PART III

FAST PARTICLE-DRIVEN INSTABILITIES: NONLINEAR EVOLUTION AND TRANSPORT OF FAST PARTICLES

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OUTLINE

- INTRODUCTION
- NONLINEAR EVOLUTION OF FAST PARTICLE-DRIVEN MODES
- FOUR MAIN SCENARIOS OF NONLINEAR EVOLUTION IN 1D BUMP-ON-TAIL NEAR-THRESHOLD MODEL
- TAE-INDUCED REDISTRIBUTION AND LOSSES OF FAST IONS
- SUMMARY

THE PROBLEM: ALFVÉN INSTABILITIES IN BURNING PLASMA

- **Alpha-particles (He^4 ions) are born in deuterium-tritium nuclear reactions with birth energy 3.52 MeV exceeding plasma temperature by ~100 times**

- **For typical B and n achievable in fusion reactors, velocities of alpha-particles exceed Alfvén velocity, $V_A = B/(4\pi\rho)^{1/2}$, so that**

$$V_{Ti} \ll V_A < V_\alpha \ll V_{Te}$$

- **During slowing-down, alpha-particles cross the resonance with Alfvén waves**

$$V_A = V_{\parallel\alpha}$$

and may excite Alfvén instabilities

- **Free energy source for such instabilities is radial pressure gradient of α's, and the instability results in a radial re-distribution / loss of alphas when the amplitude of Alfvén wave becomes high.**

- **Present-day machines with ICRH/ NBI excite Alfvén instabilities very often.**

THE EXCITATION CONDITION

A wave with frequency ω and amplitude

A~ exp(γt)·exp (iωt)

becomes linearly unstable (i.e. its amplitude starts to increase)

IF

the alpha-particle drive exceeds the wave damping due to thermal plasma,

$$\gamma \equiv \gamma_\alpha - |\gamma_{damp}| > 0.$$

↓

Waves with weak damping due to thermal plasma are easiest to excite with alpha-particles.

PARALLEL WAVE-VECTOR IN TOKAMAK PLASMAS

In a torus, the wave solutions are quantized in toroidal and poloidal directions:

$$\phi(r,\vartheta,\zeta,t) = \exp(-i\omega\, t + in\zeta) \sum_m \phi_m(r)\ \exp(-im\vartheta) + c.c.$$

n **is the number of wavelengths in toroidal direction and**
m **is the number of wavelengths in poloidal direction**

- **The parallel wave-vector for the** m **-th harmonic of a mode with toroidal mode number** n**,**

$$k_{\parallel m}(r) = \frac{1}{R}\left(n - \frac{m}{q(r)}\right),$$

is determined by the safety factor $q(r) = rB_\zeta / RB_\vartheta$ **.**

- **For a given** $q(r)$ **and** n**, one finds** $m = nq$ **giving a rational surface with** $k_{\parallel m} = 0$

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TOROIDICITY-INDUCED GAP IN ALFVÉN CONTINUUM

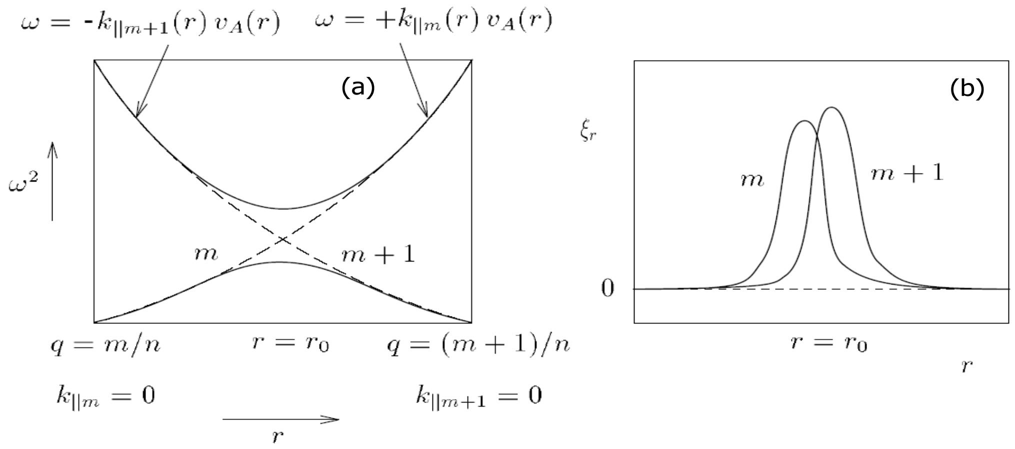
- **Cylindrical Alfvén continua, $\omega_A \equiv \pm k_{\parallel m}(r)\cdot V_A(r)$, for poloidal *m* and (*m*+1) harmonics couple at frequency satisfying**

$$\omega = +k_{\parallel m}(r)V_A(r) = -k_{\parallel m+1}(r)V_A(r)$$

and form a frequency "gap" in the Alfvén continuum. At the gap, Alfvén continuum has an extremum point,

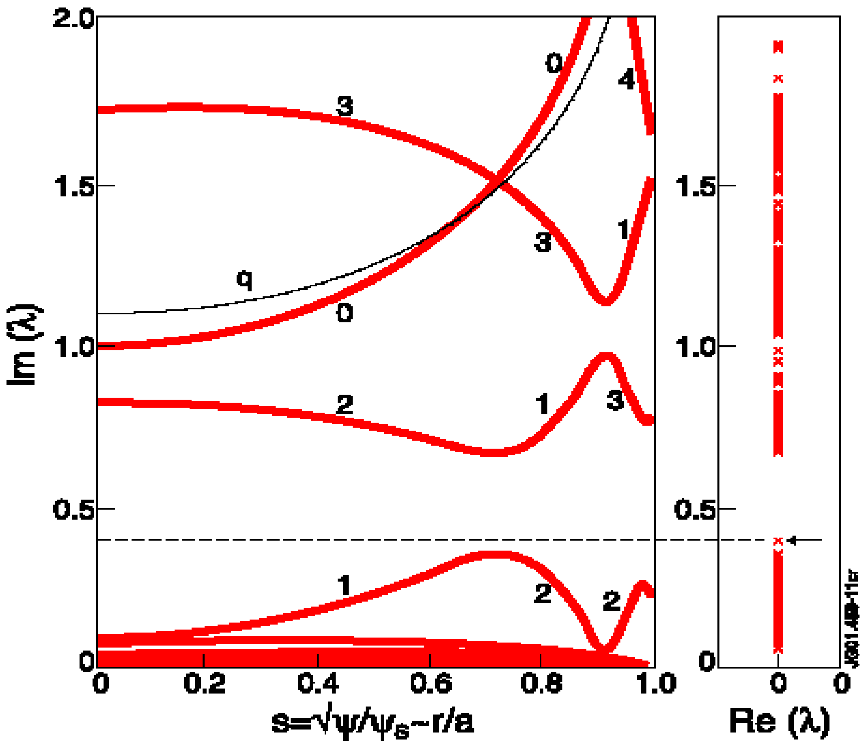
$$\left.\frac{d\omega_A(r)}{dr}\right|_{r=r_0} = 0$$

so a discrete toroidal Alfvén eigenmode could exist at $r = r_0$ and $\omega_{TAE} \simeq V_A(r_0)/(2R_0 q(r_0))$



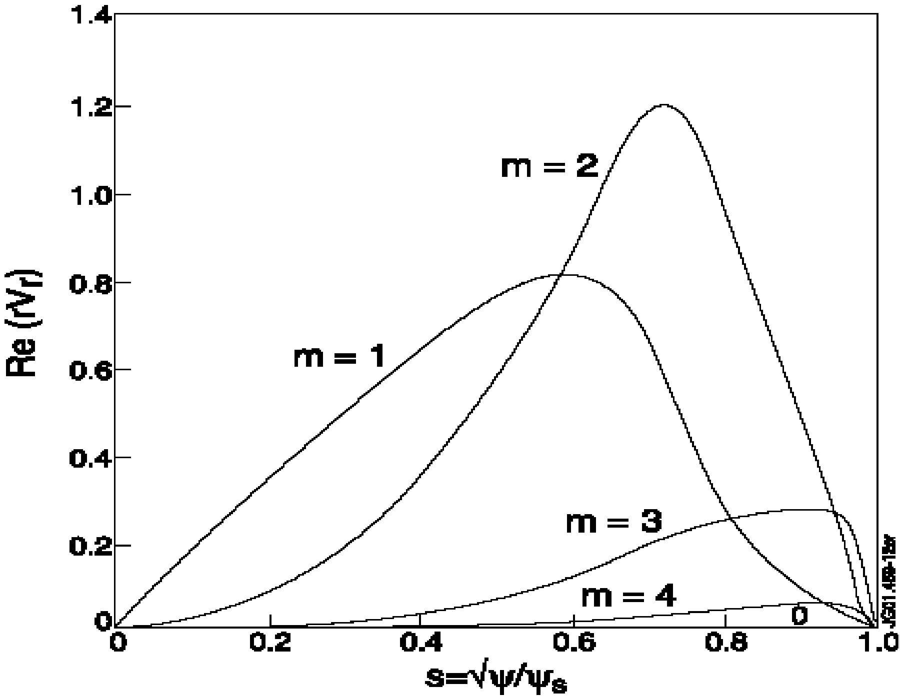
WEAKLY-DAMPED TOROIDAL ALFVÉN EIGENMMODE (TAE)

- **In addition to the continuum, Toroidal Alfvén Eigenmode (TAE) can exist with TAE-frequency *in* the Toroidicity-induced gaps**
- **At higher frequencies, Ellipticity and Triangularity induced gaps and relevant discrete eigenmodes can also exist**



WEAKLY-DAMPED TOROIDAL ALFVÉN EIGENMMODE (TAE)

- **Similarly to GAE in cylindrical plasma, TAE frequency does not satisfy local Alfvén resonance in the region of TAE localization, $\omega_{TAE} \neq \omega_A(r)$, so TAE has no singularity and does not experience strong continuum damping**



Radial dependence of the Fourier harmonics of TAE

CHARACTERISTIC PROPERTIES OF TAE

- **Start from TAE-gap coupling condition** $\omega = k_{\parallel m}(r) V_A(r) = -k_{\parallel m+1}(r) V_A(r)$ **(*)**

$$\downarrow$$

$$k_{\parallel m}(r) = -k_{\parallel m+1}(r)$$

$$\downarrow$$

$$\frac{1}{R_0}\left(n - \frac{m}{q(r)}\right) = -\frac{1}{R_0}\left(n - \frac{m+1}{q(r)}\right)$$

$$\downarrow$$

$$q(r) = \frac{m+1/2}{n}$$

- **TAE gap is localised at** $q(r_0) = (m+1/2)/n$ **determined by the values of** n **and** m
- **Substitute this value of safety factor in the starting equation (*) to obtain**

$$\omega_0 = \frac{V_A(r_0)}{2R_0 q(r_0)}$$

Note: this TAE-gap frequency does NOT depend on n **and** m

CHARACTERISTIC PROPERTIES OF TAE - 3

- **Localisation of TAE gaps determined by** $q(r_0) = (m + 1/2)/n$ **for typical** $0.75 < q < 5$ **are:**

	m=1	m=2	m=3	m=4	m=5	m=6
n=1	1.5	2.5	3.5	4.5	5.5	6.5
n=2	0.75	1.25	1.75	2.25	2.75	3.25
n=3	0.5	0.83	1.167	1.5	1.83	2.167
n=4		0.625	0.875	1.125	1.375	1.625
n=5		0.5	0.7	0.9	1.1	1.3
n=6			0.583	0.75	0.917	1.083

- **Characteristic frequencies of the TAE gap for JET parameters are:**

$$B_0 \cong 3\ T;\ n_i = 5 \times 10^{19}\ m^{-3};\ m_i = m_D$$

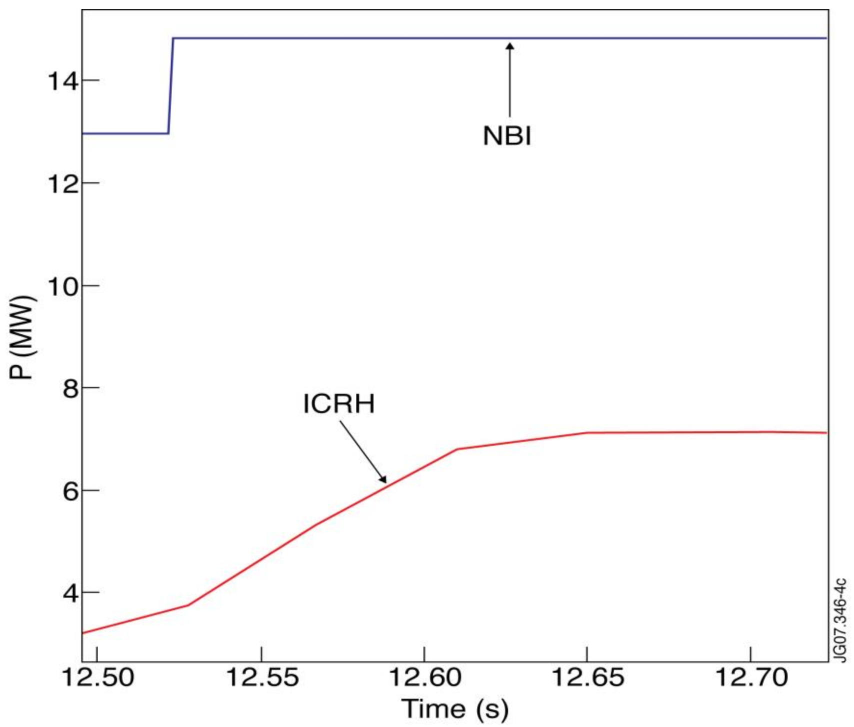
$$\downarrow$$

$$V_A \cong 6.6 \times 10^6\ m/s$$

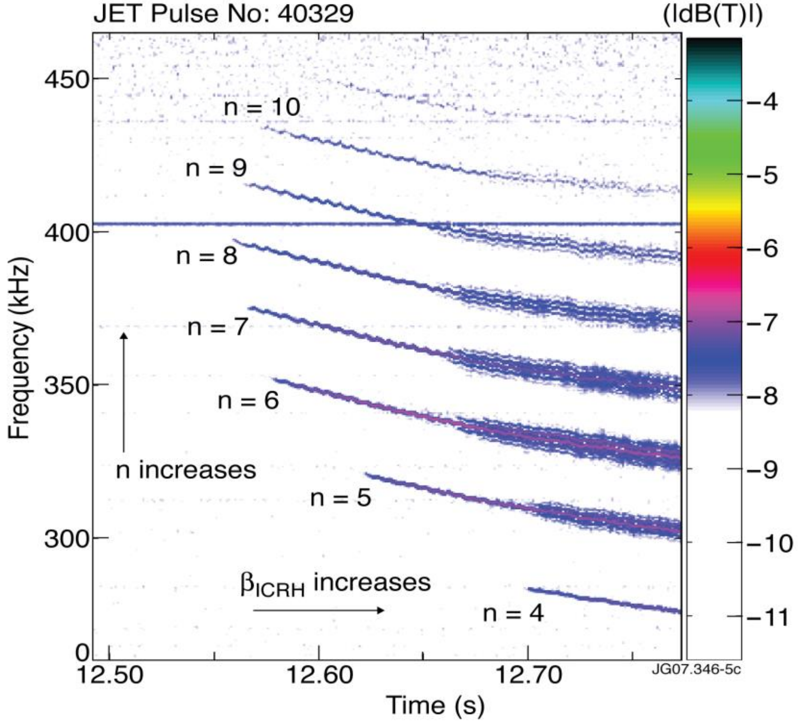
- **For typical value of q=1.1 one has the frequency estimate then:**

$$\omega_0 \cong 10^6\ \sec^{-1} \rightarrow \qquad f_0 \equiv \omega_0 / 2\pi \cong 160\ kHz$$

TAE EXCITATION DEPENDS ON THE MODE NUMBER



Power waveforms of ICRH driving TAE and NBI (NBI provides damping)



TAEs with toroidal mode numbers from n=4 to n=10 are seen separated by frequency ~ 25 kHz

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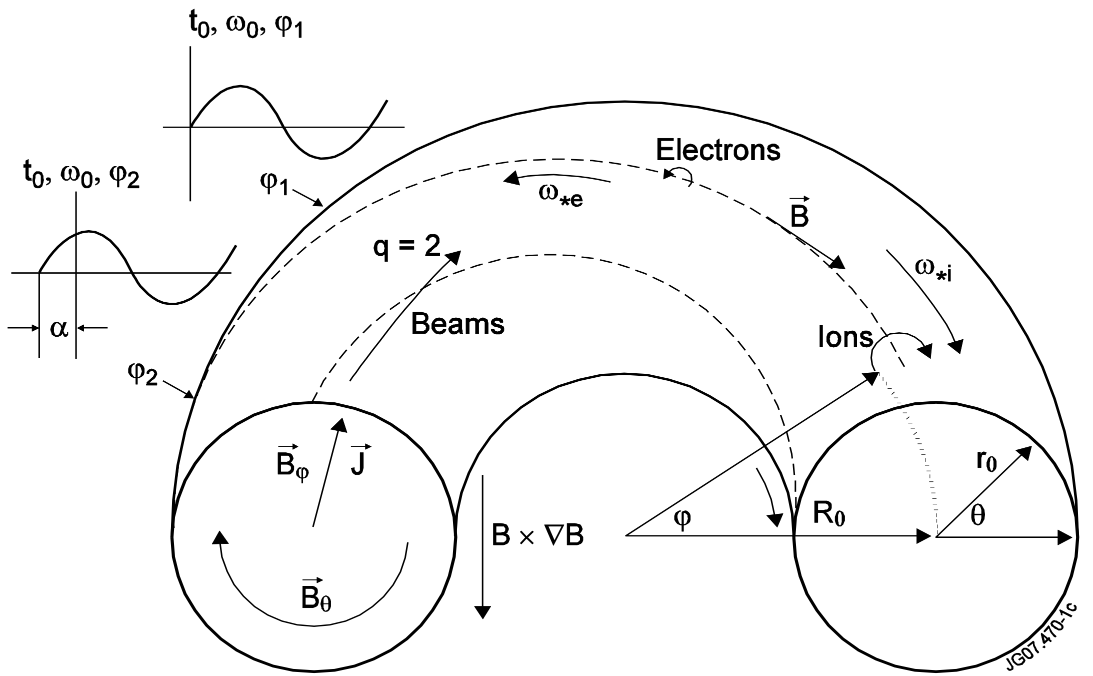
MIRNOV COILS – FOURIER TRANSFORMED ***PHASE*** DATA

- **For determining toroidal mode number n of the mode, phase shift is measured between two (or more) Mirnov coils at different toroidal angles**



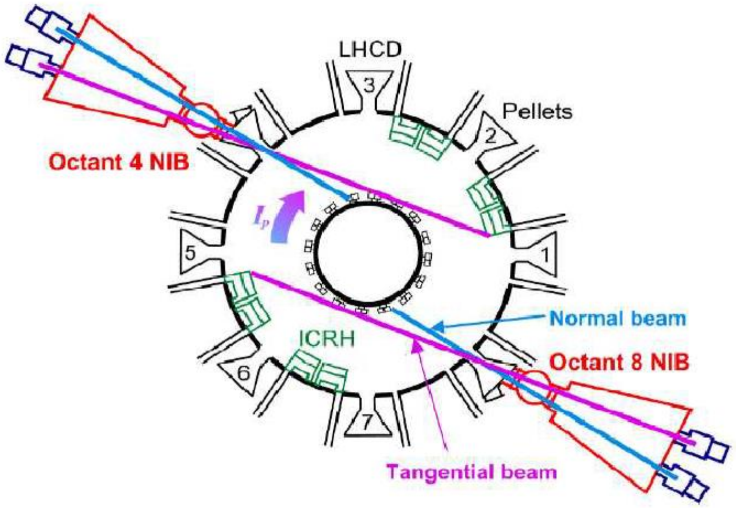
Sinusoidal signals measured at different toroidal angles at the same time and at same frequency are shifted in phase by α.

EXPERIMENTALLY OBSERVED TAE – THE QUESTIONS TO ASK

- **TAE frequencies are well above the expected 160 kHz. Why?**
- **TAE modes with $n = 6 - 9$ are excited earlier, while TAE modes with lower or higher n's need a higher pressure of fast ions. Why?**
- **The TAE spectral lines broaden and split after some time. Why?**
- **Do TAE cause significant losses and/ or redistribution of fast ions?**

DOPPLER SHIFT OF THE MODE FREQUENCY

- **Uni-directional NBI on JET drives a significant toroidal plasma rotation (up to 40 kHz)**

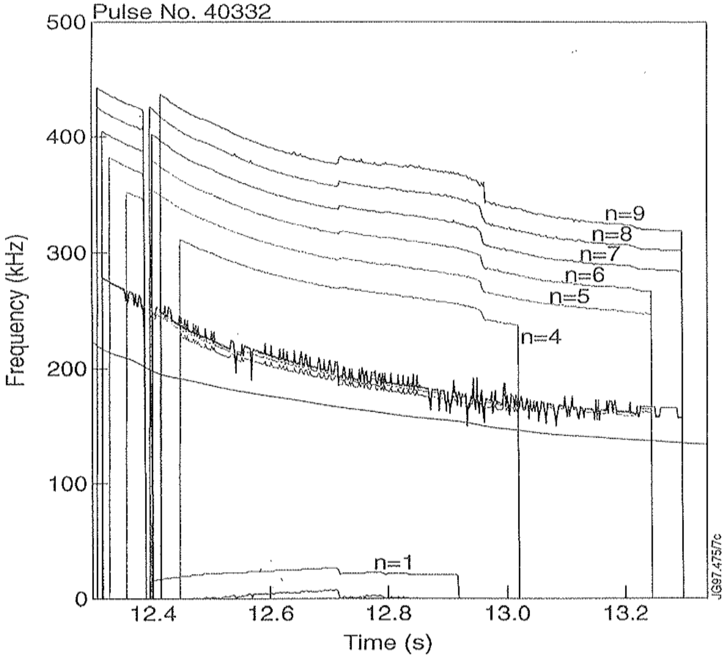


Geometry of NBI injection system on JET (view from the top of the machine)

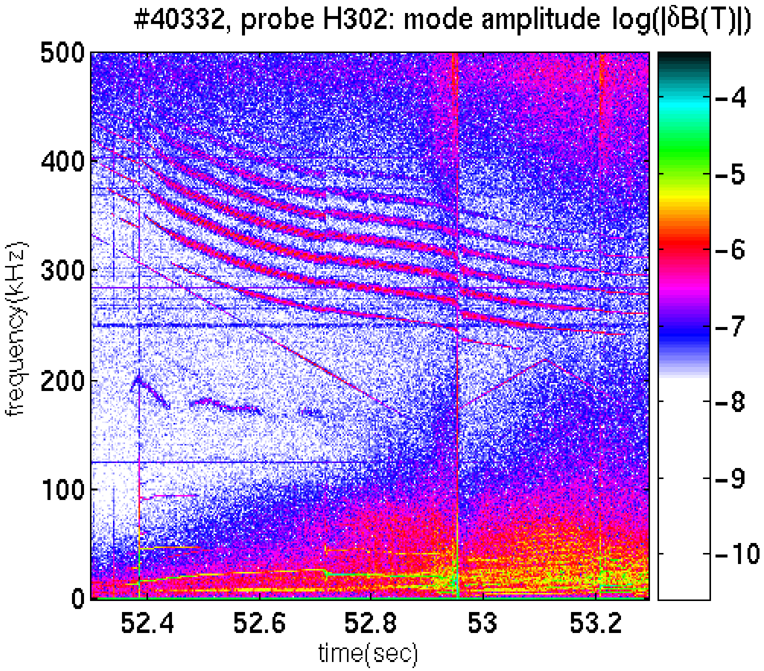
- **Frequencies of waves with mode number n in laboratory reference frame, f_n^{LAB}, and in the plasma, f_n^0, are related through the Doppler shift $nf_{rot}(r)$:**

$$f_n^{LAB} = f_n^0 + nf_{rot}(r)$$

COMPUTED VERSUS OBSERVED TAEs



Eigenfrequencies of TAEs with n=4…9 computed for equilibrium in JET discharge #40332. Added Doppler shift matches the experiment



Discrete spectrum of TAE observed in JET discharge #40332. Plasma starts at t=40 sec. Frequency changes due to plasma density increase, $f \sim B/\sqrt{n_i M_i}$.

NONLINEAR EVOLUTION OF

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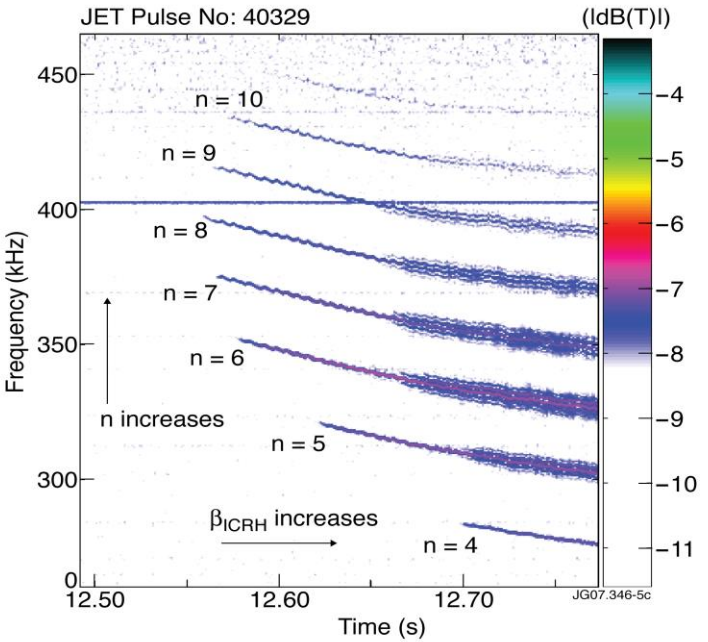
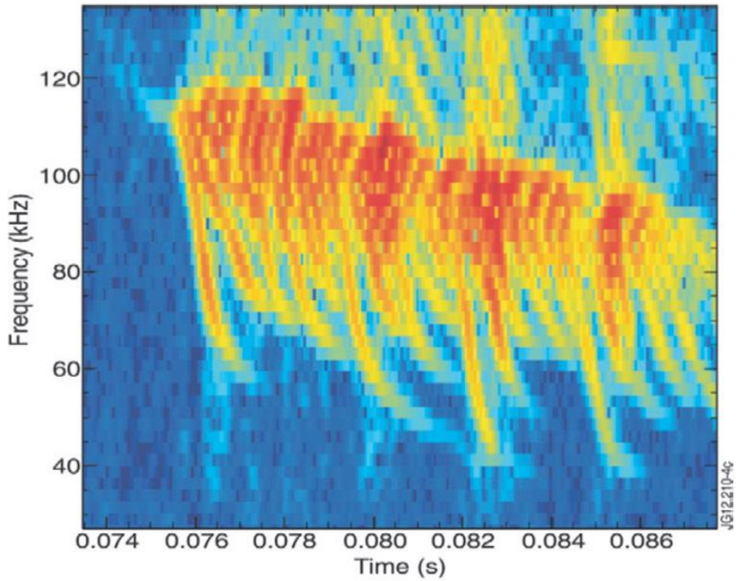
DIFFERENT REGIMES OF MODE EVOLUTION OBSERVED ON DIFFERENT MACHINES:



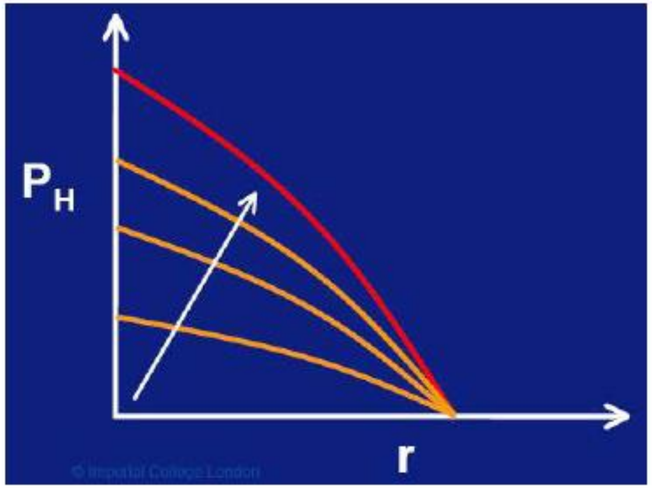
ICRH-driven TAEs during ICRH power ramp-up on JET



NBI-driven TAEs on MAST have bursting amplitudes and sweep in frequency

THE NEAR-THRESHOLD CONDITION

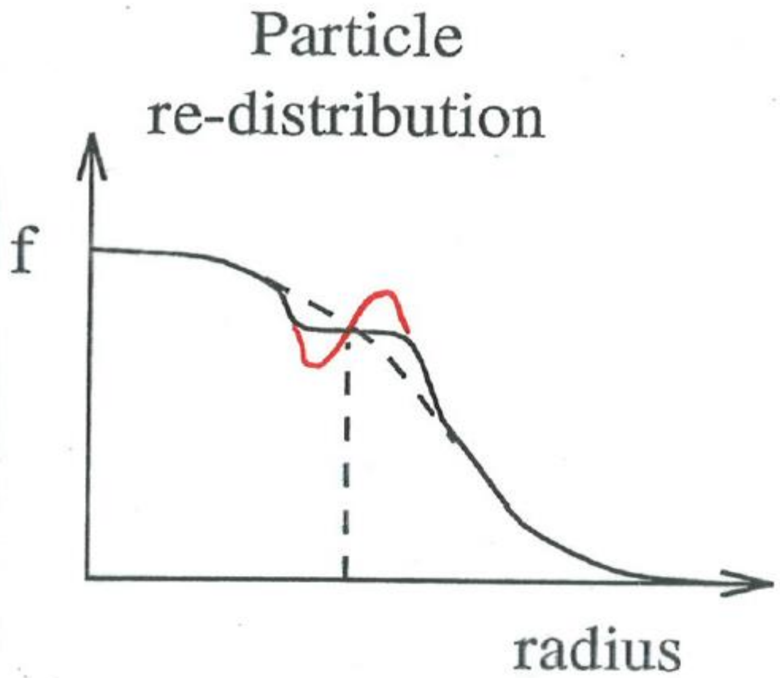
- **Consider the scenario with a *gradual build-up of fast ion pressure* so that the fast ion drive of TAE, $\gamma_\alpha(t) \propto -\beta'_\alpha(t)$, increases in time at unchanged TAE damping γ_d**



- **TAE *instability threshold*: exact balance between TAE drive and damping, $\gamma_\alpha = \gamma_d$**
- **The *near-threshold* condition:**

$$\left|\gamma_\alpha - \gamma_d\right| << \gamma_d \leq \gamma_\alpha$$

HOW TAE INSTABILITY SATURATES?



- **Non-linear TAE behaviour: competition between *the field of the mode* that tends to *flatten* distribution function near the resonance (effect proportional to the net growth rate $\gamma \equiv \gamma_L - \gamma_d$) and *the collision-like processes* that constantly *replenish* it (proportional to ν_{eff})**

THE COLLISIONALITY

- **The near-threshold regime *allows the "collisions"* restoring the unstable distribution function of fast ions *to compete with the mode growth***

$$\left|\gamma_{\alpha}-\gamma_{d}\right| \approx \nu_{eff}$$

- **Demonstrate this effect *analytically for the "bump-on-tail"* problem in a *1D velocity* space. This problem has physics similar to TAE, but is 1D.**

- **The "bump on tail" problem: consider the *nonlinear* evolution of a *marginally unstable* electrostatic wave with frequency $\omega=\omega_{pe}=\sqrt{4\pi\ n_e e^2/m_e}$ in the presence of an unstable beam distribution function $F(x,\ v,\ t)$ with *collisional operator* (*Berk et al., PRL 76 1256 (1996)*)**

- **Collisional operator that includes electron drag, diffusion, and Krook terms was found to reproduce all near-threshold nonlinear scenarios**

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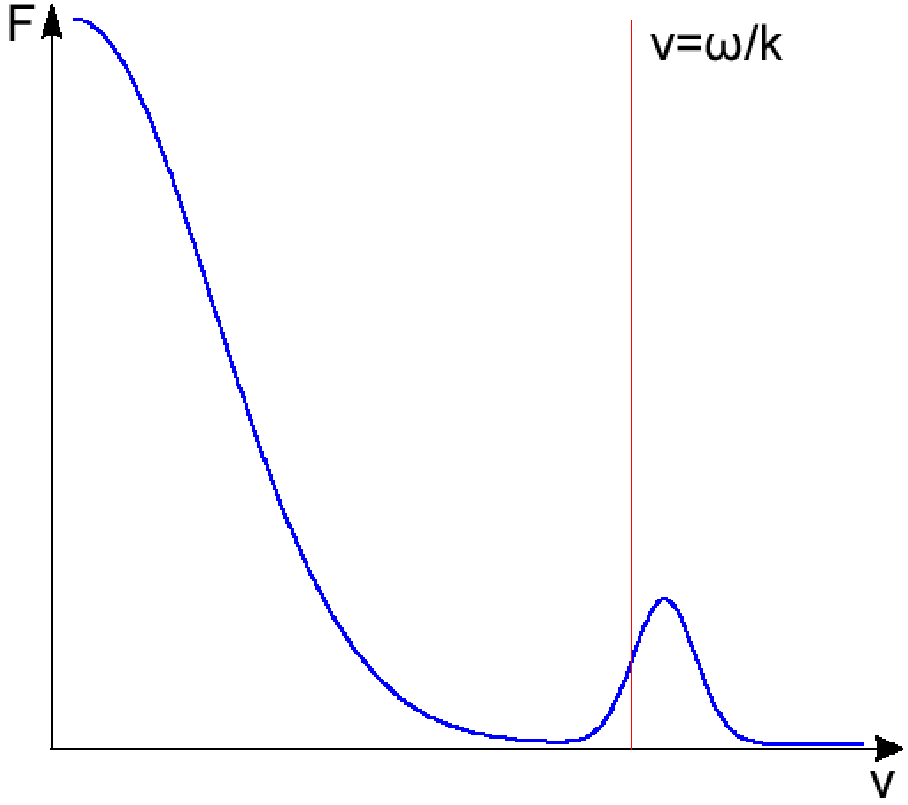


1D model: Unstable bump-on-tail distribution function.

BERK-BREIZMAN NEAR-THRESHOLD THEORY: THE WAVE AMPLITUDE EQUATION WITH DRAG, DIFFUSION, AND KROOK

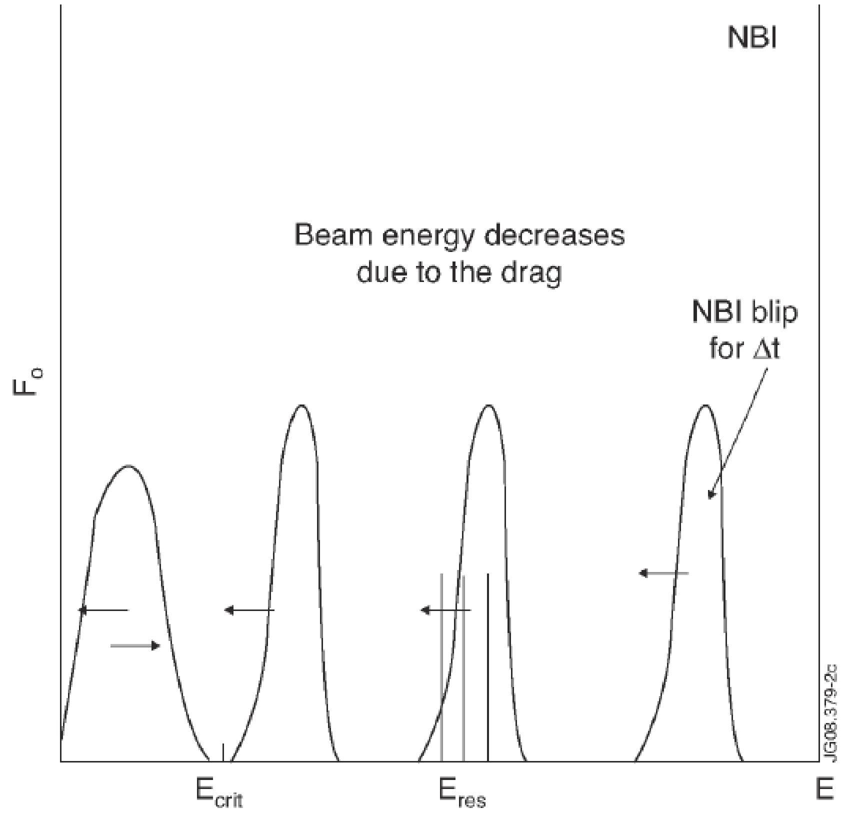
The key ingredients of the problem:

- **A bulk plasma eigenmode with a known spatial structure and eigenfrequency** ω_0
- **A weak damping rate of the mode** $\gamma_d << \omega_0$ **in the absence of energetic particles**
- **A population of energetic particles that creates a kinetic driving rate of the mode** γ_L **via wave-particle resonance with** $\gamma_d < \gamma_L << \omega_0$
- **Sources and sinks that create an unstable energetic particle distribution in the absence of the excited mode and provide free energy continuously.**

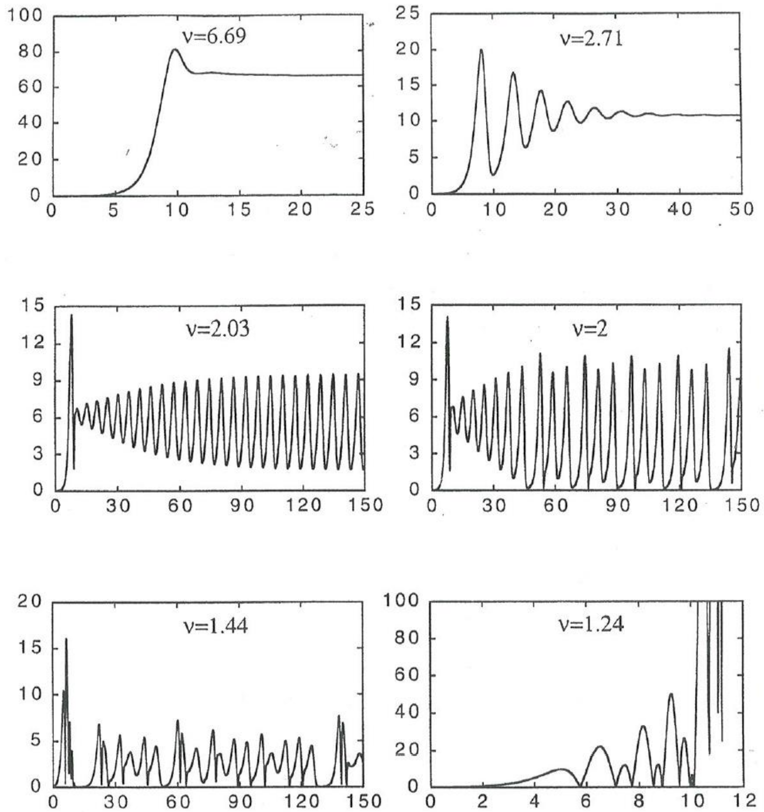
$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + \frac{e}{2m}\left[\hat{E}(t)\exp(ikx - i\omega_0 t) + \text{c.c.}\right]\frac{\partial F}{\partial u} = \left[\frac{\nu^3}{k^2}\frac{\partial^2}{\partial u^2} + \frac{\alpha^2}{k}\frac{\partial}{\partial u} - \beta\right](F - F_0)$$

$$\frac{\partial \hat{E}}{\partial t} = -4\frac{\omega_0}{k}\pi e\int\langle F\exp(-ikx + i\omega_0 t)\rangle du - \gamma_d \hat{E}$$

WHAT REPLENISHES THE UNSTABLE ENERGETIC PARTICLE DISTRIBUTION: DRAG OR DIFFUSION?



DIFFUSION ONLY CASE



Evolution of $|A|$ in time, $t \equiv \gamma t$

Nonlinear equation for the amplitude

$$\frac{dA}{dt} = A - \frac{1}{2}\int_0^{t/2} \tau^2 \int_0^{t-2\tau} \exp\left[-\nu^3 \tau^2 (2\tau/3 + \tau_1)\right]$$

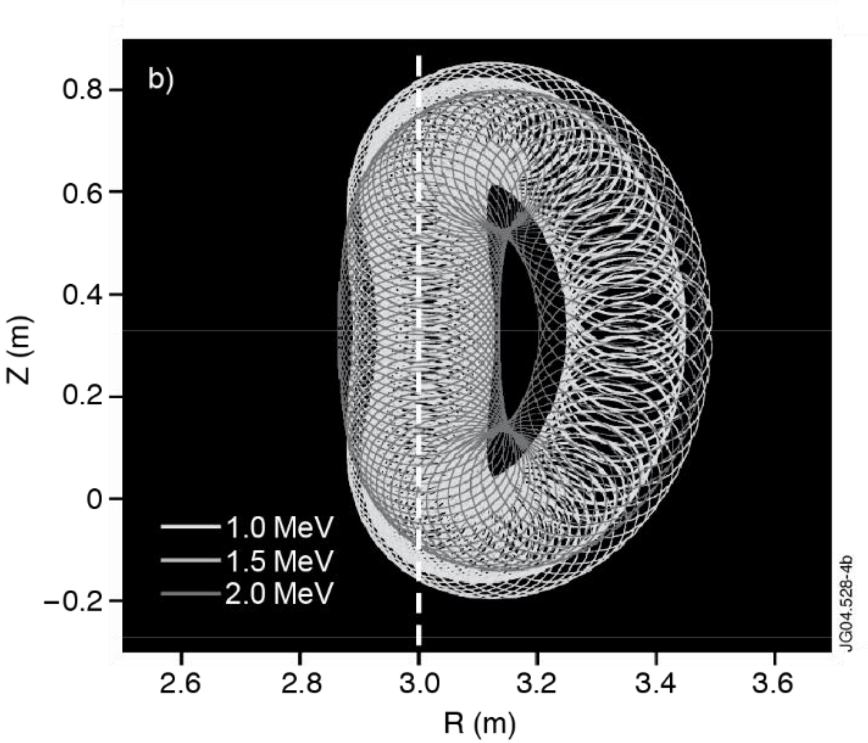
$$\times A(t-\tau)A(t-\tau-\tau_1)A^*(t-2\tau-\tau_1)d\tau_1 d\tau$$

describes *four* regimes of mode evolution:

a) Steady-state;
b) Periodically modulated;
c) Chaotic;
d) Explosive

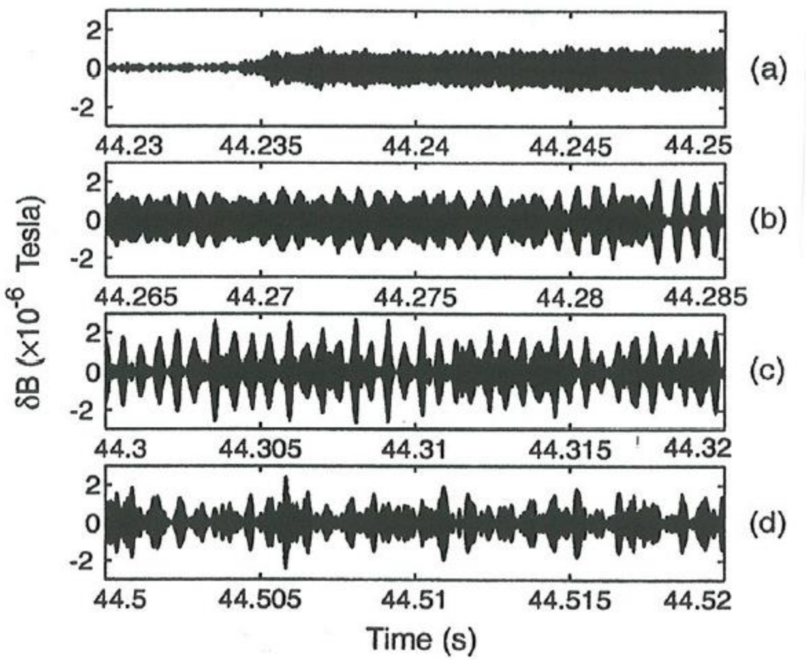
The explosive regime in a more complete non-linear model leads to frequency-sweeping 'holes' and 'clumps' on the perturbed distribution function (H.L.Berk, B.N.Breizman, and N.V.Petviashvili, Phys. Lett. A234 (1997) 213)

FAST MINORITY IONS ACCELERATED WITH ICRH ON JET

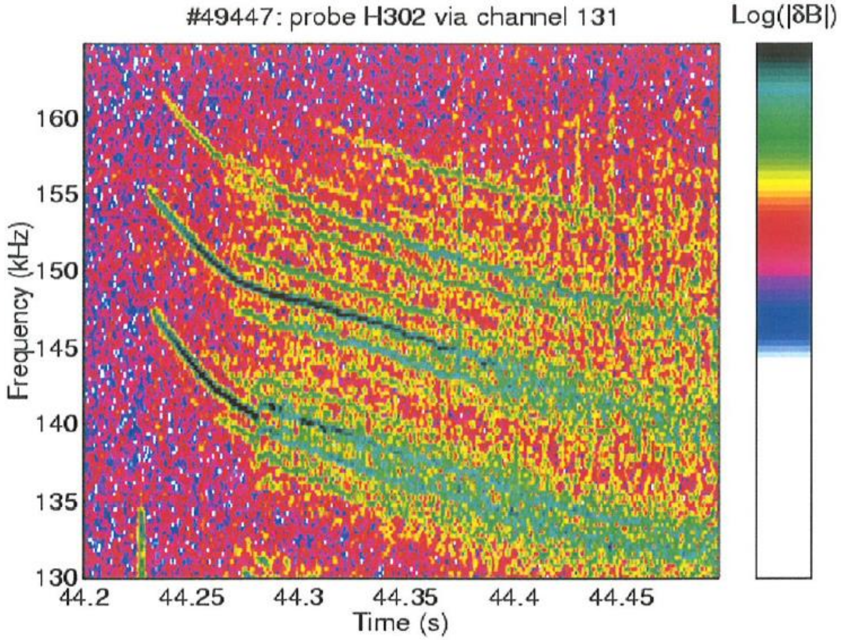


Examples of trapped orbits of He ions in the MeV energy range in JET tokamak.

THE BUMP-ON-TAIL SOLUTIONS WITH DIFFUSION ARE OBSERVED FOR ICRH-DRIVEN TAE ON JET



At gradually increasing ICRH power, TAEs exhibit steady state, periodically modulated, and chaotic regimes



Magnetic spectrogram corresponding to the left Figure with raw data. Steady state, periodically modulated (pitchfork splitting), and chaotic regimes are seen

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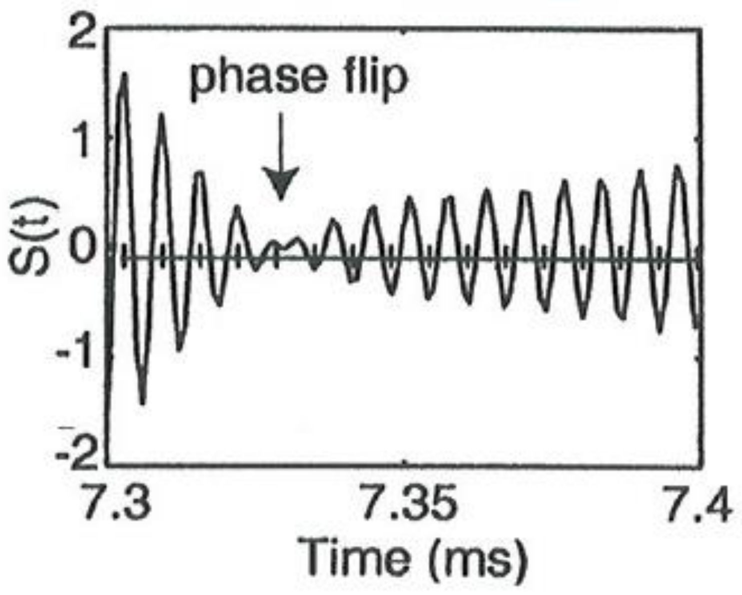
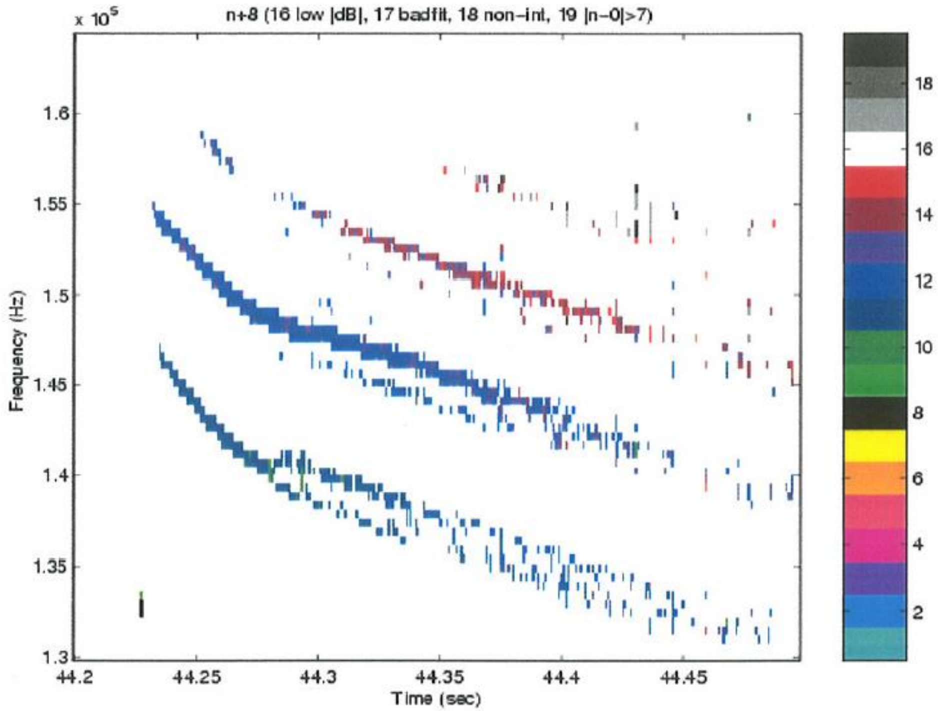
THE CHAOTIC REGIME FOR TAE



When TAE amplitude modulation becomes comparable to the amplitude, chaotic TAE evolution is observed



The chaotic TAE evolution significantly complicates the phase analysis of TAE mode numbers

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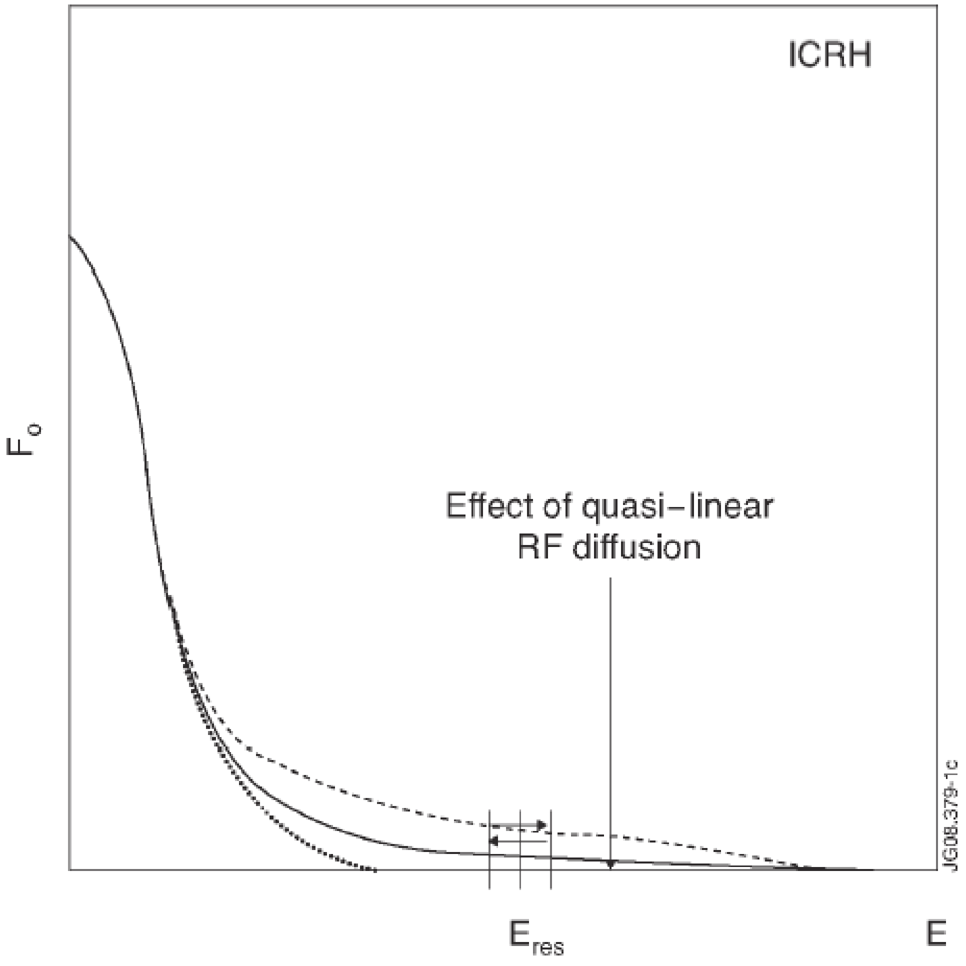
ICRH REPLENISHES FAST ION DISTRIBUTION VIA *DIFFUSION*



WHY 1D BOT THEORY WORKS FOR TAE?

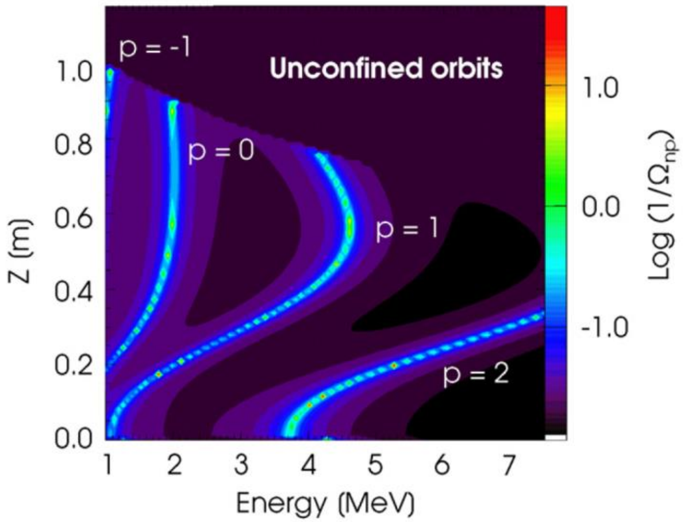
The TAE resonance condition reads:

$$\Omega \equiv \omega - n\omega_{\varphi}(E;P_{\varphi};\Lambda) - l\omega_{\theta}(E;P_{\varphi};\Lambda) = 0$$

For ICRH ions

$$\Lambda = const \ , P_{\varphi}(V_{\parallel} = 0) \rightarrow Z$$

TAE resonances are 1D lines in 5D space then:



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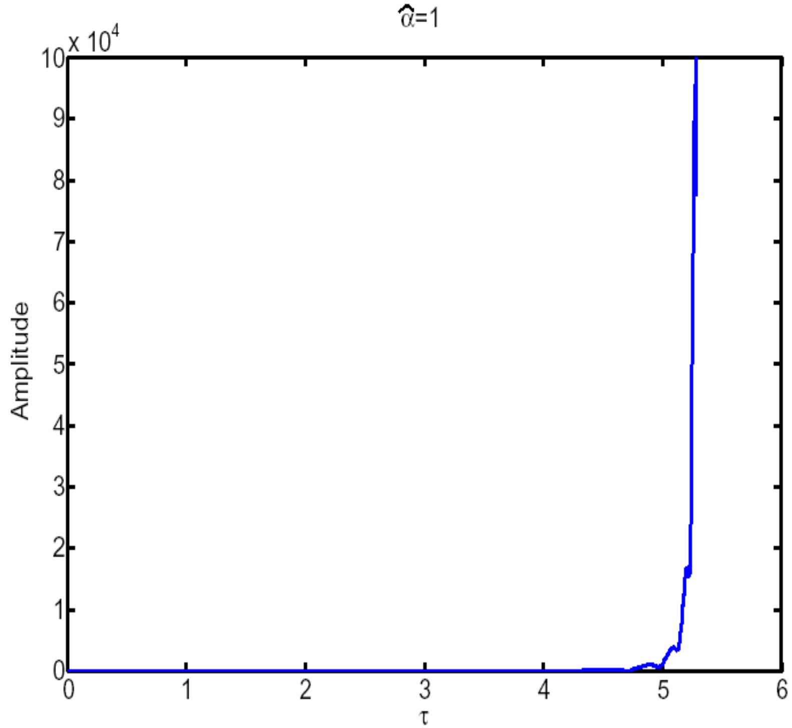
DRAG ONLY CASE



Evolution of $|A|$ in time, $t \equiv \gamma t$

Nonlinear equation for the amplitude

$$\frac{dA}{dt} = A - \frac{1}{2}\int_0^{t/2}\tau^2\int_0^{t-2\tau}\exp\left[i\hat{\alpha}^2\tau(\tau+\tau_1)\right]$$
$$\times A(t-\tau)A(t-\tau-\tau_1)A^*(t-2\tau-\tau_1)d\tau_1 d\tau$$

In contrast to the diffusion case, drag gives *oscillatory behaviour in the kernel* leading to the *explosive evolution* of the amplitude blowing up in a finite time,

$$A \propto (t-t_0)^{-p}$$

This is *the only* scenario for the drag !

M.Lilley, B.N.Breizman, and S.E.Sharapov, Phys. Rev. Lett. (2009)

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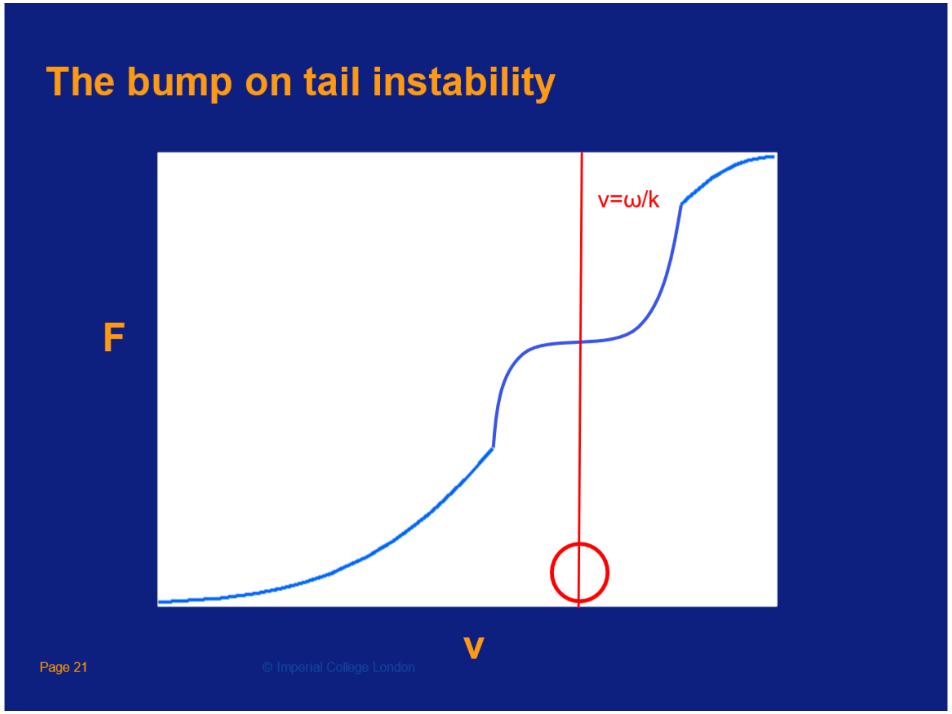
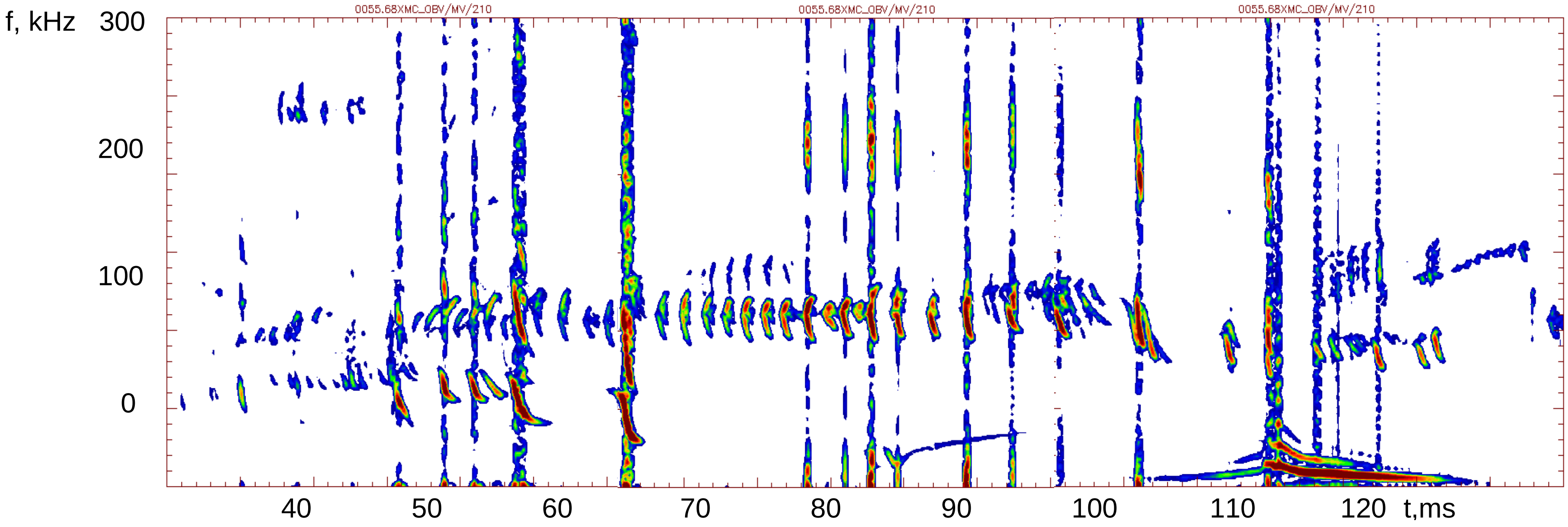
THE UNI-DIRECTIONAL EP FLOW DUE TO THE DRAG ACROSS THE RESONANCE GENERATE EXPLOSIVE MODE EVOLUTION



BACK TO TAE: NBI-DRIVEN AEs ON MAST ARE DOMINATED BY THE DRAG IN THE V_A REGION



AEs are not steady-state modes, they burst in amplitude and sweep in frequency

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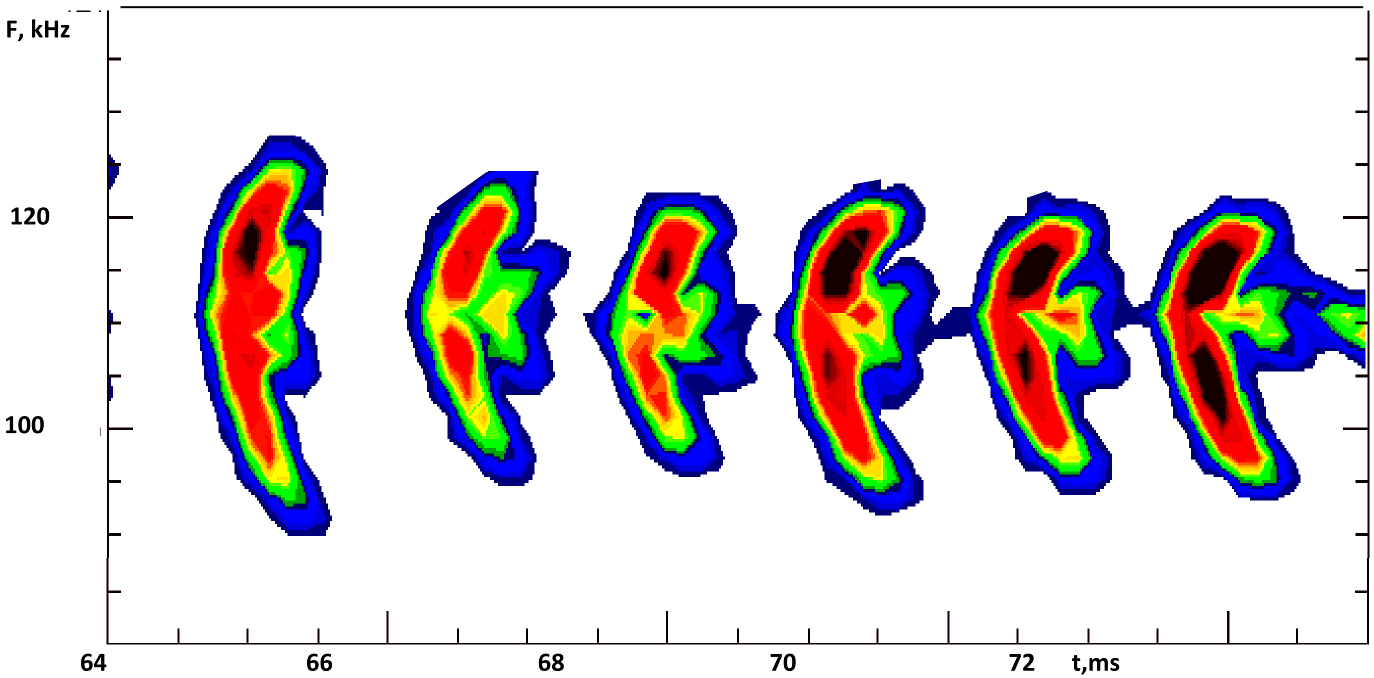
ZOOM OF TAEs DRIVEN BY NBI ON MAST



Pinches et al., PPCF 46 (2004) S47

FAST PARTICLE TRANSPORT CAUSED BY TAE

QUALITATIVE ESTIMATES – 1

- **The *unperturbed* orbit of a particle is determined by three invariants:**

$$\mu \equiv \frac{Mv_\perp^2}{2}\,; \qquad E \equiv \frac{Mv^2}{2}\,; \quad P_\varphi \equiv -\frac{e}{c}\psi(r) + RMv_\varphi$$

- **In the presence of a *single TAE* mode with perturbed quantities $\propto \exp i(n\varphi - \omega t)$, the wave-particle interaction is invariant with respect to transformation**

$$t \to t + \tau\,; \quad \varphi \to \varphi + \frac{\omega}{n}\tau$$

- **In the presence of the TAE, neither E nor P_φ is conserved for particle orbit, but *their following combination is still invariant*:**

$$E - \frac{\omega}{n}P_\varphi = const$$

- **Change in the particle energy is related to change in particle radius produced by TAE**

$$\Delta E = \frac{\omega}{n}\Delta P_\varphi \cong \frac{\omega e}{nc}\psi' \Delta r$$

- **The relative change in particle energy is much smaller than in particle radius:**

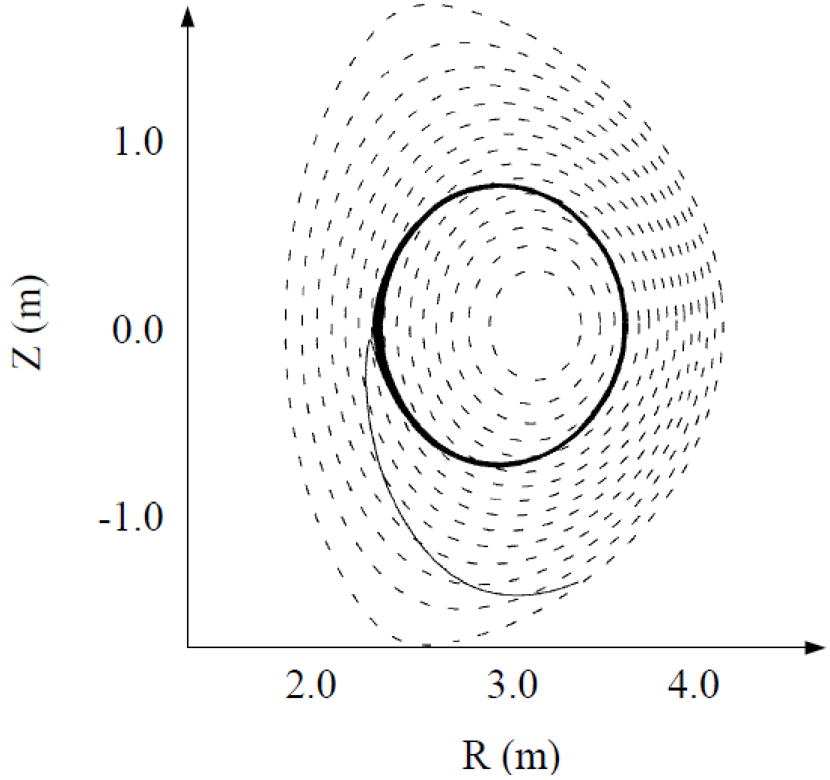
$$\frac{\Delta v}{v} = \frac{\omega}{\omega_{*\alpha}} \cdot \frac{\Delta r}{L_\alpha}\,; \quad \textbf{where } \omega_{*\alpha} \equiv \frac{nq\rho_\alpha v}{2rL_\alpha} >> \omega$$

QUALITATIVE ESTIMATES – 2

- **The interaction between TAE and fast particles causes *radial transport of the particles at nearly constant energy***
- **This type of interaction is very unpleasant as it tends to deposit a population of fusion born alphas too close to the first wall**
- **Losses of fusion born alphas must be minimised down to few percent (<5% on ITER) for avoiding the first wall damage**
- **The radial redistribution also gives a non self-consistent alpha-heating profiles etc. and may affect the burn**

TAE-INDUCED TRANSPORT IN PRESENT-DAY MACHINES

• **Fast ion orbits comparable to the machine radius, $\rho_\alpha / a \cong 10^{-1} \div 1$, typical of present-day machines, could be lost due to a *single mode perturbation.* The losses scale as $\propto \delta B_{TAE}$ in such case. Example: TAE-induced enhancement of prompt losses due to topology change of fast ion orbit:**



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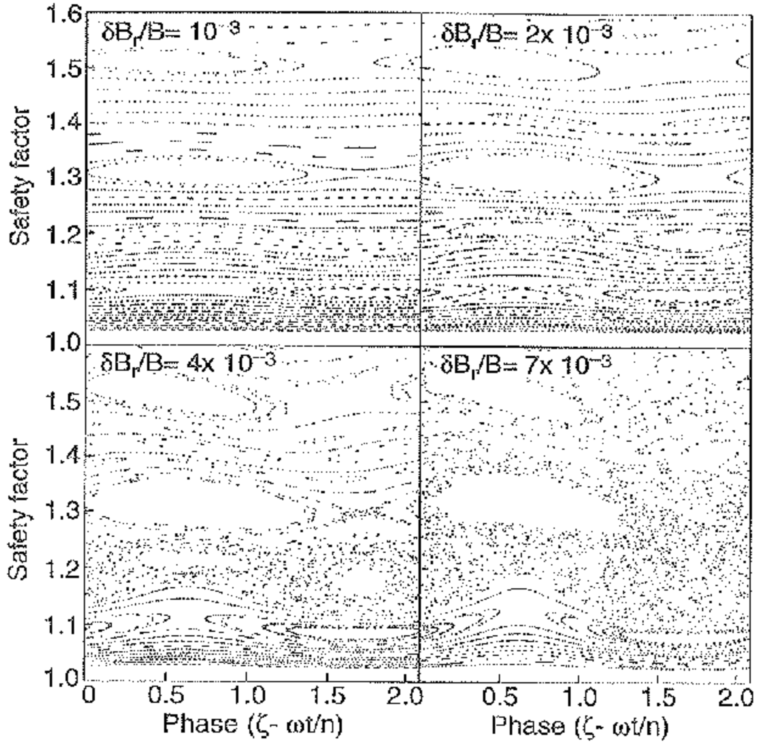
MAIN TYPE OF TAE-INDUCED TRANSPORT IN ITER

- **For ITER with parameter $\rho_\alpha / a \cong 10^{-2}$, the dominant channel of alpha-particle transport will differ from that on present-day machines.**

- **On ITER, higher-*n* (*n* > 10) TAEs will be most unstable. The radial width of a poloidal harmonic will be more narrow, $\Delta_{\mathrm{mode}} \propto r_{AE} / nq$, but the number of unstable modes may be significantly larger than in present-day tokamaks**

- **Resonance overlap will lead to a global stochastic diffusion of energetic ions over a broad region with unstable TAEs, with transport $\propto \delta B_{TAE}^2$**

DRIFT ORBIT STOCHASTICITY



- **The analytically derived stochasticity threshold (Berk et al Phys. Fluids B5, 1506, 1993) is close to that obtained numerically:**

$$\delta B_r / B_0 > r_{TAE} \cdot \left(64 m R_0 q S\right)^{-1} \cong 1.5 \times 10^{-3} / m$$

SUMMARY

- Fusion-born alpha-particles are super-Alfvénic and may excite shear Alfvén waves via the resonance $V_\alpha = V_A$ if Alfvén waves have lower damping than fast particle drive
- Shear Alfvén waves with frequencies close to frequency at $d(k_{\parallel}V_A)/dr=0$ may form a discrete spectrum and can be excited more easily with fast ions: GAE in cylinder, TAE in torus
- Experimentally, TAE frequency and mode structure seen in present-day machines agree well with linear theory
- Nonlinear 1D bump-on-tail near-threshold theory of fast particle-driven instabilities is in a remarkable agreement with the experimental data in cases of dominant diffusion or dominant drag
- Transport of fast ions on machines with $\rho_\alpha / a \cong 10^{-1} \div 1$ is mostly caused by single-mode perturbation; on ITER with $\rho_\alpha / a \cong 10^{-2}$ a multi-mode stochastic transport will dominate