Linear waves and reconnecting instabilities in slab magnetised low-beta non-relativistic electron-positron plasmas

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The properties of a non-relativistic low-beta magnetised electron-positron plasma in slab geometry are investigated. The two species are taken to be drift-kinetic while we retain Larmor radius effects in quasi-neutrality, and inertia in Ohm's law. It is shown that the system supports collisionless dispersive waves, which can greatly impact nonlinear magnetic reconnection, but only for large β, the ratio of kinetic and plasma pressure. It is found that the drift wave is not present in such plasma. Tearing modes can be driven unstable by equilibrium current density gradients. When the ordering of the fields' amplitudes is made consistent with those of Zocco & Schekochihin (2011), an improved set of nonlinear fluid-like equations is derived where magnetic compressibility must be included, and drift waves contributions do not cancel.

1. Introduction

Electron-positron plasmas have played a crucial role in the theory of magnetic reconnection. By exploiting the similarities of a simple fluid model and electron magnetohydrodynamics (EMHD) with electron inertia (Zocco *et al.* 2008*a*, 2009), Chacón *et al.* (2008) have shown that dispersive waves are not the cause of fast magnetic reconnection. This result came as a confirmation of earlier particle-in-cell simulation results (Bessho & Bhattacharjee 2005; Daughton & Karimabadi 2007). Non-relativistic electron-positron plasmas, however, are not only pathological models which are useful to settle controversies among theoreticians. There is now great excitement about the creation of a laboratory electron-positron plasma (Pedersen *et al.* 2012; Saitoh *et al.* 2014) which, by itself, justifies new investigations in this field.

In this article we revisit some fluid equations similar to those of Chacón *et al.* (2008), but in the framework of gyrokinetics for magnetic reconnection of Zocco & Schekochihin (2011). One new aspect here introduced is in the quasineutrality equation. It is proposed that charge neutralizes above the Larmor scale, $\rho_e = v_{the}/\Omega_c$, which is assumed to be much smaller than the inertial scale $d_e = \rho_e/\sqrt{\beta}$, where β is the ratio of kinetic to plasma pressure, $\Omega_c = eB/(mc)$ is the cyclotron frequency, and v_{the} is the thermal speed of both species. The inclusion of the Debye length (Helander 2014; Helander & Connor 2016), is avoided as this would require an electromagnetic relativistic treatment. Linear waves and reconnecting instabilities are first investigated adopting a simple drift-kinetic model, without ordering fields' amplitudes. In this case, the effect of drift waves cancels exactly, dispersive waves frequency does not diverge at short wavelengths, reconnecting instabilities proceeds as in a one-fluid electron-ion plasma, where the Alfvén speed is replaced by the electron Alfvén speed. An improved set of nonlinear fluid-like equations is then derived. Here the relative amplitudes of fields are made consistent with the orderings of Zocco & Schekochihin (2011), where the nonlinear $\mathbf{E}\times\mathbf{B}$ frequency is comparable to the electron passing frequency, and the spatial variation of all quantities along the total magnetic field is retained. In this new model, diamagnetic effects enter through temperature fluctuations and magnetic compressibility, and do not always cancel. Equations are presented in Sec. (2). Linear waves are discussed in Sec. (3). Current-driven instabilities are studied in Sec. (4). The new improved model is derived in Sec. (5).

2. Equations

Following Helander Helander (2014), Poisson's equation for the electrostatic potential, φ, is written in the following way,

$$\frac{T_0}{2e}\left(\frac{\delta n_{e+}}{n_0}-\frac{\delta n_{e-}}{n_0}\right)=-\rho_e^2\nabla^2\varphi, \tag{2.1}$$

where we are assuming

$$\frac{\delta n_{e+}}{n_0}-\frac{\delta n_{e-}}{n_0}\sim\beta\frac{e\varphi}{T_0}\ll\frac{e\varphi}{T_0}, \tag{2.2}$$

which, for $k_\perp d_e\sim 1$, with $d_e=c/\omega_{pl}$, $\beta=8\pi n_0T_0/B^2\ll 1$, and

$$d_e\gg\rho_e\gg\lambda_D, \tag{2.3}$$

guarantees balance in Eq. (2.1).

Density fluctuations are calculated by taking the zeroth moment of perturbed distribution function $\delta f_{e\mp}=-e_\pm\varphi F_0/T_0+h_{e\mp}$, where the non-adiabatic part of δf satisfies the electromagnetic drift-kinetic equation [the $k_\perp\rho\to 0$ limit of Frieman & Chen (1982) nonlinear gyrokinetic equation] (i)

$$\begin{aligned}\frac{dh_{e\mp}}{dt}+v_\parallel\hat{\mathbf{b}}\cdot\nabla h_{e\mp}&=\frac{e_\mp F_0}{T_0}\frac{\partial}{\partial t}\left(\varphi-\frac{v_\parallel}{c}A_\parallel\right)\\-\frac{c}{B_0}\mathbf{e}_z\cdot\nabla\left(\varphi-\frac{v_\parallel}{c}A_\parallel\right)&\times\nabla F_0+\left(\frac{dh_{e\mp}}{dt}\right)_{coll},\end{aligned} \tag{2.4}$$

where $d/dt=\partial_t+B_0^{-1}\{\varphi,\}$, $\hat{\mathbf{b}}\cdot\nabla=\partial_z-B_0^{-1}\{A_\parallel,\}$, $v_\parallel$ is the particles velocity in the direction parallel to the guide field of modulus B_0, and F_0 is the Maxwellian equilibrium. The result is the familiar continuity equation

$$\frac{d}{dt}\frac{\delta n_{e\mp}}{n_0}=-\hat{\mathbf{b}}\cdot\nabla u_{\parallel e\mp}-i\omega_{*e\mp}\frac{e^{\mp}\phi}{T_0}, \tag{2.5}$$

where we are using the local approximation

$$\mathbf{v}_E\cdot\frac{\nabla n_0}{n_0}=-i\omega_{*e\mp}\frac{e^{\mp}\phi}{T_0} \tag{2.6}$$

for the background density gradient, which introduces effects associate with the diamagnetic frequency $\omega_{*e\mp}=-iv_{the}/(2L_n)\rho_e\partial_y$, where $\nabla n_0/n_0\approx -L_n^{-1}$. Equation (2.5), if we take into account Eq. (2.1), implies that

$$\omega\sim k_\parallel v_{A,e}\ \text{ and }\ \frac{v_{the}}{c}A_\parallel\sim\sqrt{\beta}\varphi, \tag{2.7}$$

where $v_{A,e}=B/\sqrt{4\pi m_e n_0}$ is the Alfvén speed based on the electron mass.

(i) but see also Zocco & Schekochihin (2011); Zocco *et al.* (2015); Loureiro *et al.* (2016)

We calculate the $v_\parallel-$moment of Eq. (2.4), to obtain

$$\begin{aligned}&\frac{d}{dt}\left(A_\parallel+\frac{mc}{e^{\mp}}u_{\parallel e^{\mp}}\right)=-c\frac{\partial\varphi}{\partial z}-\frac{T_0c}{e^{\mp}}\hat{\mathbf{b}}\cdot\nabla\left(\frac{\delta n_{e^{\mp}}}{n_0}+\frac{\delta T_{\parallel e^{\mp}}}{T_0}\right)\\&+i\omega_{*e^{\mp}}\left(1+\eta_e\right)A_\parallel+\frac{mc}{e}\nu\left(u_{\parallel e^-}-u_{\parallel e^+}\right),\end{aligned}\tag{2.8}$$

where $\eta_e=n_0\nabla T_0/(T_0\nabla n_0)$, ν is the collision frequency, and a simple collision model operator has been used (Zocco & Schekochihin 2011). Parallel Ampère's law gives

$$\frac{e}{mc}d_e^2\nabla^2A_\parallel=u_{\parallel e^-}-u_{\parallel e^+}.\tag{2.9}$$

The system is closed with an equation for the temperature fluctuations, derived using a highly collisional fluid closure for the flux of energy (Zocco & Schekochihin 2011; Zocco *et al.* 2015)

$$\begin{aligned}&\frac{d}{dt}\frac{\delta T_{\parallel e^{\mp}}}{T_0}=\frac{v_{the}^2}{2\nu}\left(\hat{\mathbf{b}}\cdot\nabla\right)^2\frac{\delta T_{\parallel e^{\mp}}}{T_0}-i\frac{v_{the}^2}{2\nu}\hat{\mathbf{b}}\cdot\nabla\eta_{e^{\mp}}\omega_{*e^{\mp}}\frac{e^{\mp}A_\parallel}{T_0}\\&-i\eta_{e^{\mp}}\omega_{*e^{\mp}}\frac{e^{\mp}\varphi}{T_0}-2\hat{\mathbf{b}}\cdot\nabla u_{\parallel e^{\mp}}.\end{aligned}\tag{2.10}$$

This is just a choice that facilitates the forthcoming discussion. The system could easily be left completely kinetic, then Eq. (2.10) would couple to higher order moments. However, each of these moments would follow a universal equation when projected on the basis of Hermite polynomials which allow for an efficient treatment of the non-isothermal case $\delta T_\parallel\neq0$ [(Zocco & Schekochihin 2011; Zocco *et al.* 2015; Zocco 2015; Schekochihin *et al.* 2016)]. In the truly collisionless case the hierarchy of Hermite moments generates a plasma response which was proven to be equivalent to the collisionless response evaluated via Landau contour integration (Zocco 2015). The isothermal approximation instead, $\delta T_\parallel\equiv0$, would be described by the electron response of the nonlinear model of Schep *et al.* (1994). In the context of linear magnetic reconnection, the presence of temperature fluctuations is a technicality that has an impact on the transition from collisional to collisionless regimes, but it is irrelevant when one wants to estimate reconnection rates at very low and very high collisionality. The inclusion of the resonant electron response (i.e. Landau resonance) is also not necessary to obtain correct reconnection rates in the collisionless limit, since such regime is really entered as soon as the inertial scale exceeds the resistive one, and this can happen even when the amount of collisions is finite. For this reasons, we are justified to use Eq. (2.10) and yet consider a collisionless limit for linear magnetic reconnection. Nonlinearly, the role of high order moments that couple to the equation for temperature fluctuations is very important, as it was shown by Loureiro *et al.* (2013) for electron-ion plasmas.

3. Waves

Let us consider the limit of small electron thermal diffusivity. In this case,

$$\frac{d}{dt}\frac{\delta T_{\parallel e^{\mp}}}{T_0}\approx-i\omega_{*e^{\mp}}\frac{e^{\mp}\varphi}{T_0}-2\hat{\mathbf{b}}\cdot\nabla u_{\parallel e^{\mp}}.\tag{3.1}$$

We use Eq. (3.1) in Eqs. (2.8), we then add the parallel moment equations of the two species, to notice that diamagnetic effects cancel exactly. Thus, we obtain

$$A_\parallel-\frac{k_\parallel c}{\omega}\varphi=\frac{\nu}{i\omega}\left(1-i\frac{\omega}{2\nu}-\frac{3}{4}\frac{k_\parallel^2v_{the}^2}{i\omega\nu}\right)k_\perp^2d_e^2A_\parallel.\tag{3.2}$$

On the other hand, Poisson's equation, after using the continuity equations and Ampère's law, becomes

$$\phi = \frac{1}{4\beta_e}\frac{k_\parallel v_{the}}{\omega}\frac{v_{the}}{c}A_\parallel. \tag{3.3}$$

By combining Eq. (3.2) and (3.3), we obtain

$$\omega^2 = \frac{1}{4}\frac{k_\parallel^2 v_{A,e}^2}{1+\left(1+i\frac{2\nu}{\omega}\right)k_\perp^2 d_e^2/2}, \tag{3.4}$$

where we are taking the limit

$$k_\perp^2\rho_e^2 \sim \beta \ll 1. \tag{3.5}$$

Thus, we find no drift wave, a result also obtained by Helander Helander (2014). In the "collisionless" regime ($\omega \gg \nu$) we find the dispersive waves

$$\omega^2 = \frac{1}{4}\frac{k_\parallel^2 v_{A,e}^2}{1+k_\perp^2 d_e^2/2}, \tag{3.6}$$

which, at long wavelengths, becomes a shear Alfvèn wave,

$$\omega^2 \approx \frac{k_\parallel^2 v_{A,e}^2}{4}. \tag{3.7}$$

In the presence of collisions, electron thermal conduction induces a damping at short wavelengths

$$\omega \approx -i\frac{3}{4}\frac{k_\parallel^2 v_{the}^2}{\nu}. \tag{3.8}$$

Perhaps not surprisingly, Eq. (3.8) defines the semicollisional scale introduced by Drake & Lee (1977). In the subsidiary long wavelength limit, we have a shear Alfvén wave based on the electron mass

$$\omega \approx \pm\frac{k_\parallel v_{A,e}}{2}. \tag{3.9}$$

Had we retained the Debye length instead of the Larmor radius in Eq. (2.1) ($\rho_e^2 \to \lambda_D^2$), we would have found two waves travelling at the speed of light, which we prefer not to allow for. This would have been true also in the isothermal limit ($\delta T_\parallel = 0$). Then, Eq. (3.4) would have been $\omega = \pm k_\parallel c/2$, which, cannot be accepted. Had one retained the whole hierarchy of moments coupled to Eq. (2.10), valid for arbitrary collisionality, they would still have entered the dispersion relation via the $k_\perp^2\lambda_D^2$ term and yielded a wave travelling at the speed of light. We conclude that, within this ordering, a truly collisionless electromagnetic limit must be relativistic. The reason is more apparent if one ponders the consequences of allowing the electrostatic potential to vary on the Debye scale, while letting the current varying on the inertial scale. This implies (i)

$$\lambda_D \sim d_e \to v_{the} \sim c, \tag{3.10}$$

which demands a covariant description.

(i) An electromagnetic gyrokinetic theory that retains Larmor radius effects seems to suffer from a similar problem, since in this case $\lambda_D \sim \rho \to v_A \sim c$, where ρ is the Larmor radius and v_A the Alfvén speed.

4. Reconnecting instabilities

When considering a sheared slab, in the neighbourhood of a resonant surface, we have

$$k_\parallel \approx k_y \frac{x}{L_s}, \tag{4.1}$$

where L_s is the shear length. Poisson's law and Ohm's law become, respectively

$$\rho_e^2 \frac{\partial^2 \varphi}{\partial x^2} = -\frac{k_y v_{the}}{4\omega} \frac{v_{the}}{c} \frac{x}{L_s} d_e^2 \frac{\partial^2 A_\parallel}{\partial x^2}, \tag{4.2}$$

and

$$A_\parallel - \frac{k_y c}{\omega} \frac{x}{L_s} \varphi = \left(i\frac{\nu}{\omega} + \frac{1}{2} - \frac{3}{4} \frac{k_y^2 v_{the}^2}{\omega^2} \frac{x^2}{L_s^2} \right) d_e^2 \frac{\partial^2 A_\parallel}{\partial x^2}, \tag{4.3}$$

which can easily be cast in the form also presented by Zocco & Schekochihin (2011). Now, we have

$$-\frac{x}{\delta} \left(A_\parallel - \frac{x}{\delta} \tilde{\varphi} \right) \sigma \left(\frac{x}{\delta} \right) = 4\rho_e^2 \frac{\partial^2 \tilde{\varphi}}{\partial x^2}, \tag{4.4}$$

and

$$-\frac{x}{\delta} d_e^2 \frac{\partial^2 A_\parallel}{\partial x^2} = 4\rho_e^2 \frac{\partial^2 \tilde{\varphi}}{\partial x^2}, \tag{4.5}$$

where $\delta = L_s\omega/(k_y v_{the})$, $\tilde{\varphi} = (c/v_{the})\varphi$, and

$$\sigma \left(\frac{x}{\delta} \right) = \frac{1}{i\frac{\nu}{\omega} + \frac{1}{2} - \frac{3}{4}\frac{x^2}{\delta^2}}. \tag{4.6}$$

Since we are always assuming $\rho_e \ll d_e \sim \delta$, we are effectively in a one-fluid limit, the ultralow-beta discussed by the authors. We report on the collisionless case; results apply to the collisional case in a straightforward manner. The analysis is known but we reproduce some key steps for the sake of clarity. One can introduce the function $\chi(\xi) = \xi A_\parallel' - A_\parallel$, where $\xi = x/\delta_{in}$, and $\delta_{in} = \sqrt{8\delta\rho_e^2}$, to obtain one equation for $\tilde{\chi} = -1 + \chi/\chi_0$,

$$\xi^2 \frac{d}{d\xi} \left[\frac{1}{\xi^2} + \alpha^2 G \right] \tilde{\chi}' - \left(\xi^2 + \lambda^2 \right) \tilde{\chi} = \lambda^2, \tag{4.7}$$

where $\lambda^2 = 8\delta\rho_e/d_e^2$, $\alpha = 2\sqrt{\rho/\delta}$, $G = (\delta^2/x^2)(\sigma^{-1} - 2)$, and χ_0 is a constant of integration. The dispersion relation for the rescaled eigenvalue λ^2 is then

$$\int_0^\infty d\xi \frac{\tilde{\chi}'}{\xi} = -\frac{\Delta' \delta_{in}}{2}, \tag{4.8}$$

where Δ' is the parameter that measures the discontinuity of the derivative of $A_\parallel^{MHD}$ across the reconnection layer, and $A_\parallel^{MHD}$ is the stable solution found in the ideal MHD region, $x \to \infty$ s.t. $E_\parallel \to 0$ (Furth *et al.* 1963). As already pointed out by Zocco & Schekochihin (2011), there is no need to solve Eq. (4.7) to derive scaling laws for reconnection rates. We can apply to our case Eq. (B47) that the authors suggest, $\lambda^2 \sim \delta_{in}\Delta'$, and obtain

$$\frac{\gamma}{k_y v_{the}} \sim (\Delta' d_e)^2 \frac{d_e^2}{\rho_e L_s}. \tag{4.9}$$

This is the equivalent of the collisionless result of Drake & Lee (1977) found in electron-ion plasmas, where the Alfvén speed is based on the electron mass. The collisional counterpart is recovered by replacing $d_e \to \sqrt{\nu d_e^2/\gamma}$, to obtain the traditional result of Furth *et al.*

(1963) (but based on the electron Alfvén speed). When $\Delta'\delta_{in} \gg 1$, the current is limited by the scale δ_{in}, so that $\partial_x^2 A_\parallel \sim A_\parallel/\delta_{in}^2$. Then the dispersion relation becomes $\lambda^2 \sim 1$, which yields (Basu & Coppi 1981)

$$\frac{\gamma}{k_y v_{the}} \sim \frac{d_e^2}{\rho_e L_s}, \tag{4.10}$$

which gives the (Coppi *et al.* 1976) scaling $\gamma \sim (\nu d_e^2)^{1/3}$ in the collisional limit. We must notice that the absence of drift waves renders these modes purely growing, since they lack the typical diamagnetic stabilisation present in electron-ion plasmas [(Ara *et al.* 1978; Connor *et al.* 2012)].

5. Towards an improved nonlinear model

The inclusion of the Larmor scale in Eq. (2.1), instead of the Debye length, allowed us to avoid a relativistic treatment. The use of the drift-kinetic model of Zocco & Schekochihin (2011) helped us, but we did not exploit its full nonlinear potential yet. For this, fields' amplitudes must be ordered more carefully. Equation (2.4), in fact, is nothing more than a drift-kinetic equation that one could have cosidered regardless of the results on magnetic reconnection in gyrokinetics [Zocco & Schekochihin (2011)]. What we already noticed about the amplitude of the electrostatic potential , togheter with Eq (2.1), implies that

$$\frac{\delta n}{n_0} \sim \sqrt{\beta}\epsilon_{GK}. \tag{5.1}$$

So, effectively, if one considers a quasineutrality equation where $\delta n_{e^-} = \delta n_{e^+}$ for zero Larmor radius, density fluctuations are downgraded as compared to electrostatic ones, when the $\mathbf{E}\times\mathbf{B}$ nonlinearity and the streaming term $k_\parallel v_{the}$ are kept of the same order. This also implies that magnetic compressibility now must be retained, since

$$\frac{\delta B_\parallel}{B_0} \sim \beta\frac{e\varphi}{T_0} \sim \sqrt{\beta}\epsilon_{GK}. \tag{5.2}$$

Perpendicular magnetic fluctuations are ordered by balancing the electrostatic and the vector potential amplitudes of the gyrokinetic potential $\chi = \varphi - (v_\parallel/c)A_\parallel$, then

$$\frac{\delta B_\perp}{B_0} \sim \frac{u_\perp}{v_{A,e}}\frac{1}{\sqrt{\beta}}, \tag{5.3}$$

where $u_\perp \sim ck_\perp\varphi/B_0$. In many relevant situations, the spatial variation of all quantities along the exact magnetic field is required, then $k_\perp\delta B_\perp \sim k_\parallel B_0$, which implies

$$\frac{u_\perp}{v_{A,e}} \sim \sqrt{\beta}\epsilon \rightarrow k_\perp\rho_e \sim \sqrt{\beta}, \tag{5.4}$$

and is naturally consistent with our fundamental ordering $k_\perp d_e \sim 1$. We therefore use

$$\frac{\delta B_\perp}{B_0} \sim \epsilon_{GK}, \tag{5.5}$$

which is different from what the continuity equation (2.5) would have implied

$$\frac{\delta B_\perp}{B_0} \sim \beta\epsilon_{GK}. \tag{5.6}$$

Having completed the amplitues orderings, in order to obtain fluid-like equations, one can separate the first two moments of the perturbed distribution function

$$h_{e^\mp} = \left(\frac{e^\mp\varphi}{T_0} - 2\frac{v_\| u_\|}{v_{the}^2}\right) F_0 + \frac{\delta n_{e^\mp}}{n_0} F_0 + g, \tag{5.7}$$

where $\int d^3\mathbf{v} g \equiv \int d^3\mathbf{v} v_\| g \equiv 0$ to all orders in $k_\perp^2\rho_e^2 \sim \beta \ll 1$. When magnetic compressibility is taken into account, the gyrokinetic potential on the RHS of Eq. (2.4) becomes

$$\varphi - \frac{v_\|}{c}A_\| \to \varphi - \frac{v_\|}{c}A_\| + \frac{T_0}{e}\hat{v}_\perp^2\frac{\delta B_\|}{B_0}, \tag{5.8}$$

where

$$\frac{\delta B_\|}{B_0} = -\beta \sum_\mp \int d^3\mathbf{v}\hat{v}_\perp^2 h_{e^\mp}. \tag{5.9}$$

We notice that, due to its parity in velocity space, the new $\delta B_\|$ term enters in the equation for density fluctuations. Let us evaluate the density moment of Eqs. (2.4) after using Eq. (5.8), and subtract the two results obtained, one for each species. To leading order we have

$$\hat{\mathbf{b}} \cdot \nabla d_e^2 \nabla^2 A_\| = 0. \tag{5.10}$$

To next order we find

$$\frac{d}{dt}\rho_e^2\nabla^2\frac{e\varphi}{2T_0} = 0. \tag{5.11}$$

Magnetic compressibility is

$$\frac{\delta B_\|}{B_0} = -\beta\left(\frac{\delta T_{\perp,e^-}}{T_0} + \frac{\delta T_{\perp,e^+}}{T_0}\right), \tag{5.12}$$

where

$$\delta T_{\perp,e^\mp} = \frac{1}{n_0}\int d^3\mathbf{v} v_\perp^2 h_{e^\mp}, \tag{5.13}$$

and we are using $\delta n/n_0 \ll \delta T/T_0$, and Eq. (2.1). Indeed, density fluctuations cancel when Larmor radius effects are negligible, and $k_\perp d_e \sim 1$.

For high enough collisionality, we expect isotropic temperature fluctuations, that is $\delta T_\perp = \delta T_\|$. This is achieved with the simple collision operator model

$$\left(\frac{\partial h_{e^\mp}}{\partial t}\right)_{coll} = \nu\left\{\frac{1}{2}\frac{\partial}{\partial \hat{v}_\|}\left(\frac{\partial}{\partial \hat{v}_\|} + \hat{v}_\|\right)h_{e^\mp} + 2\frac{v_\| u_{\|,e^\pm}}{v_{the}^2} + \left(1 - 2\hat{v}_\|^2\right)\frac{\delta T_{\perp,e^\mp}}{T_0}F_0\right\}. \tag{5.14}$$

Then, only an equation for either $\delta T_\|$ of $\delta T_\perp$ is required. We can multiply Eq. (2.4) by $\hat{v}_\|^2$ [after using Eq. (5.8)] and integrate over velocity space. The result is an equation coupled to $\int d^3\mathbf{v} v_\|^3 h_{e^\mp}$. We then calculate the $\hat{v}_\|^3$ moment of Eq. (2.4), neglect higher order moments, invert the collisional operator, and obtain an explicit expression for $\int d^3\mathbf{v} v_\|^3 h_{e^\mp}$ to insert in the $\delta T_\|$ equation. We add the two equations obtained to find

$$\frac{d}{dt}\left(\frac{\delta T_{\|e^-}}{T_0} + \frac{\delta T_{\|e^+}}{T_0}\right) = \frac{cT_0}{eB_0}\frac{\nabla n_0}{n_0}(1+\eta_e)\partial_y\frac{e\varphi}{T_0} - \hat{\mathbf{b}}\cdot\nabla\left(u_{\|e^-} + u_{\|e^+}\right), \tag{5.15}$$

where the sum of the parallel momenta of both species is given by

$$\frac{d}{dt}\left(u_{\|e^-} + u_{\|e^+}\right) = -\frac{v_{the}^2}{2}\hat{\mathbf{b}}\cdot\nabla\left(\frac{\delta T_{\|e^-}}{T_0} + \frac{\delta T_{\|e^+}}{T_0}\right) - v_{the}^2\frac{\nabla n_0}{n_0}(1+\eta_e)\frac{\partial_y A_\|}{B_0}, \tag{5.16}$$

and we used Eq. (5.10). To summarize: Eqs. (5.10)-(5.11)-(5.15) and (5.16) are a closed nonlinear system for φ, $A_\|$, $\delta T_{\|e^-} + \delta T_{\|e^+}$, and $u_{\|e^+} + u_{\|e^-}$. Thus, to leading order in our low-beta expansion, quasineutrality is determined by setting to zero the divergence of the electric current [Eq. (5.10)]. Any unbalance of perturbed charge density occurs below the Larmor scale [Eq. (5.11)]. Magnetic compressibility is driven as a response to temperature fluctuations in order to keep pressure balance [Eq. (5.12)].

6. Conclusion

We presented a simple study of non-relativistic electron-positron plasmas in a low beta magnetised sheared slab. The two species were described by using the drift-kinetic model of Zocco & Schekochihin (2011). A quasineutrality equation is given which provides charge neutrality above the Larmor scale. Below that, charge unbalance is allowed to be of the order of the square root of the plasma beta, which is assumed to be very small, but not explicitly ordered with fields' amplitudes. The system supports linear dispersive a dispersive modification of shear Alfvén waves based on the electron mass. It has been found that the growth rates of subalfenic reconnecting modes are also based on the electron Alfvén speed and feature scalings similar to those of electron-ion plasmas. In the low beta ordering here needed, two-fluid effects on non-linear reconnection do not seem to be important. However, we can expect the non-linear collapse of the electric current density which provides super-exponential growth of magnetic reconnection instabilities in the early non-linear stage [(Ottaviani & Porcelli 1995; Zocco *et al.* 2008*b*, 2009)]. Drift waves are not present. While, on the one hand, such lack of diamagnetic effects corresponds to a lack of diamagnetic stabilisation of reconnecting modes, on the other it implies the absence of density and/or temperature gradient driven modes that can cause turbulence. However, when the fields' amplitudes are explicitly ordered so that spatial variations of all fields along the exact magnetic field are retained and the transit frequency is comparable to the nonlinear $\mathbf{E}\times\mathbf{B}$ frequency, magnetic compressibility can balance small density fluctuations, and introduce some drift-wave dynamics.

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