PART II

FAST PARTICLE INSTABILITIES IN MAGNETIC NUCLEAR FUSION: EXPERIMENT VERSUS THEORY

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OUTLINE

- INTRODUCTION: ALPHA-DRIVEN INSTABILITIES
- WEAKLY-DAMPED ALFVÉN EIGENMODES (AEs): GLOBAL AE (GAE) IN CYLINDRICAL PLASMA AND TOROIDAL AE (TAE) IN TOROIDAL PLASMA
- WEAKLY-DAMPED TAEs: EXPERIMENT VS LINEAR THEORY
- NONLINEAR EVOLUTION OF FAST PARTICLE-DRIVEN MODES
- TAE-INDUCED REDISTRIBUTION AND LOSSES OF FAST IONS
- SUMMARY



THE PROBLEM: ALFVÉN INSTABILITIES IN BURNING PLASMA

- Alpha-particles (He⁴ ions) are born in deuterium-tritium nuclear reactions with birth energy 3.52 MeV exceeding plasma temperature by ~100 times
- For typical B and n achievable in fusion reactors, velocities of alpha-particles exceed Alfvén velocity, $V_A=B/(4\pi\rho)^{1/2}$, so that

$$V_{Ti} << V_A < V_\alpha << V_{Te}$$

• During slowing-down, alpha-particles cross the resonance with Alfvén waves

$$\mathbf{V}_{\mathsf{A}} = \mathbf{V}_{\parallel \alpha}$$

and may excite Alfvén instabilities

- Free energy source for such instabilities is radial pressure gradient of α's, and the instability results in a radial re-distribution / loss of alphas when the amplitude of Alfvén wave becomes high.
- Present-day machines with ICRH/ NBI excite Alfvén instabilities very often.



THE EXCITATION CONDITION

A wave with frequency $\boldsymbol{\omega}$ and amplitude

A~ exp(γt)-exp (iωt)

becomes linearly unstable (i.e. its amplitude starts to increase)

IF

the alpha-particle drive exceeds the wave damping due to thermal plasma,

$$\gamma \equiv \gamma_{\alpha} - |\gamma_{damp}| > 0.$$

Waves with weak damping due to thermal plasma are easiest to excite with alpha-particles.



WEAKLY-DAMPED ALFVÉN EIGENMODES (AEs): GLOBAL AE IN CYLINDRICAL PLASMA AND TOROIDAL AE IN TOROIDAL PLASMA



THE SHEAR ALFVÉN WAVE

• For shear Alfvén waves the main MHD equation for the fluid displacement vector $\boldsymbol{\xi}$ is

$$\frac{\partial^2 \boldsymbol{\xi}_{\perp}}{\partial t^2} = V_A^2 \frac{\partial^2 \boldsymbol{\xi}_{\perp}}{\partial z^2}$$

which coincides with equation, e.g., for string oscillations. The "returning" force is the tension of magnetic field lines, which acts similarly to the strings

• In shear Alfvén wave the fluid displacement vector ξ and \tilde{E} are perpendicular to the magnetic field B_0 . The wave propagates along B_0 :

$$\boldsymbol{\omega} = \pm k_{\parallel} V_{A}; V_{A} = \frac{B_{0}}{\sqrt{4\pi \sum_{i} n_{i} M_{i}}}; k_{\parallel} = \mathbf{k} \cdot \mathbf{B}_{0} / B_{0}$$

 Among all the waves in plasmas, the Alfvén wave (H. Alfvén, Arkiv. Mat. Astron. Fysik 29B(2) (1942)) constitutes the most significant part of the MHD spectrum and is probably the best studied



HOW ALFVÉN WAVE PACKET EVOLVES IN *INHOMOGENEOUS* PLASMA?



Life-time τ of a radially-extended wave-packet of shear Alfvén waves is limited by "phase mixing" between modes at different radii forming the Alfvén continuum

$$au^{-1} \propto rac{d}{dr} ig (k_{\parallel} \left(r
ight) \cdot V_A (r) ig),$$

 τ could be long only if the wave-packet is localised in vicinity of an extremum point of the Alfvén continuum,

$$\frac{d}{dr} \left(k_{\parallel} \left(r \right) \cdot V_A(r) \right) = 0$$



GLOBAL ALFVÉN EIGENMODES IN CYLINDRICAL PLASMAS

• In *cylindrical* geometry, in addition to the continuous Alfvén spectrum, $\omega^2 = \omega_A^2(r) \equiv k_{\parallel}^2(r) V_A^2(r)$, a *discrete* Global Alfvén Eigenmode with frequency

 $\omega_{GAE} < \omega_A$ exists in plasma with current (D.W.Ross et al. Phys. Fluids 25, 652 (1982);K.Appert et al. Plasma Phys. 24, 1147 (1982))



The ideal plasma-coil-wall system used in the numerical investigation



DISCOVERY OF GLOBAL ALFVÉN EIGENMODE

• A new high-quality, $Q \equiv \omega/\gamma \sim 10^3$, resonance was discovered during these Alfvén antenna studies, in plasmas *with current*



Real part of the coil impedance vs normalized frequency



GLOBAL ALFVÉN EIGENMODE

• GAE with $\omega_{GAE} < \omega_A$ exists in Ideal MHD if the current profile determining dB_g/dr provides a minimum in Alfvén continuum



• The local minimum of the Alfvén continuum provides a maximum of the perpendicular refraction index $N_r = ck_r/\omega$. Similarly to fiber optics, the electromagnetic wave has to propagate in a "wave-guide" surrounding the region of the extremum refraction index.



NO CONTINUUM DAMPING FOR GLOBAL ALFVÉN EIGENMODE



Ideal MHD GAE with m=-2

• The eigenfrequency of GAE does *not* satisfy the local Alfvén resonance condition, $\omega_{GAE} \neq \omega_A(r)$, for 0 < r/a < 1. Therefore, this SA mode has *no* singularity and does *not* experience *continuum damping*



THE NEXT STEP: TOROIDAL GEOMETRY

In a torus, the wave solutions are quantized in toroidal and poloidal directions:

$$\phi(r, \vartheta, \zeta, t) = \exp(-i\omega t + in\zeta) \sum_{m} \phi_m(r) \exp(-im\vartheta) + c.c.$$

n is the number of wavelengths in toroidal direction and m is the number of wavelengths in poloidal direction

• The parallel wave-vector for the m-th harmonic of a mode with toroidal mode number n,

$$k_{\parallel m}(r) = \frac{1}{R} \left(n - \frac{m}{q(r)} \right),$$

is determined by the safety factor $q(r) = rB_{\zeta} / RB_{g}$.

- For a given q(r) and n, one finds m = nq giving a rational surface with $k_{\parallel m} = 0$
- This also gives $\omega_A(r) = k_{\parallel m}(r)V_A(r) = 0$ and the "gap" $0 < \omega < \omega_A$ cannot exist, in contrast to cylindrical geometry



WEAKLY-DAMPED TOROIDAL ALFVÉN EIGENMODES

• Could we obtain some other conditions so the Alfvén continuum can satisfy in a torus

$$\frac{d\omega_A(r)}{dr}\Big|_{r=r_0} = 0$$

• Yes, due to the poloidal dependence of the equilibrium,

$$B = B_0 (1 - (r/R)\cos \theta)$$

$$\cos \theta = (e^{i\theta} + e^{-i\theta})/2$$

the poloidal m-th and (m+1)-th harmonics of the Alfvén perturbation couple to form a gap in the Alfvén continuum at the frequency satisfying

$$\omega = k_{\parallel m}(r)V_A(r) = -k_{\parallel m+1}(r)V_A(r)$$





FREQUENCIES AS FUNCTION OF RADIUS FOR SHEAR ALFVÉN CONTINUUM AND DISCRETE WEAKLY-DAMPED TAE IN JET



Typical structure of Alfvén continuum (and SM continuum) in toroidal geometry



TOROIDAL ALFVÉN EIGENMODES COMPUTED IN IDEAL MHD:

• Similarly to GAE in cylinder, TAE frequency does not satisfy local Alfvén resonance condition in the region of TAE localization, $\omega_{TAE} \neq \omega_A(r)$, so TAE has no singularity and does not experience strong continuum damping \rightarrow are easier to excite with alpha-particles



Radial dependence of the Fourier harmonics of TAE



CHARACTERISTIC PROPERTIES OF WEAKLY-DAMPED TAE

• **TAEs** with perturbed scalar ϕ and vector δA potentials satisfy:

$$\delta B_{\parallel} \equiv \delta \mathbf{B} \cdot \mathbf{B}_{0} / B_{0} = 0 \quad \longrightarrow \quad \delta \mathbf{A} = \delta A_{\parallel} \left(\mathbf{B}_{0} / B_{0} \right) \equiv \delta A_{\parallel} \mathbf{b}$$
$$\delta E_{\parallel} \equiv \delta \mathbf{E} \cdot \frac{\mathbf{B}_{0}}{B_{0}} = 0 = -\mathbf{b} \cdot \nabla \phi - \frac{1}{c} \frac{\partial}{\partial t} \delta A_{\parallel} \quad \longrightarrow \quad k_{\parallel} \phi = \frac{\omega}{c} \delta A_{\parallel}$$

• TAEs have perturbed electric and magnetic fields perpendicular to B₀:

$$\begin{split} \delta E_r &= -\frac{\partial \phi_m}{\partial r} \exp(i[n\zeta - m\vartheta - \omega t]) + complex \ conjugate \ (c.c.\\ \delta E_\vartheta &= \frac{im}{r} \phi_m \exp(i[n\zeta - m\vartheta - \omega t]) + c.c.\\ \delta B_r &= -\frac{k_{\parallel} c}{\omega} \cdot \frac{im}{r} \phi_m \exp(i[n\zeta - m\vartheta - \omega t]) + c.c.\\ \delta B_\vartheta &= -\frac{k_{\parallel} c}{\omega} \cdot \frac{\partial \phi_m}{\partial r} \exp(i[n\zeta - m\vartheta - \omega t]) + c.c. \end{split}$$

)



WEAKLY-DAMPED TAES IN JET:

EXPERIMENT VS LINEAR THEORY



TOKAMAK JET (JOINT EUROPEAN TORUS)



Volume ~ 100 m³; $B_{max} = 4 T$; $I_{max} = 7 MA$; $P_{FUS} = 16 MW$



ENERGETIC IONS IN JET VERSUS ALPHAS IN ITER

Machine	JET	JET	JET	JET	ITER
Type of fast ions	Hydrogen	He ³	He ⁴	Alpha	Alpha
Source	ICRH tail	ICRH tail	ICRH tail	Fusion	Fusion
Mechanism	minority	minority	3 rd harm. NBI	DT nuclear	DT nuclear
Vf(0)/VA(0)	≈2	≈1.5	≈1.3	1.6	1.9
τ _s (s)	1.0	0.9	0.4	1.0	0.8
<i>P</i> _f (0) (MW/m ³)	0.8	1.0	0.5	0.12	0.55
<i>n</i> _f (0) / <i>n</i> _e (0) (%)	1.0	1.5	1.5	0.44	0.85
β _f (0) (%)	2	2	3	0.7	1.2
<β _f > (%)	0.25	0.3	0.3	0.12	0.3
max <i>Rβ'_f</i> / (%)	≈5	≈5	5	3.5	3.8

Ratio of on-axis velocities $V_f(0)/V_A(0)$, slowing down time, τ_s , heating power per volume, $P_f(0)$, ratio of the fast ion density to electron density, $n_f(0) / n_e(0)$, on-axis fast ion beta, $\beta_f(0)$, volume-averaged fast ion beta, $\langle \beta_f \rangle$, and normalised radial gradient of fast ion beta, max| $R\beta'_f$ |, in JET vs. ITER projected parameters.



TAE PARAMETERS IN JET EXPERIMENTS

• Characteristic frequencies of the TAE gap for JET parameters are:

$$B_0 \cong 3 T; n_i = 5 \times 10^{19} m^{-3}; m_i = m_D$$

$$\downarrow$$

$$V_A \cong 6.6 \times 10^6 m/s$$

• For typical value of q=1.1 one obtains the frequency estimate then:

$$\omega_{TAE} \cong 10^6 \text{ sec}^{-1} \rightarrow f_{TAE} \equiv \omega_{TAE} / 2\pi \cong 160 \text{ kHz}$$



ALFVÉN WAVES ARE BEST SEEN FROM **\delta** B PERTURBATIONS



• Mirnov coils are used for measuring magnetic flux

 $\frac{\partial}{\partial t} \delta B_{g}^{edge} \cong \omega \cdot \delta B_{g}^{edge}$

- The coils are VERY sensitive for high frequencies, e.g. for values of $\omega \approx 10^6 \text{ sec}^{-1}$ perturbed fields $\left| \delta B_g^{edge} / B_0 \right| \approx 10^{-8}$ are measured
- Measurements up to 500 kHz

JET cross-section showing the position and directivity of Mirnov coils separated in toroidal angle



TYPICAL DATA SHOWS DISCRETE SPECTRUM OF FAST ION-DRIVEN WAVES





COMPUTED VERSUS OBSERVED TAEs



Eigenfrequencies of TAEs with n=4...9 computed for equilibrium in JET discharge #40332. Added Doppler shift matches the experiment

Discrete spectrum of TAE observed in JET discharge #40332. Plasma starts at t=40 sec. Frequency changes due to plasma density increase, $f \sim B/\sqrt{n_i M_i}$.





TAE EXCITATION DEPENDS ON THE MODE NUMBER





FAST PARTICLE DRIVE: QUALITATIVE PICTURE - 1



• TAE modes are attached to magnetic flux surfaces, while the resonant ions, both passing and trapped, experience significant drift across the flux surfaces and TAE mode structure





- r/a
- When resonant ion moves radially across TAE from point A to point B, the mode and the ion exchange energy $e\Delta\phi$ as Figure shows.





The maximum of $e\Delta\phi$ occurs when the orbit size, Δ_{orbit} , is equal to the mode width, $\Delta_{TAE} \approx r_{TAE} / m$, where m is poloidal mode number. This is why energetic ion drive is expected to have a maximum at $m \approx nq \approx r_{TAE} / \Delta_{orbit}$



NONLINEAR EVOLUTION OF FAST PARTICLE-DRIVEN MODES



DIFFERENT REGIMES OF MODE EVOLUTION OBSERVED ON DIFFERENT MACHINES:





ICRH-driven TAEs during ICRH power ramp-up on JET

NBI-driven TAEs on MAST have bursting amplitudes and sweep in frequency



THE NEAR-THRESHOLD CONDITION

• Consider the scenario with a gradual build-up of fast ion pressure so that the fast ion drive of TAE, $\gamma_{\alpha}(t) \propto -\beta'_{\alpha}(t)$, increases in time at unchanged TAE damping γ_d



- TAE instability threshold: exact balance between TAE drive and damping, $\gamma_{\alpha} = \gamma_{d}$
- The near-threshold condition:

$$\left| \boldsymbol{\gamma}_{\alpha} - \boldsymbol{\gamma}_{d} \right| << \boldsymbol{\gamma}_{d} \leq \boldsymbol{\gamma}_{\alpha}$$



HOW TAE INSTABILITY SATURATES?



 Non-linear TAE behaviour: competition between the field of the mode that tends to flatten distribution function near the resonance (effect proportional to the net growth rate γ=γL-γd) and the collision-like processes that constantly replenish it (proportional to veff)



THE COLLISIONALITY

• The near-threshold regime allows the "collisions" restoring the unstable distribution function of fast ions to compete with the mode growth

$$\left|\gamma_{\alpha}-\gamma_{d}\right|\approx v_{eff}$$

- Demonstrate this effect *analytically for the "bump-on-tail"* problem in a *1D velocity* space. This problem has physics similar to TAE, but is 1D.
- The "bump on tail" problem: consider the *nonlinear* evolution of a *marginally unstable* electrostatic wave with frequency $\omega = \omega_{pe} = \sqrt{4\pi n_e e^2 / m_e}$ in the presence of an unstable beam distribution function F(x, v, t) with collisional operator (Berk et al., PRL 76 1256 (1996))
- Collisional operator that includes electron drag, diffusion, and Krook terms was found to reproduce all near-threshold nonlinear scenarios



BERK-BREIZMAN NEAR-THRESHOLD THEORY: THE WAVE AMPLITUDE EQUATION WITH DRAG, DIFFUSION, AND KROOK

$$\frac{dA}{d\tau} = A\left(\tau\right) - \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) \int_{0}^{\tau-2z} dx \, e^{-\hat{\nu}^3 z^2 (2z/3+x) - \hat{\beta}(2z+x) + i\hat{\alpha}^2 z(z+x)} \\ \times A\left(\tau - z - x\right) A^*\left(\tau - 2z - x\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) \left(\tau - 2z - x\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2} \int_{0}^{\tau/2} dz \, z^2 A\left(\tau - z\right) = \frac{1}{2}$$

where
$$A = \left[ek\hat{E}(t) / m (\gamma_l - \gamma_d)^2 \right] \left[\gamma_l / (\gamma_l - \gamma_d) \right]^{1/2}, \tau = (\gamma_l - \gamma_d) t, \hat{\nu}^3 = \nu^3 / (\gamma_l - \gamma_d)^3,$$

 $\hat{\alpha} = \alpha / (\gamma_l - \gamma_d)^2, \, \hat{\beta} = \beta / (\gamma_l - \gamma_d) \text{ and } \gamma_l = 2\pi^2 \left(e^2 \omega / mk^2 \right) \partial F_0(\omega/k) / \partial v.$





DIFFUSION ONLY CASE

Nonlinear equation for the amplitude

$$\frac{dA}{dt} = A - \frac{1}{2} \int_{0}^{t/2} \tau^{2} \int_{0}^{t-2\tau} \exp\left[-\nu^{3}\tau^{2} \left(2\tau/3 + \tau_{1}\right)\right] \\ \times A(t-\tau)A(t-\tau-\tau_{1})A^{*} \left(t-2\tau-\tau_{1}\right)d\tau_{1}d\tau$$

describes *four* regimes of mode evolution: a) Steady-state;

- b) Periodically modulated;
- c) Chaotic;
- d) Explosive

The explosive regime in a more complete nonlinear model leads to frequency-sweeping 'holes' and 'clumps' on the perturbed distribution function (H.L.Berk, B.N.Breizman, and N.V.Petviashvili, Phys. Lett. A234 (1997) 213)



THE BUMP-ON-TAIL SOLUTIONS WITH DIFFUSION ARE OBSERVED FOR ICRH-DRIVEN TAE ON JET





At gradually increasing ICRH power, TAEs exhibit steady state, periodically modulated, and chaotic regimes Magnetic spectrogram corresponding to the left Figure with raw data. Steady state, periodically modulated (pitchfork splitting), and chaotic regimes are seen



THE CHAOTIC REGIME FOR TAE





Time (sec) The chaotic TAE evolution significantly complicates the phase analysis of TAE mode numbers





ICRH REPLENISHES FAST ION DISTRIBUTION VIA DIFFUSION





DRAG ONLY CASE - 1



Evolution of |A| in time, $t \equiv \gamma t$

Nonlinear equation for the amplitude

$$\frac{dA}{dt} = A - \frac{1}{2} \int_{0}^{t/2} \tau^{2} \int_{0}^{t-2\tau} \exp\left[i\hat{\alpha}^{2}\tau(\tau+\tau_{1})\right]$$

× $A(t-\tau)A(t-\tau-\tau_{1})A^{*}(t-2\tau-\tau_{1})d\tau_{1}d\tau$

In contrast to the diffusion case, drag gives oscillatory behaviour in the kernel leading to the explosive evolution of the amplitude blowing up in a finite time,

$$A \propto \left(t - t_0\right)^{-p}$$

This is the only scenario for the drag !

M.Lilley, B.N.Breizman, and S.E.Sharapov, Phys. Rev. Lett. (2009)



DRAG ONLY CASE - 2





BACK TO TAE: NBI-DRIVEN AES ON MAST ARE DOMINATED BY THE DRAG IN THE V_A REGION



AEs are not steady-state modes, they burst in amplitude and sweep in frequency



FAST PARTICLE TRANSPORT CAUSED BY TAE



QUALITATIVE ESTIMATES – 1

• The *unperturbed* orbit of a particle is determined by three invariants:

$$\mu \equiv \frac{M v_{\perp}^2}{2}; \qquad E \equiv \frac{M v^2}{2}; \quad P_{\varphi} \equiv -\frac{e}{c} \psi(r) + R M v_{\varphi}$$

• In the presence of a single TAE mode with perturbed quantities $\propto \exp i(n\varphi - \omega t)$, the wave-particle interaction is invariant with respect to transformation

$$t \rightarrow t + \tau; \quad \varphi \rightarrow \varphi + \frac{\omega}{n} \tau$$

• In the presence of the TAE, neither E nor P_{φ} is conserved for particle orbit, but *their* following combination is still invariant:

$$E - \frac{\omega}{n} P_{\varphi} = const$$

• Change in the particle energy is related to change in particle radius produced by TAE

$$\Delta E = \frac{\omega}{n} \Delta P_{\varphi} \cong \frac{\omega e}{nc} \psi' \Delta r$$

• The relative change in particle energy is much smaller than in particle radius:

$$\frac{\Delta v}{v} = \frac{\omega}{\omega_{*\alpha}} \cdot \frac{\Delta r}{L_{\alpha}}; \quad \text{where } \omega_{*\alpha} \equiv \frac{nq\rho_{\alpha}v}{2rL_{\alpha}} >> \omega$$



QUALITATIVE ESTIMATES – 2

- The interaction between TAE and fast particles causes radial transport of the particles at nearly constant energy
- This type of interaction is very unpleasant as it tends to deposit a population of fusion born alphas too close to the first wall
- Losses of fusion born alphas must be minimised down to few percent (<5% on ITER) for avoiding the first wall damage
- The radial redistribution also gives a non self-consistent alpha-heating profiles etc. and may affect the burn



TAE-INDUCED TRANSPORT IN PRESENT-DAY MACHINES

• Fast ion orbits comparable to the machine radius, $\rho_{\alpha}/a \cong 10^{-1} \div 1$, typical of presentday machines, could be lost due to a *single mode perturbation*. The losses scale as $\propto \delta B_{TAE}$ in such case. Example: TAE-induced enhancement of prompt losses due to topology change of fast ion orbit:





MAIN TYPE OF TAE-INDUCED TRANSPORT IN ITER

• For ITER with parameter $\rho_{\alpha}/a \cong 10^{-2}$, the dominant channel of alpha-particle transport will differ from that on present-day machines.

• On ITER, higher-*n* (*n* > 10) TAEs will be most unstable. The radial width of a poloidal harmonic will be more narrow, $\Delta_{\text{mod}e} \propto r_{AE} / nq$, but the number of unstable modes may be significantly larger than in present-day tokamaks

• Resonance overlap will lead to a global stochastic diffusion of energetic ions over a broad region with unstable TAEs, with transport $\propto \delta B_{TAE}^2$



DRIFT ORBIT STOCHASTICITY



• The analytically derived stochasticity threshold (Berk et al Phys. Fluids B5, 1506, 1993) is close to that obtained numerically:

$$\delta B_r / B_0 > r_{TAE} \cdot (64mR_0 qS)^{-1} \cong 1.5 \times 10^{-3} / m$$



SUMMARY

- Magnetic fusion approaches the phase when self-heating of DT plasmas with alphaparticles becomes a dominant physics phenomenon. Confinement of the alphas and their interaction with Alfvén waves causes uncertainty in predicting transport of alphas
- Fusion-born alpha-particles are super-Alfvénic and may excite shear Alfvén waves via the resonance $V_{\alpha} = V_A$ if Alfvén waves have lower damping than fast particle drive
- Shear Alfvén waves with frequencies close to frequency at d(k_{II}V_A)/dr=0 may form a discrete spectrum and can be excited more easily with fast ions
- Experimentally, AEs seen in present-day machines agree well with linear theory
- Nonlinear near-threshold theory of fast particle-driven instabilities is in remarkable agreement with the experimental data
- Transport of fast ions on machines with $\rho_{\alpha} / a \cong 10^{-1} \div 1$ is mostly caused by single-mode perturbation; on ITER with $\rho_{\alpha} / a \cong 10^{-2}$ a multi-mode stochastic transport will dominate

