Signatures of quantum effects on radiation reaction in laser – electron-beam collisions

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Two signatures of quantum effects on radiation reaction in the collision of a $\sim$ GeV electron-beam with a high-intensity ($\sim 10^{21}$ Wcm^{-2}) laser-pulse have been considered. We show that the decrease in the average energy of the electron-beam may be used to measure the Gaunt factor g (in the cases considered here the quantum efficiency parameter $\eta \sim 0.1$ and $g \lesssim 0.5$). We derive an equation for the evolution of the variance in the energy of the electron-beam in the quantum regime, $\eta \not\ll 1$. We show that the evolution of the variance may be used to demonstrate quantitatively the quantum stochasticity of the radiation reaction and determine the parameter regime where this is observable. For example, the stochasticity results in an increase in the standard deviation in the energy of ~25% after 1 fs in the collision of a GeV electron-beam with a laser-pulse of intensity 10^{21} Wcm^{-2}.

1. Introduction

Radiation reaction is the recoil force on an accelerating charged particle caused by the particle emitting electromagnetic radiation. This effect will play an important role in laser-matter interactions at the intensities set to be reached by next generation high-intensity laser facilities ($\gtrsim 10^{23}$ Wcm^{-2}) (as shown by several authors, for example Zhidkov *et al.* (2002); Ridgers *et al.* (2012); Nakamura *et al.* (2012); Brady *et al.* (2012)), where radiation reaction can lead to almost complete absorption of the laser-pulse (for example see Nerush *et al.* (2011); Ji *et al.* (2014); Zhang *et al.* (2015); Grismayer *et al.* (2016)). At the parameters expected to be reached in these interactions the electric field in the rest frame of the ultra-relativistic electrons in the plasma created by the laser E_{RF} approaches the critical field for quantum electrodynamics $E_{\text{crit}} = 1.38 \times 10^{18}$ Vm^{-1} (Heisenberg & Euler (1936)). In this case the emission of radiation by the electrons must be described in the framework of strong-field quantum-electrodynamics (QED), using the 'Furry picture' as explicated by Furry (1951). Specifically, when the quantum efficiency parameter $\eta = E_{RF}/E_{\text{crit}} \gtrsim 0.1$ the radiation reaction force becomes stochastic (Duclous *et al.* (2011)) and the electron's dynamics are no longer well approximated by deterministic motion along a classical worldline (Shen & White (1972)).

This quantum regime has been reached in experiments at CERN SPS in the interaction of $\sim$ 100 GeV electrons with the strong fields from the atoms in a crystal lattice, as described

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by Andersen *et al.* (2012), where the Gaunt factor for synchrotron emission was measured. The analogous process of non-linear Compton scattering, more relevant to laser-plasma interactions, has been studied experimentally at the Stanford Linear Accelerator in the interaction between an electron beam of energy $E = 46.6$ GeV and a counter-propagating high-intensity ($10^{18} - 10^{19}$ Wcm^{-2}) laser-pulse as reported by Bula *et al.* (1996) (positron generation was also observed in this experiment as described by Burke *et al.* (1997)). In this experiment the laser intensity was too low to access the very non-linear regime of relevance to next generation laser-matter interactions, where $a_0 \sim \sqrt{I\lambda^2/10^{18}\ \mathrm{Wcm}^{-2}\mu\mathrm{m}^2} \gg 1$ is required (λ is the laser wavelength). Current PW laser systems can achieve focused intensities of $\sim 10^{21}$ Wcm^{-2} which is sufficient to access the $a_0 \gg 1$ non-linear regime. In the interaction of an electron-beam with energy $\mathcal{E}$ with a counter-propagating laser-pulse of intensity I, η can be parameterised as $\eta \sim 0.1(\mathcal{E}/500\ \mathrm{MeV})\sqrt{I\lambda^2/10^{21}\ \mathrm{Wcm}^{-2}\mu\mathrm{m}^2}$. The quantum, non-linear regime of Compton scattering and the resultant radiation reaction can therefore be studied by accelerating the electrons to energies greater than 500MeV. This is possible with laser wake field acceleration (first described by Pukhov & Meyer-ter Vehn (2002)) which can produce electron beams of energy $>$ 500 MeV (for example see Clayton *et al.* (2010)). Therefore, all-optical equivalents of the SLAC experiment are possible using PW lasers, indeed weakly non-linear Compton scattering (but not radiation reaction) was recently observed in such a setup by Sarri *et al.* (2014). Devising ways in which quantum effects on radiation reaction can be distinguished is therefore timely.

In this paper we will derive equations describing the effect of radiation reaction on the evolution of: (i) the expectation value of the momentum and energy of a population of electrons and (ii) the variance in the energy distribution of the population. We will use these equations to determine ways in which the two most important quantum effects may be determined, i.e. the reduction in the radiated power due to the finite recoil of an electron during the emission of a photon and the probabilistic nature of the emission commonly known as 'straggling' (see Shen & White (1972) and Duclous *et al.* (2011)) or 'quenching' (see Harvey *et al.* (2017)).

In order to simplify the treatment of quantum radiation reaction we use a quasi-classical approach described by Baĭer & Katkov (1968). Here we assume that the electromagnetic fields may be split into two types depending on their frequency scale. Fields varying on the scale of the laser frequency (including those due to collective plasma processes) are treated as classical background fields. The photons emitted by the electrons on acceleration by these background fields, i.e. those responsible for the radiation reaction force, are treated in the framework of strong-field QED (Furry (1951)). These photons are of much higher energy (typically $\gtrsim$MeV) than the laser photons ($\sim$eV). The accuracy of this quasi-classical approach has recently been demonstrated by comparison to full QED calculations for the electron energies and laser intensities considered here Dinu *et al.* (2016). Two further simplifying approximations are made (see Kirk *et al.* (2009)). By making the quasi-static approximation we assume that the formation length of the hard photons is much smaller than the scale over which the background fields vary and thus the background fields may thus be treated as constant over the space-time interval during which the emission occurs. This approximation is valid for $a_0 \gg 1$, which is the case in high-intensity laser matter interactions (Di Piazza *et al.* (2010) has shown that $a_0 \gtrsim 10$ is sufficient). By making the weak-field approximation we assume that the emission rate of photons depends entirely on η and not the field invariants $\mathcal{F} = (E^2 - c^2B^2)/E_{\mathrm{crit}}^2$ and $\mathcal{G} = c\mathbf{E}\cdot\mathbf{B}/E_{\mathrm{crit}}^2$. This is valid if these invariants are much smaller than η. For next-generation laser-matter interactions $E, cB \lesssim 10^{-3}E_s$, so this approximation is also reasonable. The weak-field approximation allows us to assume that the rate of photon emission (and the energy spectrum of the emitted photons) is well described by the well known rate in an equivalent set of constant fields as given in Ritus (1985) (for constant crossed electric and magnetic fields) and Erber (1966) (for a constant magnetic field).

Using this quasi-classical model (making the quasi-static and weak-field approximations), it is possible to include the quantum radiation reaction force in a kinetic equation describing the evolution of the electron distribution, as given by Shen & White (1972), Elkina *et al.* (2011), Sokolov *et al.* (2010), Neitz & Di Piazza (2013) and Ridgers *et al.* (2014). Although this equation has been solved numerically using a Monte-Carlo algorithm (see Duclous *et al.* (2011); Elkina *et al.* (2011); Ridgers *et al.* (2014); Gonoskov *et al.* (2015)) it has not been solved analytically for even the simplest configuration of electromagnetic fields (for example a uniform, static magnetic field as in Shen & White (1972)). On the other hand, a classical model of radiation reaction, using the prescription of Landau & Lifshitz (Landau & Lifshitz (1987) – shown to be consistent with the classical limit of strong field QED by Ilderton & Torgrimsson (2013)), can be included in the electron equation of motion straightforwardly which has been solved in several cases for example for electron motion in a rotating electric field (by Bell & Kirk (2008)) or a plane electromagnetic wave (by DiPiazza (2008)). A modified classical model, where the radiated power is reduced by the Gaunt factor, has been used to derive the dispersion relation for an electromagnetic wave moving through a plasma where the electrons experience significant radiation reaction by Zhang *et al.* (2015) (and the equivalent classical result by Bashinov & Kim (2013)). The kinetic equation can be used to show that the modified classical model of radiation reaction is sufficient to describe the average energy loss of the electrons (Ridgers *et al.* (2014)). In addition, the kinetic equation can give insight about which observables can be used to measure various aspects of quantum radiation reaction. Here we show that the measurements of the average energy loss can be used to measure the Gaunt factor associated with the emission and that the evolution of the variance of the electron energy distribution can be used to measure the degree of stochasticity of the emission. The latter was discussed by Vranic *et al.* (2015) in the limit of small η, here we extend this result to arbitrary η.

2. Radiation reaction models

In this section we describe the radiation reaction models considered here: (i) classical – using the ultra-relativistic form of the Landau & Lifshitz prescription; (ii) modified classical – as the classical model but including a function describing the reduction in the power radiated due to quantum effects, the Gaunt factor g (Blackburn *et al.* (2014); Ridgers *et al.* (2014)); (iii) stochastic – a probabilistic treatment of the emission consistent with the approximations made in the quantum emission model described above and in more detail by Ridgers *et al.* (2014). The stochastic model is the most physical as it includes both the important quantum effects (the Gaunt factor and quantum stochasticity).

Using the quasi-classical approach we may write the evolution of the single-particle electron distribution function, including the radiation reaction force, as

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t} \right)^X_{em} .$$

$fd^3\mathbf{x}d^3\mathbf{p}$ is the number of electrons at position $\mathbf{x}$ with momentum $\mathbf{p}$ (velocity $\mathbf{v}$). $\mathbf{E}$ and $\mathbf{B}$ are the large-scale electromagnetic fields (treated classically). $(\partial f/\partial t)^X_{em}$ is an emission operator which describes the radiation reaction force and is particular to which of the three cases given above is under consideration (denoted by the superscript X).

2.1. *'Classical' and 'modified classical' emission operators*

The classical and modified classical emission operators, assuming that photon emission is in the direction of propagation of the electron, take the following form

$$\left(\frac{\partial f}{\partial t}\right)_{em}^{cl} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\frac{P_{cl}}{c}f\right) \qquad \left(\frac{\partial f}{\partial t}\right)_{em}^{mod\ cl} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 g\frac{P_{cl}}{c}f\right) \tag{2.1}$$

where $g(\eta)$ is the Gaunt factor for synchrotron emission, i.e. a function that gives the reduction in the radiated power P_{cl} due to quantum modifications to the synchrotron spectrum. P_{cl} is parameterised in terms of η as

$$P_{cl} = \frac{2\alpha_f c}{3\lambda_c} m_e c^2 \eta^2$$

and $g(\eta)$ is defined as

$$g(\eta) = \frac{\int_0^{\eta/2} F(\eta,\chi)d\chi}{\int_0^{\infty} F_{cl}\left(\frac{4\chi}{3\eta^2}\right)d\chi} = \frac{3\sqrt{3}}{2\pi\eta^2}\int_0^{\eta/2} F(\eta,\chi)d\chi.$$

F_{cl} and F are the classical and quantum synchrotron spectra respectively. For completeness their forms are given in appendix A. An accurate fit to this function is $g(\eta) \approx [1 + 4.8(1+\eta)\ln(1 + 1.7\eta) + 2.44\eta^2]^{-2/3}$ (Baier *et al.* (1991)).

In section 3 we will show that the classical and modified classical emission operators given in equation (2.1) describe radiation reaction forces of the form

$$\mathbf{F}_{cl} = -\frac{P_{cl}}{c}\hat{\mathbf{p}} \qquad \mathbf{F}_{mod\ cl} = -\frac{gP_{cl}}{c}\hat{\mathbf{p}}$$

respectively.

2.2. *'Stochastic' emission operator*

The stochastic emission operator is as follows

$$\left(\frac{\partial f}{\partial t}\right)_{em}^{st} = -\lambda_\gamma(\eta)f + \frac{b}{2m_e c}\int_p^\infty dp'\lambda_\gamma(\eta')p_\chi(\eta',\chi)\frac{p'^2}{p^2}f(\mathbf{p}') \tag{2.2}$$

where the first term describes the movement of electrons out of a given region in phase space due to emission. The second term describes electrons moving into the region under consideration by leaving regions of higher energy as they emit. $\eta = \gamma b$ and have assumed the electrons are ultra-relativistic, such that $b = |\mathbf{E}_\perp + \mathbf{v}\times\mathbf{B}|/E_s$. We have also assumed that photon emission is in the direction of propagation of the electron. $\chi = (h\nu b)/(2m_e c^2)$ is the quantum efficiency parameter for an emitted photon (with energy $h\nu$). The explicit form of the photon emission rate λ_γ and the probability p_χ that an electron with energy parameterised by η emits a gamma-ray photon with energy parameterised by χ are given in appendix A.

3. Moment equations

While we have not obtained exact solutions to equation (2.1) including the stochastic emission operator given in equation (2.2), progress can be made by determining the equation for the evolution of the expectation value of the electron momentum $E[\mathbf{p}]$ and variance in the electron energy distribution $V[\gamma]$. The average over the distribution function f of a momentum dependent quantity $\psi(\mathbf{p})$ is defined as

$$\langle\psi(\mathbf{p})\rangle = \int d^3\mathbf{p}\psi(\mathbf{p})f(\mathbf{x},\mathbf{p},t).$$

3.1. *The temporal evolution of* $E[\mathbf{p}]$

The equation for the evolution of the expectation value of the momentum of the electron population $E[\mathbf{p}] = \langle \mathbf{p} \rangle$ has been derived previously by Elkina *et al.* (2011). The equation for the evolution of the average energy $E[\gamma] = \langle \gamma \rangle$ of the population has been derived by Ridgers *et al.* (2014):

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\frac{\langle gP_{cl}\hat{\mathbf{p}}\rangle}{c}. \tag{3.1}$$

Here the angled brackets represent averages over momentum, i.e. $\langle \ldots \rangle = \int \ldots f d^3\mathbf{p}$. In appendix B we show how this equation can be derived by taking the first moment of the stochastic emission operator in equation (2.2).

Taking the first moment of the classical and modified classical emission operators given in equation (2.1), as detailed in appendix B, yields

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{cl} = -\frac{\langle P_{cl}\hat{\mathbf{p}}\rangle}{c} \qquad \left(\frac{dE[\mathbf{p}]}{dt}\right)_{mod\ cl} = -\frac{\langle gP_{cl}\hat{\mathbf{p}}\rangle}{c} \tag{3.2}$$

3.2. *The temporal evolution of* $V[\gamma]$

Following the derivation in appendix B we can obtain the following equation for the evolution of the variance in the Lorentz factor γ of the electron distribution:

$$\left(\frac{dV[\gamma]}{dt}\right)_{st} = -2\frac{\langle \varDelta\gamma g P_{cl}\rangle}{m_e c^2} + \frac{\langle S\rangle}{m_e^2 c^4}. \tag{3.3}$$

$V[\gamma] = \langle\gamma^2\rangle - \langle\gamma\rangle^2$ and $\varDelta\gamma = \gamma - E[\gamma]$. Consistent with the result found by Vranic *et al.* (2015), the equation for $dV[\gamma]/dt$ contains two terms. The first is always negative. This is because electrons at higher energy radiate more energy than those at lower energy, causing a decrease in the width of the energy distribution. The second term is positive and is a result of the stochastic nature of the emission.

The function $S(\eta)$ is given by

$$S(\eta) = \frac{55\alpha_f c}{24\sqrt{3}\lambda_c b} m_e^2 c^4 \eta^4 g_2(\eta).$$

$g_2(\eta)$, which is analogous to $g(\eta)$, is defined as

$$g_2(\eta) = \frac{\int_0^{\eta/2} \chi F(\eta,\chi) d\chi}{\int_0^{\infty} \chi F_{cl}\left(\frac{4\chi}{3\eta^2}\right) d\chi} = \frac{144}{55\pi\eta^4}\int_0^{\eta/2} \chi F(\eta,\chi) d\chi.$$

As for g, it is useful to find an accurate fit to g_2. We find the following $g_2(\eta) \approx [1 + (1 + 4.528\eta)\ln(1 + 12.29\eta) + 4.632\eta^2]^{-7/6}$. This gives the correct limits for $\eta \ll 1$ and $\eta \gg 1$ ($g_2 \approx 1$ and $g_2 \approx 0.167\eta^{-7/3}$ respectively). g_2, as a function of η, along with the fit are shown in figure 1. In the limit where $\eta \ll 1$ and the energy distribution is Gaussian with $\sqrt{V[\gamma]} \ll E[\gamma]$ (and assumed to be a Gaussian at all times), equation (3.3) reduces to

$$\left(\frac{dV[\gamma]}{dt}\right)_{st} \approx \frac{\alpha_f c b^2}{\lambda_c}\left(\frac{55b}{24\sqrt{3}}E[\gamma]^4 - \frac{8}{3}V[\gamma]E[\gamma]\right), \tag{3.4}$$

which is consistent with the equation derived by Vranic *et al.* (2015) (equation (14) in their paper).

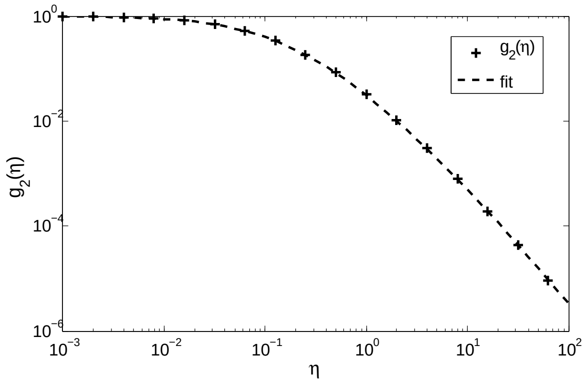

FIGURE 1. $g_2(\eta)$ (solid line) and the fit used here (dashed line).

We may also derive the corresponding expressions for $dV[\gamma]/dt$ from the classical and modified classical emission operators in equation (2.1) (the derivation is given in appendix B).

$$\left(\frac{dV[\gamma]}{dt}\right)_{cl} = -2\frac{\langle \Delta\gamma P_{cl}\rangle}{m_e c^2} \qquad \left(\frac{dV[\gamma]}{dt}\right)_{mod\ cl} = -2\frac{\langle \Delta\gamma g P_{cl}\rangle}{m_e c^2}. \tag{3.5}$$

4. Comparison to QED-PIC simulations

In order to test the validity of the expression for $V[\gamma]$ above we have simulated the interaction of an electron-beam with a counter-propagating circularly polarised plane-wave using the QED-PIC code EPOCH (Arber *et al.* (2015)). EPOCH includes the stochastic emission model using a Monte-Carlo algorithm (described in detail by Ridgers *et al.* (2014)). It also includes the classical and modified classical emission operators by directly solving equations (2.1) by first-order Eulerian integration.

The simulation parameters were as follows. The laser had a wavelength of one micron and intensity 10^{21} Wcm^{-2}, entering the simulation domain at $x = -40$ microns and the laser-pulse had a half-Gaussian temporal envelope with a rise time of 1 fs. 4000 grid cells were used to discretise a spatial domain extending from -40 microns to 40 microns. 10^5 macroparticles were used to represent an electron bunch consisting of 10^9 electrons. The electron bunch had a Gaussian spatial profile, centred on 39.7 microns, with a FWHM of 0.17 microns and had initial distribution $f(\mathbf{x}, \mathbf{p}, t = 0) = [n_e/(\sqrt{\pi}W)]\delta(p_y)\delta(p_z)\exp[-(p_x + \gamma_0 m_e c)^2/W^2]$ where $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum coordinate in phase space and n_e the number density of electrons in the beam. γ_0 was the initial average energy of the bunch and W its characteristic width.

Figure 2 shows a comparison of the spatially integrated electron energy distribution (where N is the total number of electrons in each enerrgy bin of width $\Delta\gamma = 9.8$) using classical, modified classical and stochastic emission operators with the initial spectrum at $t = 10.5$ fs, where $t = 0$ is defined as being 135 fs from the start of the simulation. We see that the modified classical and classical emission operators both give a decrease in the variance of the electron distribution whereas the stochastic emission operator gives an increase in the variance. Figure 3 shows temporal the evolution of the mean Lorentz factor $E[\gamma]$ and the standard deviation of the Lorentz factor $\sqrt{V[\gamma]}$. The QED-PIC simulations demonstrate the validity of equations (3.1), (3.2), (3.3) & (3.5).

Next we consider the ratio ξ of the classical term to the quantum term in equation (3.3) for the evolution of the variance.

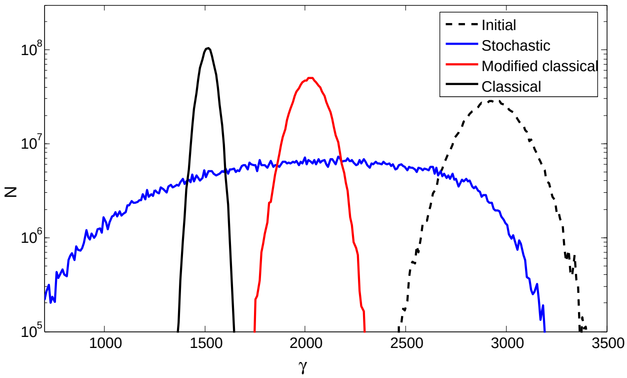

FIGURE 2. Electron distribution after $t = 10.5$ fs compared to initial distribution using the stochastic, modified classical and classical emission operators.

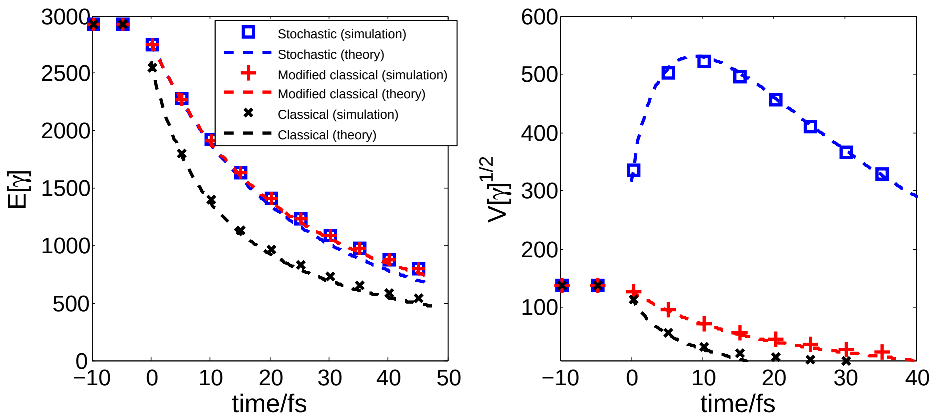

FIGURE 3. Left: mean Lorentz factor versus time using the various emission models from simulation and as predicted by equations (3.1) and (3.2). Right: standard deviation in Lorentz factor versus time from simulation and as predicted by equations (3.3) and (3.5).

$$\xi = \frac{T_Q}{T_C} \qquad T_Q = \frac{\langle S \rangle}{m_e^2 c^4} \qquad T_C = 2\frac{\langle \Delta\gamma g P_{cl}\rangle}{m_e c^2}.$$

Considering the idealised case where $f = n_e/(2f\gamma_0 m_e c)\delta(p_y)\delta(p_z)$ for $\gamma_0 m_e c(1-f) < |p_x| < \gamma_0 m_e c(1+f)$ and assuming $g = g_2 = 1$, we obtain

$$\xi = 1.6\frac{\alpha(f)}{\beta(f)}\eta_0 \tag{4.1}$$

$\eta_0 = 2\gamma_0 b$. $\alpha(f) = (1+f)^5 - (1-f)^5$ and $\beta(f) = (1+3f)(1-f)^3 - (1-3f)(1+f)^3$. Note that $\xi > 1$ indicates that the quantum term is initially dominant and we can expect the variance to increase initially. As the variance increases and the expectation value of the γ decreases we expect the classical term to eventually become dominant and so we would expect the variance to peak and then decrease after some time. This behaviour is clearly seen in figure 3.

From equation (4.1) (and also equation (3.4)) we see that ξ depends on three variables: the average Lorentz factor of the electron bunch γ_0; the width of the electron distribution $2f\gamma_0$ and the laser intensity I (which determines b). Figure 4 show ξ (including g & g_2) as a function of I & γ_0 (for $f = 0.2$) and f & I (for $\gamma_0 m_e c^2 = 1.5$ GeV). The prediction of $\xi = 1$ from equation (4.1) is shown to be reasonably accurate despite the assumption of $g = g_2 = 1$.

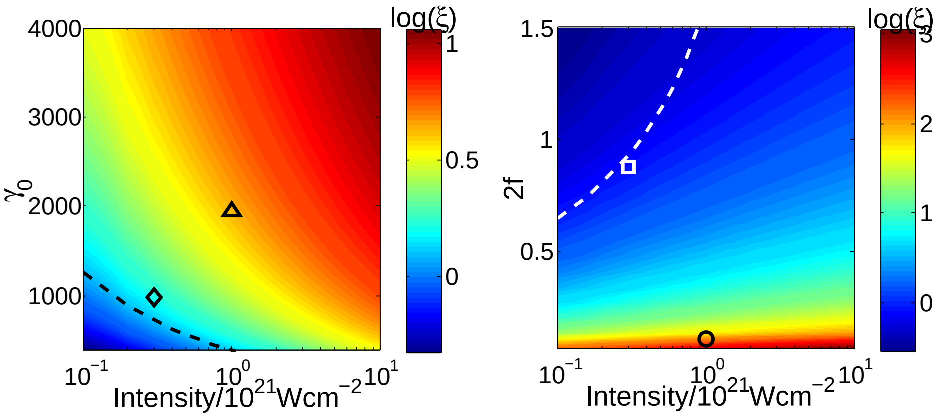

FIGURE 4. ξ as a function of: laser intensity and average Lorentz factor of the electron bunch (left); laser intensity and width of the electron energy distribution (right).

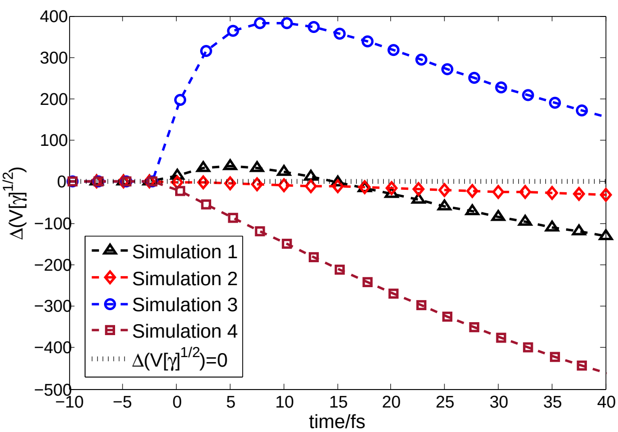

FIGURE 5. Temporal evolution of the change in standard deviation in the electron energy distribution in simulations 1–4.

In order to investigate whether the expression for ξ in equation (4.1) predicts whether the quantum or classical term dominates the evolution of the variance we performed further EPOCH simulations of the interaction of an electron-beam (again with initial distribution $f(\mathbf{x}, \mathbf{p}, t = 0) = [n_e/(\sqrt{\pi}W)]\delta(p_y)\delta(p_z)\exp[-(p_x + \gamma_0 m_e c)^2/W^2])$ and a counter-propagating plane-wave of intensity I. The following parameters were chosen:

Simulation	$I/10^{21}\mathrm{Wcm}^{-2}$	$\gamma_0 m_e c^2$/GeV	FWHM/GeV	Symbol
1	1.0	1.0	0.81	$\triangle$
2	0.3	0.5	0.21	$\diamond$
3	1.0	1.5	0.17	o
4	0.3	1.5	1.3	$\square$

We have shown where these simulations lie in the parameter space shown in figure 4 according to the symbols given in the table and assuming $f = W$. We chose the simulations so that one lies on each side of the $\xi = 1$ line. The time evolution of the change in the standard deviation of the electron energy distribution in these simulations is shown in figure 5. We see that only those simulations where equation (4.1) predicts that the quantum term is dominant show an increase in the variance (and so the standard deviation).

5. Discussion

The results of this investigation can be summarised as follows:

(i) $E[\mathbf{p}]$ evolves in the same way for the stochastic and modified classical emission operators and differently for the classical emission operator.

(ii) $V[\gamma]$ evolves differently for all operators. In particular, the stochastic emission operator can result in an increase in $V[\gamma]$ whereas the classical and modified classical operators can only cause a decrease in $V[\gamma]$ (as seen by Vranic *et al.* (2015) for $\eta \ll 1$).

Result (i) requires further explanation. Although we have shown that $(dE[\mathbf{p}]/dt)_{st}$ and $(dE[\mathbf{p}]/dt)_{modcl}$ evolve according to the same equation, from this it does not necessarily follow that the expectation values themselves are the same for these two emission models (as noted by Elkina *et al.* (2011)). We have previously shown in Ridgers *et al.* (2014) that, in fact, the expectation values of the momenta using these two models do agree to a high degree of accuracy and this was shown again for the parameters considered here in figure 3. We would expect this in the classical limit where $\eta \ll 1$. In this case the classical term in equation (3.3) dominates (from equation (4.1) we see that $\xi \propto \eta_0$) and rapidly reduces the variance of the electron bunch and the electron distribution in both the modified classical and stochastic models is $f \propto \delta(\mathbf{p} - \langle \mathbf{p} \rangle)$. The time evolution of $E[\gamma]$ depends on $\langle g P_{cl} \rangle$ which is equal to $g(\langle \eta \rangle) P_{cl}(\langle \eta \rangle)$ for both models when f is narrow in momentum-space. However, in the simulation whose results are shown in figure 3 $\eta > 0.1$. From figure 2 we see that in this case the electron energy distribution is very different when the stochastic emission operator is used compared to when the deterministic operator is used. Despite this the evolution of $E[\gamma]$ is the same due to the functional form of gP_{cl}. When $\eta \gg 1$ $gP_{cl} \propto \eta^{2/3}$. This almost linear dependence on η means that the difference in the evolution of $E[\gamma]$ between the models should be small. Finally we note that, as shown in figure 3, $E[\gamma]$ predicted by the classical emission model differs markedly from that predicted by the modified classical and stochastic models due to the neglect of the Gaunt factor g in the classical model.

$dV[\gamma]/dt$ is always negative for both the classical and deterministic emission operators. Physically, this is because electrons at higher energy radiate more energy than those at lower energy, causing a decrease in the width of the energy distribution. The classical operator predicts a more rapid decrease due to the assumption that $g = 1$ and the consequent overestimate of the scaling of the power radiated by the electrons with increasing η. For the stochastic emission operator $dV[\gamma]/dt$ can be either positive or negative and so $V[\gamma]$ can increase or decrease. The evolution of $V[\gamma]$ is determined by the balance between the quantum term T_Q (which causes $V[\gamma]$ to increase due the probabilistic nature of the emission) and the classical term T_C (which, as just described, causes $V[\gamma]$ to decrease as higher energy electrons radiate more energy). We have shown that which of these terms dominates depends on the width of the energy distribution and η. For large width the classical term increases in importance as it depends on $\Delta\gamma = \gamma - \langle \gamma \rangle$. For high η the quantum term becomes more important due to its scaling with η^4 compared to at most η^3 for the classical term (assuming $\Delta\gamma \sim \gamma$). In equation (4.1) we have provided a formula for the determination of which term is dominant.

The first of these results, i.e. that the evolution of the expectation value is the same for the modified classical and stochastic (but not classical) models, is useful in two ways. Firstly it shows that measuring the expectation value of an electron bunch after interaction with a high-intensity laser-pulse can give information about one quantum effect: the reduction of the total power emitted as expressed by g. It cannot, however, give information about the probabilistic nature of the emission, usually called 'straggling' (Shen & White (1972); Blackburn *et al.* (2014)). Secondly the result suggests that the 'modified classical' model of radiation reaction is sufficient for the calculation of laser absorption in high-intensity laser-plasma interactions. Laser absorption in this context depends on the average energy loss by the electrons (and positrons) in the plasma due to radiation reaction. The second result, i.e. the evolution of the variance differs between the models, can be used to measure straggling. An increase in the variance of the energy distribution of electrons must be due to the probabilistic nature of the emission. As further work we propose a comparison of QED-PIC simulations of laser absorption in laser-plasma interactions using the different emission models and an investigation of the use of the variance to observe straggling in 3D simulations of the interaction of a focusing laser-pulse with a counter propagating electron bunch produced by laser wake field acceleration (with a realistic energy spectrum).

6. Conclusions

We have derived equations for the evolution of the expectation value of the momentum and variance in the energy of an electron population subject to three different radiation reaction models. We have considered 'classical' and 'modified classical' models, where the radiation reaction is deterministic and the power emitted is the classical synchrotron power in the former case and in the latter case accounts for reduction to the power emitted by quantum effects (the Gaunt factor g). We have also considered a 'stochastic' model which calculates the emission using a more physically correct probabilistic treatment. We have shown that the expectation value of the energy evolves in almost the same way for the stochastic and modified classical models but differently for the classical model. The variance of the energy distribution evolves differently for all the models. This suggests that measuring the decrease in the expectation value of the energy is sufficient to measure the Gaunt factor but that a measurement of the variance is required to distinguish quantum stochastic effects.

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Appendix A. Functions describing synchrotron emission

The rate of photon emission (making the quasi-static and weak-field approximations) is

$$\lambda_\gamma(\eta) = \frac{\sqrt{3}\alpha_f c}{\lambda_c}\frac{\eta}{\gamma}h(\eta) \qquad h(\eta) = \int_0^{\eta/2} d\chi \frac{F(\eta,\chi)}{\chi}.$$

The quantum synchrotron function is given in Sokolov & Ternov (1968) eq. (6.5). In our notation it is, for $\chi < \eta/2$,

$$F(\eta,\chi) = \frac{4\chi^2}{\eta^2} y K_{2/3}(y) + \left(1 - \frac{2\chi}{\eta}\right) y \int_y^\infty \mathrm{d}t\, K_{5/3}(t)$$

where $y = 4\chi/[3\eta(\eta - 2\chi)]$ & K_n are modified Bessel functions of the second kind. For $\chi \geqslant \eta/2$, $F(\eta,\chi) = 0$. In the classical limit $\hbar \to 0$ the quantum synchrotron spectrum reduces to the classical synchrotron spectrum $F(\eta,\chi) \to F_{cl}(y_c) = y_c \int_{y_c}^\infty du K_{5/3}(u)$; $y_c = 4\chi/3\eta^2$. The probability that a photon is emitted with a given χ (by an electron with a given η) is $p_\chi(\eta,\chi)d\chi = [1/h(\eta)][F(\eta,\chi)/\chi]d\chi$.

Appendix B. Derivation of the moment equations

We obtain an equation for the evolution of the expectation value of the electron momentum by multiplying equation (2.2) by $\mathbf{p}$ and integrating over momentum.

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\int d^3\mathbf{p}\mathbf{p}\lambda_\gamma(\eta)f + \int d^3\mathbf{p}\mathbf{p}\frac{b}{2m_ec}\int_p^\infty dp'\lambda_\gamma(\eta')p_\chi(\eta',\chi)\frac{p'^2}{p^2}f(\mathbf{p}').$$

In spherical polars $d^3\mathbf{p} = p^2dpd^2\Omega$. We also write $\mathbf{p} = p\hat{\mathbf{p}}$. Therefore,

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\int d^3\mathbf{p}\mathbf{p}\lambda_\gamma(\eta)f + \int d^2\Omega\frac{b\hat{\mathbf{p}}}{2m_ec}\int_0^\infty dpp\int_p^\infty dp'\lambda_\gamma(\eta')p_\chi(\eta',\chi)p'^2f(\mathbf{p}').$$

We may flip the order of integration over p and p' in the second term on the right-hand side

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\int d^3\mathbf{p}\mathbf{p}\lambda_\gamma(\eta)f + \int d^2\Omega\frac{b\hat{\mathbf{p}}}{2m_ec}\int_0^\infty dp'\lambda_\gamma(\eta')f(\mathbf{p}')p'^2\int_0^{p'} dppp_\chi(\eta',\chi).$$

Here the p dependence of p_χ is in $\chi = [(p'-p)b]/(2m_ec)$ (where we have assumed the electrons are ultra-relativistic). To simplify the identification of gP_{cl} we define $p_{h\nu}$ as the probability that an electron with energy parameterised by η emits a photon with energy $h\nu$. $p_\chi = p_{h\nu}(dh\nu/d\chi) = p_{h\nu}(2mc^2)/b$. We may therefore write

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\int d^3\mathbf{p}\mathbf{p}\lambda_\gamma(\eta)f + \int d^2\Omega\hat{\mathbf{p}}\int_0^\infty dp'\lambda_\gamma(\eta')f(\mathbf{p}')p'^2\int_0^{p'c} dh\nu\left(p'-\frac{h\nu}{c}\right)p_{h\nu}(\eta',h\nu).$$

Now we use

$$\int_0^{p'c} dh\nu p_{h\nu}(\eta',h\nu) = 1 \qquad \int_0^{p'c} dh\nu p_{h\nu}(\eta',h\nu)h\nu = (h\nu)_{av}$$

to get

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\int d^3\mathbf{p}\mathbf{p}\lambda_\gamma(\eta)f + \int d^3\mathbf{p}\hat{\mathbf{p}}\lambda_\gamma(\eta)f(\mathbf{p})\left(p-\frac{(h\nu)_{av}}{c}\right).$$

Cancelling the appropriate terms and identifying $gP_{cl} = \lambda_\gamma(h\nu)_{av}$ yields equation (3.1),

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{st} = -\frac{\langle gP_{cl}\hat{\mathbf{p}}\rangle}{c}.$$

The equation for the evolution of $V[\gamma]$ (3.3) is obtained by using the same procedure to obtain an equation for $(d\langle\gamma^2\rangle/dt)_{st}$, i.e. we multiply equation (2.2) by γ^2 and integrate over momentum,

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{st} = -\int d^3\mathbf{p}\gamma^2\lambda_\gamma(\eta)f + \int d^3\mathbf{p}\gamma^2\frac{b}{2m_ec}\int_p^\infty dp'\lambda_\gamma(\eta')p_\chi(\eta',\chi)\frac{p'^2}{p^2}f(\mathbf{p}').$$

Which can be written as

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{st} = -\int d^3\mathbf{p}\gamma^2\lambda_\gamma(\eta)f+\int d^2\Omega\int_0^\infty dp'\lambda_\gamma(\eta')f(\mathbf{p}')p'^2\int_0^{p'c} dh\nu\left(\gamma'-\frac{h\nu}{m_ec^2}\right)^2 p_{h\nu}(\eta',h\nu).$$

where we have assumed $\gamma' = p'/m_ec$. Defining

$$\int_0^{p'c} dh\nu p_{h\nu}(\eta', h\nu)(h\nu)^2 = [(h\nu)^2]_{av}$$

gives

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{st} = -\int d^3\mathbf{p}\gamma^2\lambda_\gamma(\eta)f + \int d^3\mathbf{p}\lambda_\gamma(\eta)f(\mathbf{p})\left(\gamma^2 - 2\gamma\frac{(h\nu)_{av}}{m_ec^2} + \frac{[(h\nu)^2]_{av}}{m_e^2c^4}\right).$$

We again cancel the appropriate terms and this time identify $S = \lambda_\gamma[(h\nu)^2]_{av}$ as well as $gP_{cl} = \lambda_\gamma(h\nu)_{av}$ to get

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{st} = -2\frac{\langle\gamma gP_{cl}\rangle}{m_ec^2} + \frac{\langle S\rangle}{m_e^2c^4}.$$

To get an equation for $(dV[\gamma]/dt)_{st}$ we identify $V[\gamma] = \langle\gamma^2\rangle - E[\gamma]^2$. Therefore,

$$\left(\frac{dV[\gamma]}{dt}\right)_{st} = \left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{st} - \left(\frac{d\{E[\gamma]^2\}}{dt}\right)_{st} = \left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{st} - 2E[\gamma]\left(\frac{dE[\gamma]}{dt}\right)_{st}.$$

Substituting the results for $(d\langle\gamma^2\rangle/dt)_{st}$ and $(dE[\gamma]/dt)_{st} = \langle gP_{cl}\rangle/(m_ec^2)$ (the latter is obtained by taking the dot product of equation (3.1) with $\hat{\mathbf{p}}$ and assuming $p = \gamma m_ec$) gives the result in equation (3.3):

$$\left(\frac{dV[\gamma]}{dt}\right)_{st} = -2\frac{\langle\gamma gP_{cl}\rangle}{m_ec^2} + \frac{\langle S\rangle}{m_e^2c^4} + 2E[\gamma]\frac{\langle gP_{cl}\rangle}{m_ec^2} = -2\frac{\langle\varDelta\gamma gP_{cl}\rangle}{m_ec^2} + \frac{\langle S\rangle}{m_e^2c^4}.$$

Here we have used $\varDelta\gamma = \gamma - E[\gamma]$.

The moments of the classical and modified classical emission operators are straightforwardly obtained by integration by parts. To obtain equation (3.2) for $(dE[\mathbf{p}]/dt)_{mod\ cl}$ we multiply the emission operator $(\partial f/\partial t)_{em}^{mod\ cl}$ in equation (2.1) by $\mathbf{p}$ and integrate over momentum

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{mod\ cl} = \int d^3\mathbf{p}\frac{\mathbf{p}}{p^2}\frac{\partial}{\partial p}\left(p^2g\frac{P_{cl}}{c}f\right).$$

Substituting $d^3\mathbf{p} = p^2dpd^2\varOmega$ and $\mathbf{p} = p\hat{\mathbf{p}}$ and integrating by parts yields

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{mod\ cl} = \int d^2\varOmega\hat{\mathbf{p}}\left(\left[p^3g\frac{P_{cl}}{c}f\right]_0^\infty - \int_0^\infty dpp^2g\frac{P_{cl}}{c}f\right) = -\int d^2\varOmega\hat{\mathbf{p}}\int_0^\infty dpp^2g\frac{P_{cl}}{c}f.$$

We have used the fact that $f \to 0$ as $p \to \infty$ (faster that p^5 diverges) to get the last result. We have now derived equation (3.2)

$$\left(\frac{dE[\mathbf{p}]}{dt}\right)_{mod\ cl} = -\int d^3\mathbf{p}g\frac{P_{cl}}{c}\hat{\mathbf{p}}f = -\frac{\langle gP_{cl}\hat{\mathbf{p}}\rangle}{c}.$$

To derive equation (3.5) for $(dV[\gamma]/dt)_{mod\ cl}$ we first multiply the emission operator $(\partial f/\partial t)_{em}^{mod\ cl}$ in equation (2.1) by γ^2 and integrate over momentum

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{mod\ cl} = \int d^3\mathbf{p}\frac{\gamma^2}{p^2}\frac{\partial}{\partial p}\left(p^2g\frac{P_{cl}}{c}f\right).$$

Substituting $d^3\mathbf{p} = p^2dpd^2\varOmega$, $\gamma = p/(m_ec)$ and integrating by parts yields

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{mod\ cl} = \int d^2\Omega\left(\left[\frac{p^4}{m_e^2c^2}g\frac{P_{cl}}{c}f\right]_0^\infty - 2\int_0^\infty dp\frac{p^3}{m_e^2c^2}g\frac{P_{cl}}{c}f\right) = -\int d^2\Omega\hat{\mathbf{p}}\int_0^\infty dpp^2\gamma g\frac{P_{cl}}{m_ec^2}f.$$

Again, we have used the fact that $f \to 0$ as $p \to \infty$ (this time faster that p^6 diverges) to get the final result. We may write this more compactly as

$$\left(\frac{d\langle\gamma^2\rangle}{dt}\right)_{mod\ cl} = -2\int d^3\mathbf{p}\gamma g\frac{P_{cl}}{m_ec^2}\hat{\mathbf{p}}f = -2\frac{\langle\gamma g P_{cl}\rangle}{m_ec^2}.$$

We get equation (3.5) by identifying $V[\gamma] = \langle\gamma^2\rangle - E[\gamma^2]$ and $\Delta\gamma = \gamma - E[\gamma]$,

$$\left(\frac{dV[\gamma]}{dt}\right)_{mod\ cl} = -2\frac{\langle\gamma g P_{cl}\rangle}{m_ec^2} + 2E[\gamma]\frac{\langle gP_{cl}\rangle}{m_ec^2} = -2\frac{\langle\Delta\gamma g P_{cl}\rangle}{m_ec^2}.$$

REFERENCES

Andersen, K. K., Esberg, J., Knudsen, H., Thomsen, H. D., Uggerhøj, U. I., Sona, P., Mangiarotti, A., Ketel, T. J., Dizdar, A. & Ballestrero, S. 2012 Experimental investigations of synchrotron radiation at the onset of the quantum regime. *Phys. Rev. D.* **86**, 072001.

Arber, T. D., Bennett, K., Brady, C. S., Lawrence-Douglas, A., Ramsay, M. G., Sircombe, N. J., Gillies, P., Evans, R. G., Schmitz, H., Bell, A. R. & Ridgers, C. P. 2015 Contemporary particle-in-cell approach to laser-plasma modelling. *Plasma Phys. and Control. Fusion* **57**, 113001.

Baier, V. N., Katkov, V. M. & Strakhovenko, V. M. 1991 Quasiclassical theory of radiation and pair creation in crystals at high energy. *Rad. Eff.* **527**, 122–123.

Bashinov, A. V. & Kim, A. V. 2013 On the electrodynamic model of ultra-relativistic laser-plasma interactions caused by radiation reaction effects. *Physics of Plasmas* **20** (11), 113111.

Baĭer, V. N. & Katkov, V. M. 1968 Quasiclassical theory of bremsstrahlung by relativistic particles. *Sov. Phys. JETP* **26**, 807–813.

Bell, A. R. & Kirk, J. G. 2008 Possibility of prolific pair production with high-power lasers. *Phys. Rev. Lett.* **101**, 200403.

Blackburn, T. G., Ridgers, C. P., Kirk, J. G. & Bell, A. R. 2014 Quantum radiation reaction in laser˘electron-beam collisions. *Phys. Rev. Lett.* **112**, 015001.

Brady, C. S., Ridgers, C. P., Arber, T. D., Bell, A. R. & Kirk, J. G. 2012 Laser absorption in relativistically underdense plasmas by synchrotron radiation. *Phys. Rev. Lett.* **109**, 245006.

Bula, C., McDonald, K. T., Prebys, E. J., Bamber, C., Boege, S., Kotseroglou, T., Melissinos, A. C., Meyerhofer, D. D., Ragg, W., Burke, D. L., Field, R. C., Horton-Smith, G., Odian, A. C., Spencer, J. E., Walz, D., Berridge, S. C., Bugg, W. M., Shmakov, K. & Weidemann, A. W. 1996 Observation of nonlinear effects in compton scattering. *Phys. Rev. Lett.* **76**, 3116–3119.

Burke, D. L., Field, R. C., Horton-Smith, G., Spencer, J. E., Walz, D., Berridge, S. C., Bugg, W. M., Shmakov, K., Weidemann, A. W., Bula, C., McDonald, K. T., Prebys, E. J., Bamber, C., Boege, S. J., Koffas, T., Kotseroglou, T., Melissinos, A. C., Meyerhofer, D. D., Reis, D. A. & Ragg, W. 1997 Positron production in multiphoton light-by-light scattering. *Phys. Rev. Lett.* **79**, 1626–1629.

Clayton, C. E., Ralph, J. E., Albert, F., Fonseca, R. A., Glenzer, S. H., Joshi, C., Lu, W., Marsh, K. A., Martins, S. F., Mori, W. B., Pak, A., Tsung, F. S., Pollock, B. B., Ross, J. S., Silva, L. O. & Froula, D. H. 2010 Self-guided laser wakefield acceleration beyond 1 gev using ionization-induced injection. *Phys. Rev. Lett.* **105**, 105003.

Di Piazza, A., Hatsagortsyan, K. Z. & Keitel, C. H. 2010 Quantum radiation reaction effects in multiphoton compton scattering. *Phys. Rev. Lett.* **105**, 220403.

Dinu, V., Harvey, C., Ilderton, A., Marklund, M. & Torgrimsson, G. 2016 Quantum radiation reaction: From interference to incoherence. *Phys. Rev. Lett.* **116**, 044801.

DiPiazza, A. 2008 Exact solution of the landau-lifshitz equation in a plane wave. *Lett. Math. Phys.* **83**, 305.

Duclous, R., Kirk, J. G. & Bell, A. R. 2011 Energy straggling and radiation reaction for magnetic bremsstrahlung. *Plasma Phys. Control. Fusion* **53**, 015009.

Elkina, N. V., Fedotov, A. M., Kostyukov, I. Y., Legkov, M. V., Narozhny, N. B., Nerush, E. N. & Ruhl,

H. 2011 Qed cascades induced by circularly polarized laser fields. *Phys. Rev. ST Accel. Beams* **14**, 054401.

ERBER, T. 1966 High-energy electromagnetic conversion processes in intense magnetic fields. *Rev. Mod. Phys.* **38**, 626–659.

FURRY, W. H. 1951 On bound states and scattering in positron theory. *Phys. Rev.* **81**, 115–124.

GONOSKOV, A., BASTRAKOV, S., EFIMENKO, E., ILDERTON, A., MARKLUND, M., MEYEROV, I., MURAVIEV, A., SERGEEV, A., SURMIN, I. & WALLIN, E. 2015 Extended particle-in-cell schemes for physics in ultrastrong laser fields: Review and developments. *Phys. Rev. E* **92**, 023305.

GRISMAYER, T., VRANIC, M., MARTINS, J. L., FONSECA, R. A. & SILVA, L. O. 2016 Laser absorption via quantum electrodynamics cascades in counter propagating laser pulses. *Physics of Plasmas* **23** (5), 056706.

HARVEY, C. N., GONOSKOV, A., ILDERTON, A. & MARKLUND, M. 2017 Quantum quenching of radiation losses in short laser pulses. *Phys. Rev. Lett.* **118**, 105004.

HEISENBERG, W. & EULER, H. 1936 Consequences of dirac theory of the positron. *Z. Phys.* **98**, 714.

ILDERTON, A. & TORGRIMSSON, G. 2013 Radiation reaction in strong-field qed. *Phys. Lett. B* **725**, 481.

JI, L. L., PUKHOV, A., NERUSH, E. N., KOSTYUKOV, I. Y., SHEN, B. F. & AKLI, K. U. 2014 Energy partition, -ray emission, and radiation reaction in the near-quantum electrodynamical regime of laser-plasma interaction. *Physics of Plasmas* **21** (2), 023109.

KIRK, J. G., BELL, A. R. & ARKA, I. 2009 Pair production in counter-propagating laser beams. *Plasma Phys. Control. Fusion* **52**, 085008.

LANDAU, L. D. & LIFSHITZ, E. M. 1987 *The Classical Theory of Fields*, *The Course of Theoretical Physics*, vol. 2. Butterworth-Heinemann, Oxford.

NAKAMURA, T., KOGA, J. K., ESIRKEPOV, T. Z., KANDO, M., KORN, G. & BULANOV, S. V. 2012 High-power γ-ray flash generation in ultraintense laser-plasma interactions. *Phys. Rev. Lett.* **108**, 195001.

NEITZ, N. & DI PIAZZA, A. 2013 Stochasticity effects in quantum radiation reaction. *Phys. Rev. Lett.* **111**, 054802.

NERUSH, E. N., KOSTYUKOV, I. Y., FEDOTOV, A. M., NAROZHNY, N. B., ELKINA, N. V. & RUHL, H. 2011 Laser field absorption in self-generated electron-positron pair plasma. *Phys. Rev. Lett.* **106**, 035001.

PUKHOV, A. & MEYER-TER VEHN, J. 2002 Laser wake field acceleration: the highly non-linear broken-wave regime. *J. Appl. Phys. B* **74**, 355.

RIDGERS, C. P., BRADY, C. S., DUCLOUS, R., KIRK, J. G., BENNETT, K., ARBER, T. D., ROBINSON, A. P. L. & BELL, A. R. 2012 Dense electron-positron plasmas and ultraintense γ rays from laser-irradiated solids. *Phys. Rev. Lett.* **108**, 165006.

RIDGERS, C. P., KIRK, J. G., DUCLOUS, R., BLACKBURN, T. G., BRADY, C. S., BENNETT, K., ARBER, T. D. & BELL, A. R. 2014 Pair production in counter-propagating laser beams. *J. Comput. Phys.* **260**, 273.

RITUS, V. I. 1985 Quantum effects of the interaction of elementary particles with an intense electromagnetic field. *J. Russ. Laser Res.* **6**, 497.

SARRI, G., CORVAN, D. J., SCHUMAKER, W., COLE, J. M., DI PIAZZA, A., AHMED, H., HARVEY, C., KEITEL, C. H., KRUSHELNICK, K., MANGLES, S. P. D., NAJMUDIN, Z., SYMES, D., THOMAS, A. G. R., YEUNG, M., ZHAO, Z. & ZEPF, M. 2014 Ultrahigh brilliance multi-mev γ-ray beams from nonlinear relativistic thomson scattering. *Phys. Rev. Lett.* **113**, 224801.

SHEN, C. S. & WHITE, D. 1972 Energy straggling and radiation reaction for magnetic bremsstrahlung. *Phys. Rev. Lett.* **28**, 455.

SOKOLOV, A. A. & TERNOV, I. M. 1968 *Synchrotron Radiation*. Akademie-Verlag, Berlin.

SOKOLOV, I. V., NAUMOVA, N. M., NEES, J. A. & MOUROU, G. A. 2010 Pair creation in qed-strong pulsed laser fields interacting with electron beams. *Phys. Rev. Lett.* **105**, 195005.

VRANIC, M., GRISMAYER, T., FONSECA, R. A. & SILVA, L. O. 2015 Quantum radiation reaction in head-on laser-electron beam interaction. *New J. Phys.* **18**, 073035.

ZHANG, P., RIDGERS, C. P. & THOMAS, A. G. R. 2015 The effect of nonlinear quantum electrodynamics on relativistic transparency and laser absorption in ultra-relativistic plasmas. *New J. Phys.* **27**, 043051.

ZHIDKOV, A., KOGA, J., SASAKI, A. & UESAKA, M. 2002 Radiation damping effects on the interaction of ultraintense laser pulses with an overdense plasma. *Phys. Rev. Lett.* **88**, 185002.