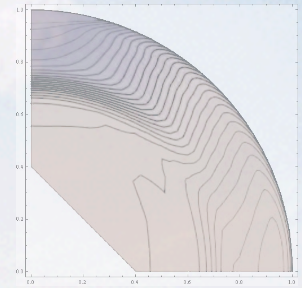
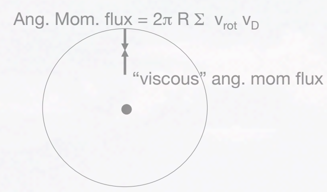
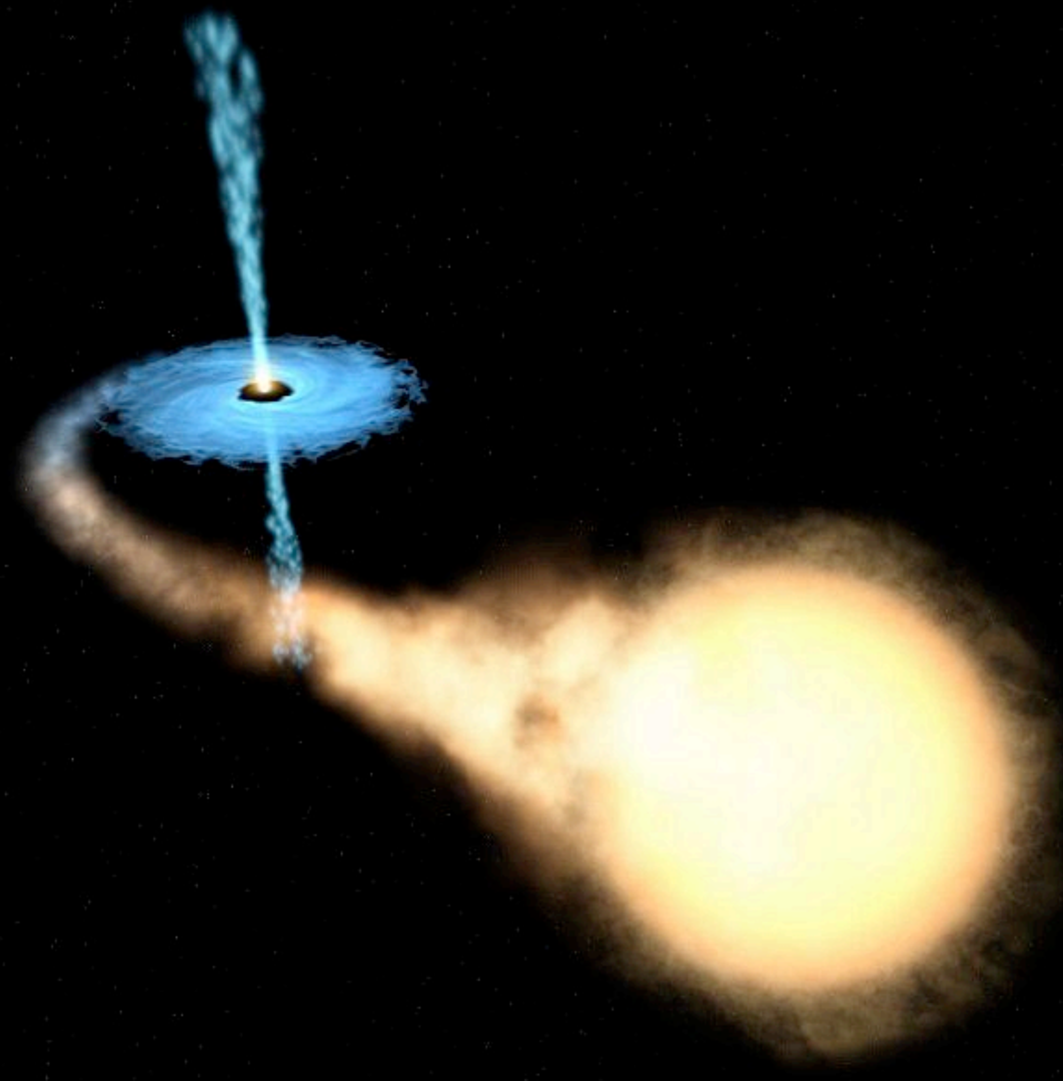
Transport in Disks and Stars





Steven Balbus
Ecole Normale Supérieure
Newton Institute 22.7.10



Accretion disks have acquired the status of an icon in modern astrophysics.

THE PROBLEM:

- *Accretion involves loss of angular momentum from fluid elements. How?*

- Ordinary viscosity is much too inefficient.

- Turbulence must be present.

This permits broad correlations of the form:

$$< \delta v_r \ \delta v_\varphi >$$

as opposed to: $\lambda c \, dv_\varphi/dr$

Angular Momentum Flux:

$$< \rho R^2 \quad v_R \, v_\varphi > \; = \; \rho R^2 \quad < \delta v_R \;\; \delta v_\varphi > \;\; + \rho R^3 \, \Omega \quad v_{DRIFT}$$

This angular momentum flux must be constant in a steady disk, so:

$$v_{DRIFT} \; = - < \delta v_R \;\; \delta v_\varphi > / \, R\Omega$$

A fluctuation dissipation relation.

Energy Flux:

$$< 0.5\ \rho R v_R v_\varphi^2 > =$$

$$\rho R v_\varphi < \delta v_R\ \delta v_\varphi > + 0.5\ \rho R v_\varphi^2 v_{DRIFT}$$

$$= -0.5\ \rho R v_\varphi^2 v_{DRIFT}, \quad \text{plus}$$

the potential energy drift, $-\rho R v_\varphi^2 v_{DRIFT}$, gives

$$-1.5\ \rho R v_\varphi^2 v_{DRIFT}.$$

Energy must be dissipated by this transport:

Energy Flux = -3/2 $\rho R v_\varphi^2 v_{DRIFT}$

Accretion rate $2 \pi \rho R v_{DRIFT}$ is constant, so the Energy Flux is *NOT* conserved:

The energy dissipated per unit area in the disk is

$$1.5 (\rho H) v_\varphi^2 v_{DRIFT} = (3/4\pi) GM\, Mdot/R^3$$

Lies, damned lies, and turbulence phenomenology

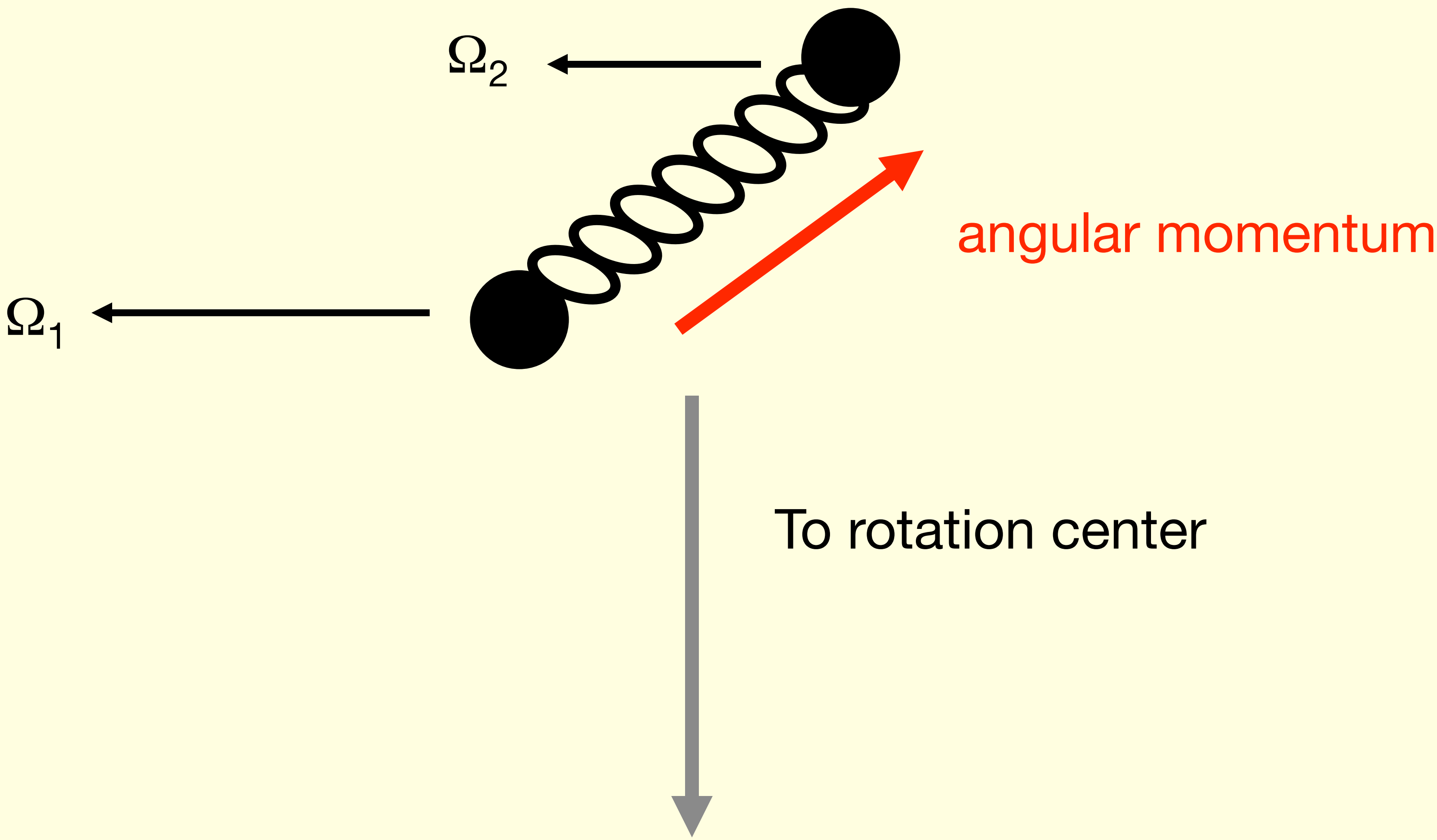
All well and good..... but δv_r and δv_φ lack phase coherence in a Keplerian disk. The correlation $< \delta v_r \delta v_\varphi >$ is ~ zero (Ji et al. 2006).

That is why hydrodynamical shear turbulence theories of accretion disks don't work. Lack of phase coherence is often a reason why large turb. flucs. don't transport well.

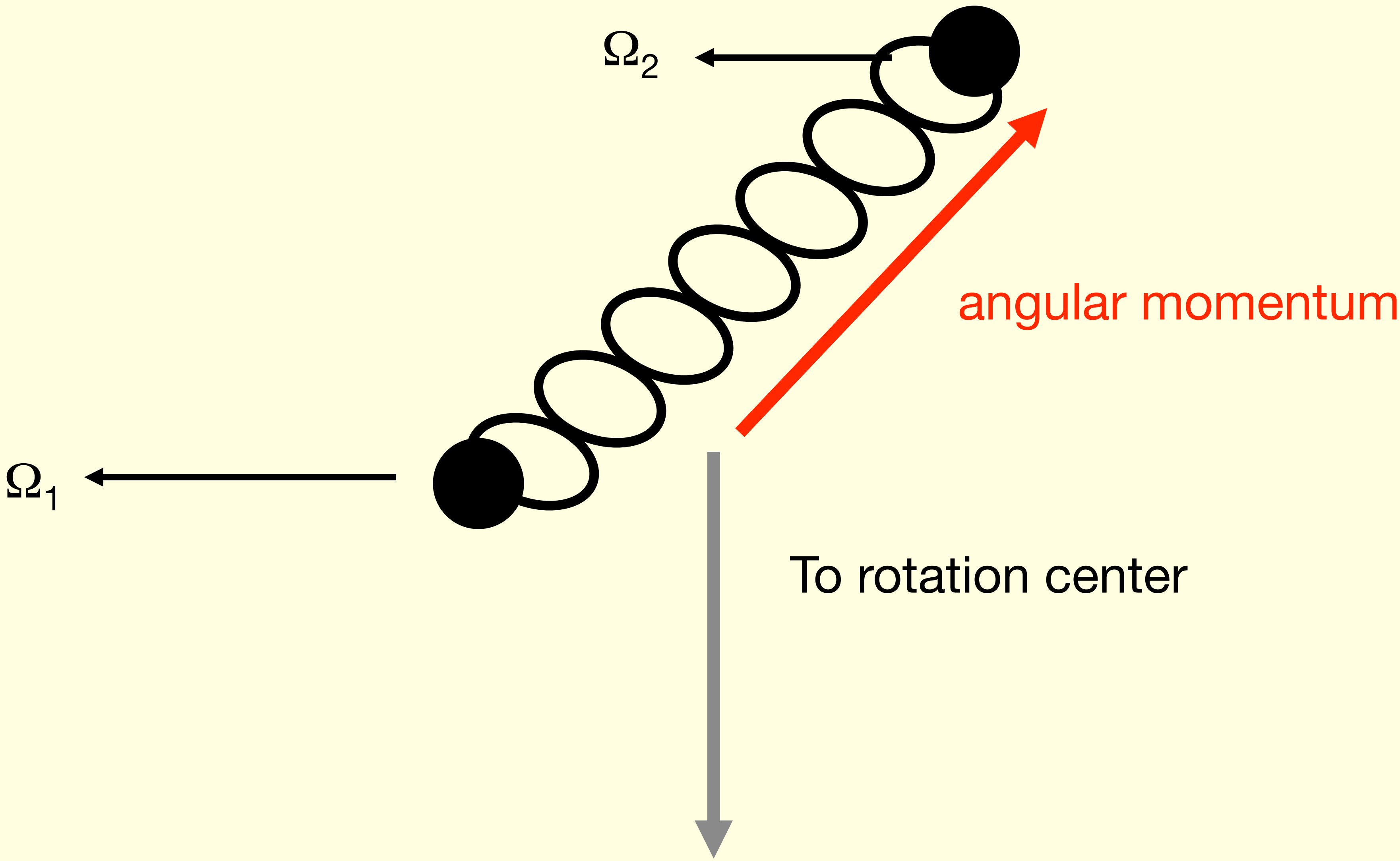
Lies, damned lies, and turbulence phenomenology

All of this changes when a magnetic field is added...

Schematic MRI



Schematic MRI



MRI Stability Criterion:

If perturbations $\sim e^{ikz}$,

$$\boxed{(kv_A)^2 + d\Omega^2/d \ln R > 0}$$

for stability ($v_A{}^2 = B^2/\mu\rho$) .

MRI:

Introduces powerful correlations between (δv_R & δv_φ) as well as (v_{AR} & $v_{A\varphi}$)

$v_A^2 = B^2/\mu\rho$.

Upshot:

Flux arguments go through just as before, but change $<\delta v_R \; \delta v_\varphi>$

to $<\delta v_R \; \delta v_\varphi - v_{AR} \, v_{A\varphi}>$.

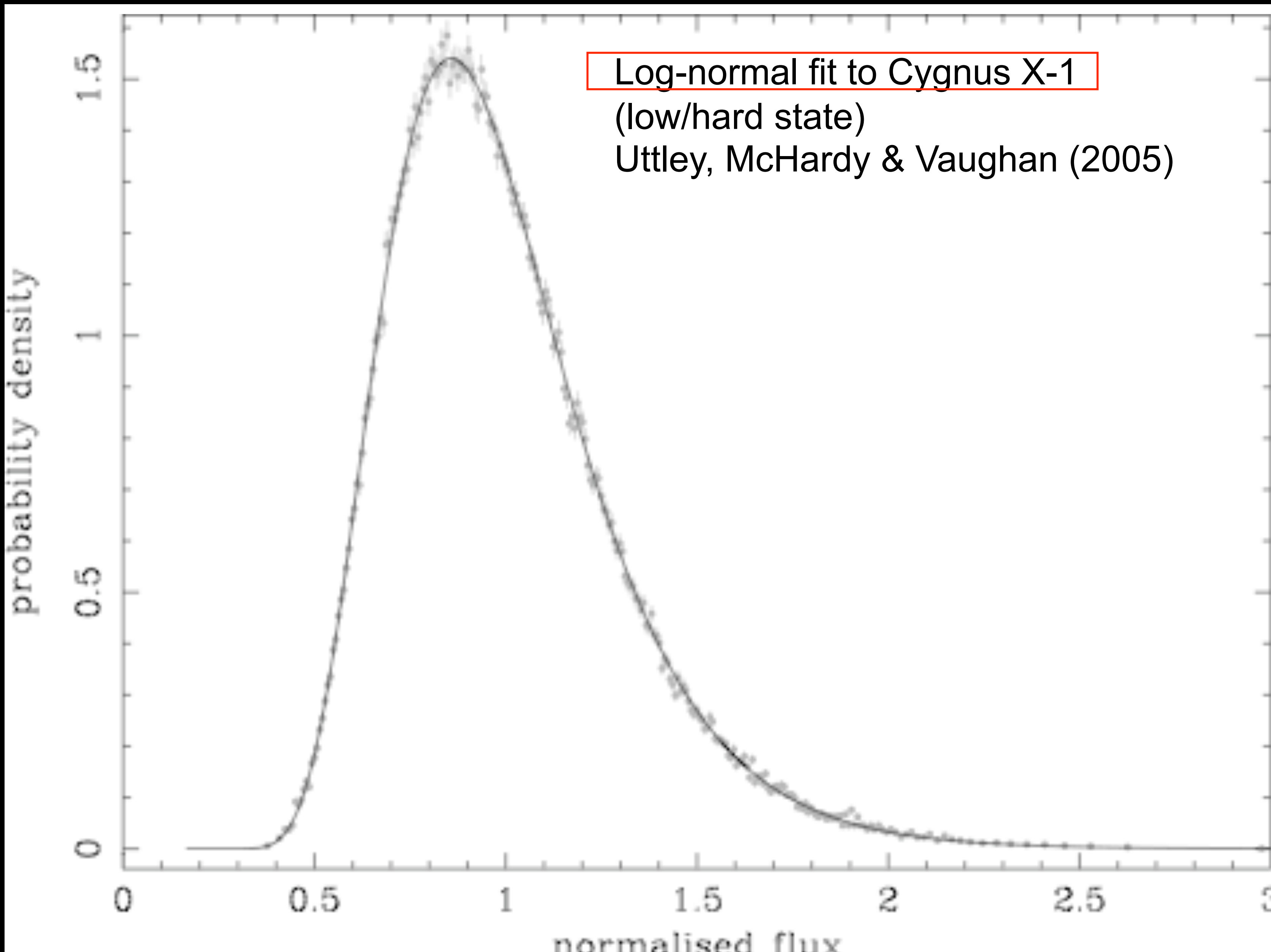
The Stress tensor:

$$T_{R\varphi} = < \rho \left(\delta v_R \, \delta v_\varphi - v_{AR} \, v_{A\varphi} \right) >$$

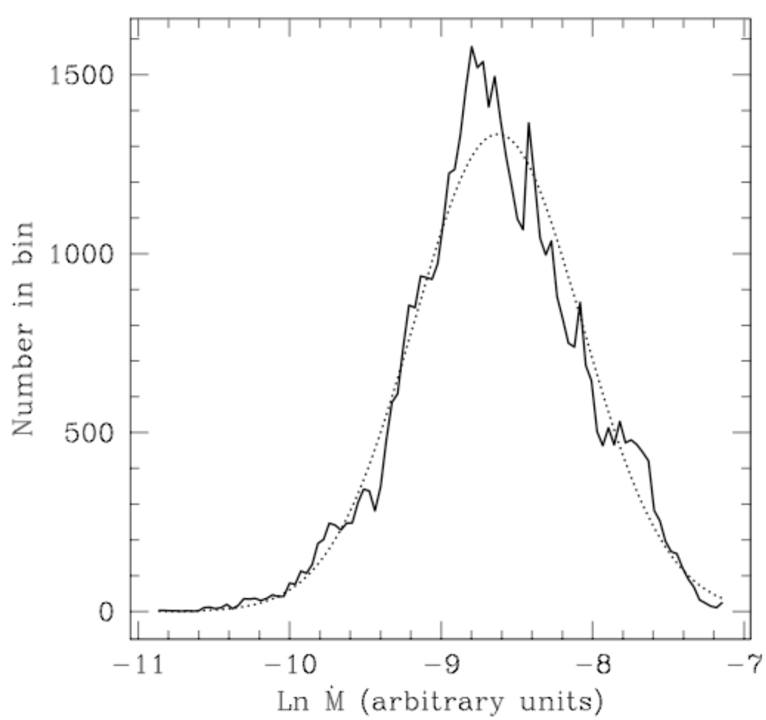
Energy Extraction:

$$dE/dt = -\, T_{R\varphi} \quad d\Omega/d \ln R$$

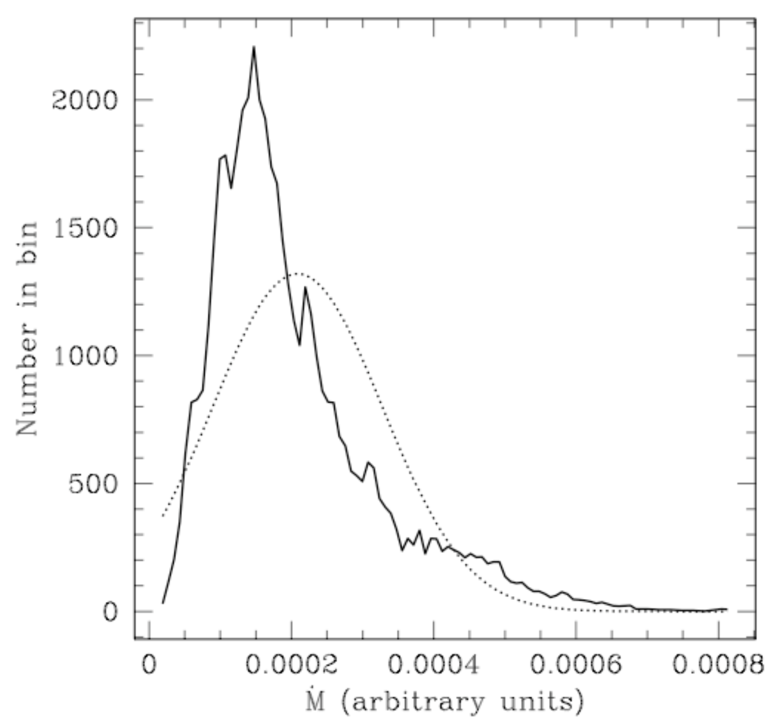
This is the rate at which energy is exchanged with differential rotation. For local turb., it is also the rate of dissipation. For waves, the energy need not be dissipated.


Log-normal fit to Cygnus X-1
(low/hard state)
Uttley, McHardy & Vaughan (2005)
probability density
normalised flux
0
0.5
1
1.5
2
2.5
3

Non-Gaussianity in numerical simulations.



Log-normal fit



Gaussian fit

(From Reynolds et al. 2008)

Why is MRI lognormal?

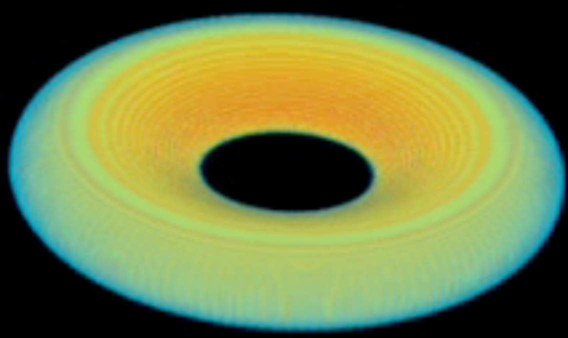
• The MRI exhibits linear local exponential growth that is abruptly terminated when fluid elements are mixed.

• Lifetime of linear growth is a random gaussian (symmetric bell-shaped) variable, *t.*

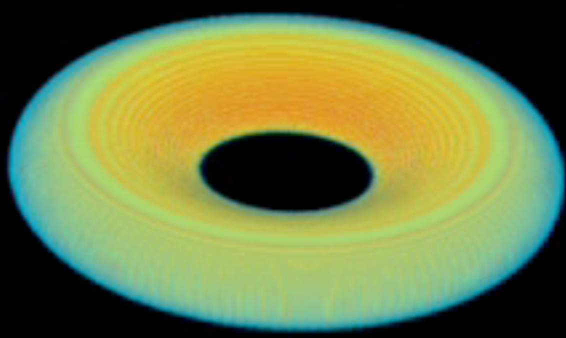
• *Local* amplitudes of fields grow like *exp(at),* then themalized and radiated; responsible for luminosity.

• If *t* is a gaussian random variable, then *exp(at)* is a *lognormal random variable.*

Global Simulations of the MRI, *Hawley 2000*



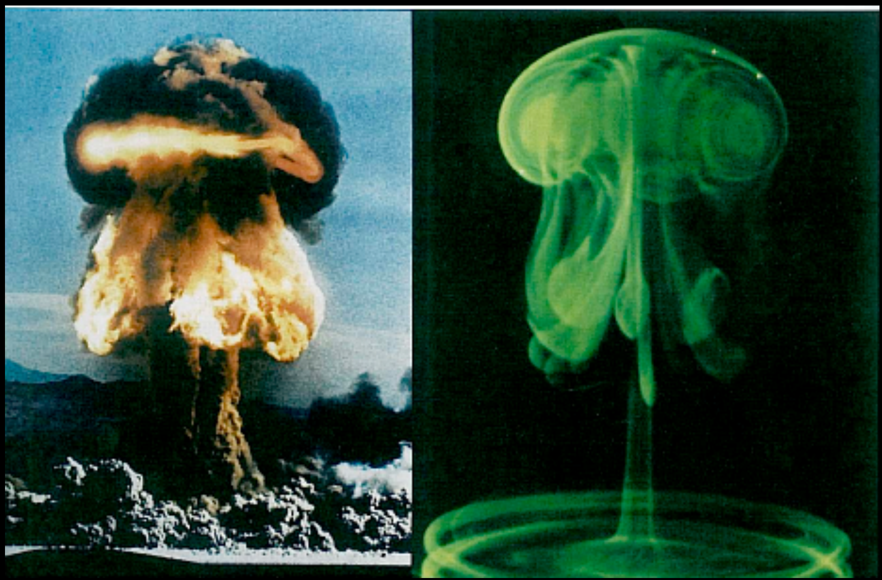
Meridional Plane



Equatorial Plane

MHD Turbulence ≠ Hydro Turbulence

The Kolmogorov picture of hydrodynamical turbulence (large scales insensitive to small scale dissipation) …



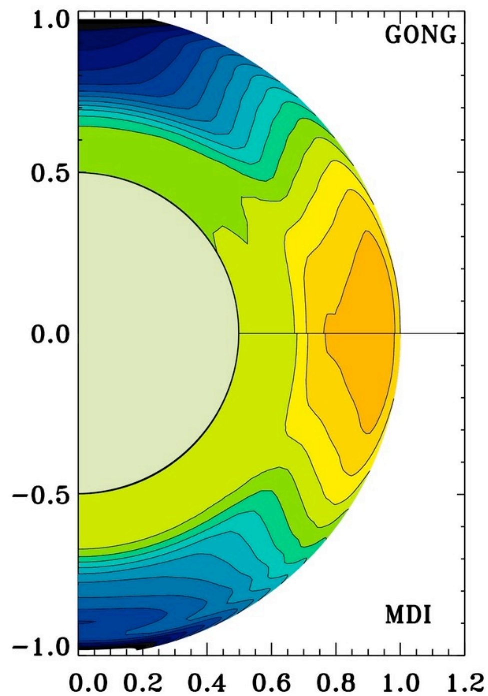
Re=10^{11} Re=10^{4}

…appears not to hold for MHD turbulence.

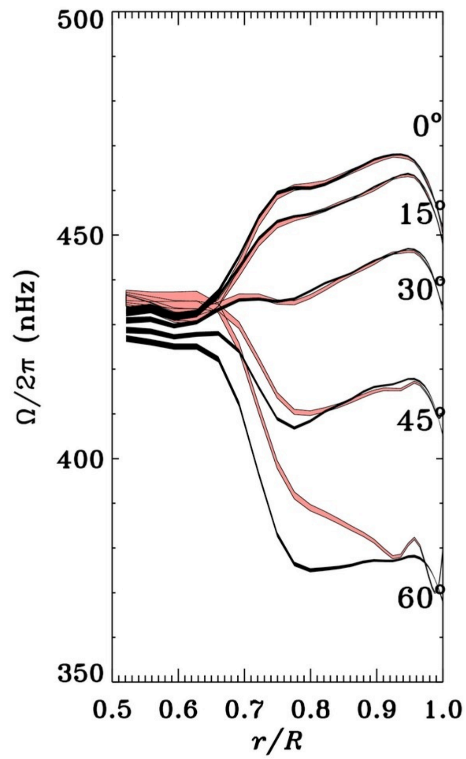
Disk turbulence folklore/issues:

1. Alpha models: $T = \alpha P$
2. What determines T? (Pm, Rm)
3. Is the turbulence local?
4. How does MHD turb. relate to astrophysical phenomenology? (Dead zones, disk states, QPOs)

STARS



GONG
MDI
1.0
0.5
0.0
−0.5
−1.0
0.0 0.2 0.4 0.6 0.8 1.0 1.2

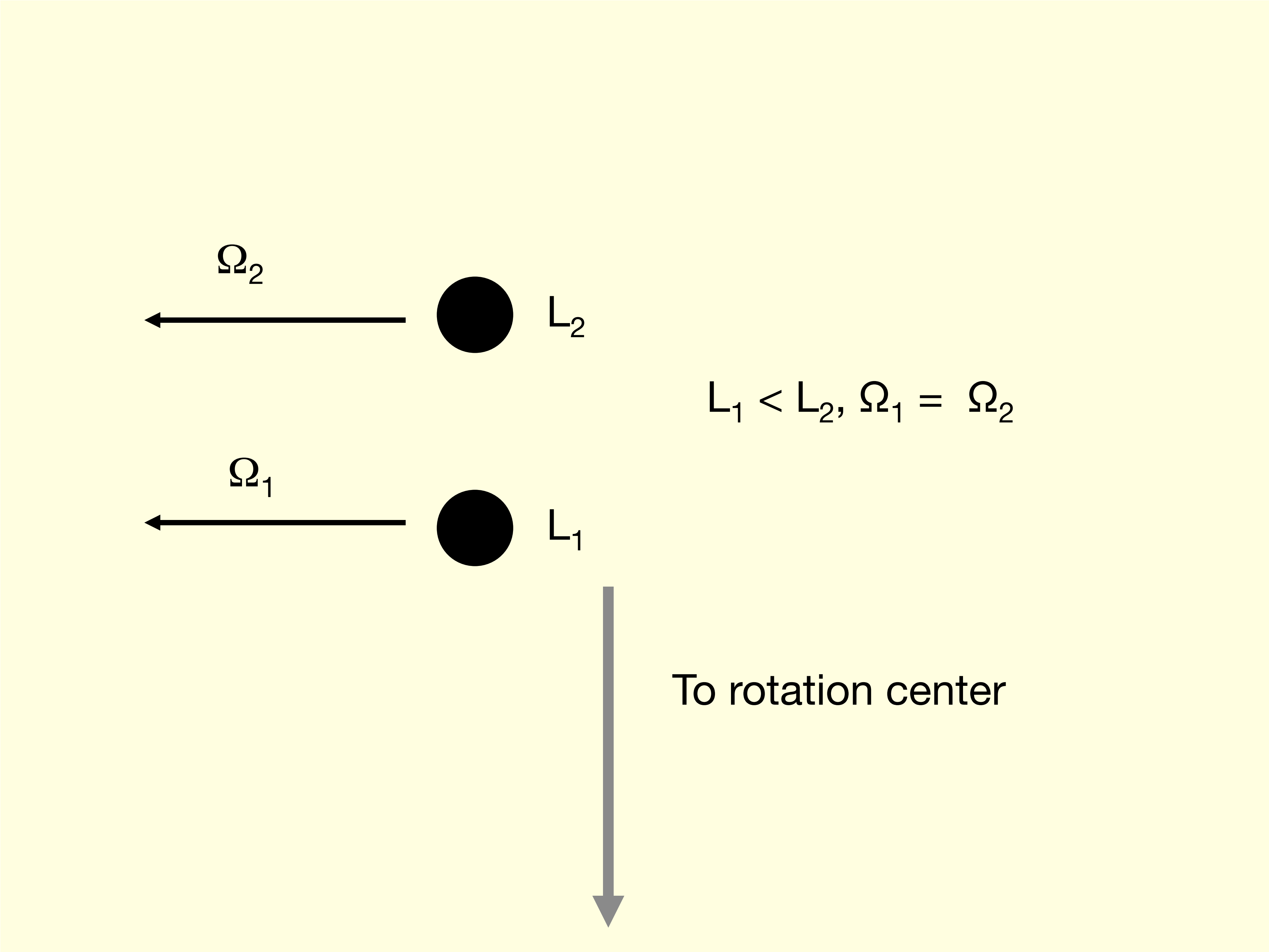


500
450
400
350
Ω/2π (nHz)
0°
15°
30°
45°
60°
0.5 0.6 0.7 0.8 0.9 1.0
r/R

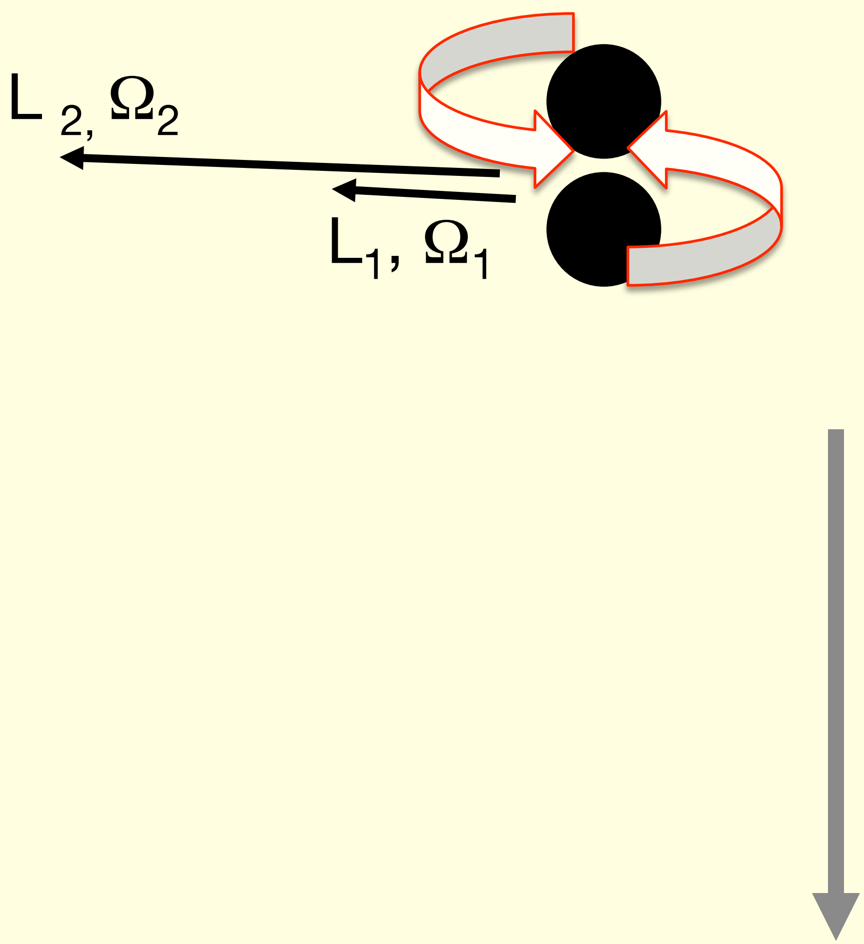
STELLAR CONVECTIVE ZONES:

Convective stars spontaneously develop differential rotation.

The $< \delta v \ \delta T>$ fluctuations are designed by nature to transport heat, not angular momentum.   Heat goes out, while angular momentum tends to go in ....

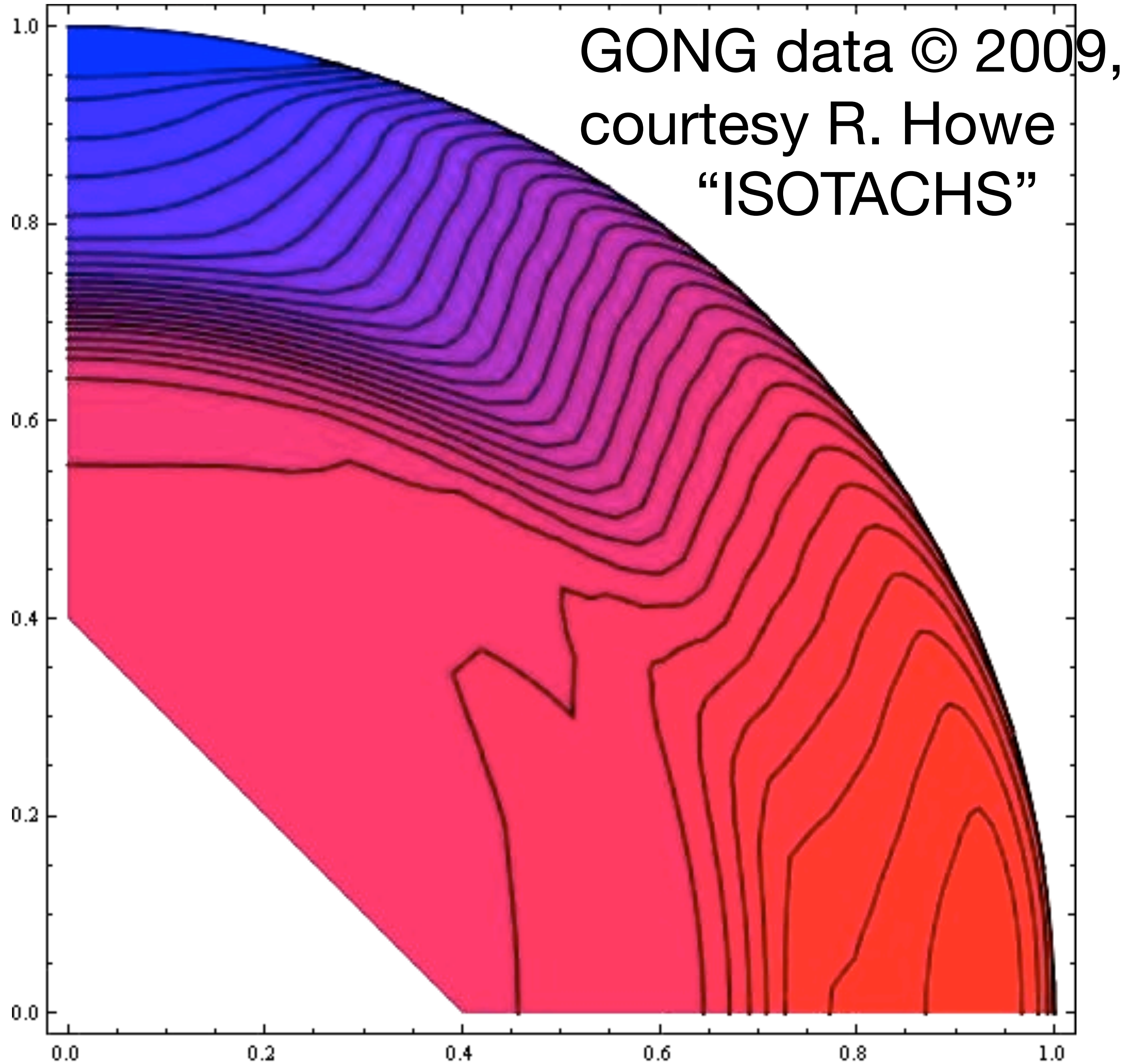


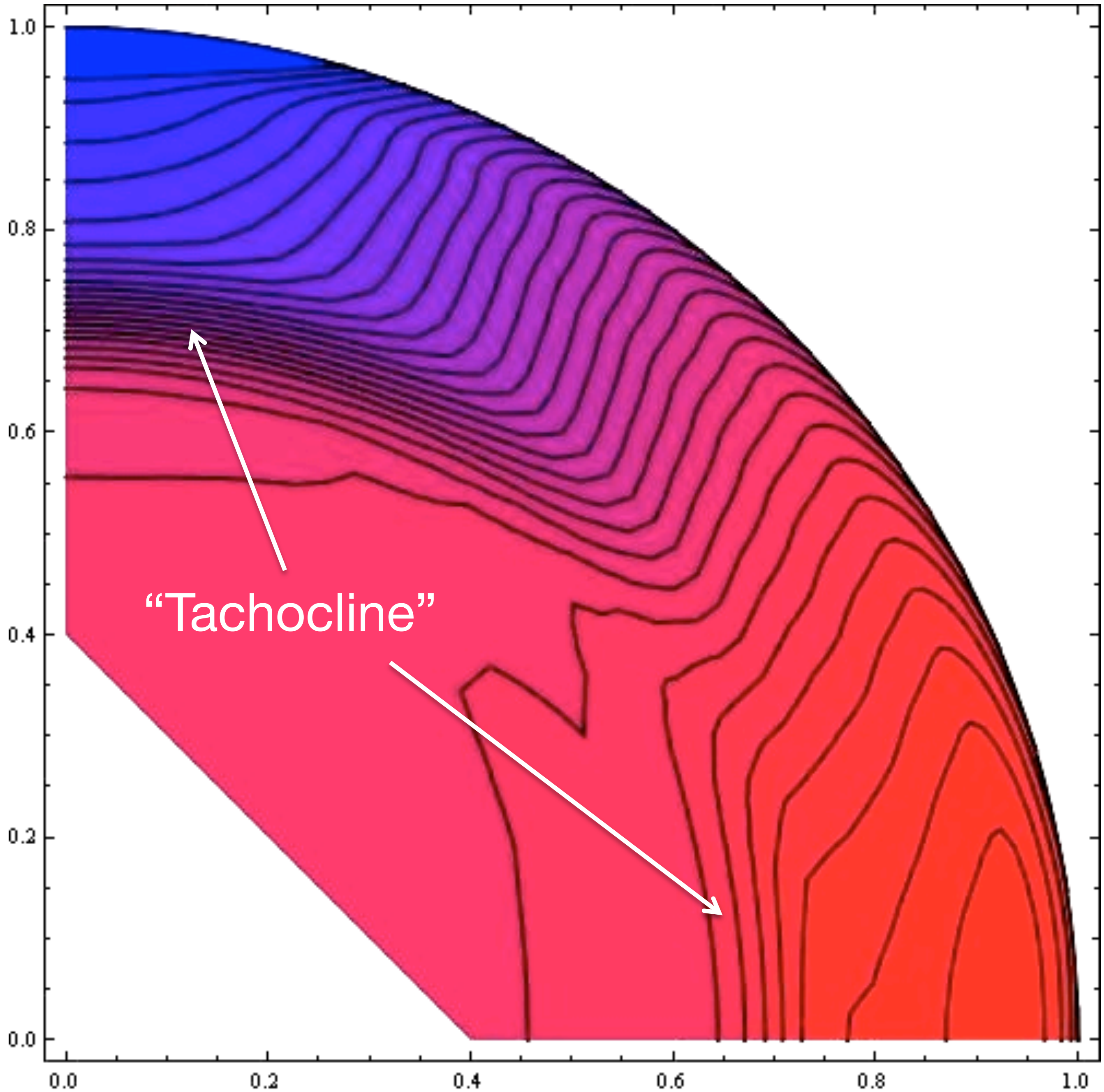
Ω_2
L_2
$L_1 < L_2, \Omega_1 = \Omega_2$
Ω_1
L_1
To rotation center

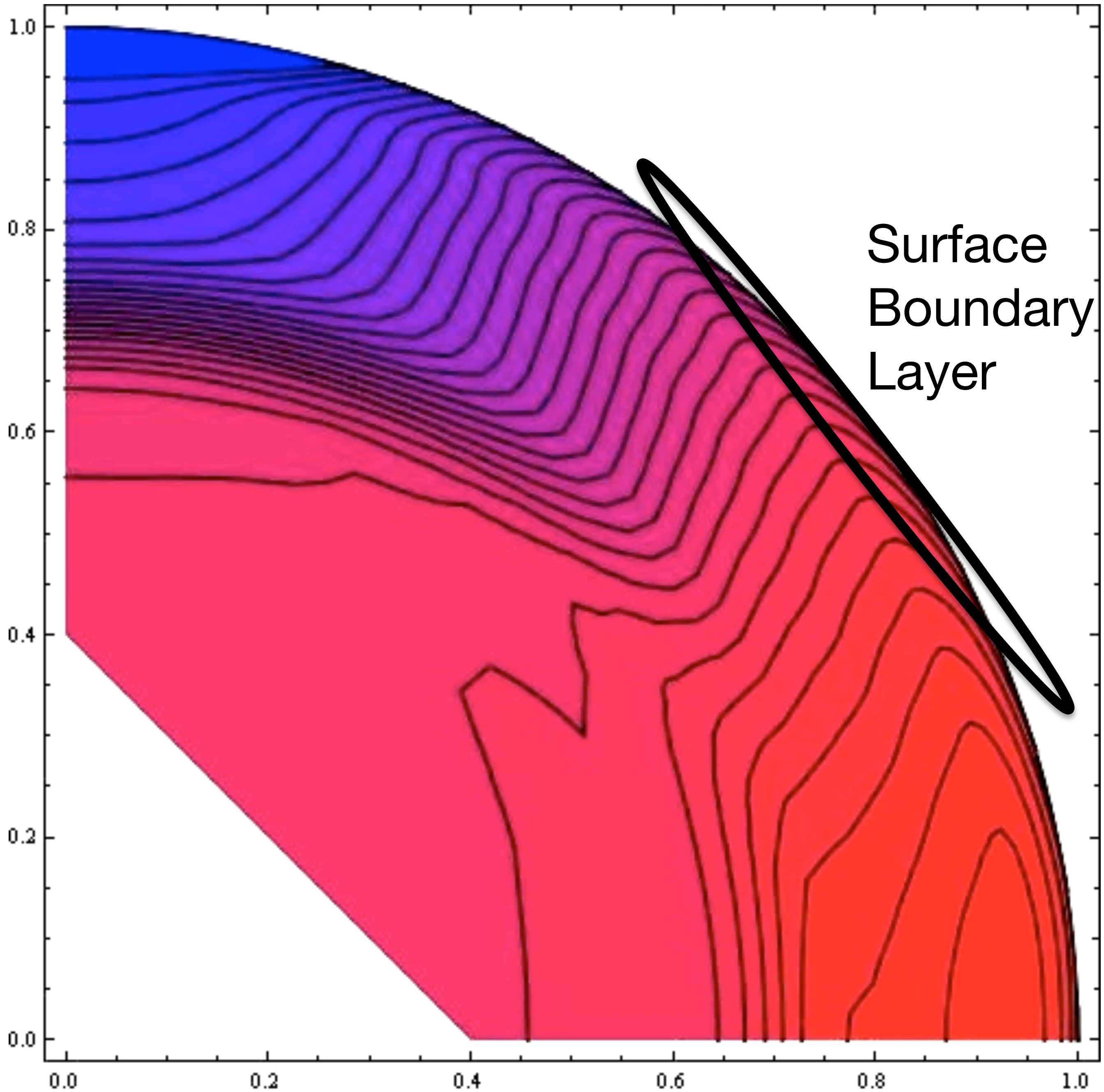


With L preserved, $L_1 < L_2$, and when elements mix, $\Omega_1 < \Omega_2$. Dissipation then requires inward transport.

To rotation center


GONG data © 2009,
courtesy R. Howe
"ISOTACHS"
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


"Tachocline"
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0



Surface
Boundary
Layer
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Convective transport and shear:

1. Convection mixes entropy S along the rolls. So. . .

2. In a spherical star, why is there a dS/ dr in steady-state?
Answer: require *some* dS/dr to maintain convective turbulence.
Sustained by radial, radiative driving from below. But . . .

3. With rotation, Coriolis induces *steady* 2D profile S = S(r, θ).
How can this be? Radiative driving from below can maintain only a 1D *radial profile,* and convection *is always mixing entropy.*

Excess entropy is well-mixed along "convective rolls"

Answer must be that the excess entropy

$S' = S(r, \theta) - S(r)$

is *indeed* well-mixed: it is

constant "along the convective rolls…"

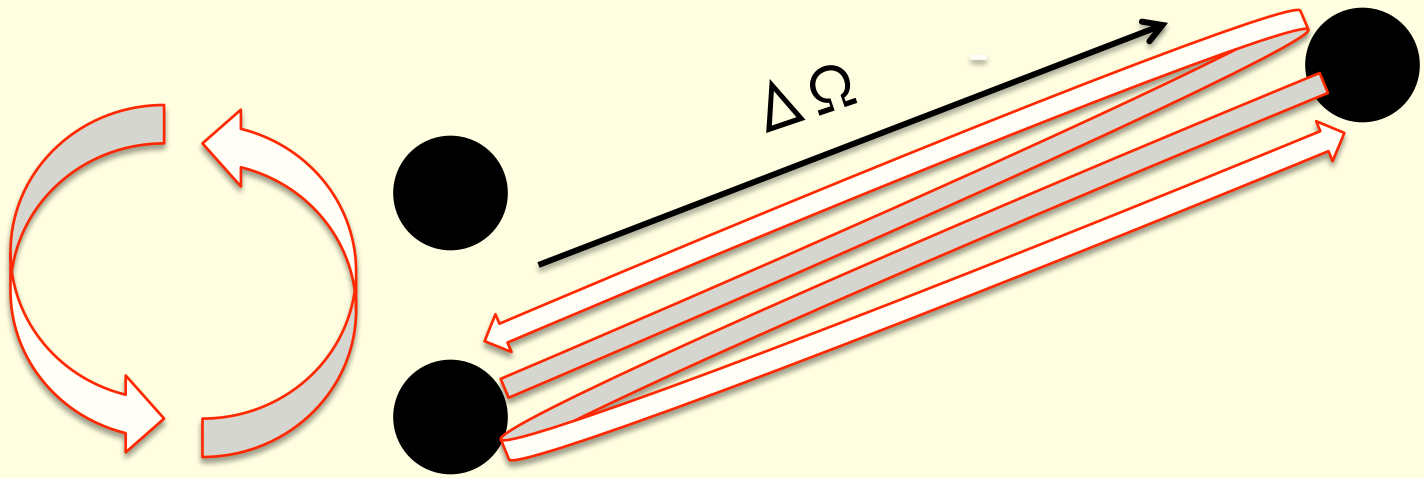
$$\mathbf{v} \cdot \nabla [\, S - S(r)\,] = 0.$$

Convective rolls live in --and maintain-- constant excess entropy surfaces.

BUT:

Convective rolls, ***especially coherent, long lived structures,*** are sheared into---and therefore also *live in*---surfaces of ***constant Ω .***

Shearing Convective Roll



A “Proof:

$\nabla \cdot \mathbf{v} = 0,$

$\mathbf{v} = \sum \mu(k) \exp(i\mathbf{k}\cdot\mathbf{x} - \mathbf{i}\omega t)$

$\mathbf{k} = \mathbf{k}(0) - mt\ \nabla\Omega,$

$\mathbf{k}\cdot\mu = 0,$

$\mu\cdot\nabla\Omega = 0.$

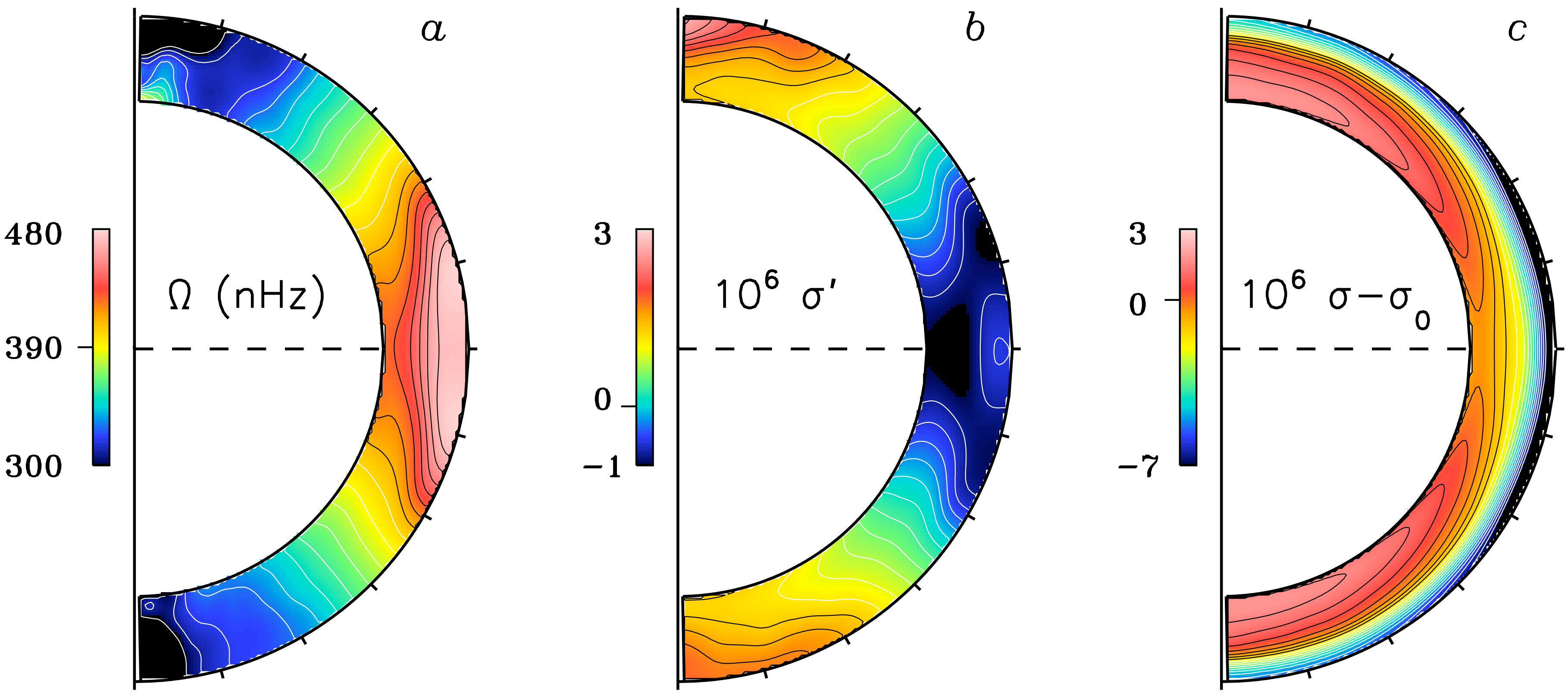
SO:

If the same convective rolls live in surfaces of constant excess sntropy, and they live in surfaces of constant Ω,

these MUST then be the ***same*** surfaces.

$$S' = f(\Omega^2)$$

Miesch 2009, private communication



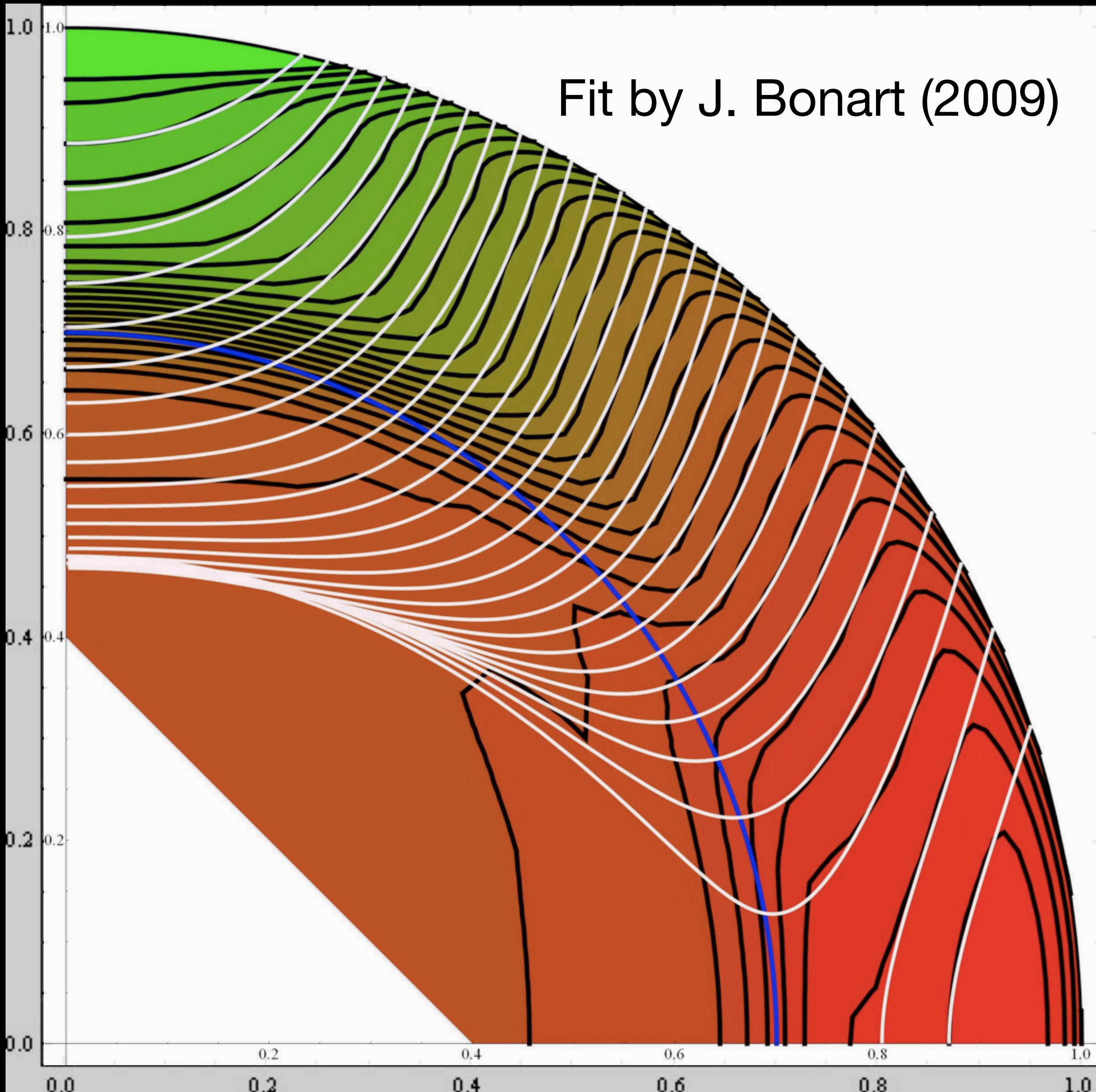
Angular Velocity Residual entropy Entropy

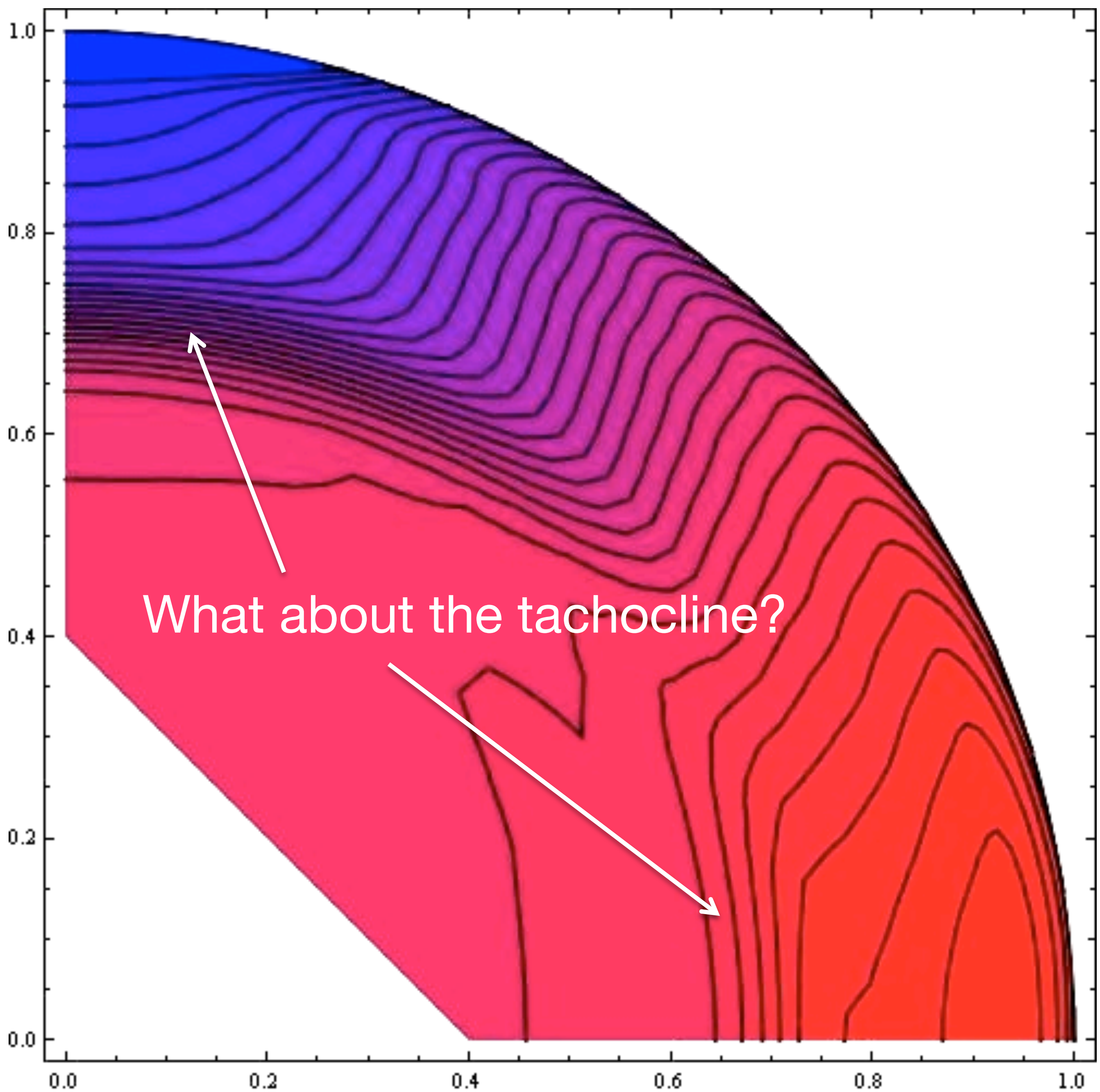
WHY IS THIS IMPORTANT?

Because in stars, the vorticity flux is more important than the angular momentum flux. The vorticity equation is

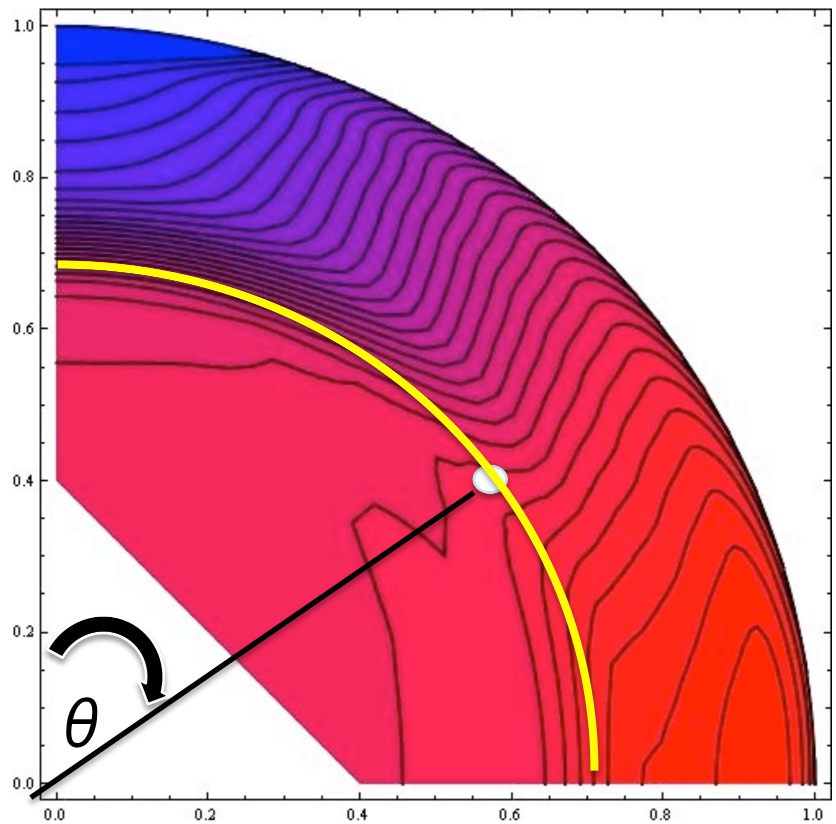
$$\nabla . (1/r \sin \theta) (\mathbf{v} \omega_{\phi} - \omega \mathrm{v}_{\phi}) = (1/\rho^2) \nabla \rho \times \nabla P,$$

and if **S' = f(Ω²),** ***an explicit solution is possible.***

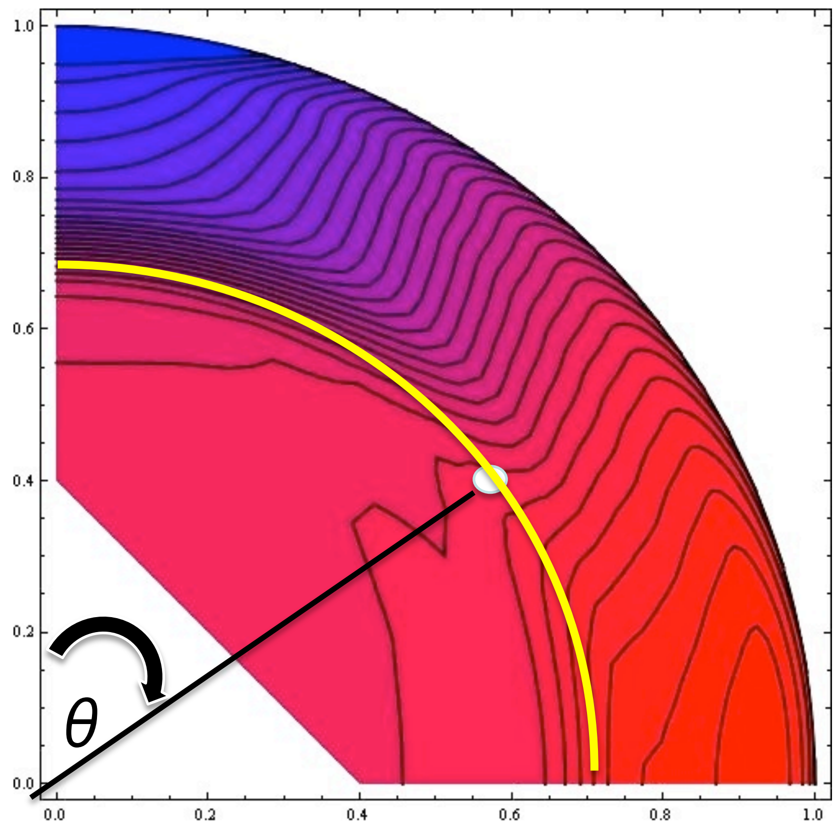

Fit by J. Bonart (2009)
0.0
0.2
0.4
0.6
0.8
1.0



What about the tachocline?
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0



$\theta = 54.7^{\circ}$



$$P_2(\cos\,\theta) = 0, \quad \theta = 54.7^{o}$$

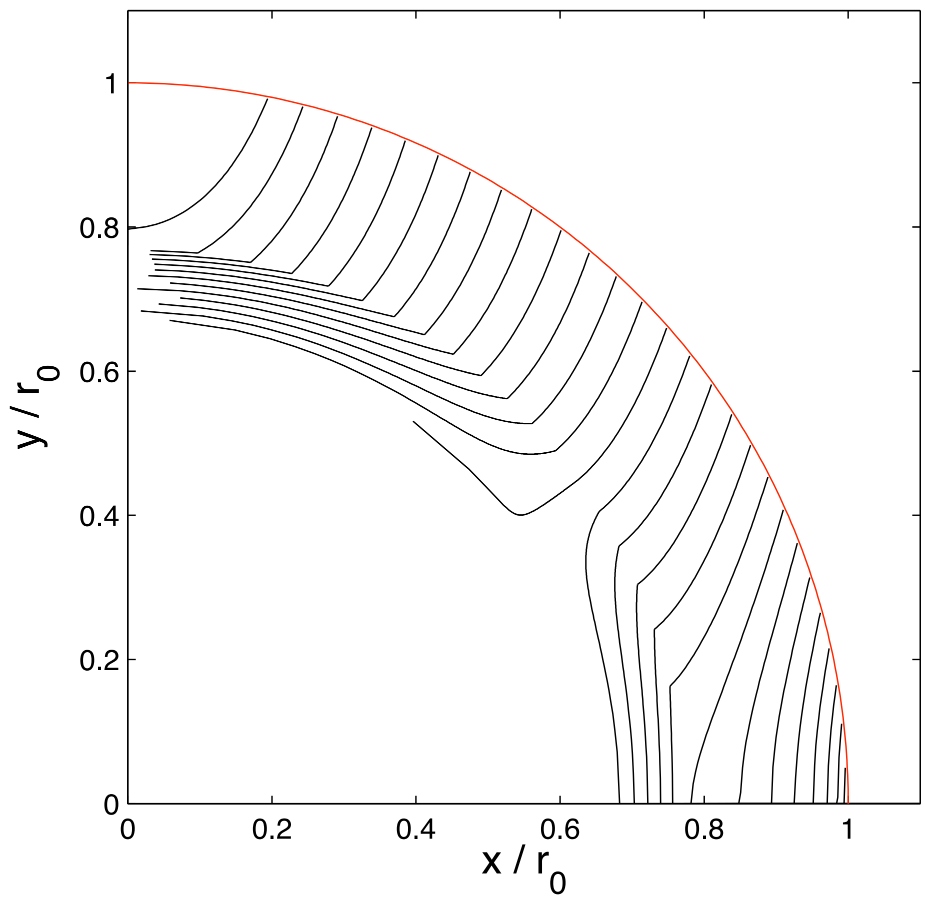
The tachocline displays a dominant quadrupolar structure.

WHY IS THIS IMPORTANT?

Vortex advection: outer layer, tachocline

$$\nabla . (1/r \sin \theta) (\mathbf{v} \omega_{\Phi} - \boldsymbol{\omega} v_{\Phi}) = (1/\rho^2) \nabla \rho \times \nabla P,$$

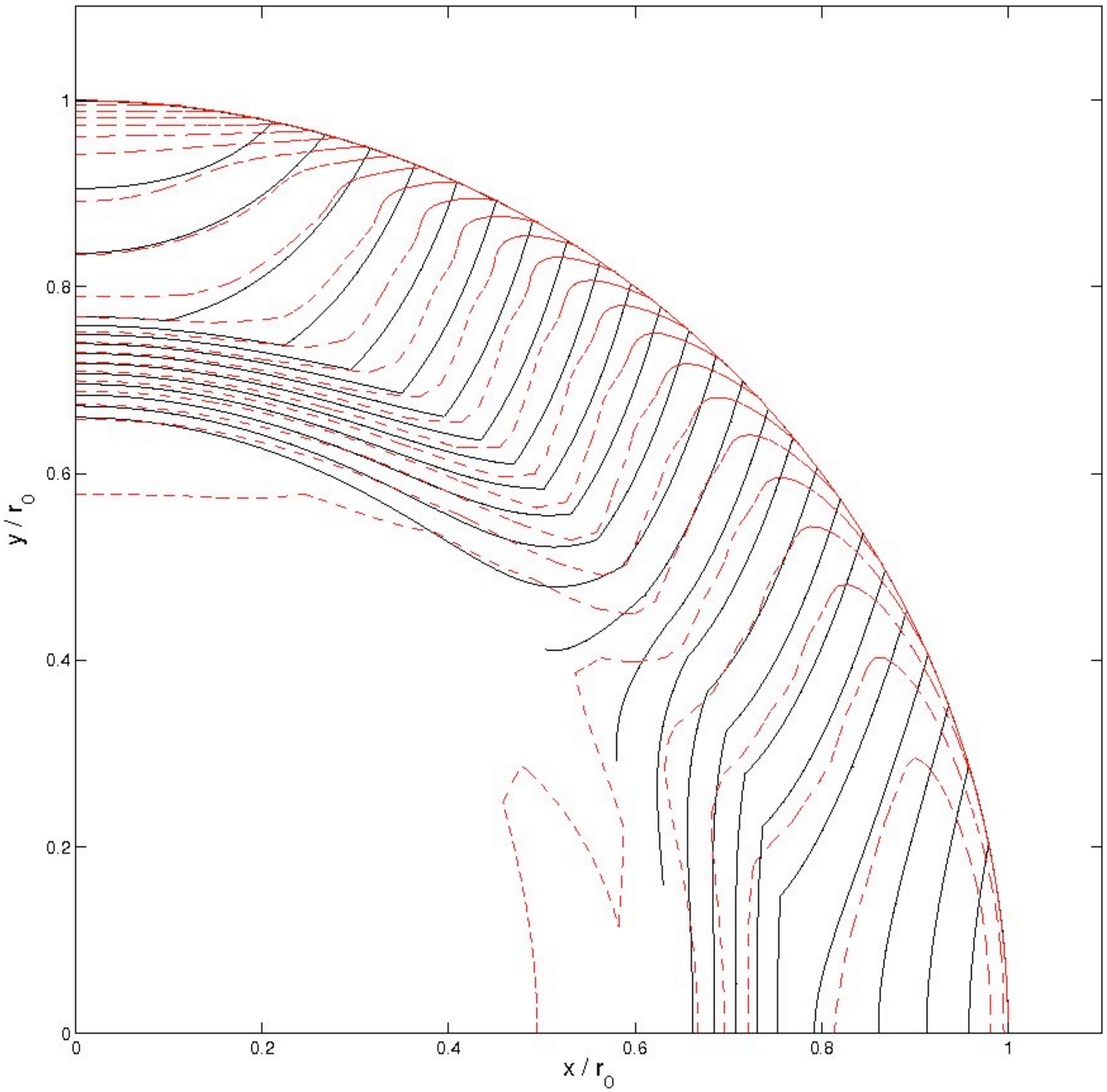
Vortex stretching: bulk of convective zone



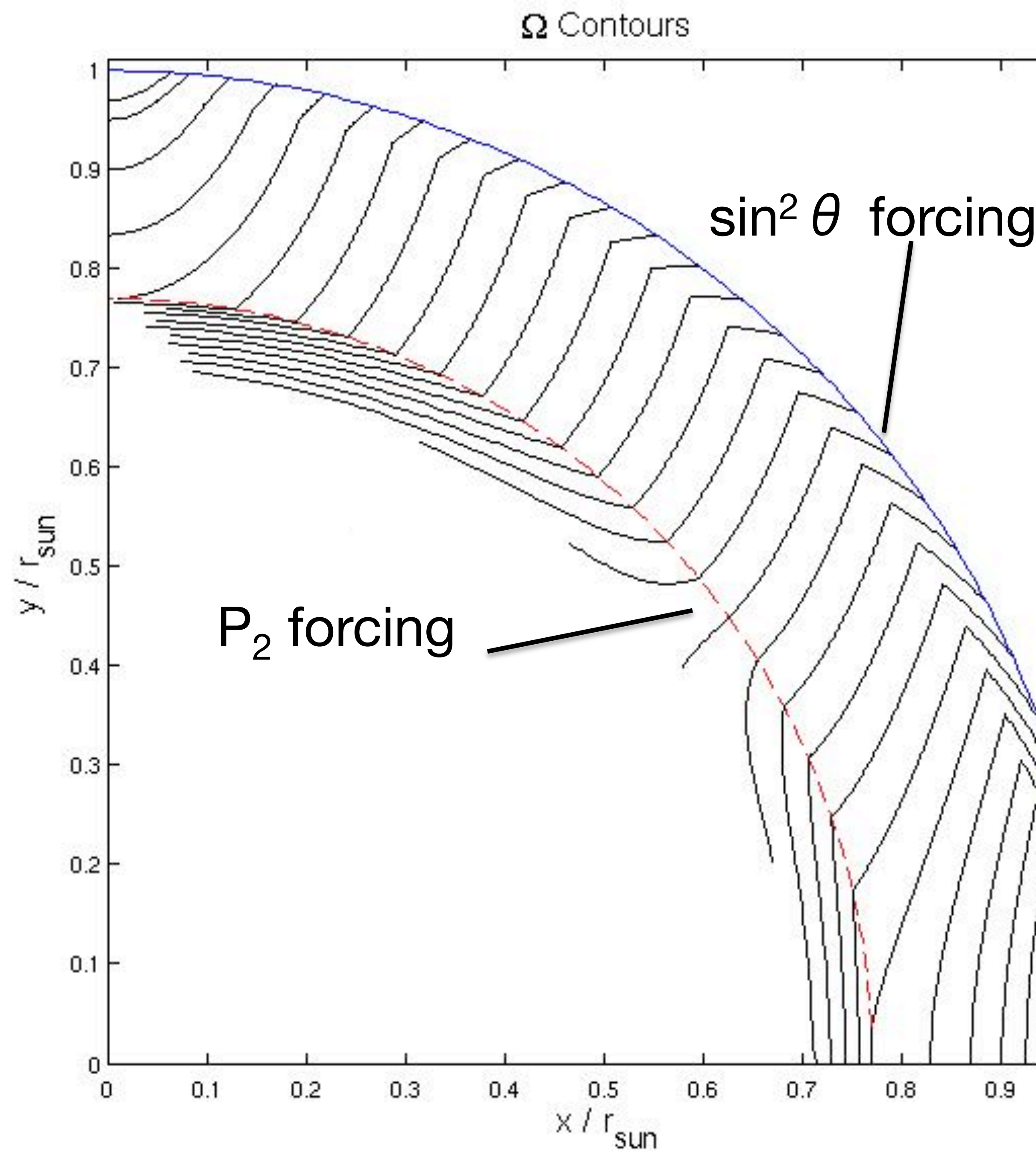
Global Solution (Balbus & Latter 2010)

GONG DATA

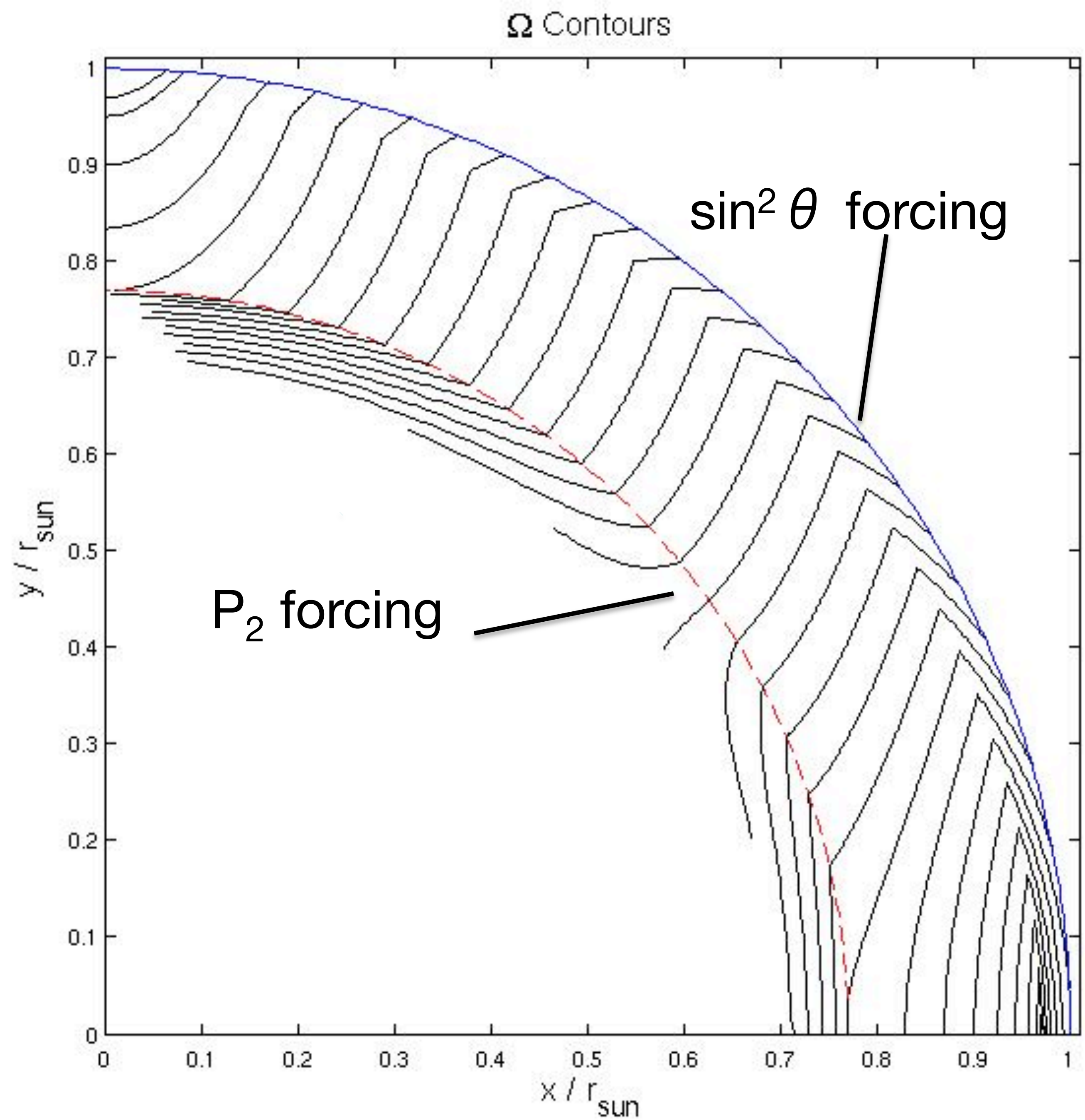
Vortex Advection:

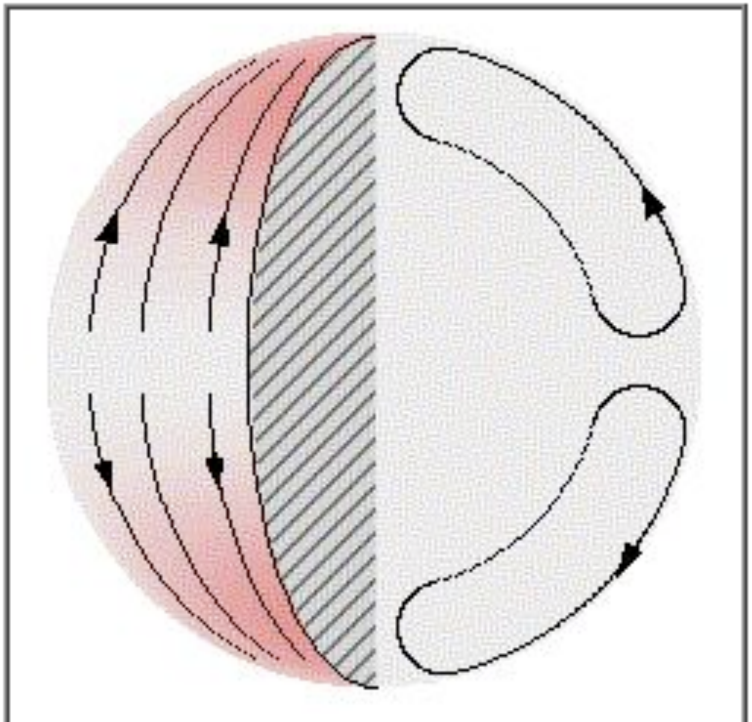


P_2 (cos θ) forcing in tachocline



Ω Contours
sin² θ forcing
P₂ forcing
y / r_sun
x / r_sun
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Ω Contours
sin² θ forcing
P₂ forcing
y / r_sun
x / r_sun
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Click on image for larger version.

The Meridional Flow

The Sun's meridional flow - the flow of material along meridian lines from the equator toward the poles at the surface and from the poles to the equator deep inside - must also play an important role in the Sun's magnetic dynamo. At the surface this flow is a slow 20 m/s (40 mph) but the return flow toward the equator deep inside the Sun where the density is much higher must be much slower still - 1 to 2 m/s (2 to 4 mph). This slow return flow would carry material from the polar regions to the equator in about 20 years.

Meridional flow has long been known to be present in the solar convective zone....

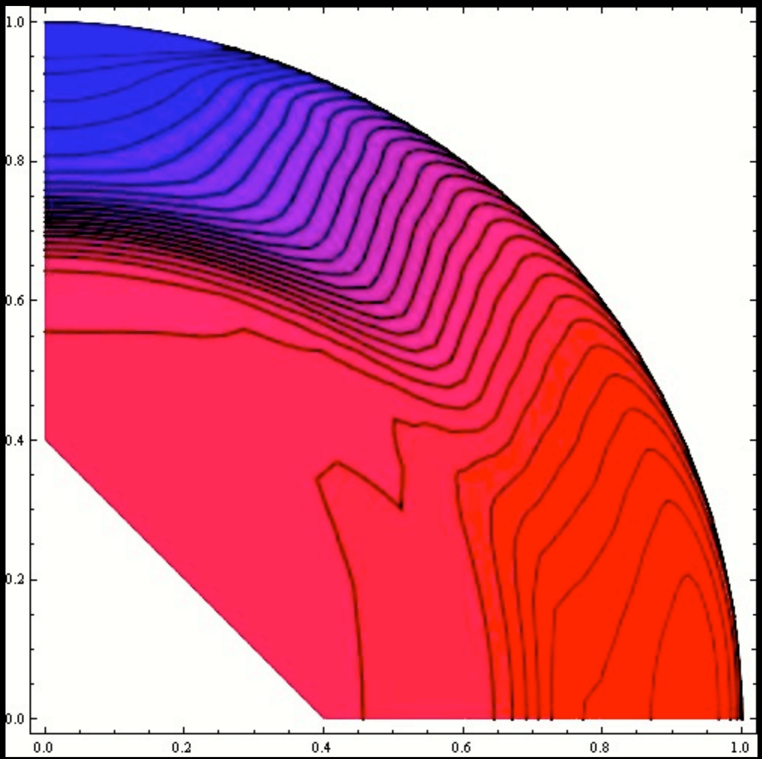
Less well known is the fact that it produces $\sin^2\theta$ and $P_2(\cos\theta)$ forcing, and nothing else, via vortex advecton...

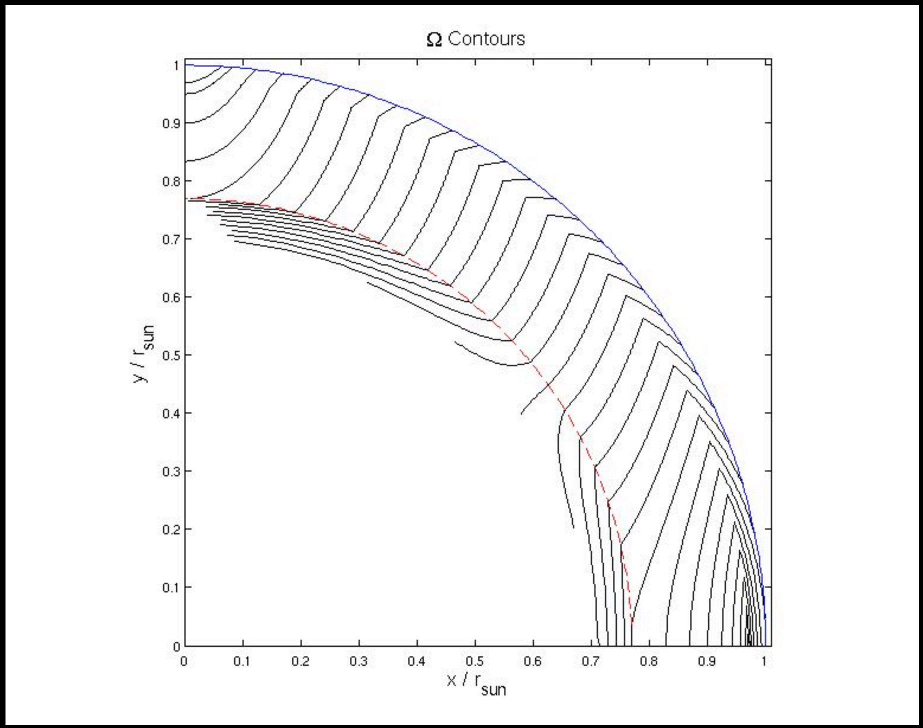
Vortex advection: outer layer, tachocline

$$\nabla . (1/r \sin\ \theta)\,(\mathbf{v}\omega_{\phi} - \omega \mathrm{v}_{\phi}) = (1/\rho^{2})\nabla\rho\times\nabla P,$$

What the precise physical interpretation of these secondary global flows is and how they contribute to the vorticity flux divergence is an active area of ongoing research.

A (fairly) SIMPLE DYNAMICAL THEORY IS COMPATIBLE WITH THE OBSERVATIONS.





1.) Accretion Disks are MHD turbulent, MRI driven by differential rotation; ang. mom. out, mass in. Secondary drift velocity corresponds to mass accretion. DR to fluc. to diss. all local.

2.) Convective stars driven by entropy gradient; vorticity flux is crucial. Rotation related to residual entropy. Advected vorticity important in outer layers, tachocline, not in bulk of CZ. Secondary flow is circulatory, probably driven by acquisition and loss of vorticity and ensuing equatorial/polar drift.