

Radiation Reaction studies in an all-optical setup: experimental limitations

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(Dated: April 9, 2017)

The recent development of ultra-high intensity laser facilities is finally opening up the possibility of studying high-field quantum electrodynamics in the laboratory. Arguably, one of the central phenomena in this area is that of quantum radiation reaction experienced by an ultra-relativistic electron beam as it propagates through the tight focus of a laser beam. In this paper, we discuss the major experimental challenges that are to be faced in order to extract meaningful and quantitative information from this class of experiments using existing and near-term laser facilities.

INTRODUCTION

Radiation Reaction (RR) is arguably one of the most fundamental and intriguing unsolved problems of electromagnetism [1–5]. Classically, it corresponds to determining the equation of motion of a charged particle in an external field by taking consistently into account self-interactions with the electromagnetic radiation emitted by the particle whilst it is accelerated. The first attempt at addressing this problem in the classical realm resulted in the so-called Lorentz-Abraham-Dirac equation, whereby this effect is modelled via the existence of an ad-hoc damping force [4]. This equation is affected by significant physical inconsistencies, including runaway solutions even in the absence of an external field and diverging terms due to the point-like nature of the electron [4]. Whilst the latter problem can be circumvented by invoking a classical renormalization principle, the first problem is only suppressed if the damping term is treated perturbatively [5]. This approach leads to the Landau-Lifshitz equation (LL) [4, 5], which is valid if the external field F in the electron rest frame is relatively small ($F \ll F_{cr}/\gamma_e$ where F_{cr} is the critical or Schwinger field, and γ_e is the laboratory Lorentz factor of the electron) and if the electromagnetic field varies on a spatial scale much larger than the electron classical radius (assumption of a localised electron wavefunction). It has been recently demonstrated that the LL equation is the self-consistent perturbative equation of motion for a charge without electric or magnetic momenta [5]. In Quantum Electrodynamics (QED), the LL equation can be retrieved as the first order of the perturbative expansion of the theory [5] and the higher order terms form what is generally referred to as quantum RR. In this scenario, a self-consistent solution to the problem is still lacking, with the main issues related to the fact that an infinite series of events can be triggered even within a single formation length [3, 5].

Even though RR effects are often invoked to explain peculiar aspects of the emission properties of ultra-powerful astrophysical objects such as pulsars and quasars [6], the LL equation, together with the quantum corrections at high field amplitudes, has never been experimentally validated; however laser-based experiments have been recently proposed by a number of theoretical groups as possible platforms to study it in the laboratory (see, for instance, Refs. [5, 7–10]). In its simplest conception, these experiments involve studying the interaction of a laser-wakefield accelerated electron beam [11] with the tight focus of a high intensity laser. This class of experiments have been performed only at relatively low electron energies and laser intensities, leading to a detailed experimental characterisation of linear and non-linear Thomson scattering (see, for instance, Refs. [12–16]). For this class of experiments it is useful to define the laser dimensionless intensity a_0 as the work done on an electron by the laser electric field divided by the electron rest energy: $a_0 = eF/(\omega_L m_e c)$, where ω_L is the central laser frequency. Non-linear effects in Thomson scattering and, at higher intensities, in RR, are non-perturbative if $a_0 > 1$ [12].

In this head-on electron-laser collisions, the analytical solution of the LL equation, assuming the laser to be a plane wave with a Gaussian temporal profile, is given by [7, 9]:

$$\frac{\Delta\gamma_e}{\gamma_e} = \frac{\sqrt{\pi/2}\tau_0 t_L \omega_L^2 \gamma_e a_0^2}{1 + \sqrt{\pi/2}\tau_0 t_L \omega_L^2 \gamma_e a_0^2}, \quad (1)$$

with $\tau_0 = 6.4 \times 10^{-24}$ s, and t_L the laser duration.

The lack of experimental data in this regime is easily understood if we consider that the electrostatic Schwinger field has an impressively high value of the order of $F_{cr} \approx 1.3 \times 10^{18}$ V/m, which is currently not attainable in the laboratory frame. However, this titanic experimental task can be simplified by studying the dynamics of an ultra-relativistic electron in an external field, since in this case the field in the rest frame of the electron is boosted by the electrons Lorentz factor γ_e . This approach was adopted in an experimental campaign carried out at SLAC [17]. By studying the interaction of a 46.6 GeV electron beam ($\gamma_e \simeq 9 \times 10^4$) with a laser field with $F_L \simeq 2 \times 10^{12}$ V/m, an electric field in the rest frame of the electron of the order of $0.2 F_{cr}$ was achieved. Nonlinearities in Compton scattering and pair production were observed, even though no clear sign of RR was reported. The only other possible experimental signature of RR could in principle be found in modelling the motion of electron beams in synchrotron machines [18]. However, in this case it is sufficient to subtract the energy emitted by the electron after each revolution, without the necessity of taking into account radiative losses self-consistently during the electron motion. Strictly speaking, the corrections are so small that they do not form part of a RR problem.

Providing experimental data on RR is also of practical importance, since the next-generation of multi-PW laser systems (such as ELI-NP [19], Apollon [20], and XCELS [21]) is expected to achieve intensities in the laboratory frame exceeding 10^{23} W/cm² ($F_L > 10^{15}$ V/m) within the next few years. These lasers will have a transformational effect on the physics community, since they will boost our capability of accelerating particles [22], generating novel

radiation sources [19], and understanding high energy density physics [23]. At these intensities, RR effects will play a dominant role in the plasma dynamics; for instance, particle-in-cell (PIC) simulations have shown how RR significantly affects laser-driven ion acceleration [24] and electromagnetic field generation [25]. The models introduced in PIC codes are either classical (i.e., largely based on the LL equation), semi-classical (i.e., introducing a phenomenological function g to account for reduced emission in the quantum regime [8]), or quantum. The latter has to be intended as a simplified QED model that accounts only for instantaneous stochastic photon emission (strictly valid only for ultra-high intensities) and pair production. These models have not been experimentally tested, even though they are routinely used to predict the performance of this new class of lasers.

The recent development in laser-driven electron acceleration, via the so-called wakefield acceleration regime [11] is now allowing for the laser-driven acceleration of electron beams with energy per particle in the GeV regime, with the current record exceeding 4 GeV [26]. This, in conjunction with achieved focussed laser intensities routinely exceeding 10^{20} W/cm² (electric field amplitude in the laboratory frame of $F > 10^{14}$ V/m) allows in principle to achieve conditions where quantum RR effects can be detected. It is useful to define a quantum parameter χ as $\chi = \gamma_e F / F_{cr}$ implying that quantum effects in the electron dynamics are relevant if $\chi \geq 1$. In this case, for a 800nm laser, the quantum parameter can be expressed as $\chi \approx 61 \times 10^{-6} a_0 \gamma_e$. For existing laser facilities it is thus realistic to reach values of χ approaching unity, with $\chi > 1$ within reach of near-term laser facilities. As an example, we plot in Fig. 1 the parameter space achievable by current and near-term high-intensity laser facilities.

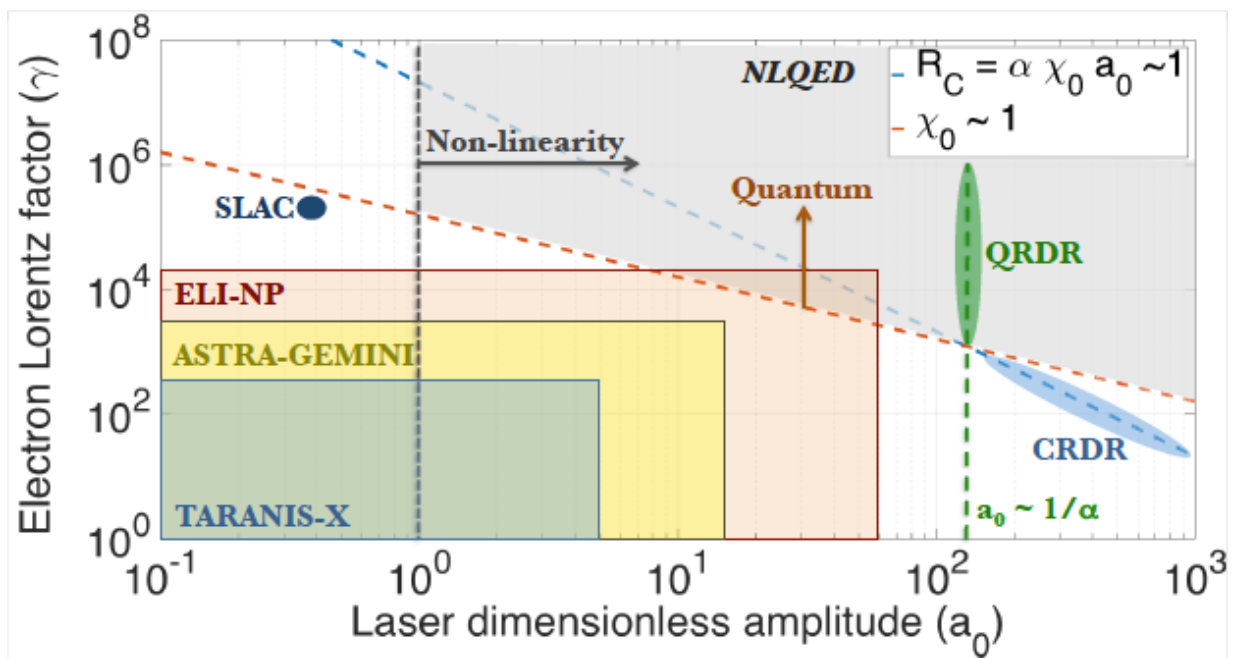


Figure 1. Electron Lorentz factors and dimensionless laser intensities achievable by different laser facilities: ELI-NP (red square, representative of near-term multi-PW laser facilities), Astra-Gemini (yellow square, representative of state-of-the-art PW-scale laser facilities), and TARANIS-X (blue square, representative of smaller University-scale laser facilities). The orange and grey dashed lines represent $\chi = 1$ and $a_0 = 1$, so that non-linear quantum electrodynamics is accessed in the grey-shaded area delimited by these two lines. Regions of Quantum Dominated (QRDR) and Classical Dominated (CRDR) Radiation Reaction [7, 27] are also highlighted. For reference, the parameter space achieved by SLAC [17] is indicated by a blue dot.

This would naturally indicate that quantum RR is now within the grasp of experimental studies. However, several other factors must be accounted for if a meaningful experimental study of these phenomena is to be carried out. It is the scope of this article to outline the main experimental issues to be addressed in this class of experiments. As a caveat, it is important to stress that the material presented in this article must be intended only as a qualitative overview of the main issues that must be addressed in these experiments and it has by no means to be taken as an exhaustive discussion of the topic. The article is organised as follows: in section 2 a discussion of the general spatial

and temporal properties of focussed laser beams is given. In section 3, issues related to the typical spatial properties of laser-driven electron beams are shown whilst the problem of reliably and consistently synchronising and spatially overlapping the focussed laser with the relativistic electron beam is discussed in section 4. Section 5 discusses the shot-to-shot reproducibility of these results whilst conclusive remarks are given in Section 6.

THE SPATIAL AND TEMPORAL PROPERTIES OF A TIGHT FOCUSED LASER BEAM

In order to achieve the highest intensity in focus and, therefore, the highest peak electric field, it is necessary to produce laser pulses with the highest possible energy contained within the shortest pulse duration. These high power pulses are routinely obtained using Ti:Sapphire systems, which operate at a central wavelength of the order of 800 nm (laser period of 2.7 fs). Laser pulses with a power in the PetaWatt regime within a duration of the order of 30 fs can now be generated in several laser facilities worldwide. Fast-focussing optics with a focal aperture of $F/2$ are then routinely used to generate the highest intensity, with the only drawback that these intensities can be maintained over a gaussian spatial profile with a Full Width Half Maximum (FWHM) of the order of 2-3 microns. Due to diffractive effects and a non-perfect spatial and spectral phase of the pulse, it is customarily not possible to encircle more than 60% of the laser energy in this focal area, resulting in peak laser intensities, lasting for tens of femtosecond and extending over a few microns, not higher than 10^{21} W/cm². It must also be pointed out that maximum electron-laser interaction is achieved for face-on collisions. This implies that short-focal length parabolas must be holed in their centre, for the two-fold reason of both avoiding back-reflections in the laser chain and allowing for clean propagation of the electrons after being scattered.

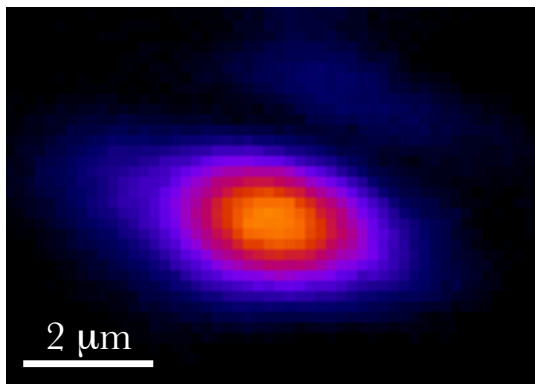


Figure 2. Measured intensity distribution in the focus of an $F/2$ parabola. The colourscale is linear and its peak value will depend on the laser energy used. The highest laser intensity is only maintained over a FWHM of the order of 2 microns.

A holed parabola will induce aberrations in the focal spot, resulting in a significant portion of the laser energy being confined in an area that is broader than that expected from a pure $F/2$ focus. Given that laser wakefield acceleration is achieved with a long focal length parabola (such as $F/20$) and that scattered light from the gas target will cover larger focal apertures (even exceeding $F/10$), the hole in the centre must be larger than $F/7$ [12]. This does not only result in a loss in laser energy that can be focussed, but also in the emergence of a complex spatial distribution of the laser intensity at focus. This distribution is, in a first approximation, typically consisting of a narrow gaussian with peak intensity of 10^{21} W/cm² and a FWHM of 2-3 microns superimposed on a broader gaussian with peak intensity in the range of 10^{20} W/cm² and a FWHM of approximately 10 microns. Moreover, it must be noted that the intensity distribution at the focus of the laser beam is routinely measured at a power that is significantly lower than the power used in full experimental shots. It is generally not correct to assume that low-power and full-power shots will retain the same focal spot distribution, since thermal loading of the laser amplification chain will introduce aberrations in the beam that will result in a generally broader focal spot at full power. While this correction is generally small for joule-class lasers, [28, 29], this effect will become significant in the near term high-energy laser systems, such as ELI-NP, which are designed to deliver hundreds of joules in a duration of few tens to few hundreds of femtoseconds.

Another important aspect that must be carefully taken into account in performing this class of experiments is that of the temporal distribution of the focussed laser. Whilst most of the theoretical work in modelling laser-driven RR assumes either a plane wave or a perfect gaussian (see, for instance, Refs. [1, 2, 5]), a real laser focus will present a more complex temporal structure. This mainly arises from two factors: the spectral phase of a high-power laser not being entirely flat, and the occurrence of longitudinal fields in the tight focus of the laser, which make the laser mode depart from being purely transverse electromagnetic. Each laser system will have different spectral phases, which can be partially controlled using active system such as a DAZZLER, and it is thus difficult to draw general conclusions on this aspect. However, it must be noted that a structured spectral phase can have significant effects on the Compton scattering of the electron in the laser field and, thus, on RR. For instance, theoretical work describing this effect can be found in Refs. [30]. On the other hand, a detailed analysis of the transverse and longitudinal fields that are present in the tight focus of a laser can be found in Ref. [31]. Given that RR is related to the transverse oscillations of the electron in the laser field, it is not correct to calculate the normalised laser intensity assuming that the laser energy is all converted into transverse fields, because this will generally lead to an overestimate of a_0 and, thus, to an overestimate of the RR experienced by the electron.

SPATIAL PROPERTIES OF LASER-ACCELERATED ELECTRON BEAMS

We have seen that laser intensities of interest for meaningful RR studies in the quantum regime require focussing the laser down to a few micron focal spot. Ideally, one would be interested in generating electron beams with similar transverse dimension, in order to ensure the highest level of overlap. Unfortunately, this is difficult to achieve in a laser-driven electron accelerator, due to the finite source size and divergence of the beam. Without entering into the physical details of laser-driven wakefield acceleration, which are beyond the scope of this article (the interested reader can refer to a recent review on the topic [11]), the electron beam diameter at source can be estimated as: $D_e \propto n_e^{-3/8} a_W^{3/4} \gamma_e^{-1/4}$, where n_e is the electron plasma density of the gaseous target, a_W is the dimensionless intensity of the laser used to drive the wakefield, and γ_e is the Lorentz factor of the accelerated electrons [32]. Betatron measurements carried out in conditions of interest to this article indicate this source size to be of the order of a few microns [32, 33]. Moreover, the electron beam will present a non-negligible divergence, intrinsic of the wakefield acceleration mechanism. This divergence will be strongly dependent on the specific electron acceleration mechanism chosen and the specific laser and gas parameters but, generally speaking, it will be of the order of a few mrad. Moreover, it is generally energy-dependent, with the highest electron energies showing a narrower divergence.

It would thus be ideal to have the electron-laser interaction as close as possible to the electron source. However, a trade-off has to be found since it is desirable to prevent the scattering laser from damaging the gaseous target rig, especially if gas-cells are used [34, 35]. For typical experiments, a minimum distance of the order of a few mm between the gaseous target and the high-intensity laser focus is thus recommended, leading to electron beam diameter at the interaction point exceeding 10 microns. This value is much greater than the FWHM of the high-intensity focus of the laser (of the order of 2-3 microns, see Fig.2), implying that only a small percentage of the electrons will effectively experience the highest laser intensity. This effect, combined with the focal spot of the laser having a structured distribution, leads to having different percentages of electrons experiencing different laser intensities.

LASER-ELECTRON TEMPORAL SYNCHRONISATION

In order to achieve a maximised interaction of the laser with the electron beam, it is necessary to synchronise the two within a temporal window comparable to the longest of the two events. High-intensity laser systems usually operate at a temporal duration of the order of 30-40 fs but, on the other hand, it is harder to provide a general estimate of the duration of the laser-accelerated electron beam, since it will be strongly dependent on the kind of acceleration chosen. Without going into too much detail (see Ref. [11] and references therein, for an overview of laser-driven electron acceleration) electron beams with durations as short as a few fs have been experimentally obtained (see, for instance, Ref. [36]). Arguably, the easiest injection mechanism for electron acceleration is the so-called ionisation injection [37]. In this case, a full loading of the plasma bubble driving the acceleration can lead to the electron beam duration to be as long as tens of fs. It is thus clear that a maximised interaction between the laser and the electrons is then achieved only if temporal synchronisation is achieved with a precision of tens of fs.

This timescale lies well below the resolution of conventional electronic devices, with the fastest photodiodes operating at a resolution of 50 - 100 ps. One has then to rely on optical methods that exploit the fastest probes available in

the experiment, i.e., the laser itself. Recently, Corvan et al. [38] have developed a simple optical technique, based on spectral interferometry, that is able to synchronise the two lasers (i.e., the scattering laser and the one driving the wakefield acceleration) with a resolution of the order of a few tens of fs. This technique was successfully applied to generate high-brightness and high-energy γ -ray beams resulting from Compton scattering of the electrons in the laser beam [12].

It must be noted that this technique allows for a fine synchronisation between the two lasers but not necessarily between the scattering laser and the wakefield-accelerated electron beam. This is mainly because the accelerated electrons are created in the wake of the driving laser, i.e., there is a non-zero time delay between them. Whilst it is hard to provide a general expression for this time delay, since it strongly depends on the specific laser and gas parameters, an order of magnitude estimate can be given. Assuming that the acceleration operates in the blow-out regime [11], the accelerating structure in the plasma will have a roughly spherical shape (bubble), with a typical radius given by: $R_b \approx 2\sqrt{a_W}c/\omega_p$ [22], with $\omega_p \approx 5.6 \times 10^4 \sqrt{n_e}$ the frequency of the background plasma having an electron density n_e expressed in electrons per centimetre cubed. Assuming the bubble to be half filled by the accelerating electrons, the time delay between the driving laser and the accelerated electron beam will be of the order of: $\Delta_t \approx R_b/2c$. For typical experiments, this time can be of the order of tens of fs, a non-negligible delay for this class of experiments.

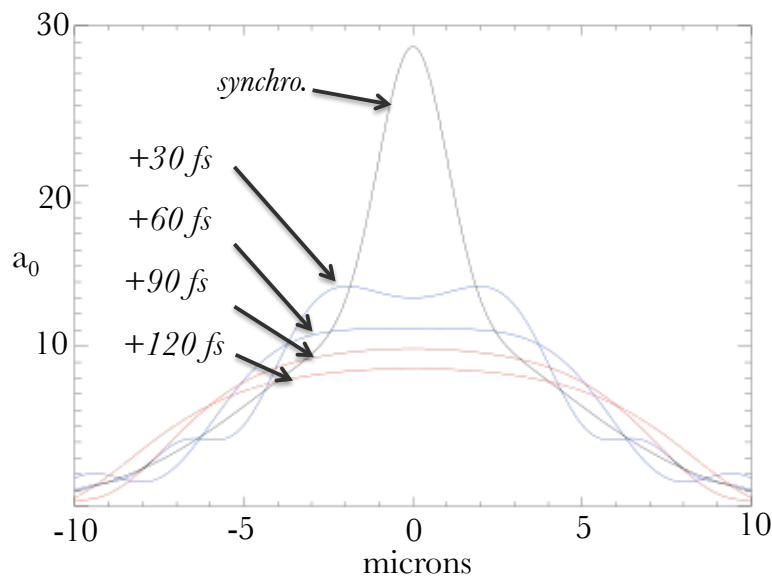


Figure 3. Example of a typical laser intensity distribution at tight focus (black line) and at different time delays. The original distribution is what to be expected by a high-intensity laser beam being focussed by an F/2 parabola with an F/7 hole in the middle.

Ideally, one would like to have a system that is able to directly synchronise the scattering laser with the electron beam itself. This is in principle possible using electro-optic crystals. In a nutshell, an electron beam propagating in a crystal will induce transient electromagnetic fields. If the scattering laser propagates through the crystal at the same time, these induced fields will cause a slight birefringence causing a rotation of polarisation in the laser. This kind of systems are currently under study (see, for instance, [39, 40]).

A slight temporal delay between the electron beam and the scattering laser is extremely problematic if a meaningful extraction of RR effects from the spectrum of the scattered electron is sought. One must consider that a delayed arrival of the electron beams will result in them experiencing a lower laser intensity (laser being slightly defocused). As an example we plot calculated intensity profiles of an F/2 focussed laser beam as a function of time in Fig. 3. Due to the fast-focussing optics required to achieve high intensities, even a 30fs delay induces a dramatic drop in peak intensity that the electrons will experience, which will naturally result in a smaller RR. This effect is of particular practical importance, since a smaller effect of RR at a nominal laser maximum intensity might mislead the experimenter in believing that quantum effects in RR are being detected (see, for instance, Ref. [41]).

SHOT-TO-SHOT REPRODUCIBILITY OF LASER-ACCELERATED ELECTRON BEAMS

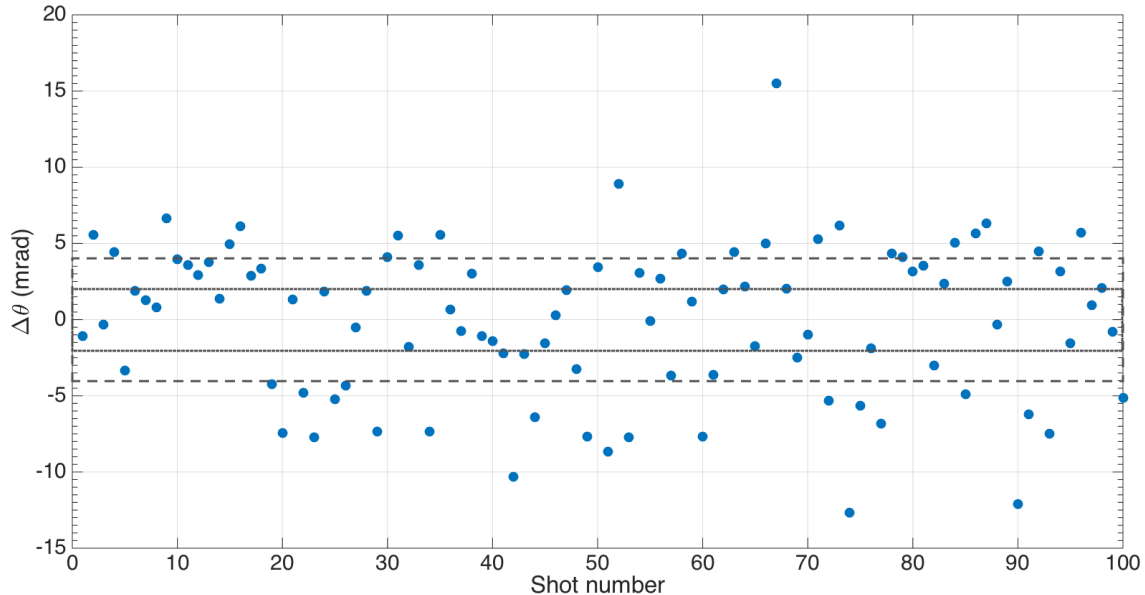


Figure 4. Angular deviation of the laser-accelerated electron beam from the axis of the driver laser as measured over 100 consecutive shots using the Astra-Gemini laser.

Another fundamental limitation in RR studies using a full-optical setup is that both the scattering laser and the laser-driven electron beam will present non-negligible shot-to-shot fluctuations in pointing. Whilst the fluctuation in the scattering laser focal spot position can easily be controlled to be within a few μrad , the pointing fluctuation in the electron beam is typically more substantial, of the order of a few mrad. As an example, we plot in Fig. 4 the measured pointing of electron beams from 100 consecutive laser shots. The angular deviation from the propagation axis of the driver laser is seen to be randomly distributed over a range of ± 10 mrad. Applying a Kolmogorov-Smirnov test on the data indicates a 95% confidence that these deviations follow a Gaussian distribution centred at the laser axis with a standard deviation of 3.2 ± 0.8 mrad. Assuming that the scattering laser is focussed 1cm away from the electron source, this indicates a fluctuation in pointing of the order of a few microns, i.e., comparable to the focal spot size of the laser. It must be noted that this fluctuation is strongly dependent on the laser and gas parameters and on the particular acceleration mechanism used and the results reported here should only be seen as indicative. For reference, different studies on shot-to-shot pointing stability have been reported in the literature (see, for instance, Ref. [34]).

It is also possible that the electron beam might present a solid offset from the driving laser (non-zero mean in the Gaussian distribution discussed above). This has been identified in being caused predominantly by residual chirp in the laser beam, due to imperfect compression, that results in a phase front tilt of the laser pulse. Details of this phenomenon can be found, for instance, in Ref. [42]. This latter phenomenon can anyway be in principle controlled and minimised optically.

As a final remark, one could minimise pointing fluctuations in the electron beam by employing active focussing techniques, such as the plasma lensing techniques recently introduced and studied by a number of research groups (see, for instance, Ref. [43, 44]). Nevertheless, this phenomenon requires particular attention if RR has to be studied in an all-optical setup since, unless independent measurements of electron-laser overlap are performed on a shot-to-shot basis, it is impossible to deduce it only from the behaviour of the scattered electrons.

CONCLUSION

We presented some of the main experimental issues related in studying high-field radiation reaction experienced by a laser-wakefield accelerated electron beam propagating through the focus of a high intensity laser. It must be noted that there are many other possible sources of uncertainty in this class of experiments, which are not discussed here. For instance, electron spectra are routinely measured using magnetic spectrometers, generally consisting of a dipole magnet followed by a scintillator screen. One must take extreme cautions in interpreting raw data from these devices since the electron spectrum extraction will be strongly dependent on the uniformity of the magnetic field, the divergence and source size of the electron beam, and on the shot-to-shot variation of the electron beam pointing. However, a detailed description of these issues is not presented here, since they are not intrinsically related to the physical limitations of laser-driven experiments. Moreover, one might wish to study the spectrum of the γ -ray photons emitted via Compton scattering during the electron-laser interaction [10]. This is experimentally not a trivial task, since it involves spectrally resolving multi-MeV photons with a high flux. Whilst γ -ray spectrometers working in this regime up to an energy of approximately 30 MeV have been developed [12, 45], there is currently no simple setup operating in a regime of hundreds of MeV.

ACKNOWLEDGEMENTS

The authors wish to acknowledge support from EPSRC (grants: EP/N022696/1, EP/N027175/1, and EP/P010059/1). The authors acknowledge fruitful discussions with A. Di Piazza, M. Tamburini, and Ch. H. Keitel.

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