# Rotational shear in tokamak plasmas

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# Fusion: how to spin it

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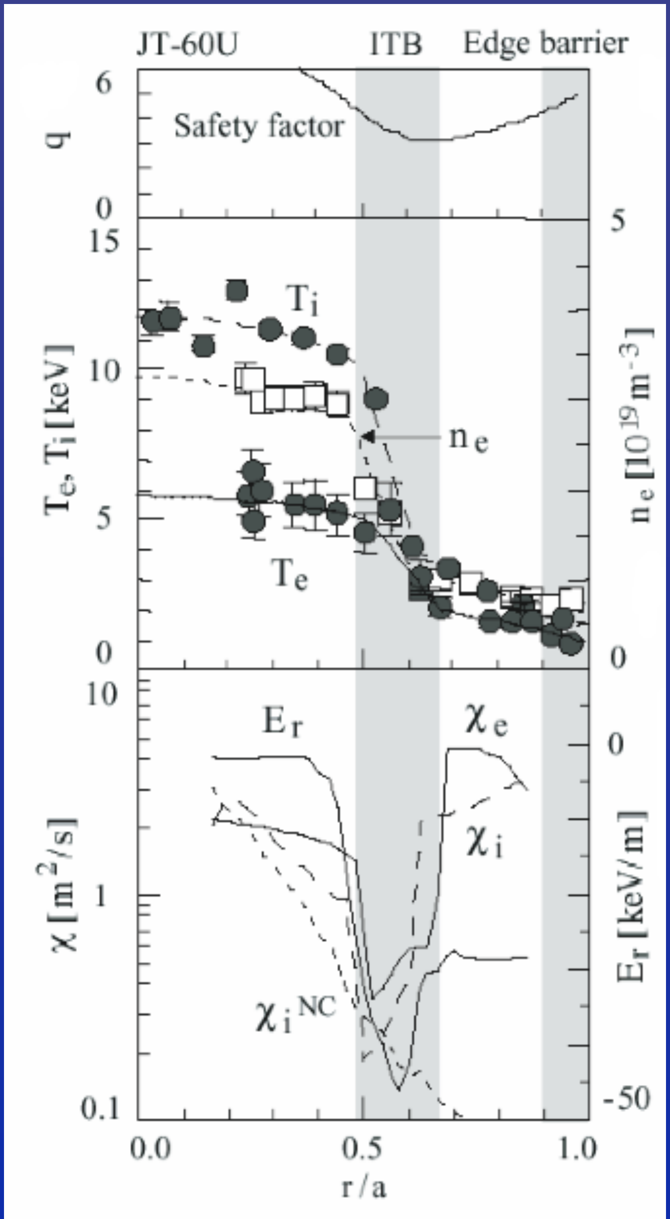
# Overview

- Motivation
- Effect of rotational shear on turbulent transport
- Implications for local gradients (0D)
- Extension to radial profiles (1D)

# Objective




- Identify mechanism(s) for achieving enhanced confinement
- Internal transport barriers observed with temperature gradients well above threshold
- Often accompanied by large E x B shear and low or negative magnetic shear
- Experimentally observed power threshold for formation

Connor et al. (2004)

# Overview

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# Model

GK equation with mean flow satisfying $\frac{\rho}{L} \ll M \ll 1$

but : $\nabla u \sim v_{th}/L$

$$\frac{dh}{dt} + \left(\mathbf{v}_{\parallel} + \mathbf{v}_D + \langle \mathbf{v}_E \rangle\right) \cdot \nabla h - \langle C[h] \rangle$$

$$= \frac{eF_0}{T}\frac{d\langle\varphi\rangle}{dt} - \langle \mathbf{v}_E \rangle \cdot \nabla\psi \left(\frac{dF_0}{d\psi} + \frac{mv_{\parallel}}{T}\frac{RB_\phi}{B}\frac{d\omega}{d\psi}F_0\right)$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + R\omega(\psi)\hat{\mathbf{e}}_\phi \cdot \nabla$$

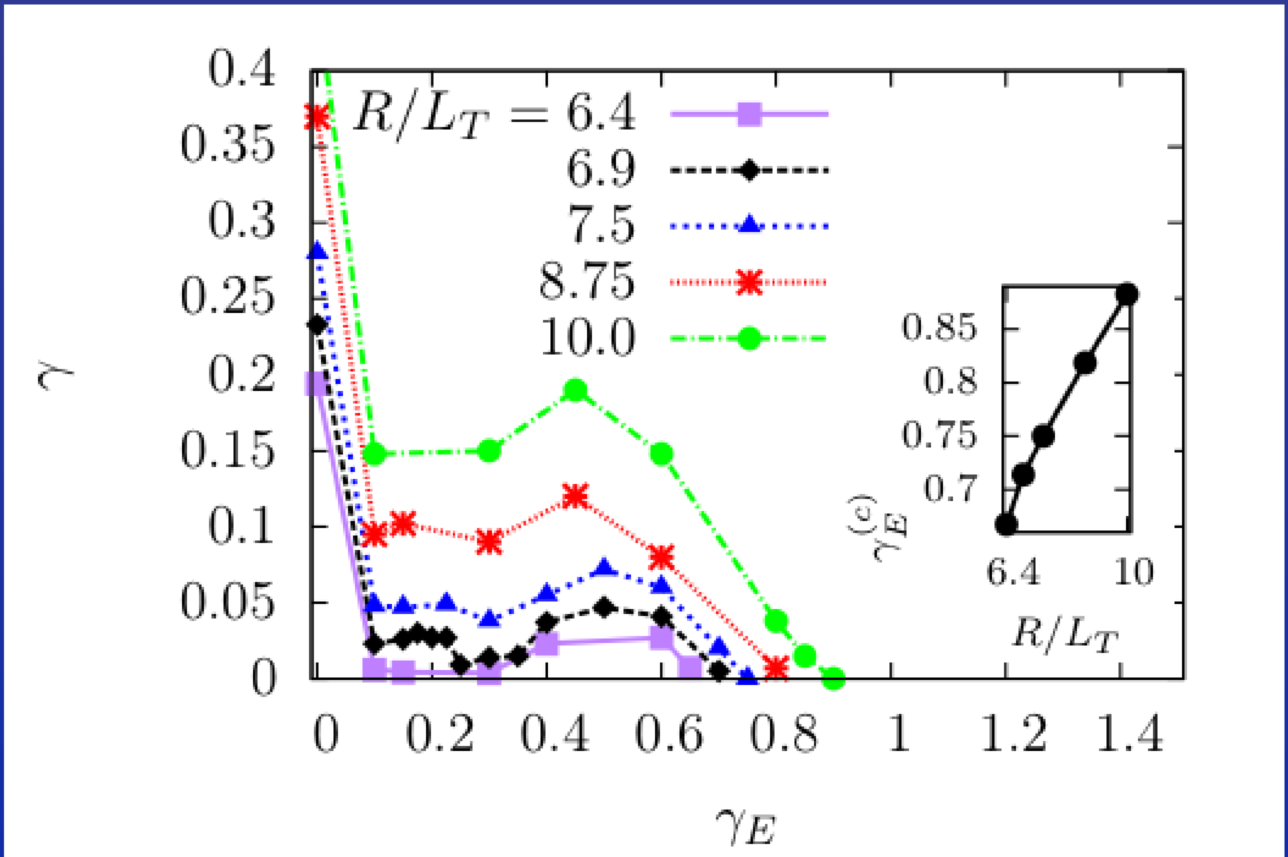
Local approximation:

$$\omega(\psi) \approx \omega(\psi_0) + (\psi - \psi_0)\left.\frac{d\omega}{d\psi}\right|_{\psi_0}$$

$$\gamma_E \equiv \frac{\psi}{q}\frac{d\omega}{d\psi}\frac{R_0}{v_{th}}$$

# Linear stability

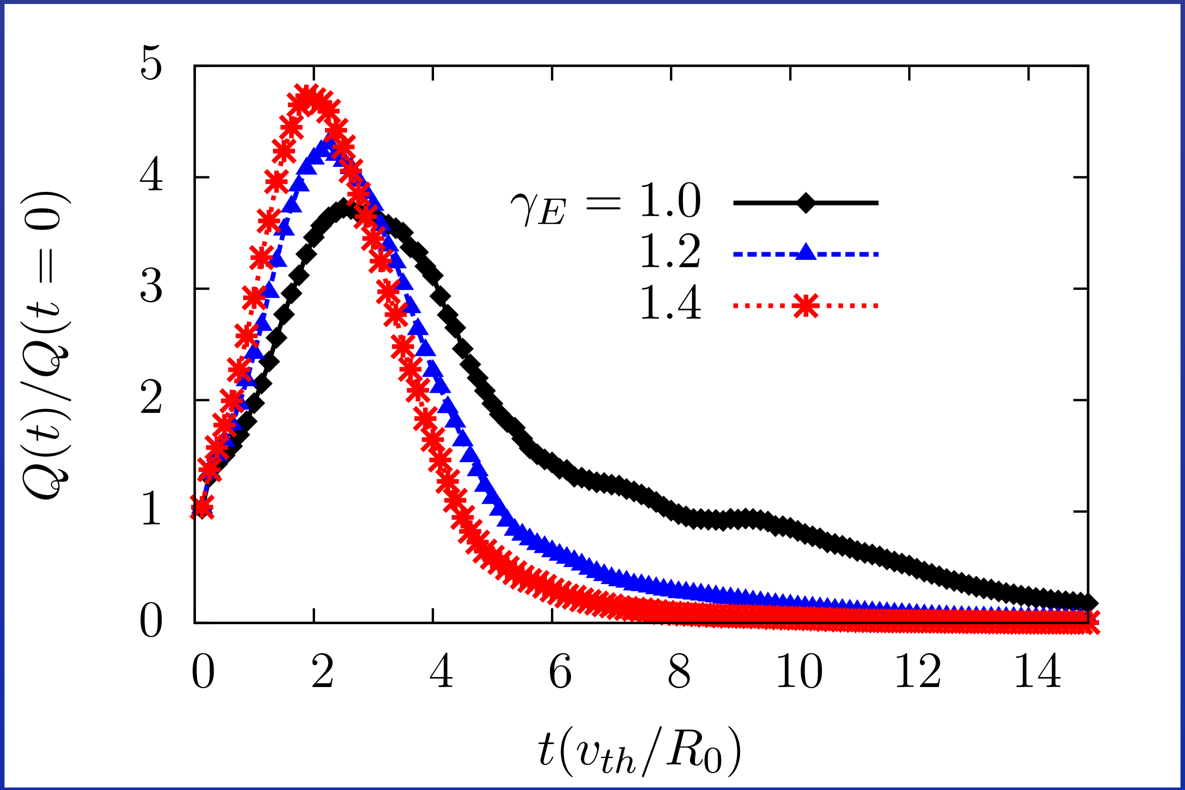
Cyclone base case: $r/R = 0.18 \quad q = 1.4 \quad \hat{s} = 0.8$




Barnes et al., 2010

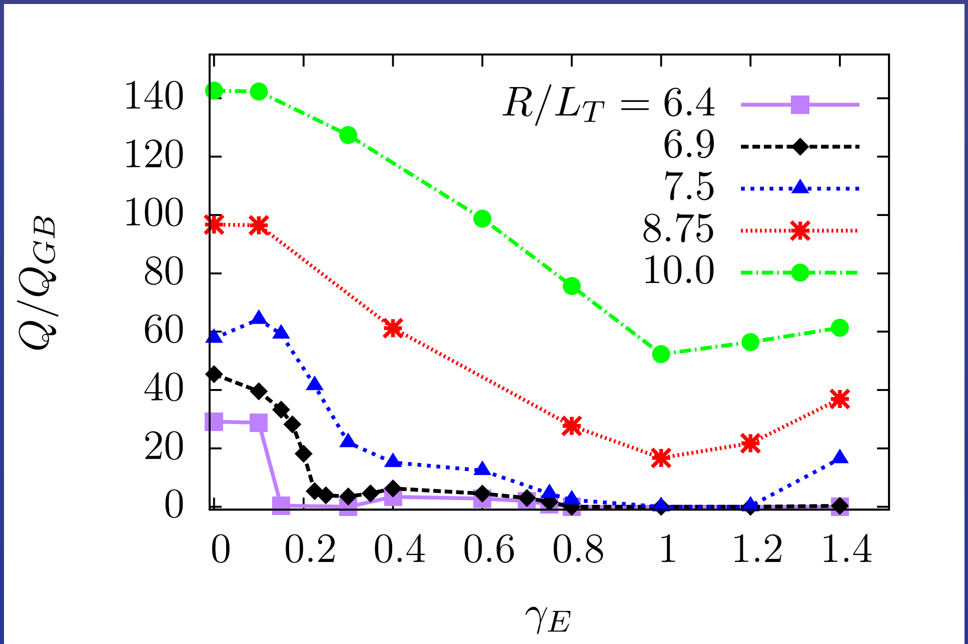
- ITG drive at small shear
- ITG/PVG drive at moderate shear
- Stabilization at large shear
- Roughly linear dependence of critical flow shear on R/LT

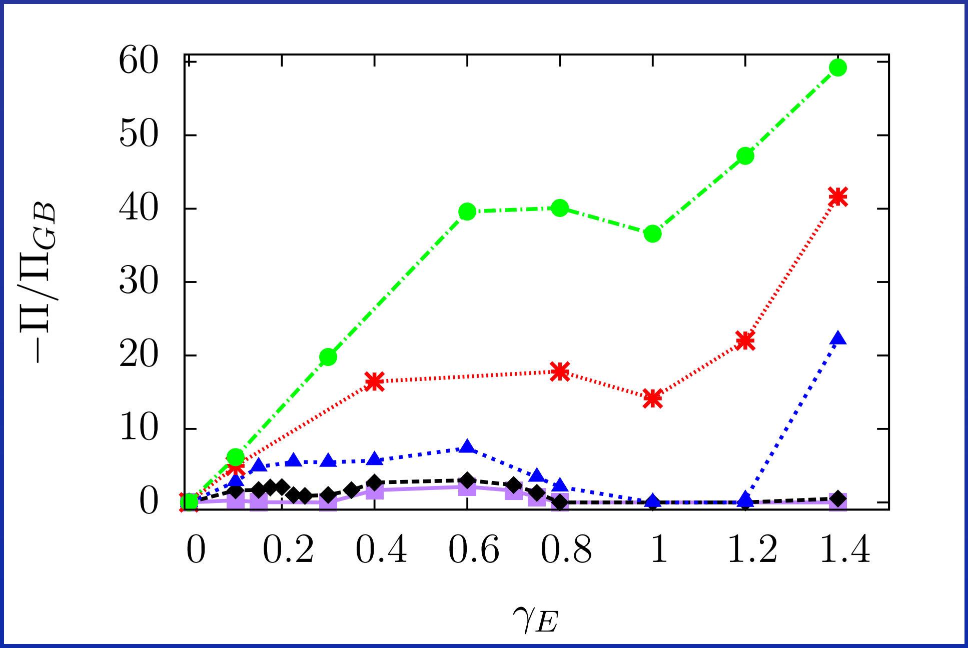
# Transient growth




Barnes et al., 2010

- Beyond critical shear value, transient linear growth
- Amplification of initial amplitude increases with shear
- Cf. Newton et al., 2010




$$Q = \overline{\int d^3v \, \frac{mv^2}{2} \left( \mathbf{v}_E \cdot \frac{\nabla\psi}{|\nabla\psi|} \right) \delta f}$$

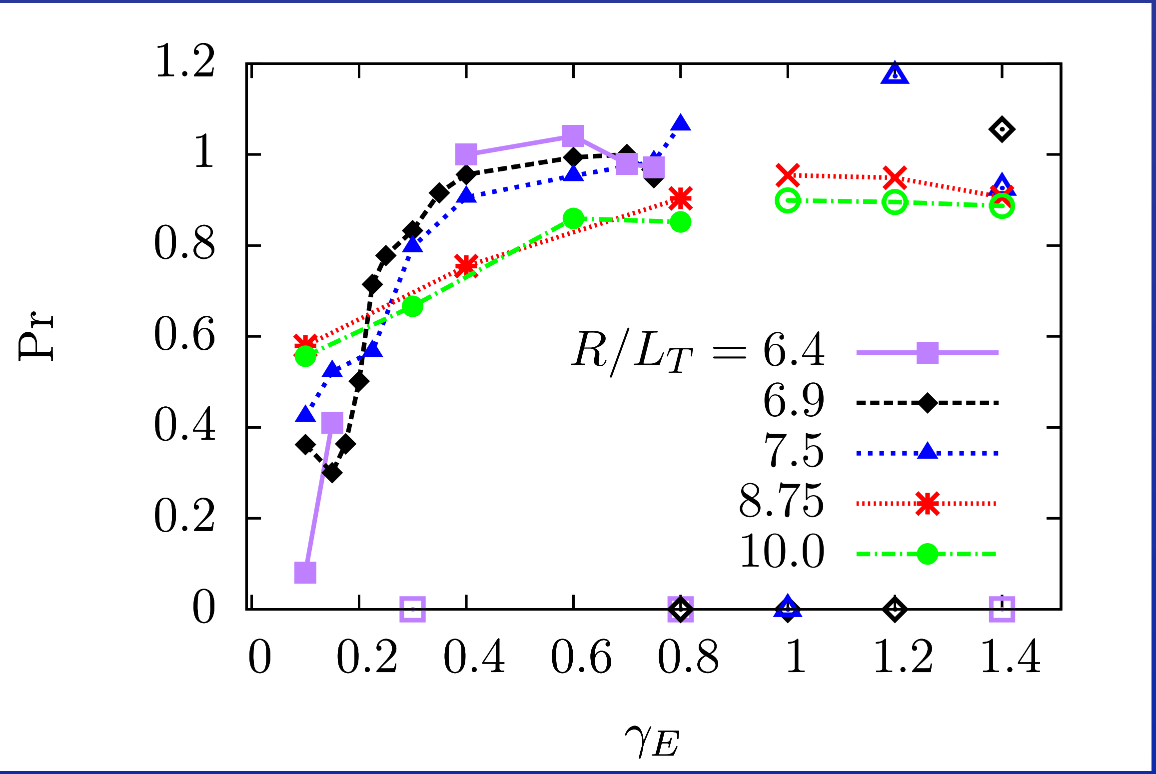
$$\Pi = \overline{\int d^3v \, mR^2 \, (\mathbf{v} \cdot \nabla\phi) \left( \mathbf{v}_E \cdot \frac{\nabla\psi}{|\nabla\psi|} \right) \delta f}$$

- Fluxes follow linear trends up to linear stabilization point
- Subcritical (linearly stable) turbulence beyond this point
- Optimal flow shear for confinement
- Possible hysteresis
- Maximum in momentum flux => possible bifurcation

# Turbulent Prandtl number

$$\mathrm{Pr} = \frac{\nu_i}{\chi_i} \qquad \Pi_i = -m_i v_{th}(qR_0/r)\nu_i\gamma_E$$

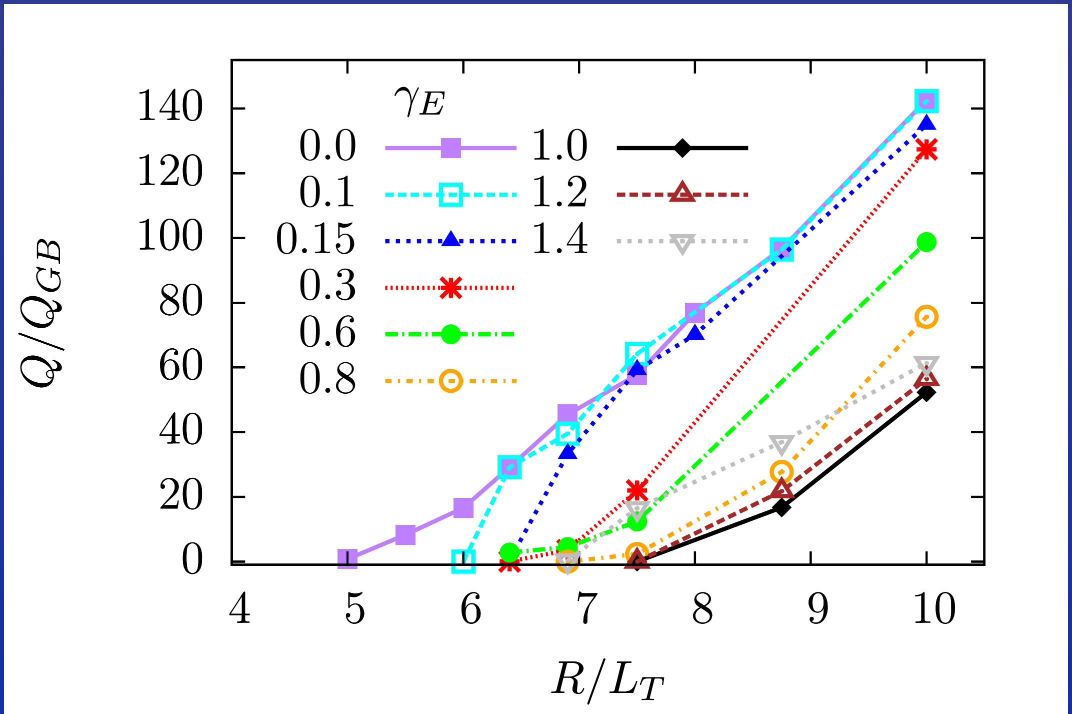
$$Q_i = -\chi_i dT_i/dr$$




- Prandtl number tends to shear- and R/LT-independent value of order unity (in both turbulence regimes)

Barnes et al., 2010

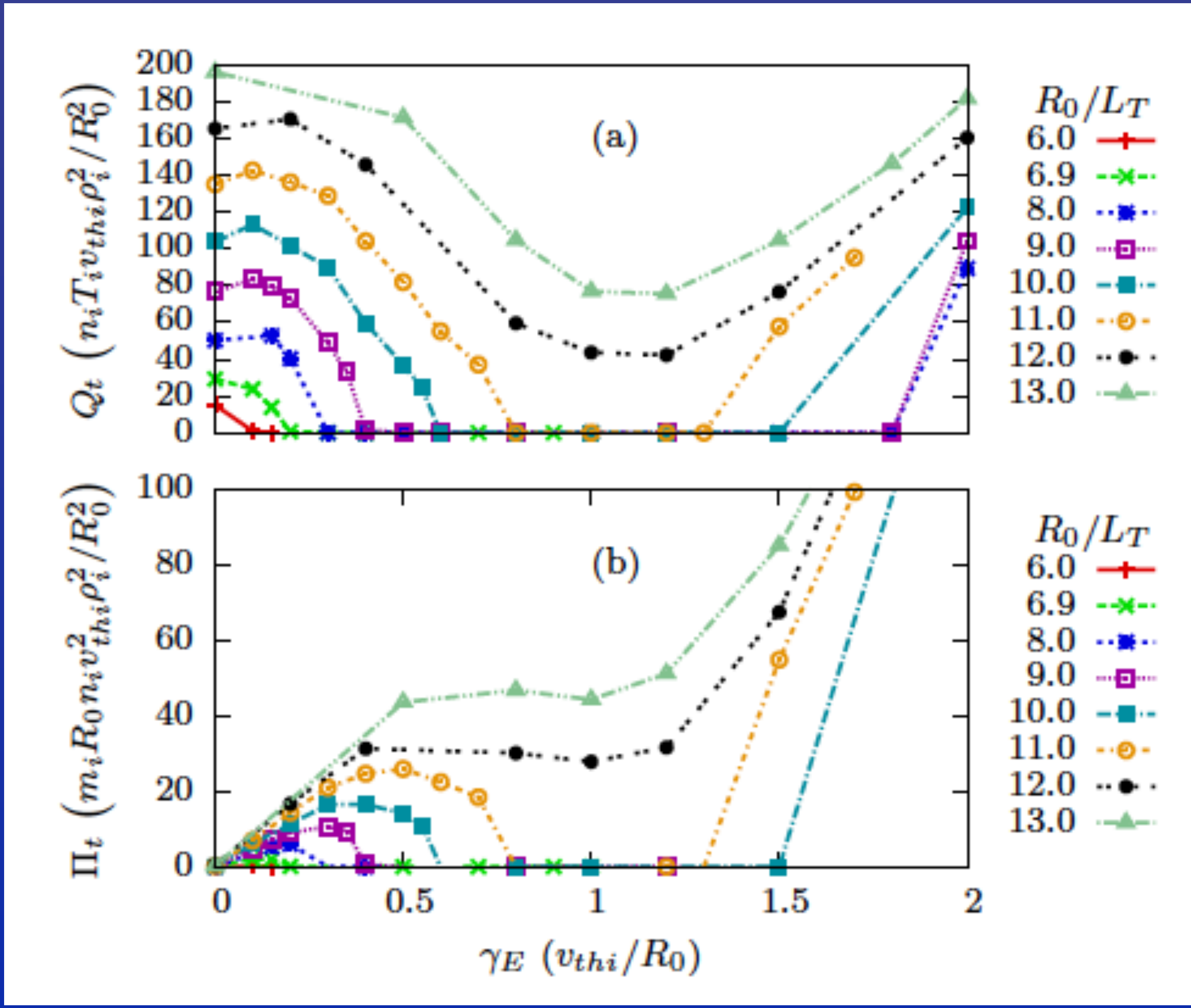
# Stiffness




Barnes et al., 2010

- Complicated dependence on shear
- Generally, critical gradient shifts higher and stiffness increases at low shear
- Critical gradient shifts lower and stiffness decreases at high shear (when turbulence driven by shear instead of R/LT)

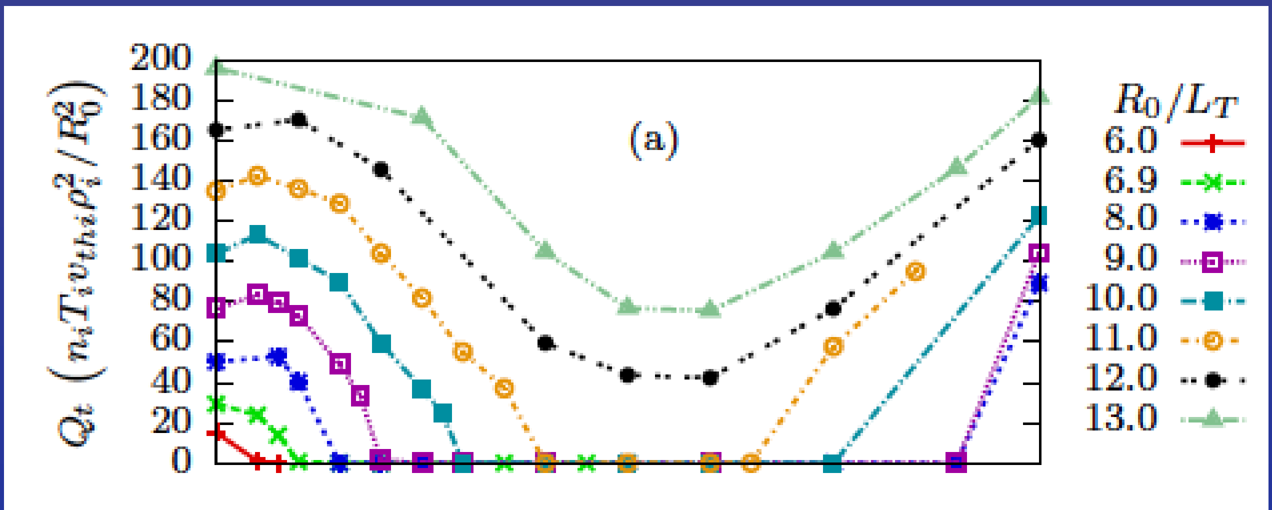
# Zero magnetic shear

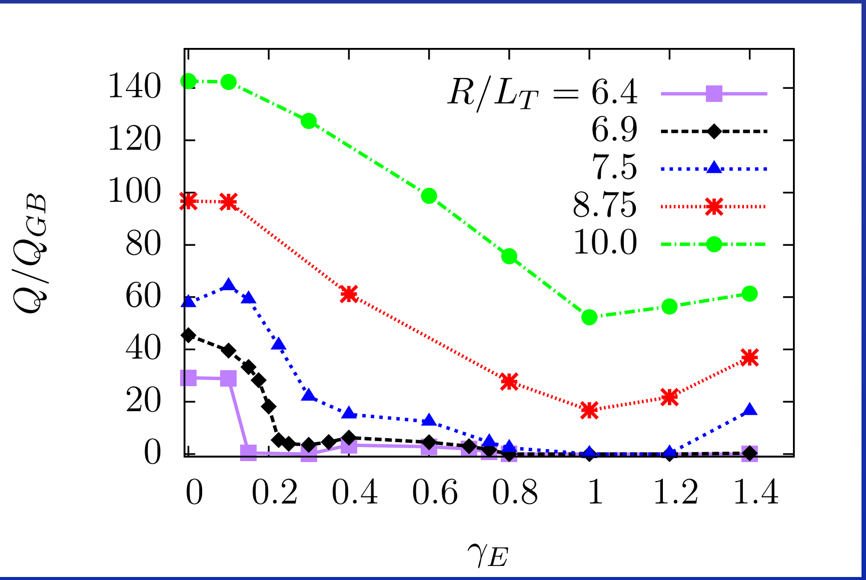



- Similar…sort of
- All turbulence subcritical

Highcock et al., 2010

# Zero magnetic shear




- Similar…sort of
- All turbulence subcritical
- Very different critical flow shear values

# Overview

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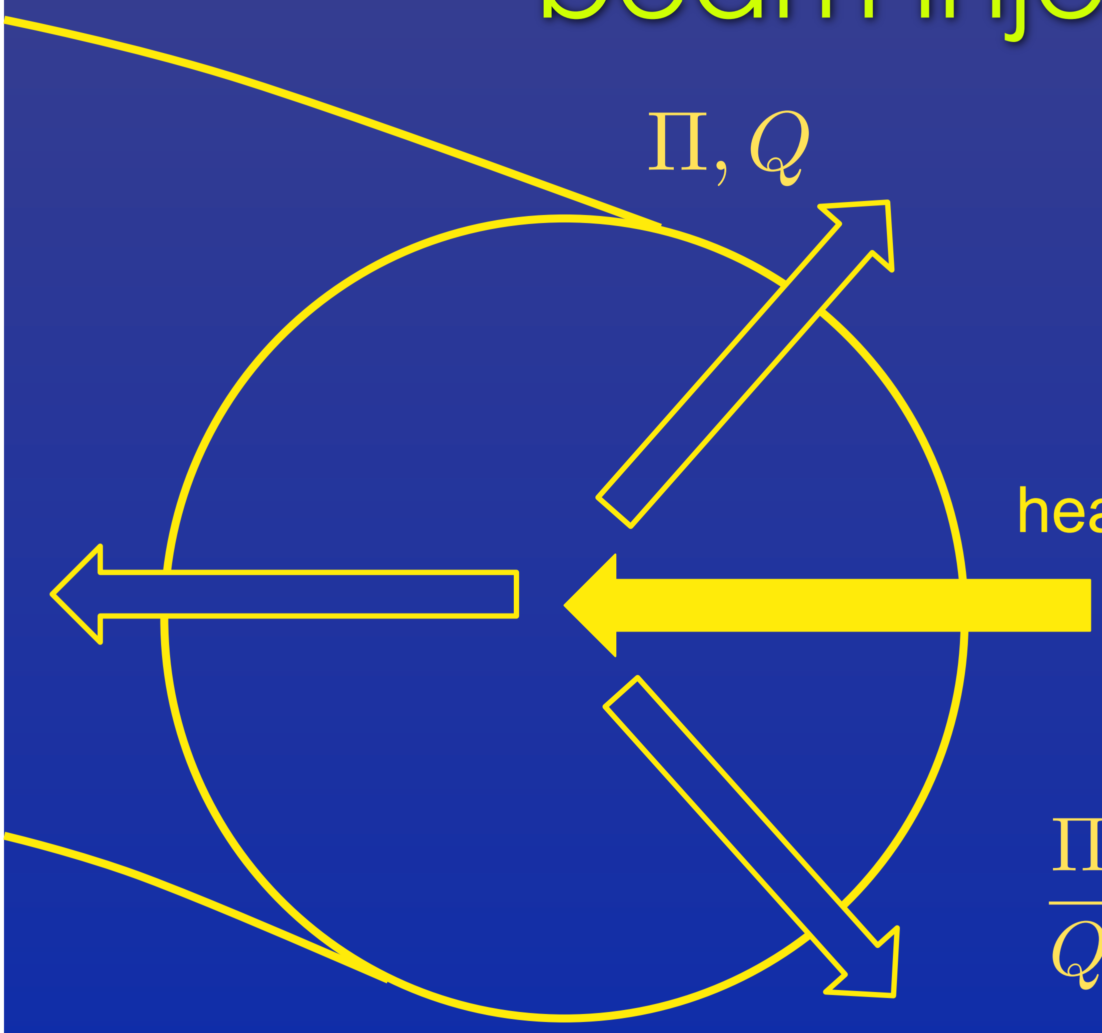
# Power/Torque balance for beam injection

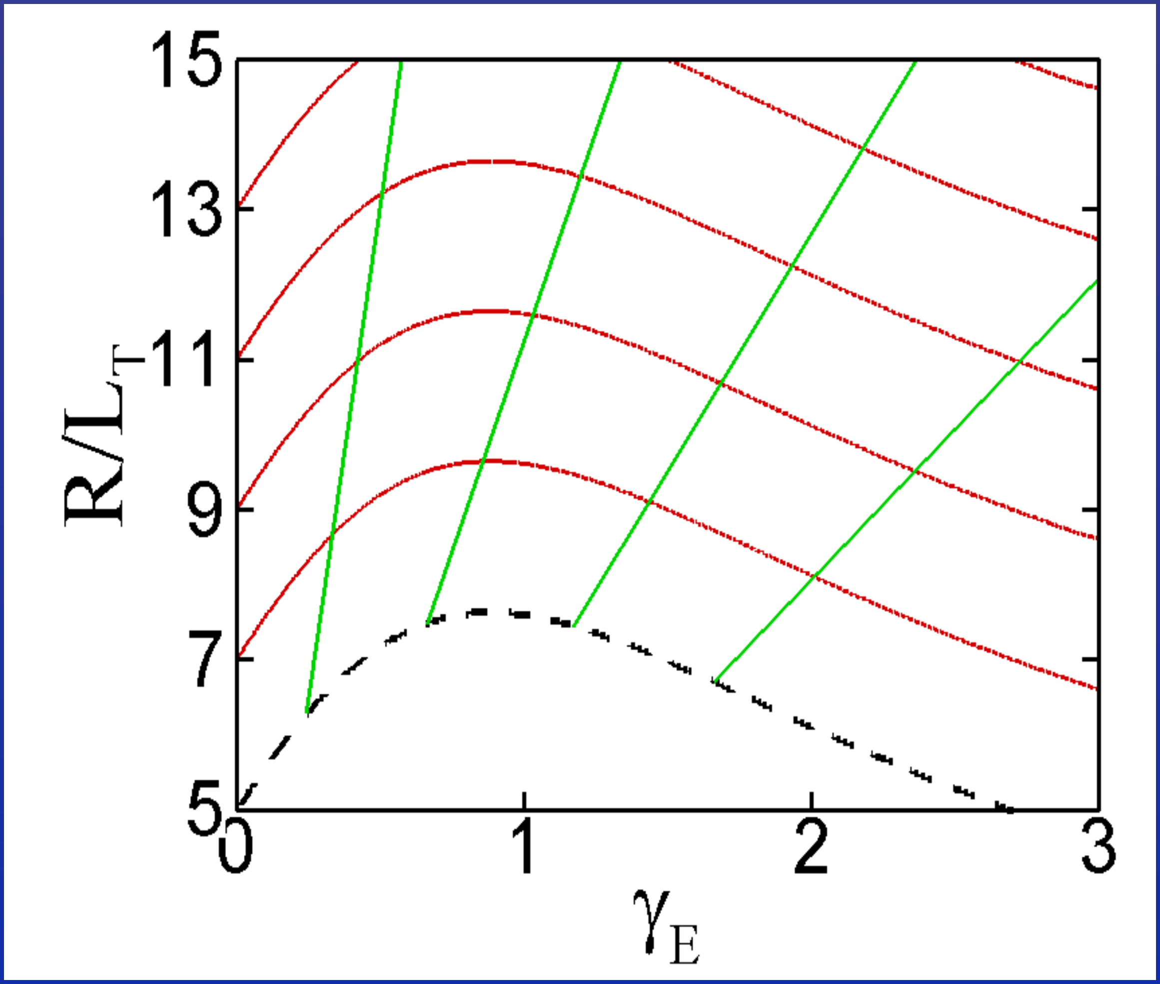



$$\frac{\Pi}{Q} \sim \frac{n_B R_B m_B v_B^2}{n_B m_B v_B^3} = \frac{R_B}{v_B}$$

# Two possible bifurcations

- Adding neoclassical leads to a bifurcation
  - It seems to contradict phenomenology
- Adding intrinsic rotation leads to another type of bifurcation
  - Speculative

# Balance w/o neoclassical



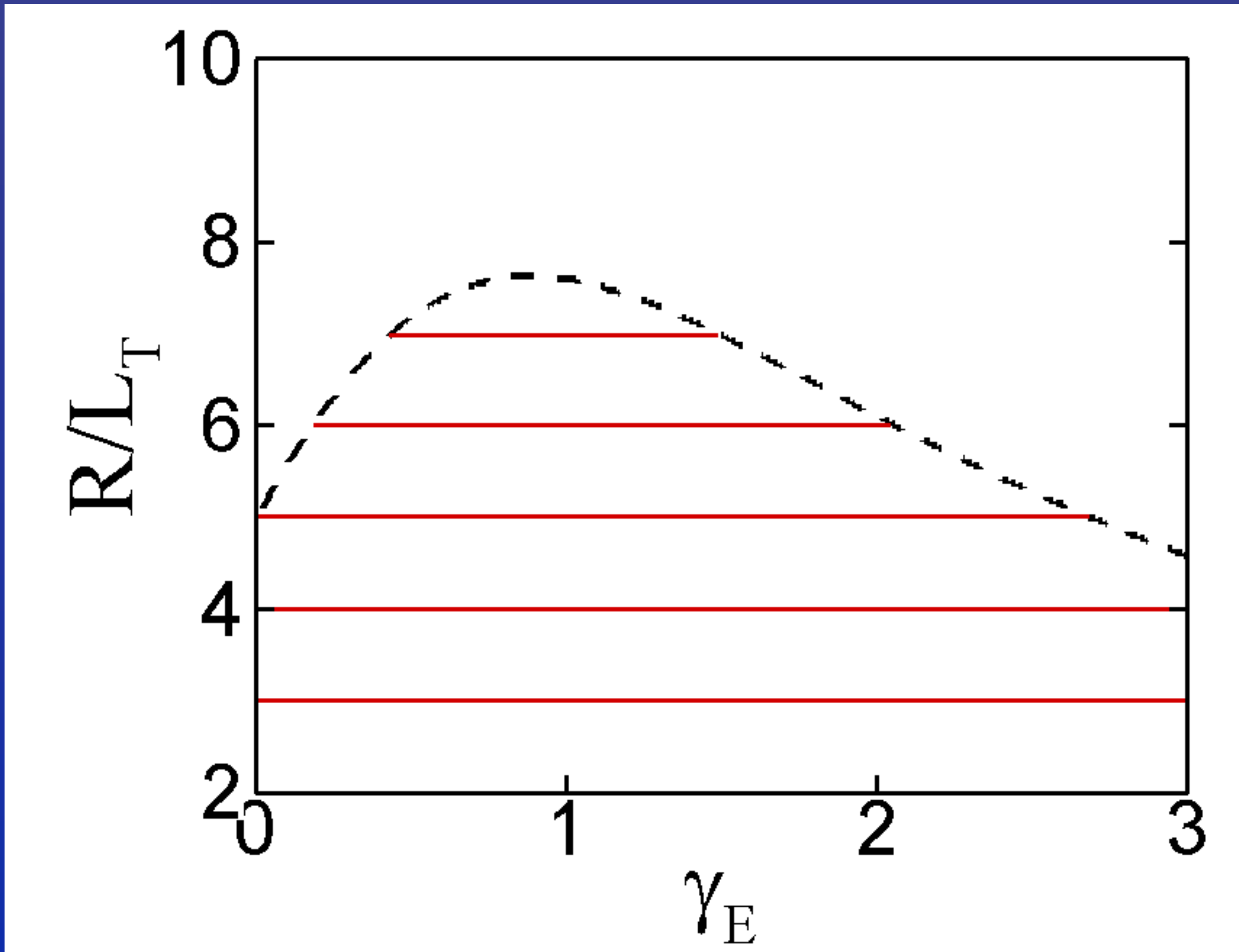

- $Q$ = red lines
- $\Pi/Q$ = green lines

$$\frac{R}{L_T} = \frac{Pr_t}{\Pi/Q}\gamma_E$$

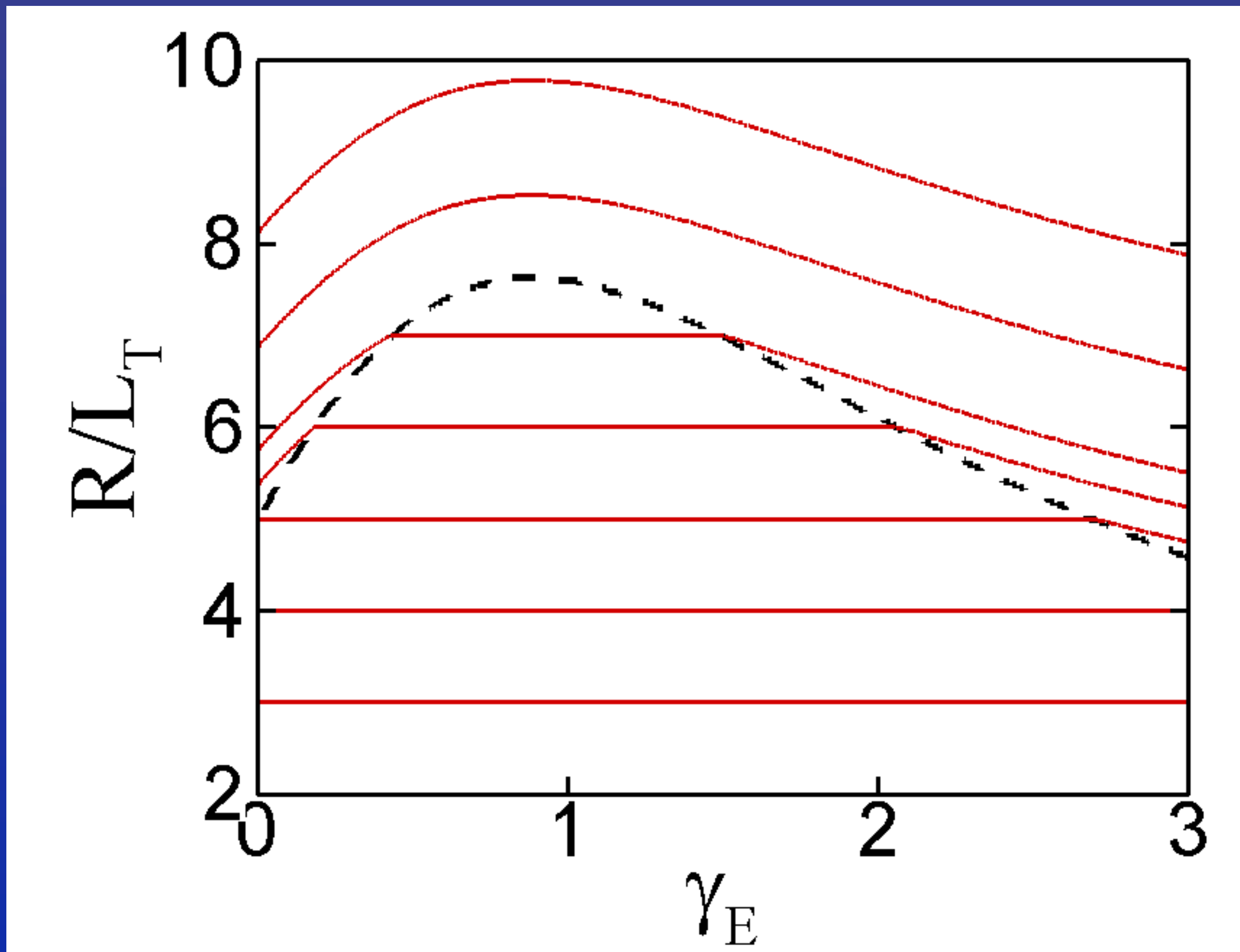
- Critical gradient = dashed line
- For given $\Pi/Q$ and $Q$, only one solution

  No bifurcation!

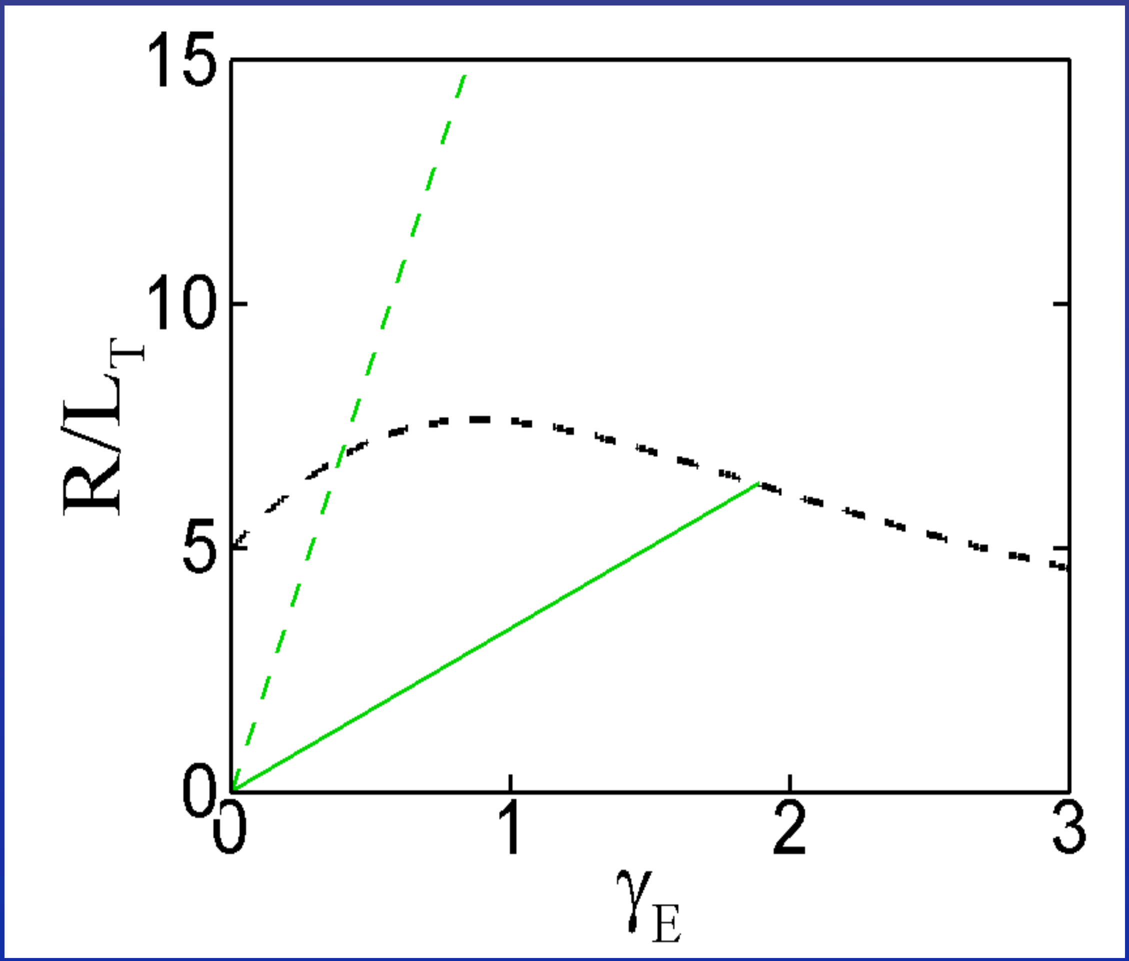
# Neoclassical energy flux



# Neoclassical energy flux



# Curves of constant $\Pi/Q$




- Neoclassical

$$\frac{R}{L_T} = \frac{Pr_n}{\Pi/Q}\gamma_E$$

- Turbulent

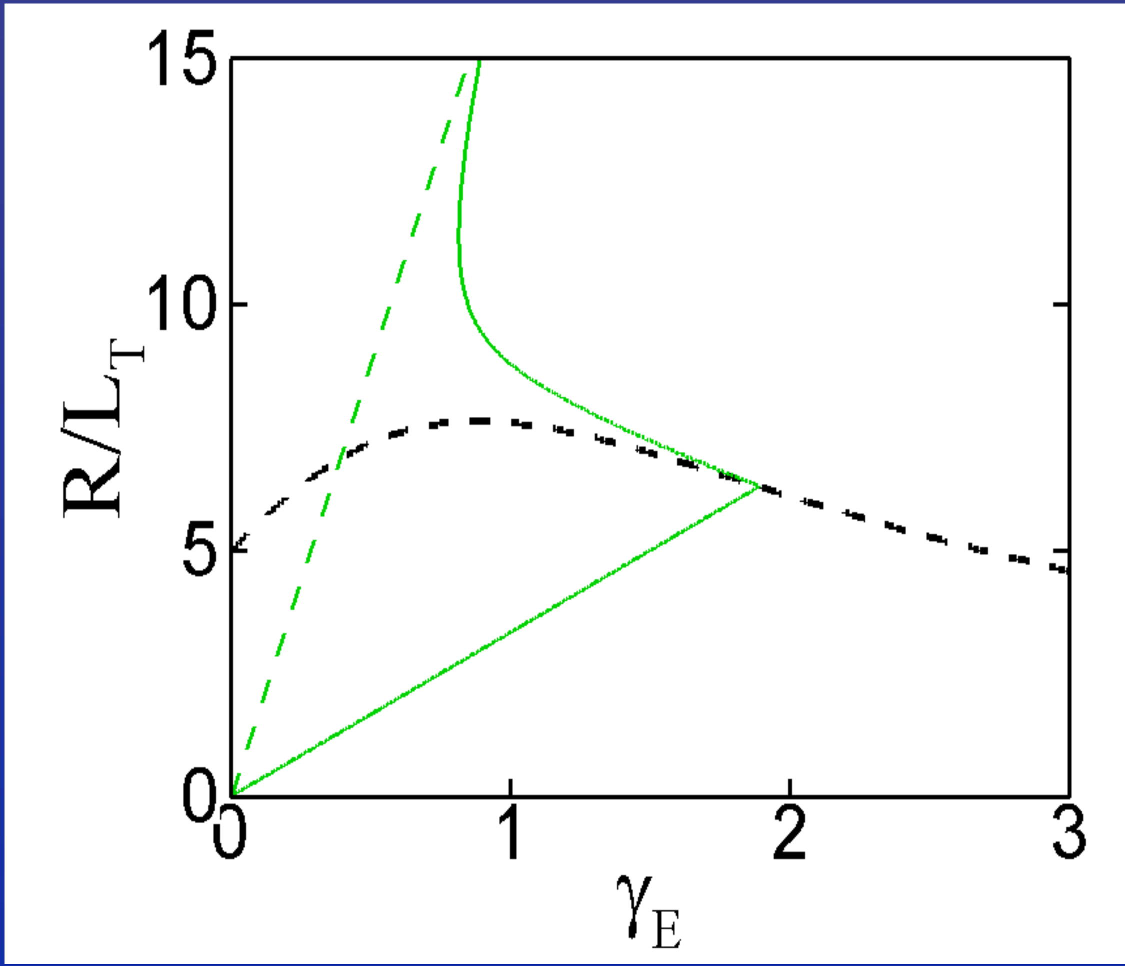
$$\frac{R}{L_T} = \frac{Pr_t}{\Pi/Q}\gamma_E$$

- Prandtl numbers

$$Pr_n < Pr_t$$

Banana orbits give energy flux, not momentum

# Curves of constant $\Pi/Q$

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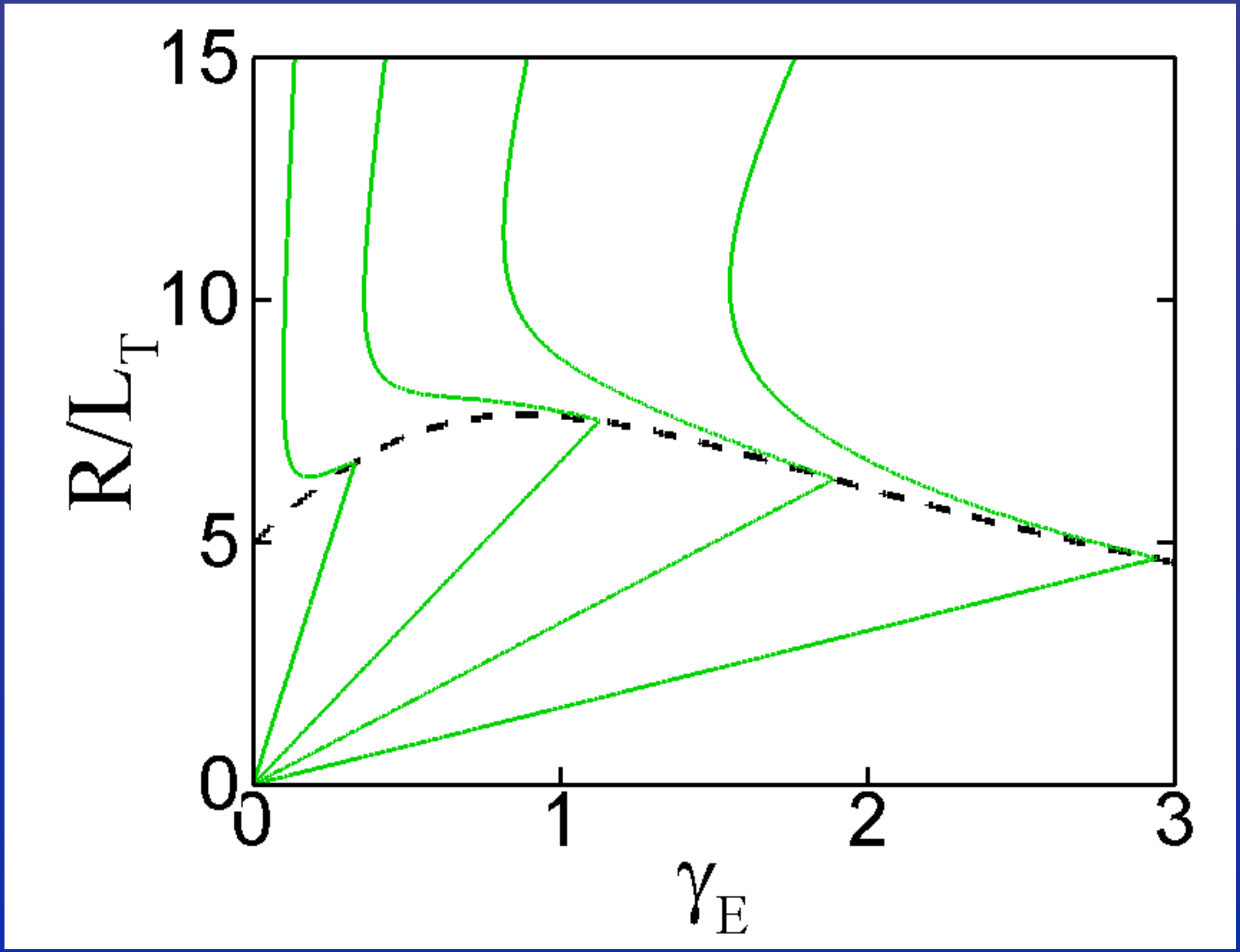
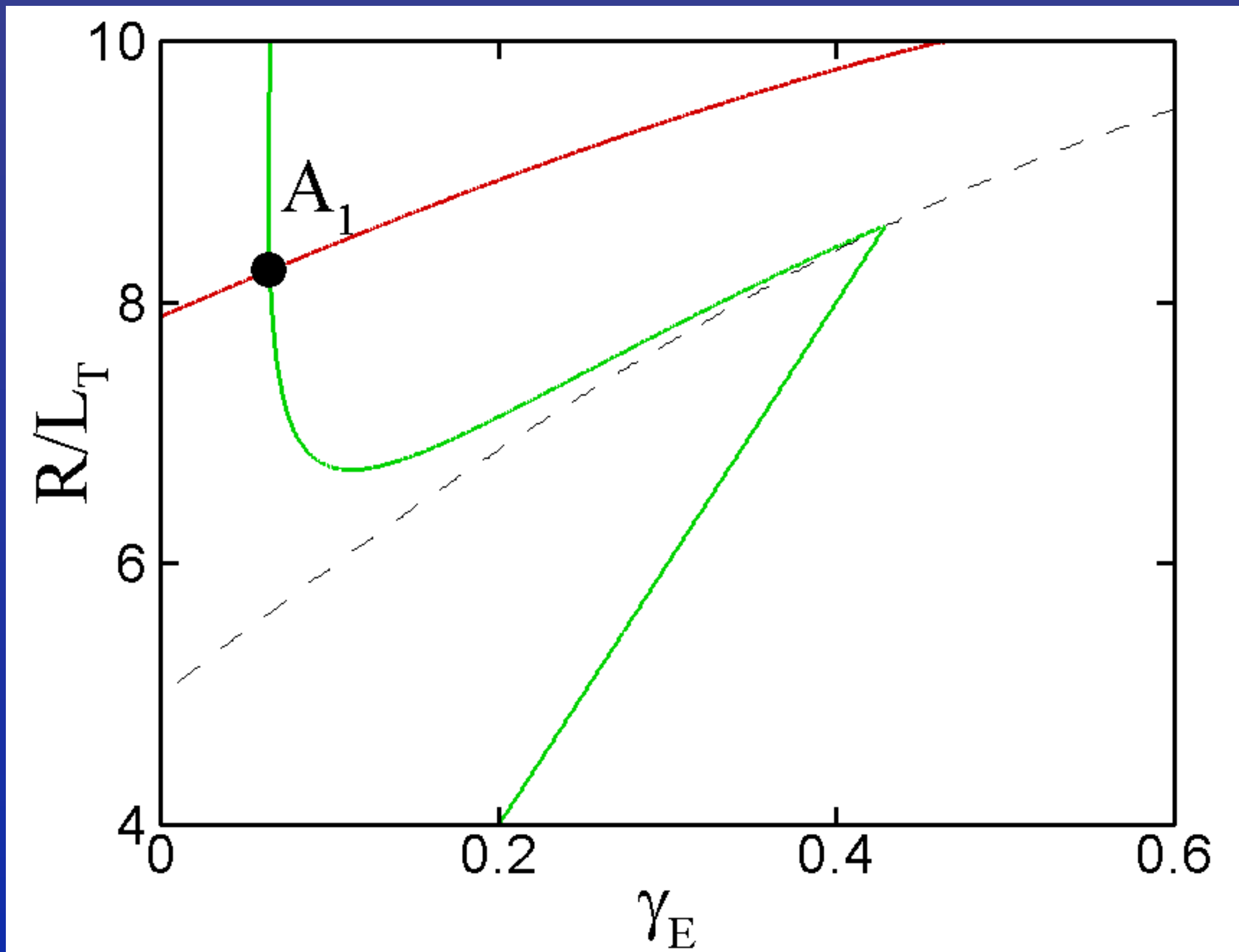


# Possible solutions



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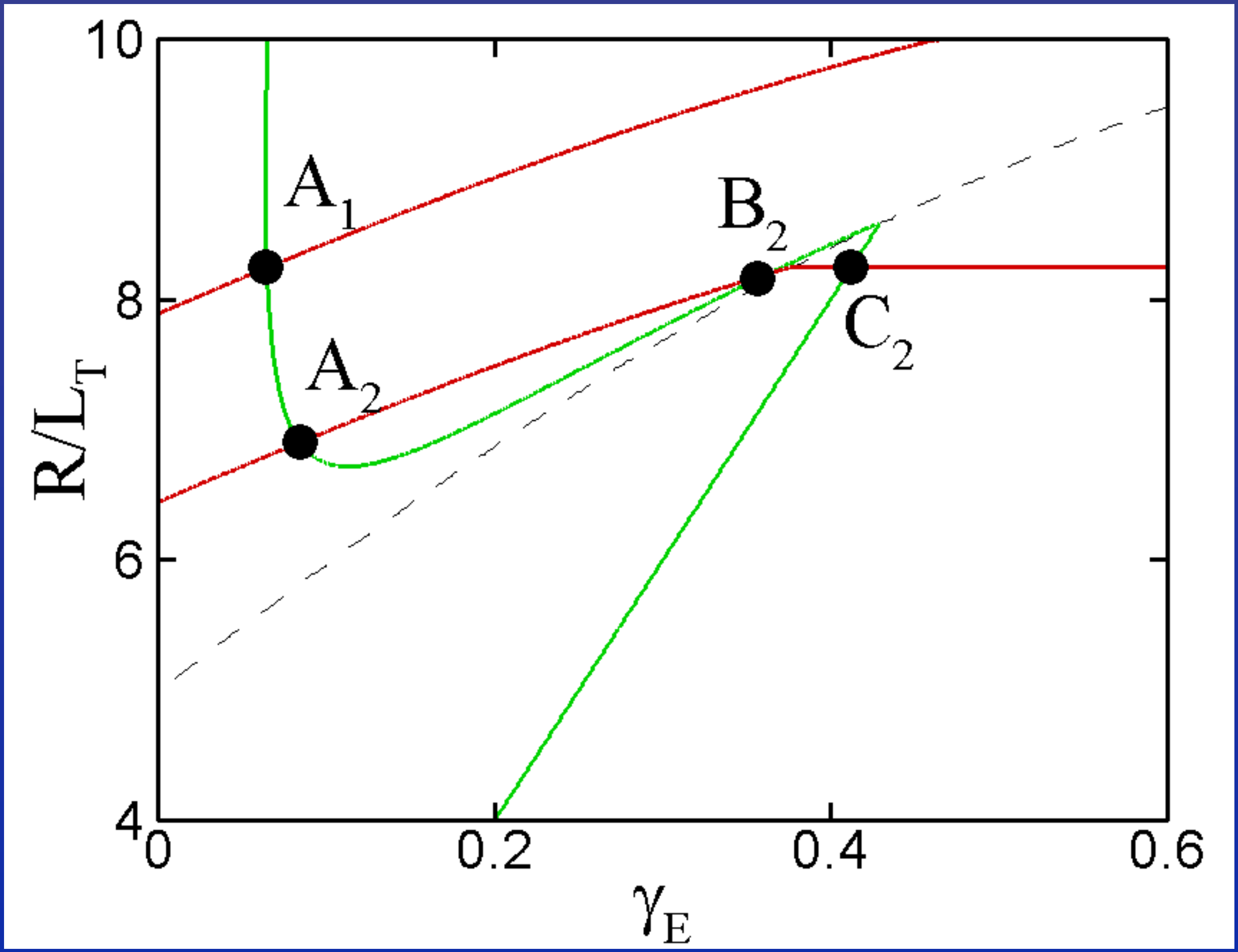

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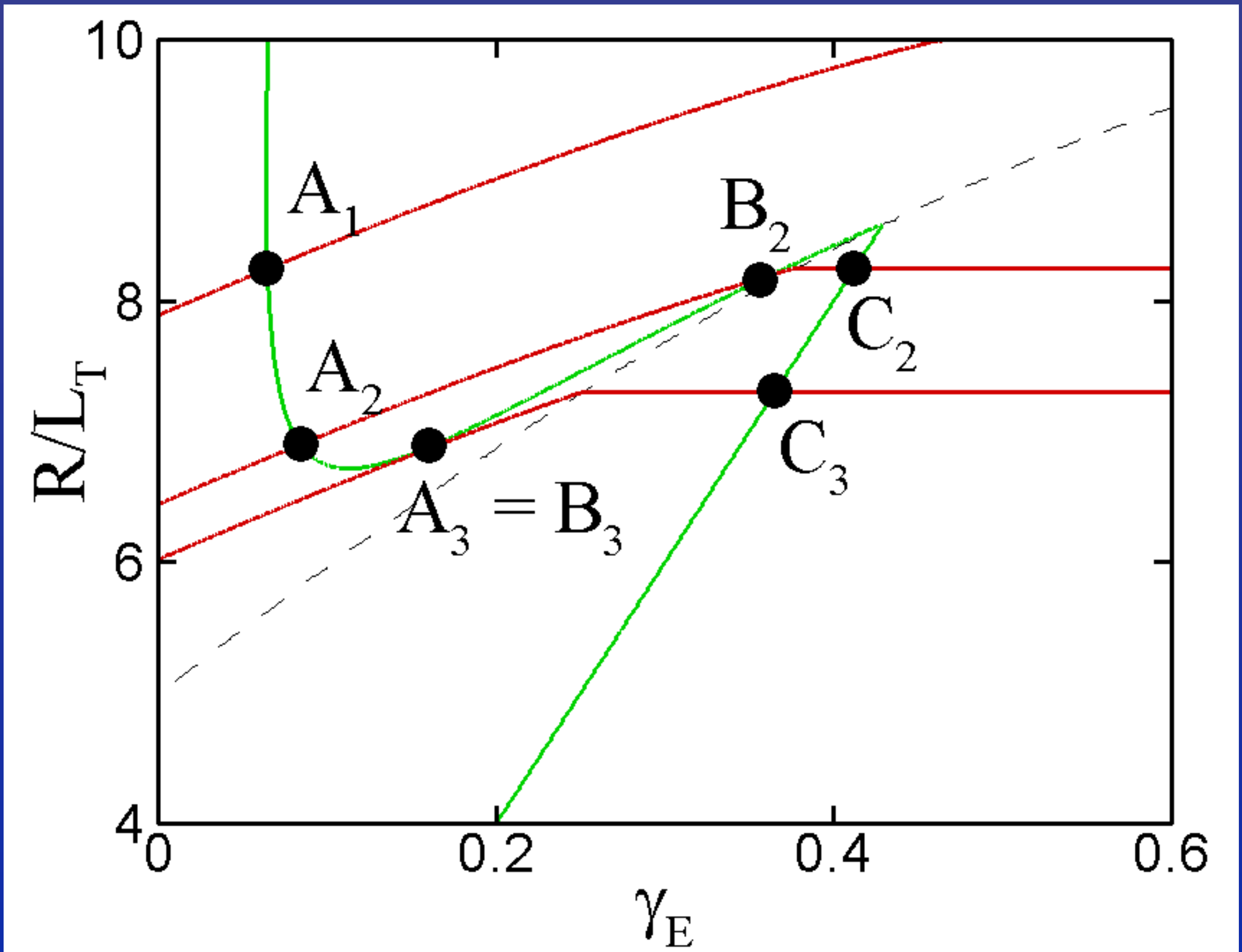
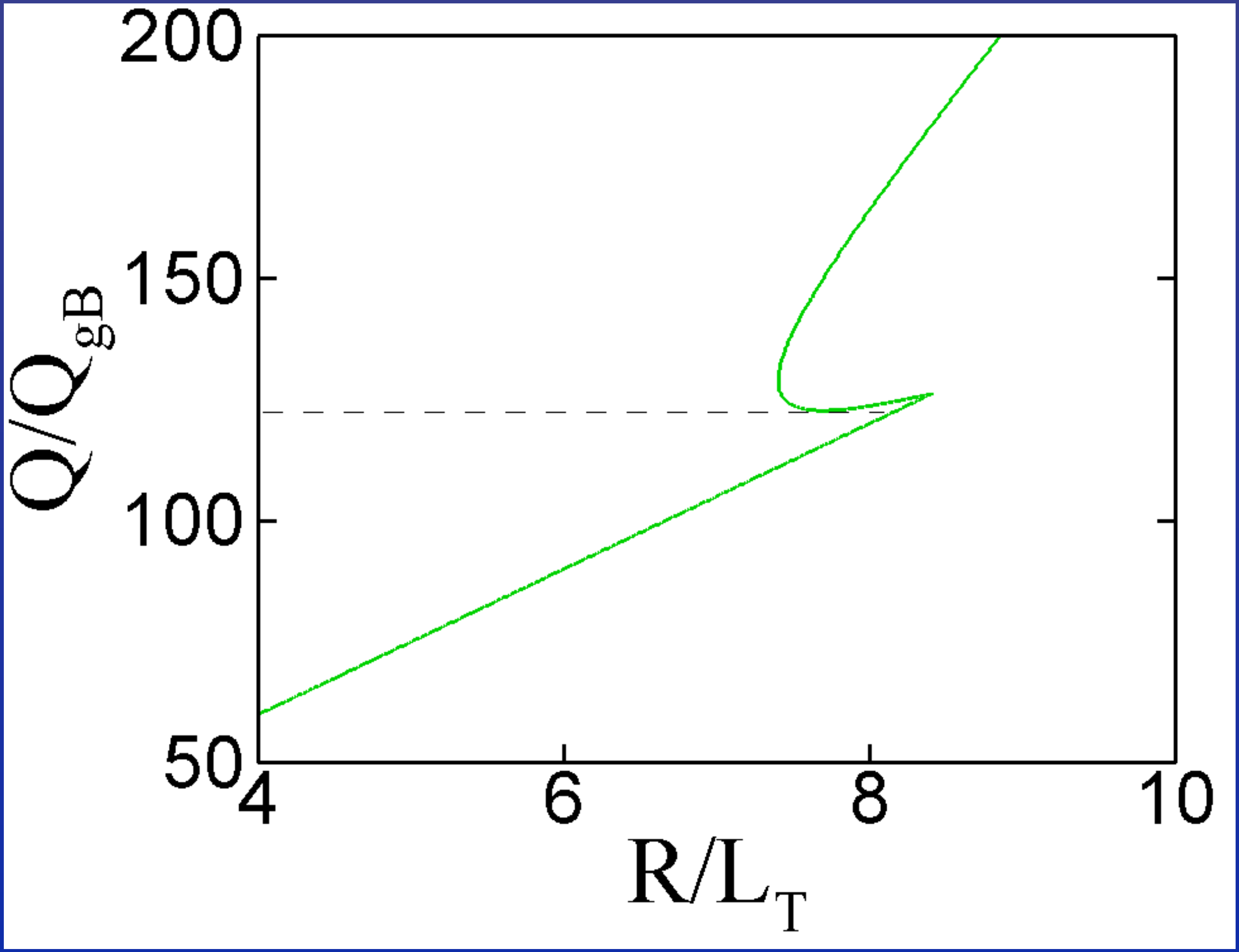
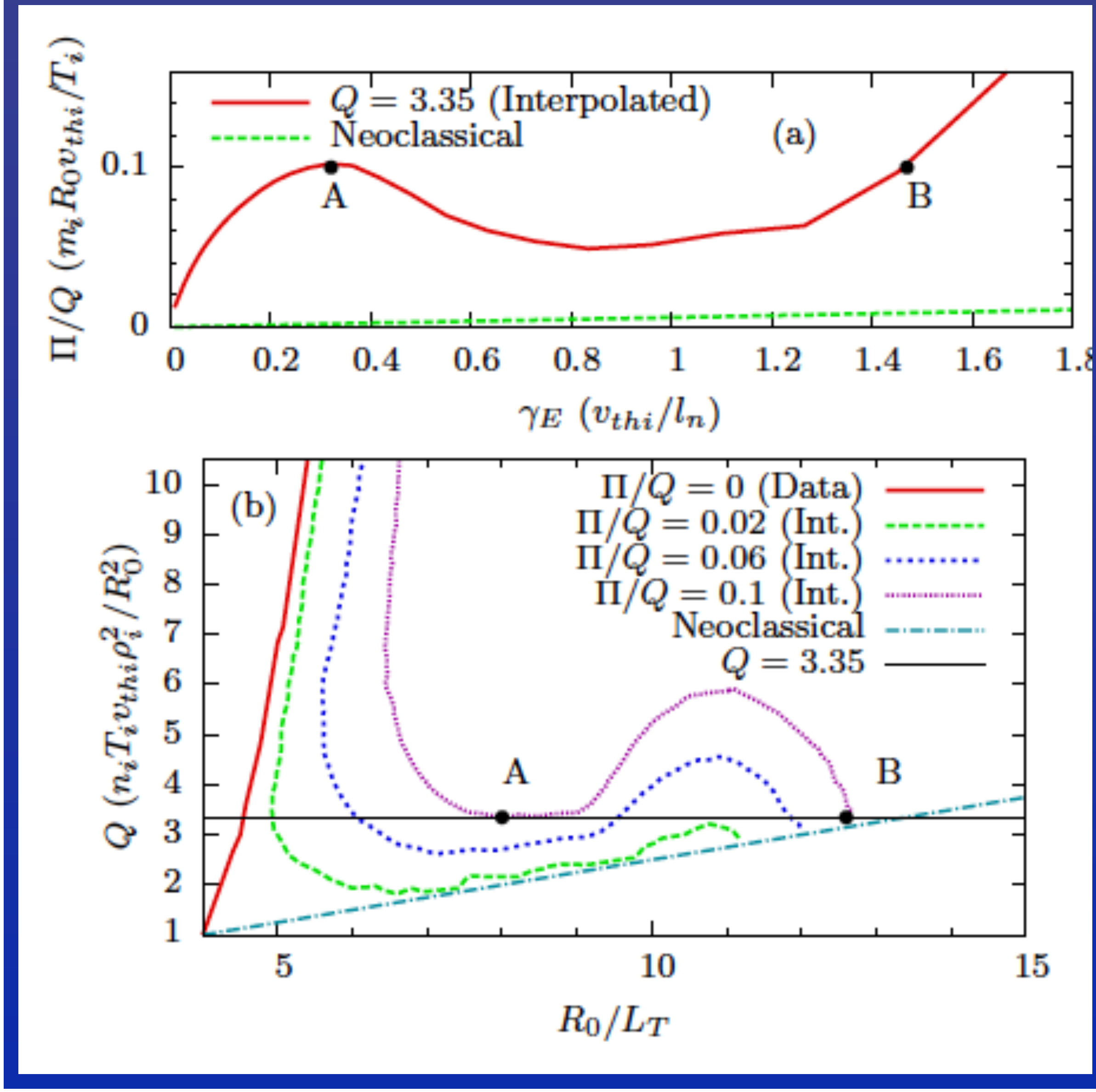


# Total energy flux



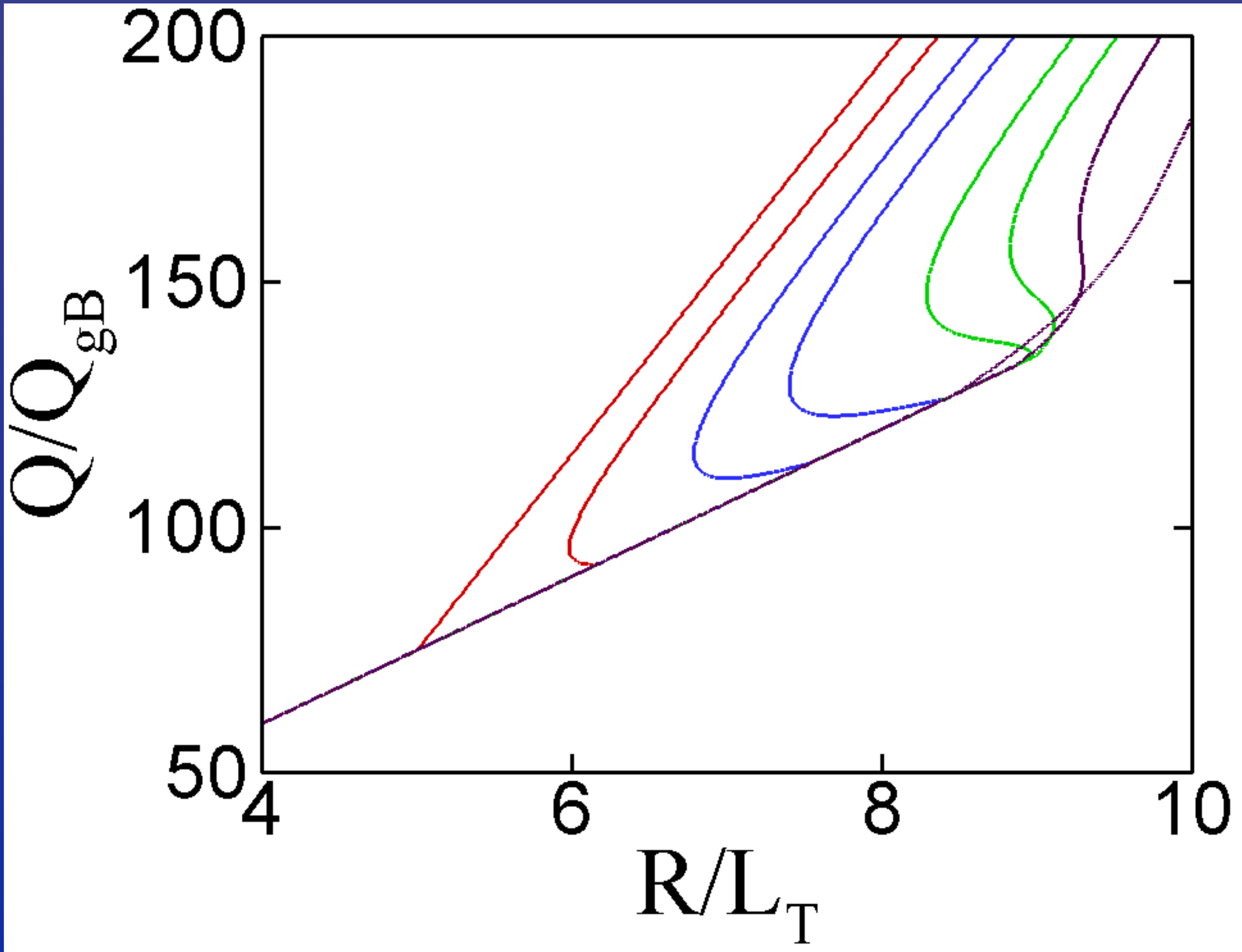
# Bifurcations



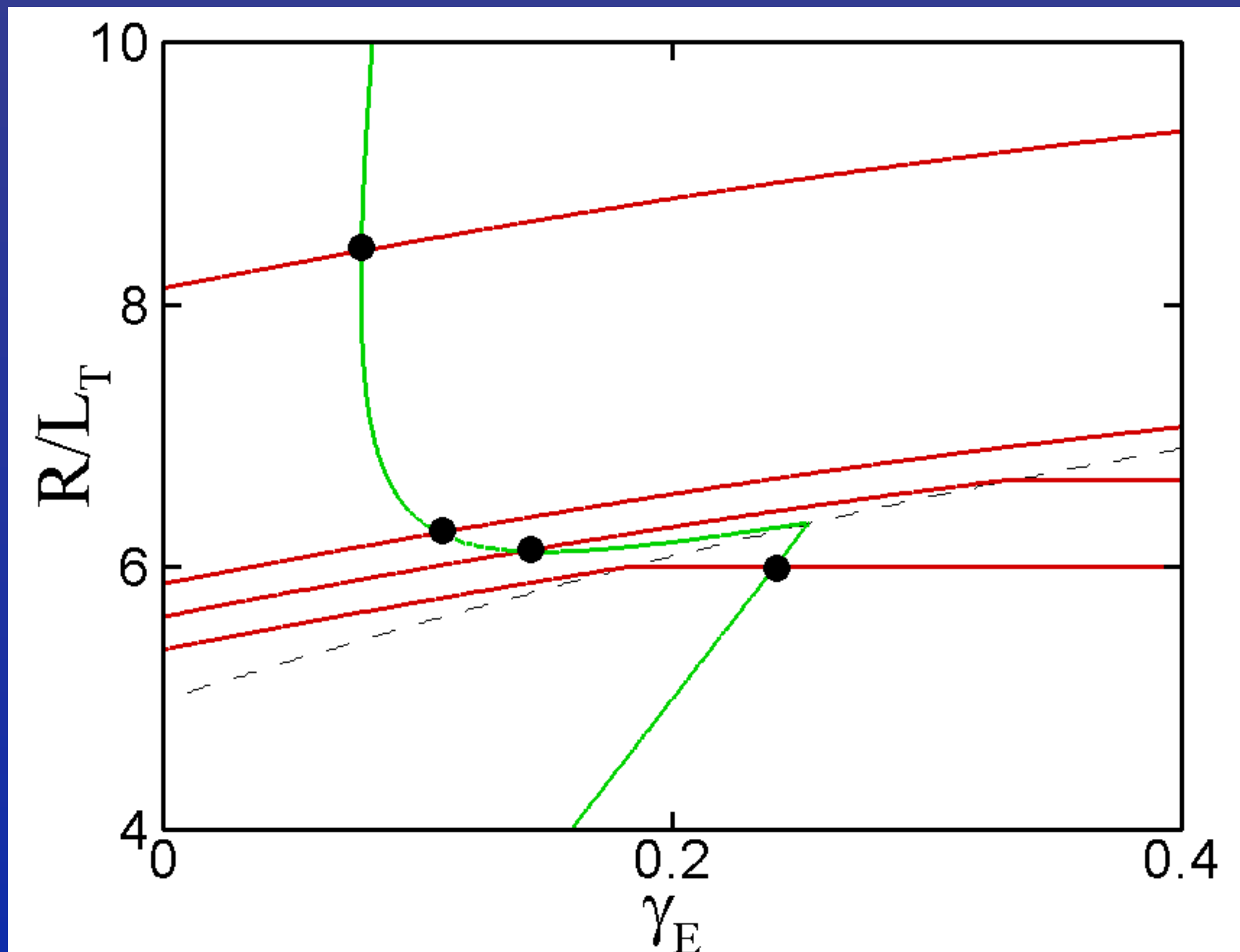

- Consider inverse problem: for fixed fluxes, what are gradients?
- With inclusion of neoclassical fluxes, we see bifurcation to much larger flow shear and R/LT

Highcock et al., 2010

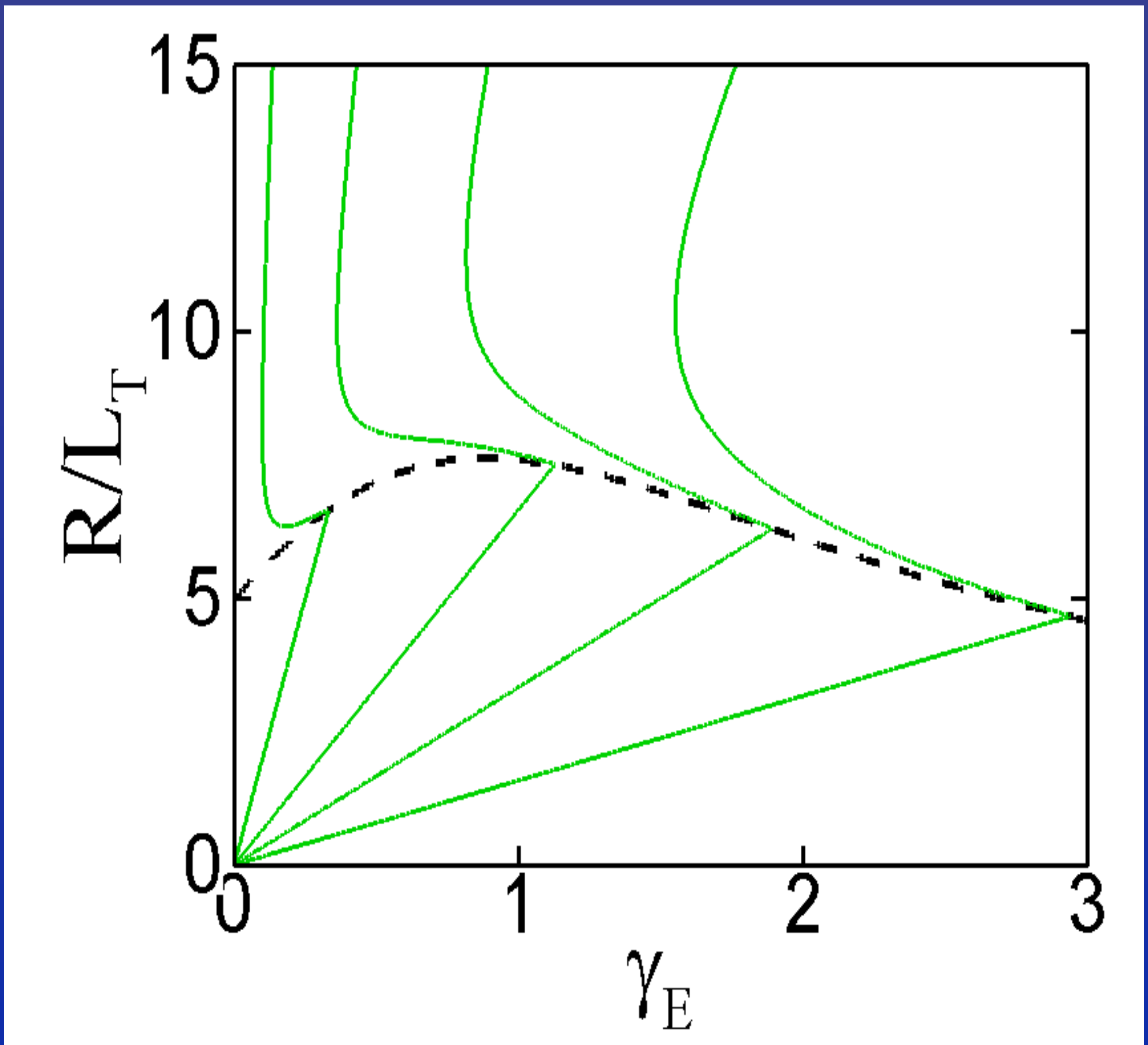
# Total energy flux



# Smooth transitions to neoclassical



# Curves of constant $\Pi/Q$




Slope increases for

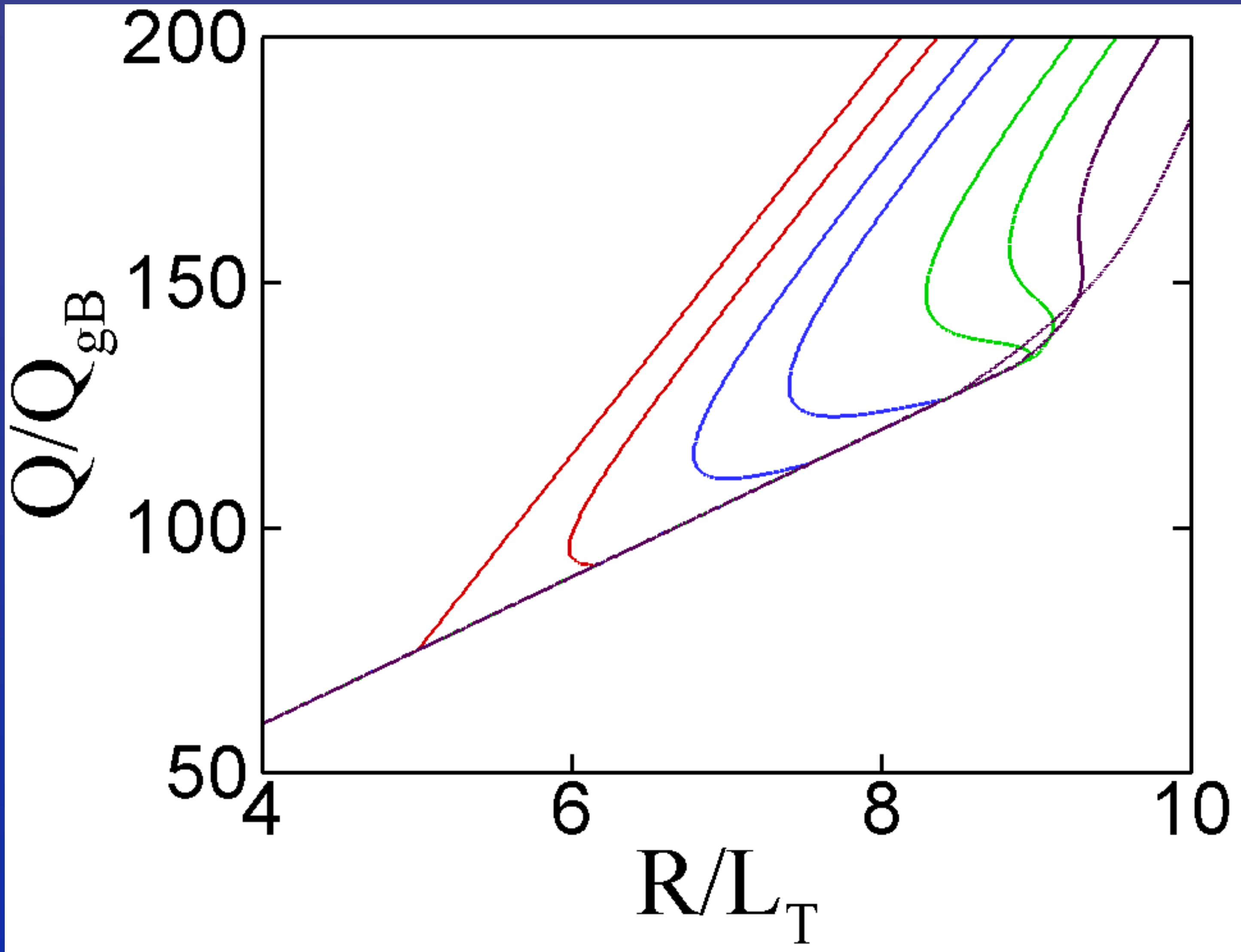
- Small $\Pi/Q$

  Favors neutral beam heating

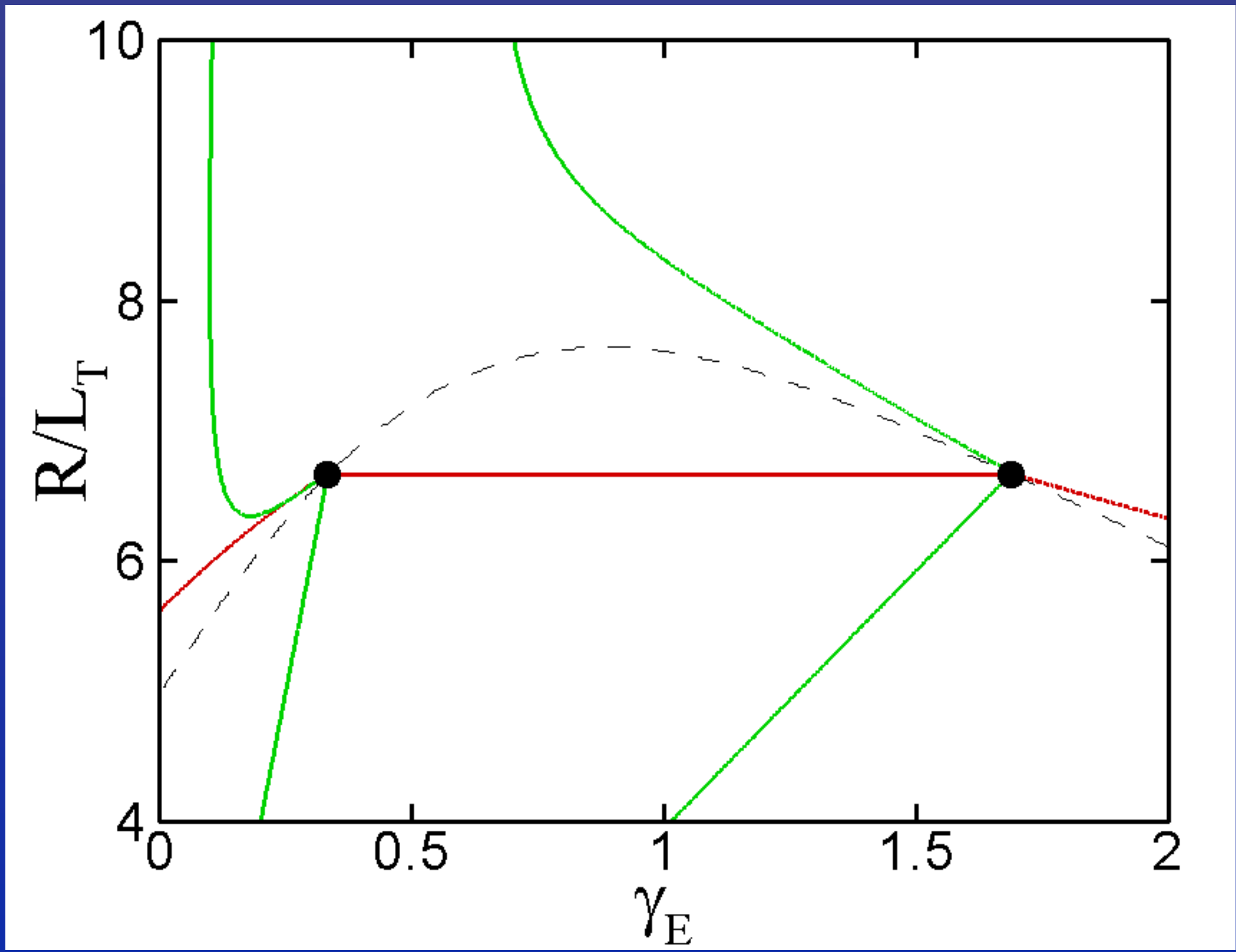
- Small $\dfrac{d(R/L_T|_c)}{d\gamma_E}$

  Favors small magnetic shear regions

# Total energy flux



# Intersecting lines



# Neoclassical bifurcation

- Bifurcation happens due to lower neoclassical Prandtl number
- Numerically tested
- Possible to obtain large jumps
- Favors neutral beams and low magnetic shear
- It is easier at lower power!

# Intrinsic rotation terms

- Idea: expansion on poloidal gyroradius

$$\Pi = \Pi_0 + \alpha \rho_p \frac{\partial^2 T}{\partial r^2} + \beta \rho_p \frac{\partial^2 \omega}{\partial r^2} + \dots$$

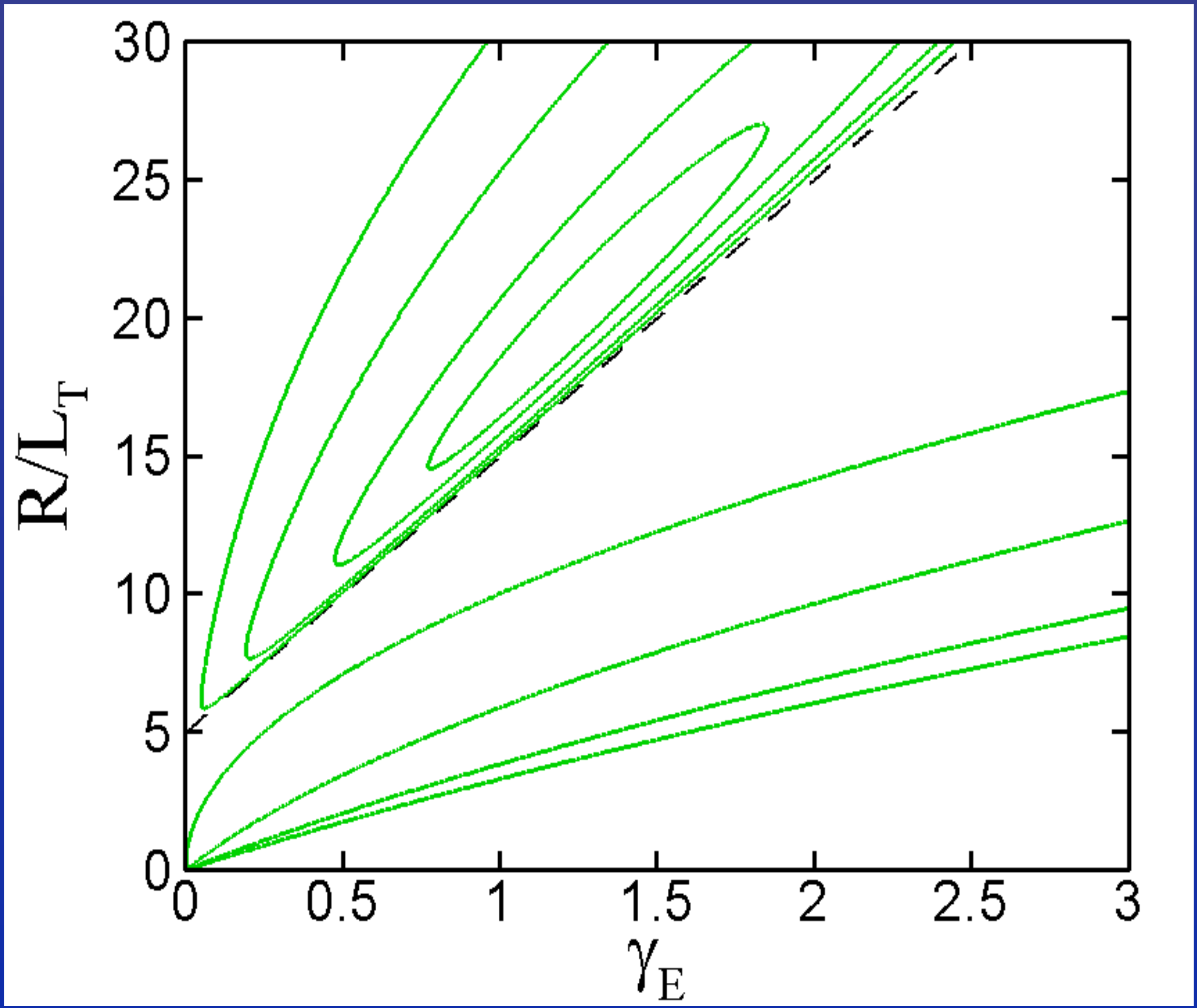
- For low flow, only temperature matters

$$\frac{\Pi_{t,n}}{Q_{t,n}} = Pr_{t,n} \frac{\gamma_E}{R/L_T} + \alpha_{t,n} \rho_p \frac{\partial^2 T/\partial r^2}{\partial T/\partial r}$$

  – Generation of intrinsic rotation (Parra & Catto, PPCF 2010)

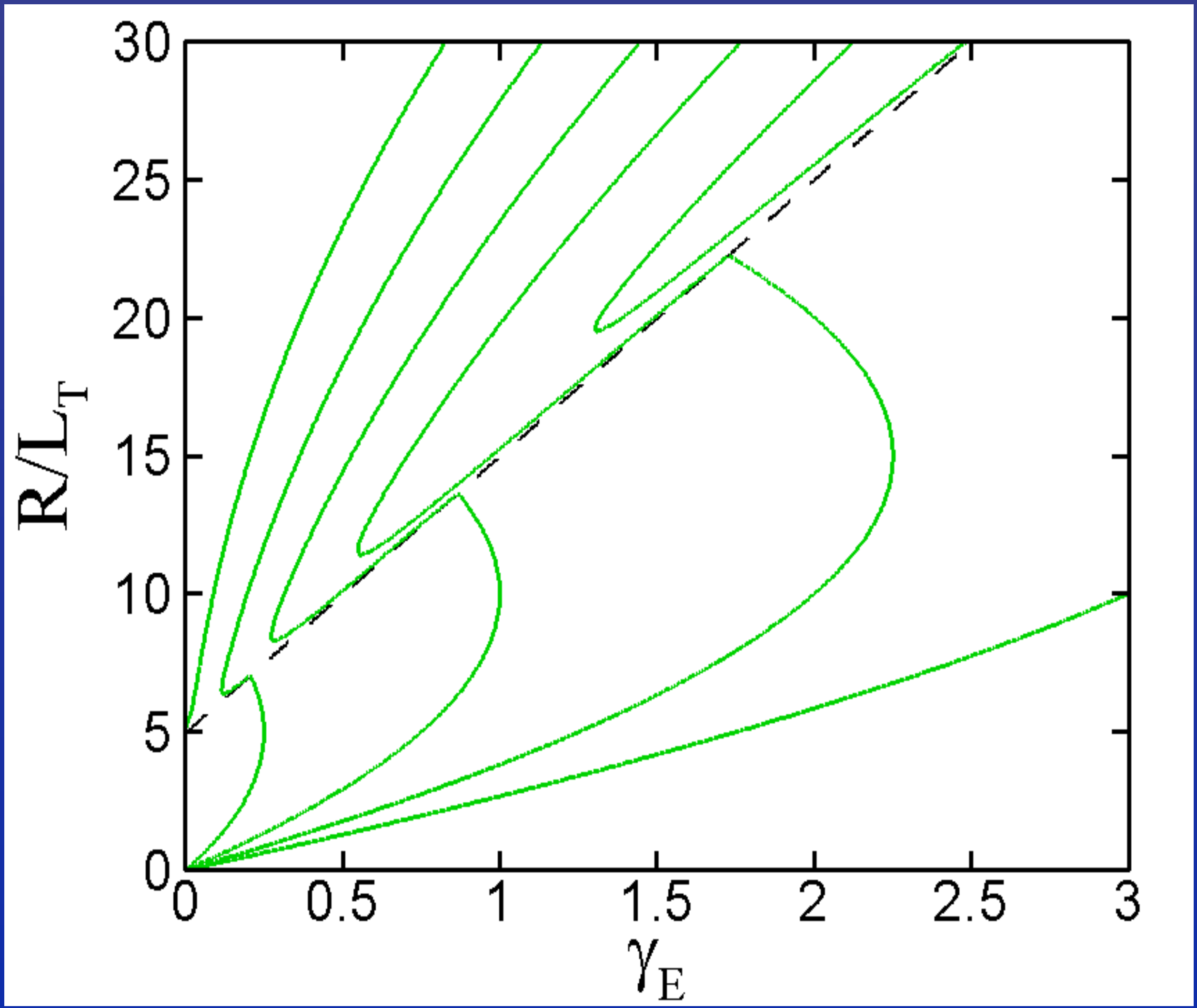
- Assume $\dfrac{\partial^2 T/\partial r^2}{\partial T/\partial r} \sim \pm \dfrac{R}{L_T}$

# Curves of constant $\Pi/Q$



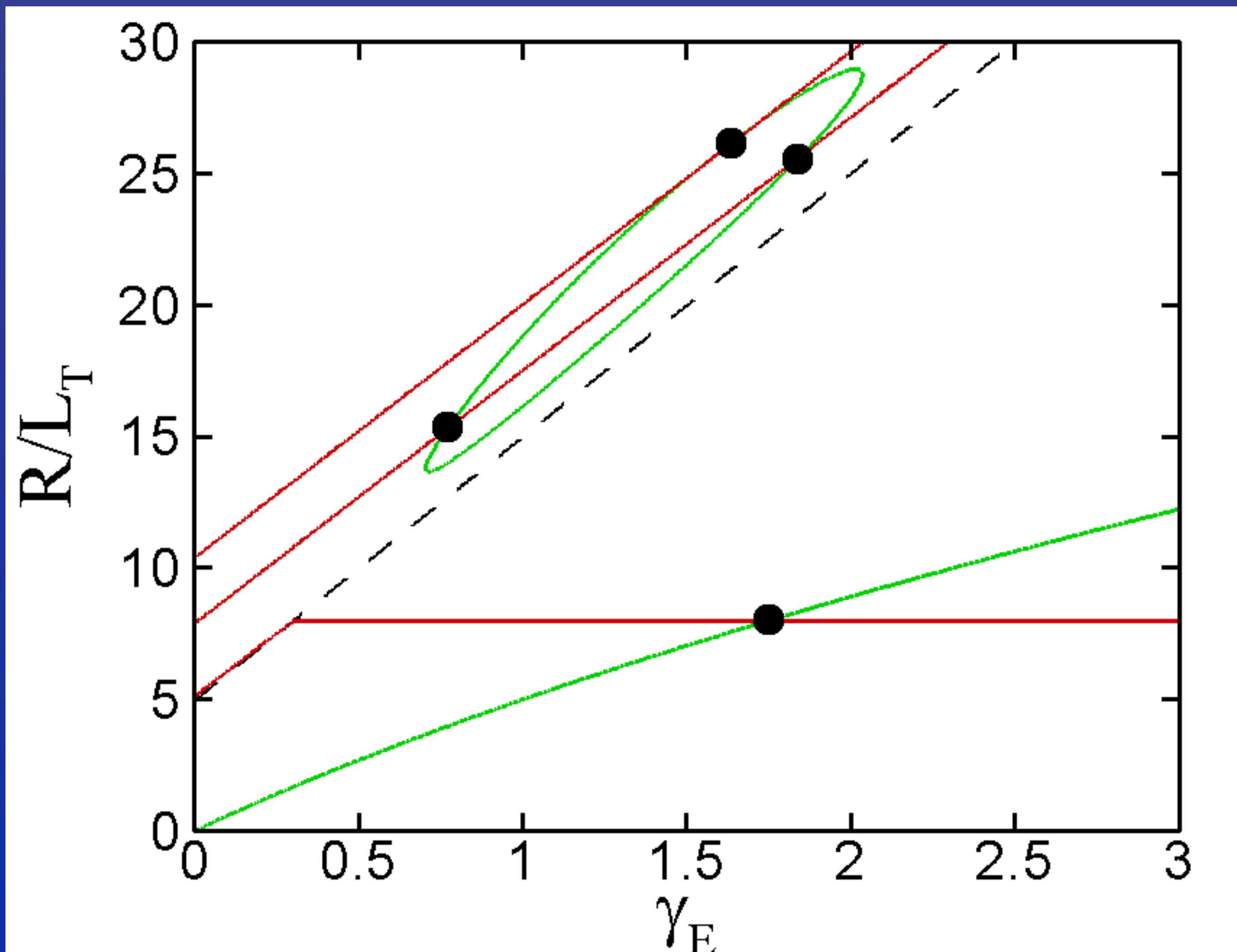

$\alpha_t \alpha_n > 0$

# Curves of constant $\Pi/Q$

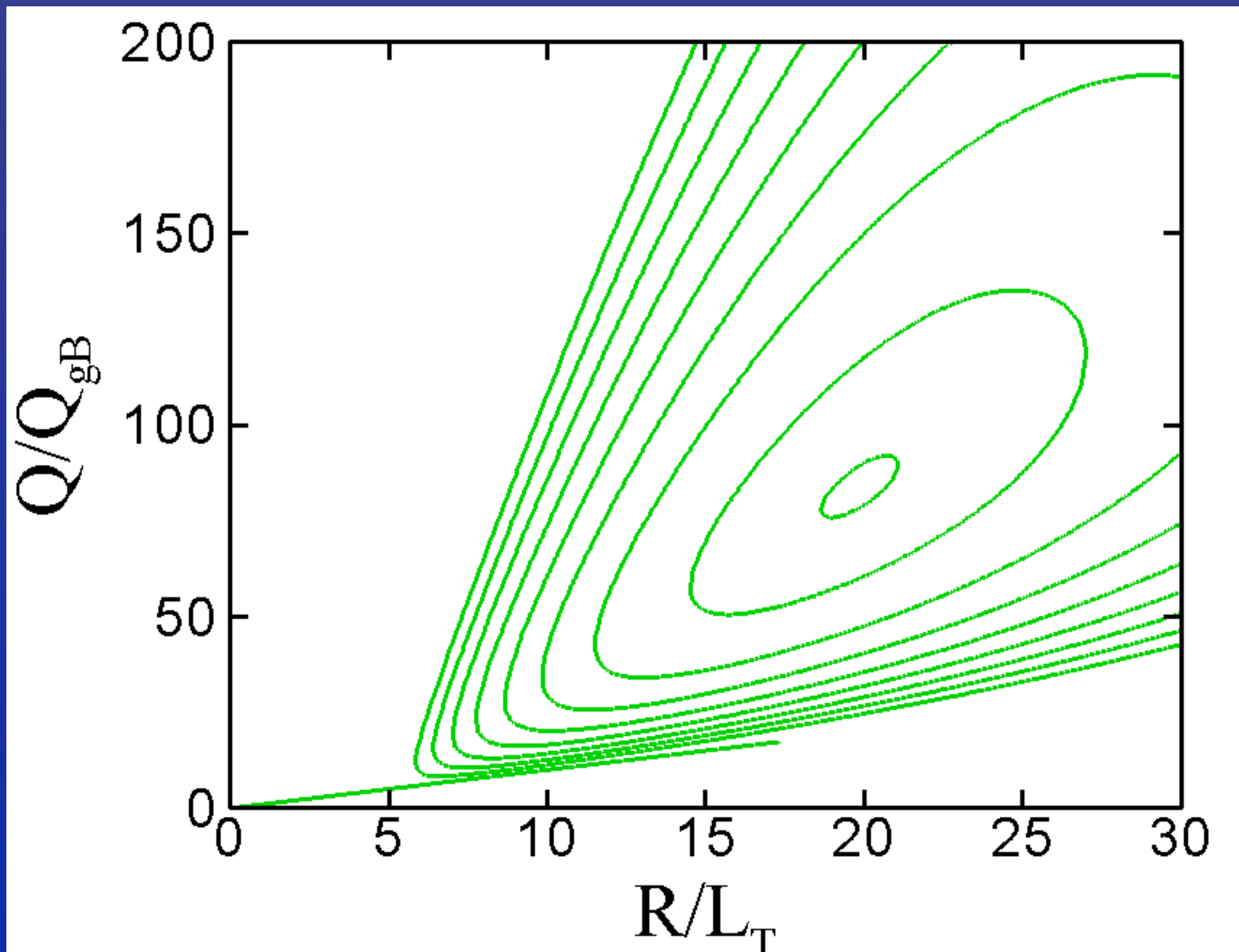



$$\alpha_t \alpha_n < 0$$

# Intrinsic rotation bifurcation



# Total energy flux



# Intrinsic rotation bifurcation

- There is a power threshold
- Very speculative
- Requires high energy input

# Overview

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- Extension to radial profiles (1D)

# Why do radial profile analysis?

- Ultimately, we want to predict mean profiles
- Magnetic geometry varies radially. Want to know where barrier forms
- Can address Coriolis pinch, turbulent and viscous heating, temperature equilibration, etc
- Inverse problem more forgiving (stiffness phenomenon reversed)

# Multiple scale problem

| Physics | Perpendicular spatial scale | Temporal scale |
|---|---|---|
| Turbulence from ETG modes | $k_\perp^{-1}$ ~ 0.005 – 0.05 cm | $\omega_*$ ~ 0.5 - 5.0 MHz |
| Turbulence from ITG modes | $k_\perp^{-1}$ ~ 0.3 - 3.0 cm | $\omega_*$ ~ 10 - 100 kHz |
| Transport barriers | Measurements suggest width ~ 1 - 10 cm | 100 ms or more in core? |
| Discharge evolution | Profile scales ~ 200 cm | Energy confinement time ~ 2 - 4 s |

simulation cost: $(L_\parallel/\Delta_\parallel) \times (L_\perp/\Delta_\perp)^2 \times (L_v/\Delta_v)^2 \times (L_t/\Delta t) \sim 10^{21}$

# Transport equations in GK

Moment equations for evolution of mean quantities:

$$\frac{\partial n_s}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \mathbf{\Gamma}_s \cdot \nabla\psi\right\rangle\right) + S_n$$

$$\begin{aligned}\frac{3}{2}\frac{\partial n_s T_s}{\partial t} &= -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \mathbf{Q}_s \cdot \nabla\psi\right\rangle\right) \\ &+ T_s\left(\frac{\partial \ln n_s}{\partial \psi} - \frac{3}{2}\frac{\partial \ln T_s}{\partial \psi}\right)\left\langle \mathbf{\Gamma}_s \cdot \nabla\psi\right\rangle + \frac{\partial \ln T_s}{\partial \psi}\left\langle \mathbf{Q}_s \cdot \nabla\psi\right\rangle \\ &- \left\langle \int d^3v \frac{h_s T_s}{F_{0s}}\left\langle C[h_s]\right\rangle_{\mathbf{R}}\right\rangle + n_s \nu_\epsilon^{su}\left(T_u - T_s\right) + S_p\end{aligned}$$

$$\frac{\partial L}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\sum_s \left\langle \pi_s\right\rangle\right) + S_L$$

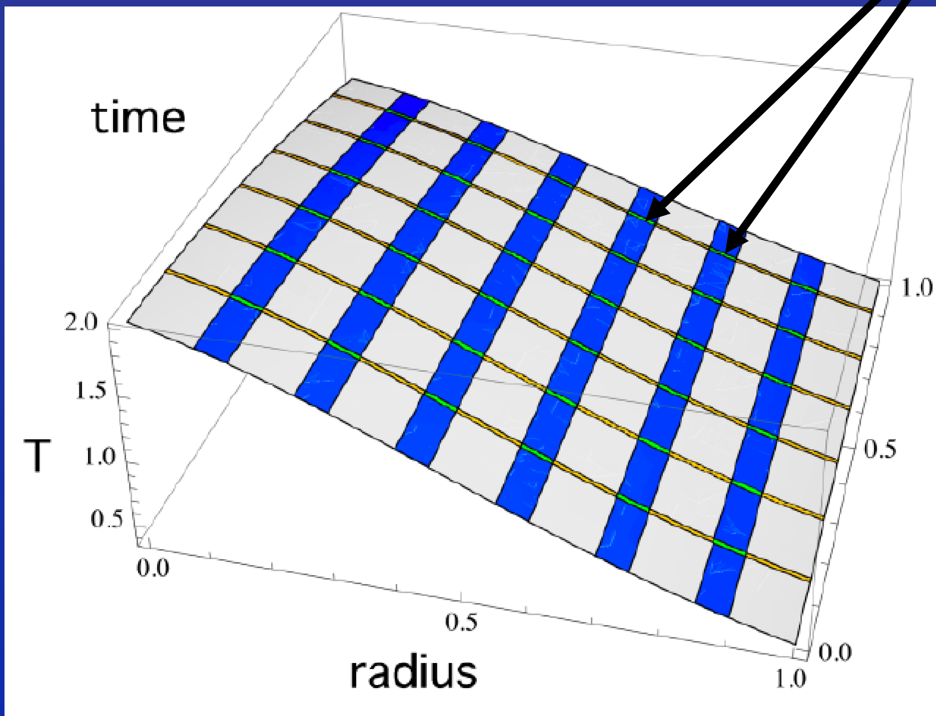
...depend on fluctuations

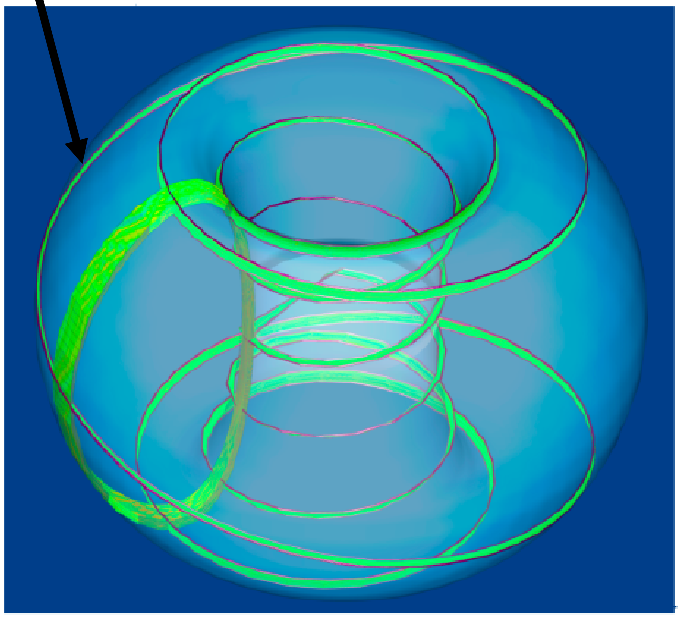
Sugama (1997), Abel (2010)

# Multiscale grid

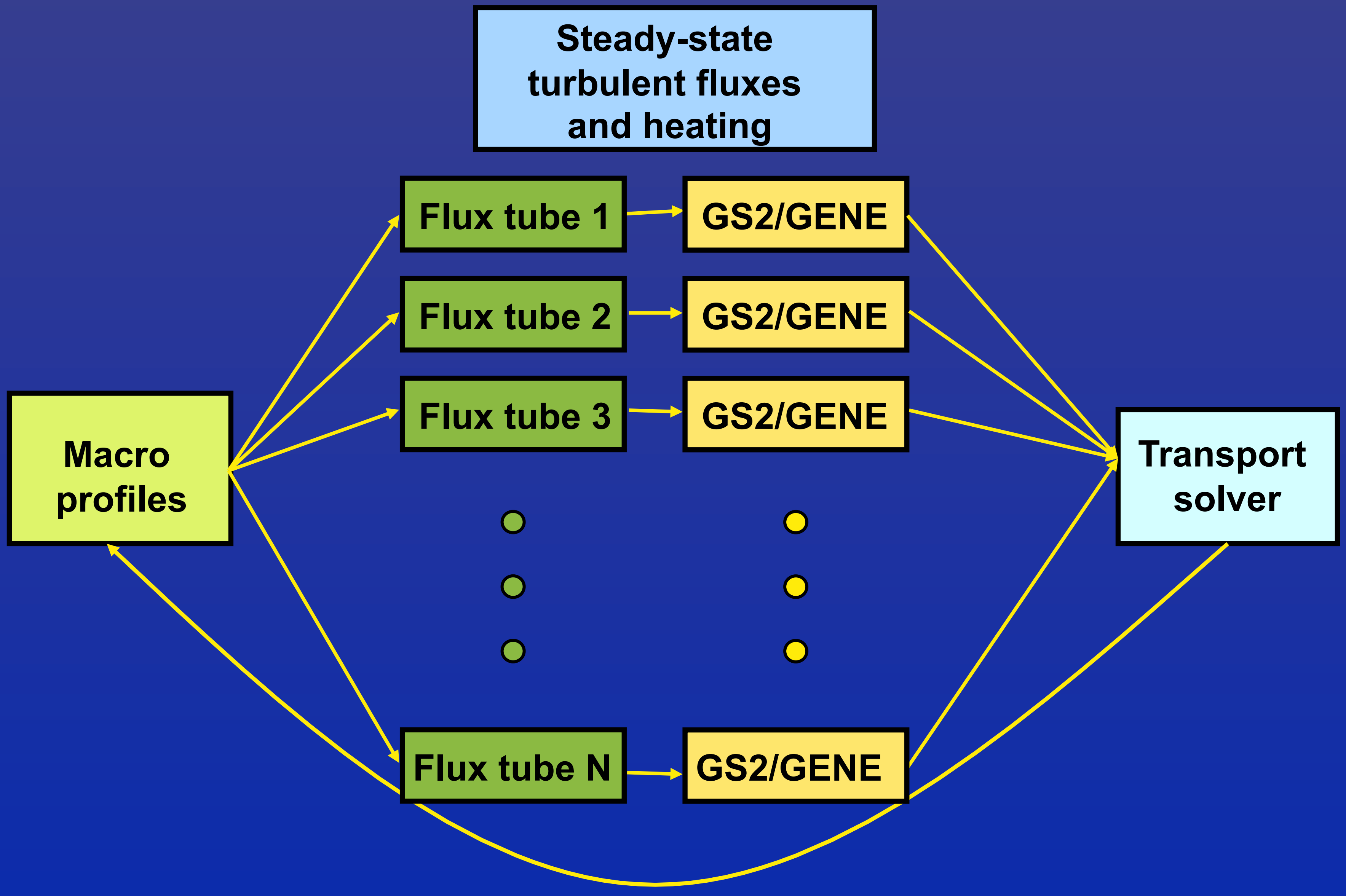
- Turbulent fluctuations calculated in small regions of fine space-time grid embedded in “coarse” grid (for mean quantities)

Flux tube simulation domain

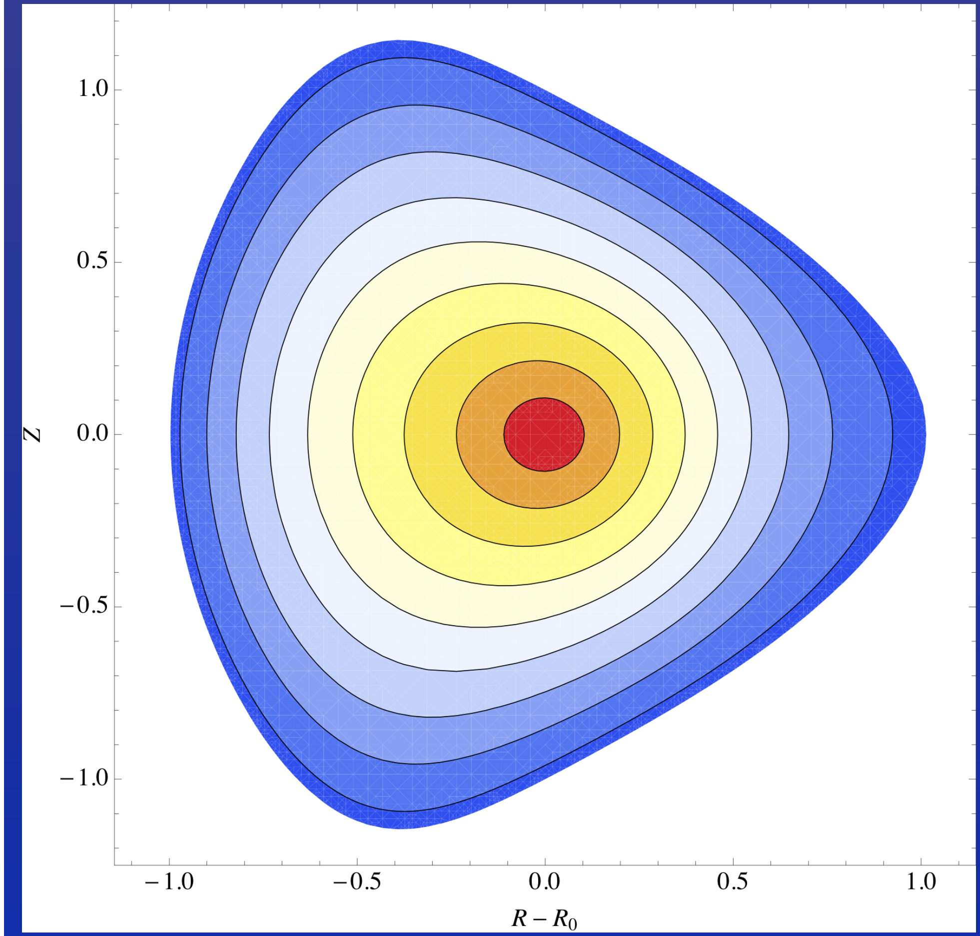


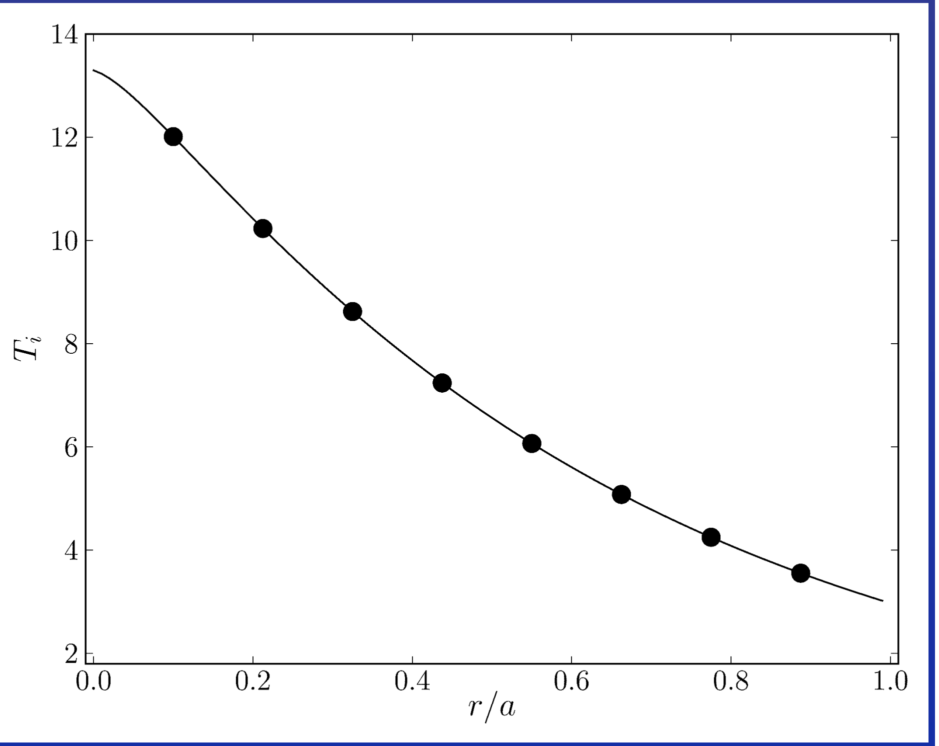



# TRINITY schematic



# Sampling profile with flux tubes



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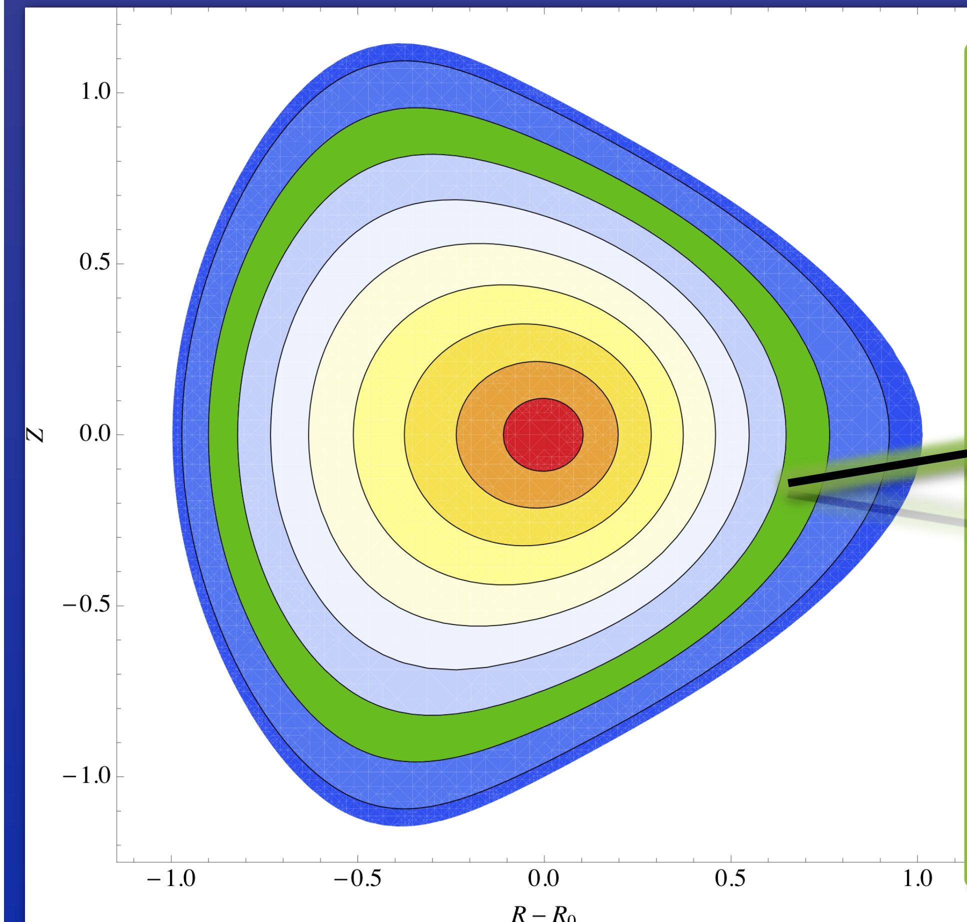
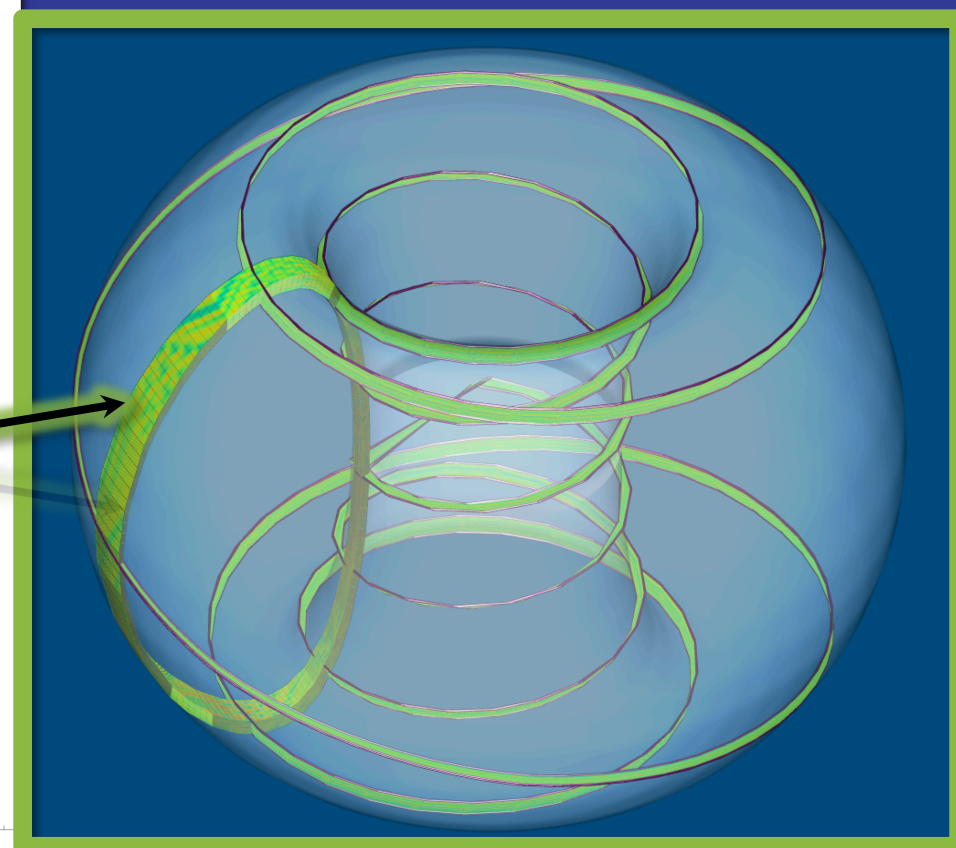


Simulation volume reduced by factor of ~10s

# TRINITY transport solver

- Transport equations are stiff, nonlinear PDEs. Implicit treatment via Newton's Method (multi-step BDF, adaptive time step) allows for time steps ~0.1 seconds (vs. turbulence sim time ~0.001 seconds)
- Challenge: requires computation of quantities like

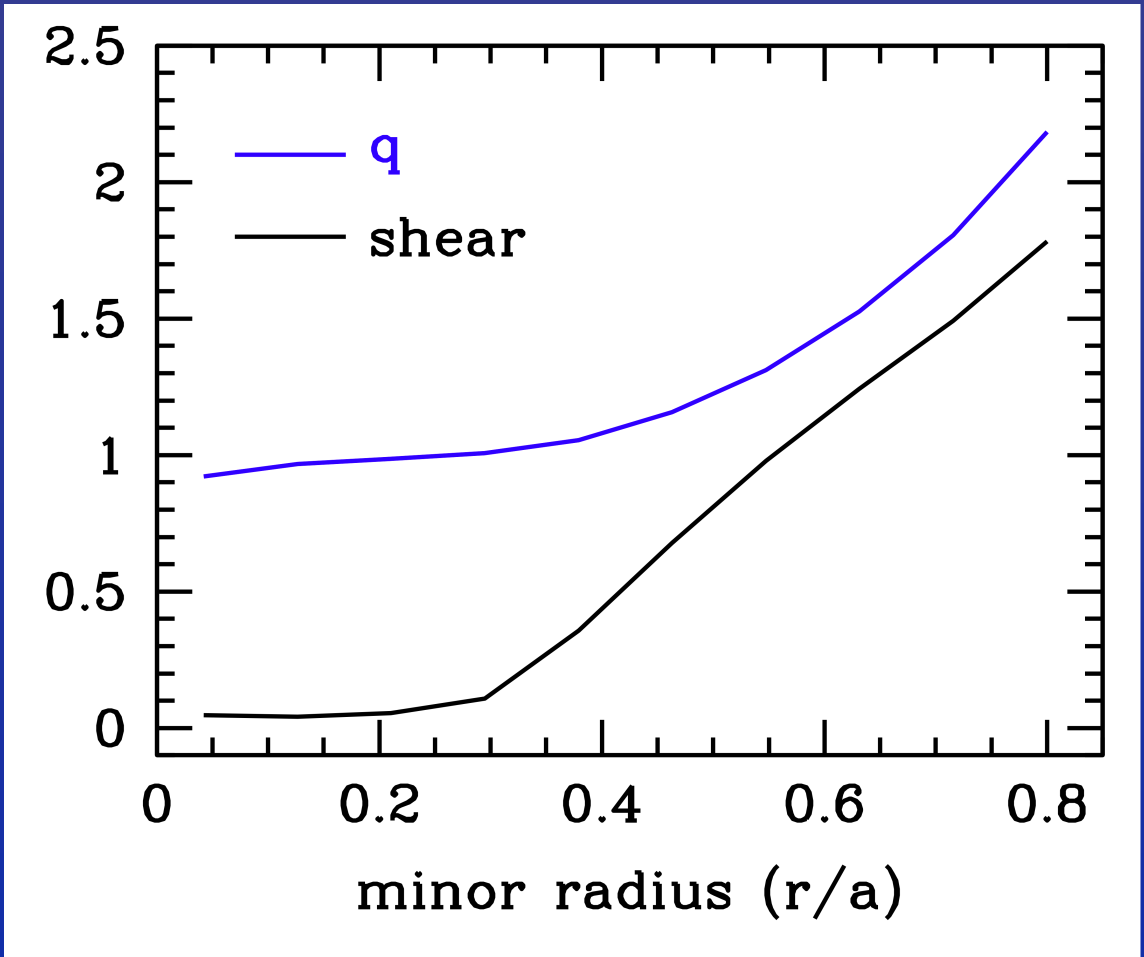
$$\Gamma_j^{m+1} \approx \Gamma_j^m + \left(\mathbf{y}^{m+1} - \mathbf{y}^m\right) \left.\frac{\partial \Gamma_j}{\partial \mathbf{y}}\right|_{\mathbf{y}^m} \qquad \mathbf{y} = \left[\{n_k\}, \{p_{i_k}\}, \{p_{e_k}\}\right]^T$$

- Local approximation: $\displaystyle \frac{\partial \Gamma_j}{\partial n_k} = \frac{\partial \Gamma_j}{\partial n_j} + \frac{\partial \Gamma_j}{\partial (R/L_n)_j} \frac{\partial (R/L_n)_j}{\partial n_k}$
- Simplifying assumption: normalized fluxes depend primarily on gradient scale lengths

# TRINITY scaling

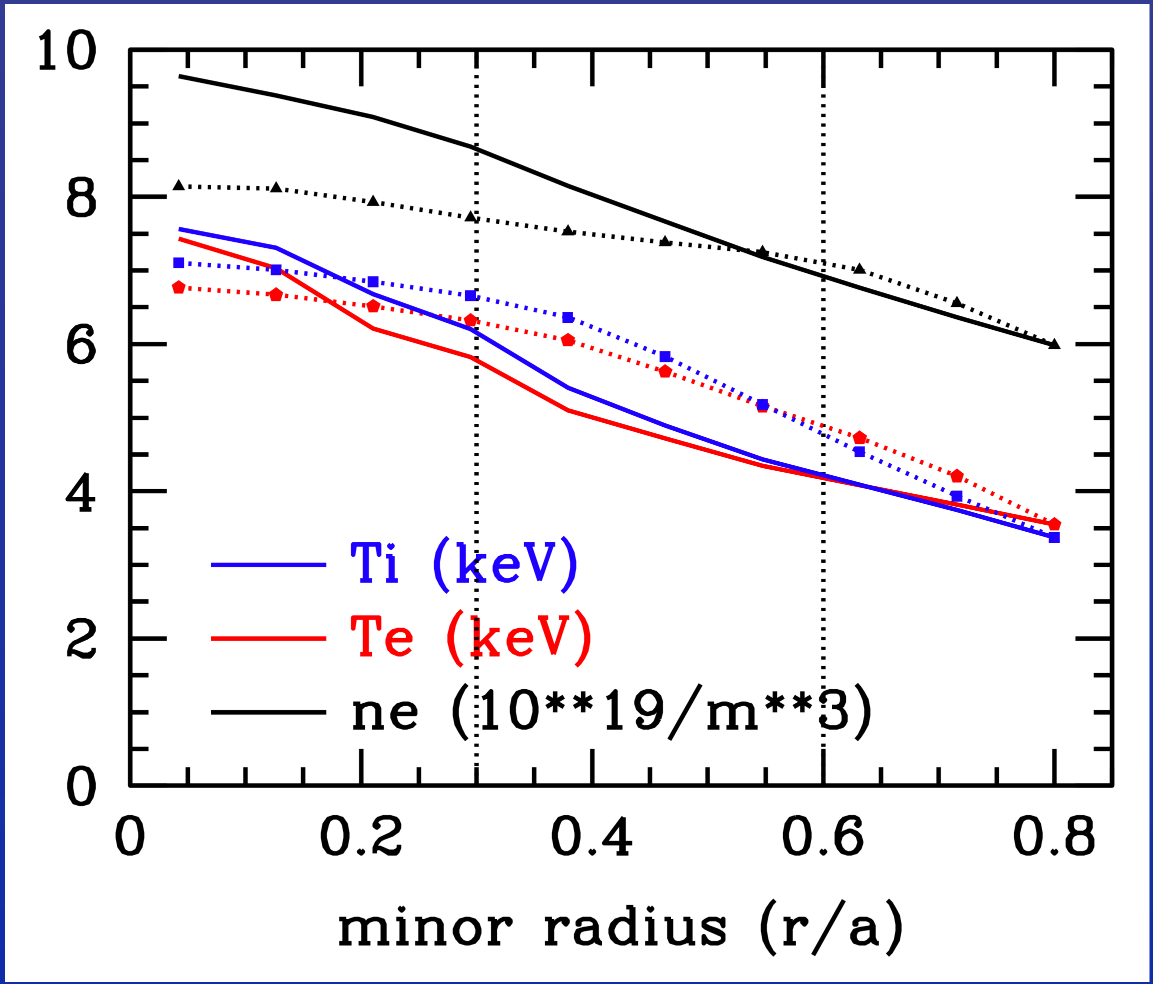
- Example calculation with 10 radial grid points:
  - evolve density, toroidal angular momentum, and electron/ion pressures
  - simultaneously calculate fluxes for equilibrium profile and for 4 separate profiles (one for each perturbed gradient scale length)
  - total of 50 flux tube simulations running simultaneously
  - ~2000-4000 processors per flux tube => scaling to over 100,000 processors with high efficiency

# JET shot #42982



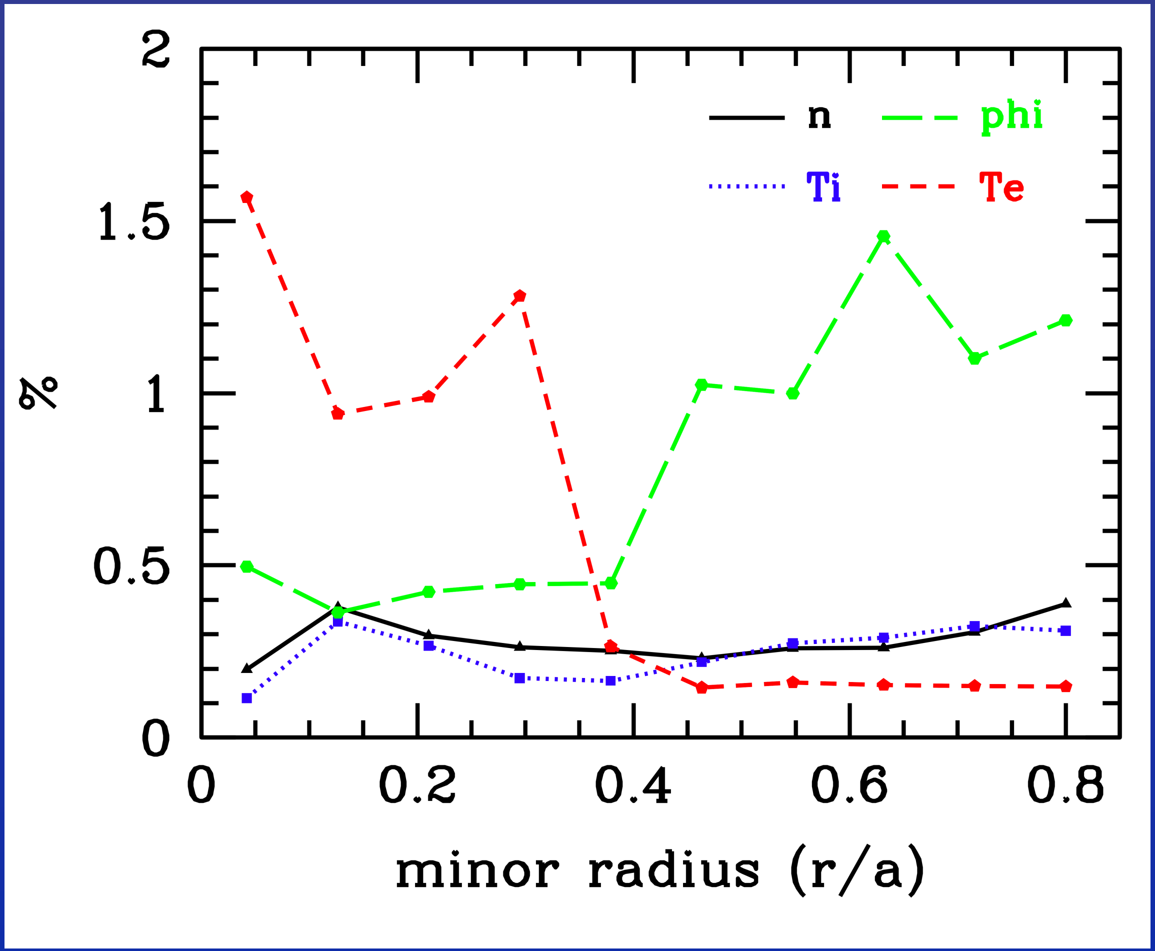

- ITER demo discharge
- H-mode D-T plasma, record fusion energy yield
- Miller local equilibrium model: q, shear, shaping
- B = 3.9 T on axis
- TRANSP fits to experimental data taken from ITER profile database

# Evolving density profile




- 10 radial grid points
- Costs ~120k CPU hrs (<10 clock hrs)
- Dens and temp profiles agree within ~15% across device
- Energy off by 5%
- Incremental energy off by 15%
- Flow shear absent

# Fluctuations



# Conclusions

- Maximum temperature gradient for given heat flux. Occurs at finite flow shear.
- Turbulent Prandtl number is constant of order unity for moderate to large flow shear values.
- Stiffness modestly decreased for high flow shear (PVG driven turbulence). Main effect at low flow shear is upshift of critical temperature gradient
- Two possible bifurcation types in 0D model:
  - Neoclassical bifurcation (observed from GS2 simulations)
  - Intrinsic rotation bifurcation (demonstrates power threshold)
- Current work focuses on extension to self-consistent, 1D transport simulations

# TRINITY transport solver

- Calculating flux derivative approximations:
  - at every radial grid point, simultaneously calculate $\Gamma_j[(R/L_n)_j^m]$ and $\Gamma_j[(R/L_n)_j^m + \delta]$ using 2 different flux tubes
  - use 2-point finite differences:

$$\frac{\partial \Gamma_j}{\partial (R/L_n)_j} \approx \frac{\Gamma_j[(R/L_n)_j^m] - \Gamma_j[(R/L_n)_j^m + \delta]}{\delta}$$

  - possible because flux tubes independent (do not communicate during calculation)
  - perfect parallelization (almost)