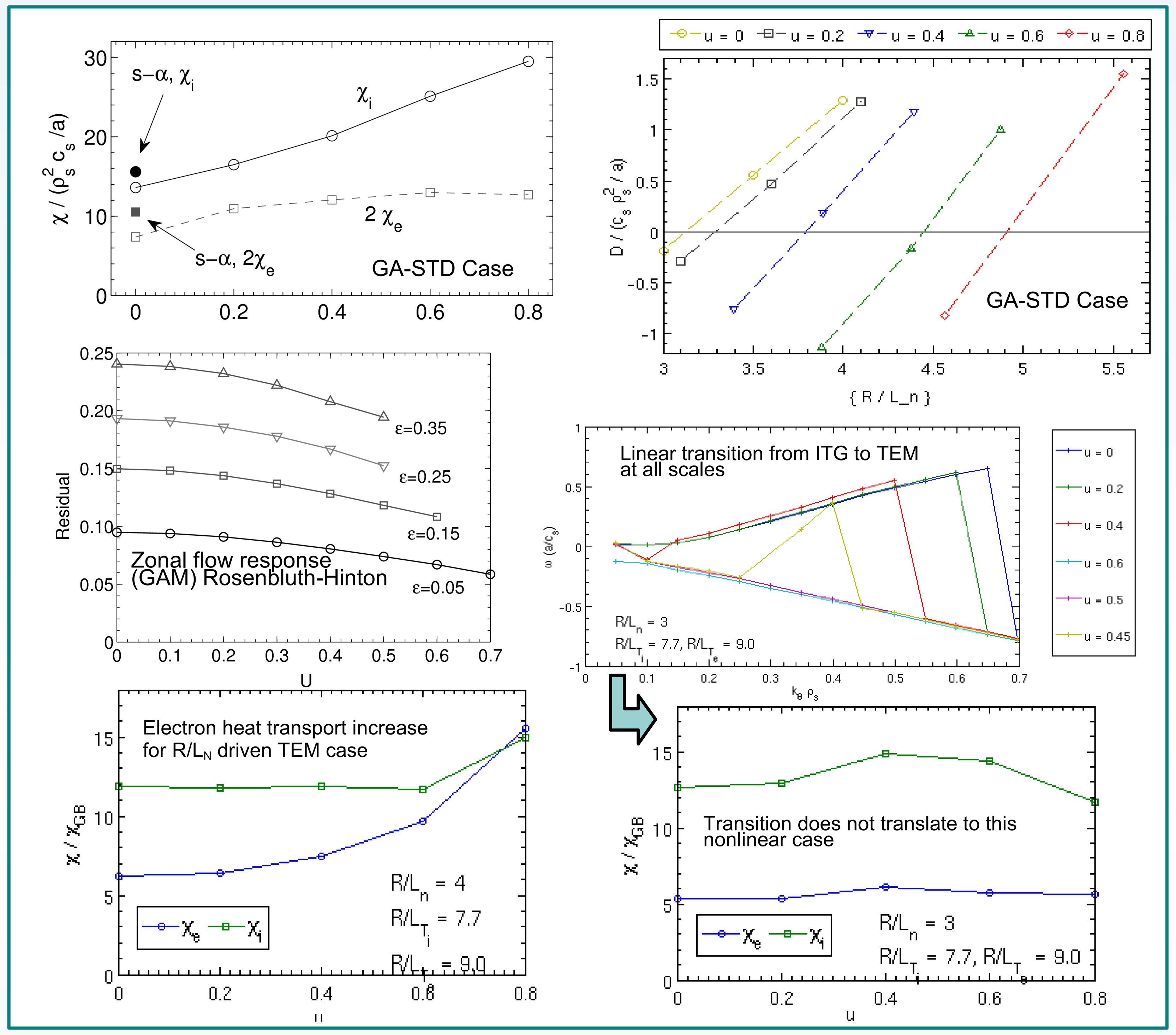


Trace impurities show stronger convective particle pinch for ITG:

$$\Gamma_s = n_s D_s\left(\frac{R}{L_n^{R0}} + C_T\frac{R}{L_T} + C_p\right) \qquad \text{Ref [5]}$$

$$u = v_{\mathrm{toroidal}}/v_{\mathrm{th}} \qquad v_{\mathrm{th}} = \sqrt{2T/m}$$

Nonlinear results



- ITG dominated GA-STD case shows increased ion heat flux due to increased trapped electron drive, and reduced zonal flow (but note that no shearing in the rotation is included for these results)
- Cases with linear transition to TEM at all scales behave differently
- Increase in electron heat transport for TEM case with stronger R/L_N
- Linear threshold for TEM dominance does not translate to nonlinear
- Null particle flux state is independent of choice for fixed density point
- Particle pinch with increasing rotation due to increased inward contribution from slower trapped electrons [6]

This work has been submitted to Physics of Plasmas (2010), preprint may be given on request.
Simulations performed using resources of HPC-FF (FSCFIM / FSCENU) and HECTOR (EP/H002081/1)

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