The dissipation of turbulent fluctuations at electron scales: PIC simulations

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Motivations

- The mechanisms that govern the dissipation of turbulent fluctuation in the Solar Wind are still unknown.
- It is generally acknowledged that kinetic effects play a role at scales smaller than ion gyroradius, and will certainly affect the nonlinear turbulent cascade.
- Observations have suggested that a nonlinear cascade might proceed further to electron scales (Sahraoui et al. PRL 2009).
- It is not yet clear whether any linear theory argument is applicable in the overall turbulent scenario of the solar wind, even at small scales.
Open questions

• Would a nonlinear cascade be able to reach electron scales, or the turbulent fluctuations are always rapidly dissipated at such small scales?

• Is the use of Vlasov linear theory justified to predict the dispersion/dissipation properties of small amplitudes fluctuations?

• Which linear mode (if any) is predominant and responsible of the dissipation of turbulent fluctuations at small scales?
Methodology

- We have performed fully-kinetic Particle-in-Cell simulations, with the code PARSEK2D.
- The code uses an implicit moment method. At the present time only an implicit algorithm allows fully-kinetic simulations that do not compromise on the value of electron-to-ion mass ratio, and on plasma-to-gyro frequencies.
- The code is 2D in physical space, 3D in velocity space.
- Mass ratio $m_i/m_e = 1836$; $\omega_{pi}/\Omega_{ci} = 1650$;
- The box includes wavevectors in the range from $k\rho_e = 0.1$ to $k\rho_e = 10$ or, equivalently, from $k\rho_i = 4.28$ to $k\rho_i = 428$ ($\rho_{e,(i)}$ is the electron (ion) gyroradius)
Methodology (2)

- We initially perturb the magnetic field in the first few modes with longest wavelength. The phase relation between modes is random.
- The plasma is initially loaded with an isotropic Maxwellian distribution.
- The initial perturbation does not correspond to any particular “normal mode”. In this way we expect to excite many of the possible modes that the plasma can support, and we do not choose *a priori* the dominant mode.
- We run the simulations until a (quasi) steady-state in the turbulent cascade is reached.
- Notice that the initialization is self-consistent with Vlasov-Maxwell equations. The plasma responds to the applied magnetic perturbation by generating currents and by cascading the excess of magnetic energy at smaller scales.
### Comparison with previous work

<table>
<thead>
<tr>
<th>Paper</th>
<th>kp&lt;sub&gt;i&lt;/sub&gt;</th>
<th>ω&lt;sub&gt;i&lt;/sub&gt;/Ω&lt;sub&gt;i&lt;/sub&gt;</th>
<th>m&lt;sub&gt;i&lt;/sub&gt;/m&lt;sub&gt;e&lt;/sub&gt;</th>
<th># particles per cell</th>
<th>Type</th>
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<tbody>
<tr>
<td>This work</td>
<td>4.28-428.48</td>
<td>1650</td>
<td>1836</td>
<td>6400</td>
<td>2D-3V PIC</td>
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<tr>
<td>Saito et al. POP (2008)</td>
<td>0.83 - 425</td>
<td>96</td>
<td>1836</td>
<td>64</td>
<td>2D-3V PIC</td>
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<td>Howes et al. PRL (2008)</td>
<td>0.4 - 8.4</td>
<td>N/A</td>
<td>1836</td>
<td>N/A</td>
<td>3D-2V Gyrokinetic (pseudospectral)</td>
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<tr>
<td>Markovskii et al. ApJ (2010)</td>
<td>0.0095 - 1.21</td>
<td>192.3</td>
<td>N/A</td>
<td>1000</td>
<td>2D Hybrid</td>
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<tr>
<td>Svidzinski et al. POP (2009)</td>
<td>0.03 - 66.7</td>
<td>15</td>
<td>100</td>
<td>&gt;100</td>
<td>2D-3V PIC</td>
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<tr>
<td>Valentini et al. PRL (2010)</td>
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<td>100</td>
<td>N/A</td>
<td>2D-3V Hybrid Vlasov</td>
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<td>Parashar et al. POP (2009)</td>
<td>0.139 - 35.7</td>
<td>N/A</td>
<td>25</td>
<td>100</td>
<td>2D Hybrid</td>
</tr>
</tbody>
</table>

ω<sub>i</sub>/Ω<sub>i</sub> is the ratio of ion plasma to cyclotron frequency (a typical value at 1AU is around 4000)  
N/A is either Not Applicable or Not Available;
Results (benchmark for cascade)

When only a single mode is excited with a large enough amplitude, a 3-waves interaction \((k=k_1+k_2)\) is able to transfer energy to higher harmonics. The only requirement for cascading to small scales is to have injection of enough energy at the largest scale.

The figure shows the amplitude of \(\delta B/B_0\) (log scale) in Fourier space, at late times (run for about 100 electron gyroperiods)
In a second run we have excited counterpropagating modes (the modes initially excited are in circles). The same kind of wave-wave interactions happen. Clearly higher harmonics are now spread at different directions.

Although this might seem a trivial result, and just a confirmation of basic plasma phenomena captured by the code, it raises an important question:

How can we incorporate in a turbulent cascade model a description consistent with linear theory for modes with small wavelength (where certainly $\delta B/B_0 \ll 1$), if nonlinear effects at larger scales persistently inject energy at such small scales?
Turbulent Cascade: perpendicular direction

In this simulation we have excited a spectrum of modes that covers the range \([0,3] \times [0,3]\) in \((k_\parallel, k_\perp)\) space.

The figure shows the spectrum of \(\delta B/B_0\) at the end of the run, and the power-law fit. A break in the spectrum appears at \(k_\rho_e \perp \approx 1\).

The slope above \(k_\rho_e \perp = 1\) is consistent with observations.

We have verified that this result is weakly dependant on the initial slope of the injected spectrum of fluctuations.
Turbulent Cascade: parallel direction

Same as above, for parallel direction.

Notice that the noise level does not affect the result until $k\rho_e > 4$.

This noise is due to particle discreteness and grid size, and it is intrinsic in any PIC code. In order to reduce it to the level shown we have used 6400 particle per cells. This is a very large number not usually needed for standard PIC simulations.

The high computational cost paid to lower the noise raises questions on the effectiveness and the robustness of results obtained with full-f PIC codes, for these studies.
In our simulations we have verified the existence of the well-known anisotropy, which naturally develops in plasma turbulence.

Most of the fluctuations energy resides in quasi-perpendicular modes.
The current debate on the linear wave 'responsible' of the damping of fluctuations is focused on Kinetic Alfven Waves and Whistler modes.

The figure shows the dispersion relation for different angles of these two modes.

Notice however, that at this wavelength, the identification of a single mode is quite hard, and the linear Vlasov equation yields many more linear modes, not identifiable with MHD-like modes.
We have used the electron compressibility $|\delta n_e / \delta B|$ as diagnostic to identify any normal mode present when the simulation reaches a quasi-steady evolution.

The results from simulations (for three different angles of propagation) are shown with symbols, and the prediction from linear theory are in solid lines, with corresponding colours.

Neither the whistler or the KAW predictions are in good agreement with the simulation result. This might be due to a mixed contribution of both modes, or even of other modes. One can probably say that the fluctuations are predominantly 'alfvenic', but they certainly do not correspond to pure KAW modes.
Conclusions

- We have performed PIC simulations of damping fluctuations at electron scales;
- The use of an implicit scheme (Courant condition $c\Delta t/\Delta x \sim 9$) allows to use physical parameters;
- We have found that a nonlinear cascade is able to proceed at electron scales, well below $k\rho_e=1$.
- The power spectra of magnetic fluctuations shows a break at about the electron gyroradius.
- The fluctuations at small scales do not have the electron compressibility predicted from the Vlasov linear theory neither for Whistler or KAW modes (interestingly, the Langmuir mode gives the best fit);
- Therefore the results question the possibility of using linear theory, or alternatively suggest the co-presence of many modes.