Nonlinear zonal flow decay: tertiary instability, energetic coupling, and the effect of magnetic geometry

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1 Introduction

"Zonal" $E \times B$ flows (ZFs) arise spontaneously in fusion devices where the magnetic guide field traces out topologically toroidal surfaces. The flows are due to the component of the electrostatic potential that is constant across these surfaces. They are linearly stable, being driven nonlinearly by interaction with turbulence, and subsequently react back on the turbulence, suppressing it by a shearing mechanism. Thus, it would seem that understanding ZFs requires a full understanding the the turbulence. However, it is useful to divide this large problem into smaller, more manageable problems.

First, one can focus on linear aspects. On timescales comparable to the ion bounce frequency, ZFs are damped by geodesic-curvature-induced coupling to ion acoustic waves, with the full mode, including its zonal and non-zonal parts, being labeled the geodesic acoustic mode (GAM). This happens in both tokamaks [1, 2] and stellarators [3, 4]. On timescales longer than the ion bounce time, ZFs can also decay, both exponentially and algebraically, but only if there is non-zero radial magnetic drift, averaged over the trapped particle orbit [5, 6]; this is generally true in stellarators . In the end, a zero-frequency residual flow is reached [7], which has received special attention as it is completely undamped, and therefore presumably can enjoy unobstructed accumulation of energy.

Nonlinearly, there are both growth and decay mechanisms. Linearization of the nonlinear problem leads to instabilities. ZF growth can thereby be estimated via secondary [8] and modulational instabilities [9]. These closely-related instabilities are fundamentally local in Fourier space, in the sense that the interaction involves a very small number of Fourier modes, so a Fourier-truncated description is accurate [10]. Zonal flow decay can be estimated by the tertiary instability [8], which, in contrast to the growth mechanisms, is localized in position-space, and also involve smaller scales than the secondary mode. We argue, therefore, that it is sensible to study growth and decay separately because, although the processes must coexist in a turbulent plasma, they may be possible to distinguish by their characteristic signatures. The present paper is concerned with nonlinear decay; work on nonlinear growth by secondary instabilities, which can be viewed as complimentary to the present work, has recently been published [11].

We note that, although it is straightforward to separate nonlinear and linear effects in theoretical studies, both seem to be important in actual experimentally relevant scenarios, depending on plasma parameters and magnetic field shaping [12, 13]. Nevertheless, the separation can help with understanding. The goal of the present work is to conduct theoretical and numerical studies of nonlinear zonal flow decay, clarifying what occurs in simple settings, and extending previous works to handle some of the complexities present in general (stellarator) magnetic geometries. Our two main theoretical tools are analytical work on the tertiary mode, and nonlinear energy transfer analysis of turbulence simulations.

This manuscript, intended for distribution to participants of "The 1st JPP Frontiers in Plasma Physics Conference", contains a detailed derivation of the tertiary mode, and describe the current direction of numerical simulations.

2 Nature of the tertiary mode

As found originally by [8], the tertiary mode arises in the two-dimensional long-wavelength gyrofluid limit in the presence of strong zonal flows, but only when there is also a significant zonal temperature component. This condition translates in the more general two-dimensional gyrokinetic setting to the condition that the ratio of the free energy to electrostatic energy of the zonal mode must exceed a minimum amount [14].

This mode has been observed to be localized around extrema in the $E\times B$ flow, allowing it to avoid the effect of $E \times B$ shearing [8, 15]. However, in the present work, we find also another branch that resides in regions of maximum $E \times B$ shear. In either case, it requires FLR effects because perpendicular phase mixing provides the destabilizing resonance. The drive (free energy source) is inhomogeneity of the zonal perpendicular temperature. The mode can be studied either with or without background linear drive terms (due to gradient of the bulk plasma, and inhomogeneity of the background magnetic field). Theoretical treatment is simpler in the absence of these linear terms. However, importantly, the physical affect of coupling between stable and unstable linear modes by zonal shearing is absent, which explains why the purely nonlinear tertiary mode is not stabilized by $E \times B$ shearing.

It is fair to ask why the parallel ion resonance could not also drive an instability, competing with the resonance mentioned above. The answer may be that such a mode requires either a zonal parallel ion flow or zonal parallel temperature to destabilize it. The vorticity equation for the zonal flow, however, shows instead nonlinear coupling to the perpendicular temperature, and consequently the secondary mode involves the perpendicular temperature, and thus there is a physical basis for driving the perpendicular zonal temperature. However, it is unclear how the other zonal components would be driven. It seems therefore that the parallel ion resonance should be stabilizing. In any case, it depends on the relative size of the zonal components; a slab instability is also theoretically possible.

Note that neglecting parallel ion motion with the conventional adiabatic electron response (neglecting trapped electrons), leads to a "local" dispersion relation. That is, the mode dependence along the field line is left undetermined, and the question of whether the local solution corresponds to some "global" normal mode, is left open. However, with electron trapping, we obtain an integral equation, which can in principle be solved for the mode structure. Additionally, the mode dependence in the radial direction could also be determined from the differential equation that we find. We do not attempt here to solve for the "global" mode explicitly, but instead focus on the simpler local mode. (Indeed, it is not always true that the global mode is relevant nonlinearly [16].)

3 Derivation of tertiary mode in general magnetic geometry, including electron trapping

We treat the tertiary mode problem following [8]. We use magnetic coordinates ψ, α, such that $\mathbf{B} = \boldsymbol{\nabla}\psi \times \boldsymbol{\nabla}\alpha$, and the field line following coordinate l, measuring the arc length along the field line. We also include electron trapping, which has not been previously considered. This makes the theory applicable to the trapped electron mode (TEM) turbulence problem, and also shows the influence of electron trapping on electrostatic ion temperature gradient (ITG) turbulence (not taken into account by the normal "adiabatic" electron model). We apply the quasi-2D ($k_\parallel v_{\mathrm{th}i} \ll 1$) limit to the gyrokinetic system, and neglect linear drive terms. The nonlinear ion gyrokinetic equation for the tertiary mode is thus

$$\gamma g + \langle \mathbf{v}_E \rangle_{\mathbf{R}} \cdot \boldsymbol{\nabla} g = 0. \tag{1}$$

where γ is the tertiary mode growth rate, $\mathbf{v}_E = \hat{\mathbf{b}} \times \boldsymbol{\nabla}\phi/B$, and $g = h - (q \langle \phi \rangle_{\mathbf{R}} / T_i) F_{0i}$ is the gyro-averaged δf_1. For velocity variables, we use v, and pitch angle $\lambda = v_\perp^2/(v^2 B)$. We assume $k_\perp^2 \rho^2 \ll 1$, with $\rho \equiv v_{\mathrm{th}}/\Omega_c$, $v_{\mathrm{th}} = \sqrt{T/m}$, and retain terms of order $k_\perp^2 \rho^2$ (where species labels are absent, ions are to be assumed). In this limit, nonlinear phase mixing causes coupling between different $v_\perp$-moments of the ion distribution function, but the moment hierarchy naturally closes due to the ordering we take. We need not distinguish between trapped and passing ions since we assume that the ion transit frequency is low compared to the frequency of the tertiary mode. We use the approximation (and a similar one for the angle average in quasi-neutrality)

$$\langle \phi \rangle_{\mathbf{R}} \approx \left(1 + \frac{v_\perp^2}{4{v_{\mathrm{th}}}^2} \rho^2 \nabla_\perp^2 \right) \phi \tag{2}$$

The electron transit frequency is assumed to be large, $k_\parallel v_{\mathrm{the}} \gg \omega$. The passing electrons therefore satisfy the usual response

$$h_e^p = -\frac{e\overline{\phi}}{T_e} F_{0e}, \tag{3}$$

where we note that the orbit average $\overline{\cdots}$

$$\overline{\phi} = \frac{\oint dl\phi/\sqrt{1-\lambda B}}{\oint dl/\sqrt{1-\lambda B}} \tag{4}$$

covers the entire surface for passing electrons, so the l integral can be written as a double integral over field line label α and finite arc lengths in l, *i.e.* $\oint dl = \int dl \int d\alpha$. For a mode that has a finite binormal wavenumber (k_α), the α-average is zero and therefore $h_e^p = 0$ for the tertiary mode. For a zonal mode, ϕ depends on ψ but not on α or l, so $\phi = \overline{\phi}$.

The trapped electrons satisfy

$$\gamma h_e^{tr} = -\overline{\mathbf{v}}_E \cdot \boldsymbol{\nabla} h_e - \gamma \frac{e\overline{\phi}}{T_e} F_{0e}, \tag{5}$$

where $\overline{\cdots}$ denotes the orbit average of trapped electrons, taken between consecutive bounce points l_1 and l_2 such that $B(l_1) = B(l_2) = 1/\lambda$. Based on these solutions, we write the quasi-neutrality equation for the zonal and non-zonal (tertiary mode) parts

$$\frac{1}{n_0}\int d\mathbf{v}\left(1 + \frac{v_\perp^2}{4{v_{\rm th}}^2}\rho^2\nabla_\perp^2\right) g_z = \left(\tau f_{tr} - \rho^2\nabla_\perp^2\right)\frac{q\phi_z}{T_i} + \frac{1}{n_0}\int_{tr} d\mathbf{v} h_{ez}^{tr}, \tag{6}$$

$$\frac{1}{n_0}\int d\mathbf{v}\left(1 + \frac{v_\perp^2}{4{v_{\rm th}}^2}\rho^2\nabla_\perp^2\right) g = \left(\tau - \rho^2\nabla_\perp^2\right)\frac{q\phi}{T_i} + \frac{1}{n_0}\int_{tr} d\mathbf{v} h_e^{tr}, \tag{7}$$

where we use the subscript z for the zonal component, and omit the subscript for the tertiary component. Note that 'tr' in the subscript of the velocity integrals emphasizes that the integral is performed over only the trapped portion of velocity space. Note also the appearance of a term proportional to the trapped particle fraction f_{tr} (here $f_{tr}(l) = (1 - B(l)/B_{\max})^{1/2}$, where $B_{\max}$ is the maximum value of the guide field), which reflects the imperfect shielding of flux-surface-average density in the presence of electron trapping. Later we will need to consider the ordering for the trapped fraction f_{tr}, so that its effect does not overwhelm the usual FLR terms that drive the tertiary.

For the tertiary mode, we assume $\phi \propto \exp(ik_\alpha\alpha + \gamma t)$. For the zonal mode we define an informal radial wavenumber k_ψ via $\nabla_\perp\phi_z \sim k_\psi|\boldsymbol{\nabla}\psi|\phi_z$. Benefitting from previous work on the tertiary, we can state the ordering for the calculation at the outset, in terms of the ordering parameter δ:

$$\delta \sim k_\psi|\boldsymbol{\nabla}\psi|\rho \sim (k_\alpha|\boldsymbol{\nabla}\alpha|\rho)^2 \ll 1. \tag{8}$$

We now integrate the ion gyrokinetic equation (for the tertiary mode) over velocity, using Eqn. 7, retaining all $\mathcal{O}(k_\perp^2\rho^2)$ terms initially (we will next examine and discard some terms based on the above ordering), yielding

$$\bar{\gamma}\left[\left(\tau+\rho^2\nabla_\perp^2\right)\frac{q\phi}{T}-\rho^2\nabla_\perp^2\frac{q\chi}{T}+\frac{1}{n_0}\int_{tr}d\mathbf{v}h_e^{tr}\right]=$$
$$(i\rho^2\nabla_\perp^2\omega_E)\frac{q\chi}{T}+ik_\alpha n_z'\phi-ik_\alpha\frac{q\chi_z'}{T}\nabla_\perp^2\phi, \quad (9)$$

where $n_z(\psi,l)=\int d\mathbf{v}g_z$, and we have defined $\bar{\gamma}=\gamma+i\omega_E$, and $\omega_E=k_\alpha\partial\phi_z/\partial\psi=k_\alpha\phi_z'$ (we will find $\bar{\gamma}\sim\mathcal{O}(\delta\omega_E)$ in the end). We have also introduced a potential χ related to the perpendicular pressure:

$$\chi=\frac{1}{qn_0}\int d\mathbf{v}\frac{mv_\perp^2}{4}g. \quad (10)$$

We obtain from the $v_\perp^2$ moment of the gyrokinetic equation an additional equation for χ

$$\bar{\gamma}\chi=ik_\alpha\chi_z'\phi. \quad (11)$$

From Eqn. 5 we obtain also an expression for the trapped electron density

$$\bar{\gamma}\int_{tr}d\mathbf{v}h_e^{tr}=ik_\alpha\phi\int_{tr}d\mathbf{v}\frac{\partial h_{ez}^{tr}}{\partial\psi}+(i\omega_E-\bar{\gamma})\int_{tr}d\mathbf{v}\frac{e\overline{\phi}}{T_e}F_{0e}, \quad (12)$$

We will substitute the above expressions into Eqn. 9 to obtain a dispersion relation. But first let's discuss which terms need to be retained according to our ordering. First we note that since $\bar{\gamma}\sim\mathcal{O}(\delta\omega_E)$, and $\chi_z\sim\phi_z$, Eqn. 11 implies that $\chi\sim\delta^{-1}\phi$. This will promote the order of terms proportional to χ in Eqn. 9. Next, we can neglect the two FLR terms in Eqn. 6 since they are small, *i.e.* $k_\psi^2|\boldsymbol{\nabla}\psi|^2\rho^2\sim\mathcal{O}(\delta^2)$. To evaluate one of the non-negligible FLR terms on the left-hand-side of Eqn. 9 we derive the following equation from Eqn. 11 (noting that $\bar{\gamma}=\gamma+i\omega_E(\psi)$):

$$\bar{\gamma}\rho^2\nabla_\perp^2\frac{q\chi}{T}\approx ik_\alpha\chi_z'\rho^2\nabla_\perp^2\frac{q\phi}{T}-i(\rho^2\nabla_\perp^2\omega_E)\frac{q\chi}{T}-2i\rho^2(\boldsymbol{\nabla}_\perp\omega_E)\cdot\left(\boldsymbol{\nabla}_\perp\frac{q\chi}{T}\right), \quad (13)$$

where we have neglected terms small in our ordering. In the case that the mode is localized radially to where the $E\times B$ shear is zero, we may neglect terms proportional to $\phi_z''\approx 0$. This implies that we should neglect the final term in Eqn. 13. However, motivated by numerical findings, we also consider the case where the mode is localized to extrema in χ_z and ϕ_z (we assume that they are in phase or π out of phase). In that case, the first two terms can be neglected and the final term retained. In summary, all these terms must be retained to capture a more general tertiary mode.

Substituting Eqns. 6, 7, 11, 12, and 13 into Eqn. 9 we find

$$
(1-f_{tr})\tau\bar{\gamma}^2\frac{q\phi}{T} = \bar{\gamma}\left[\frac{ik_\alpha}{n_0}\int_{tr} d\mathbf{v}(\phi-\overline{\phi})\frac{\partial h_{ez}^{tr}}{\partial\psi} + \frac{i\tau\omega_E}{n_0}\int_{tr} d\mathbf{v}\frac{q(\phi-\overline{\phi})}{T}F_{0e}\right]
$$

$$
+\bar{\gamma}2ik_\alpha\chi_z'\rho^2\nabla_\perp^2\frac{q\phi}{T} + 2k_\alpha\chi_z'(\rho^2|\boldsymbol{\nabla}\psi|^2\omega_E'')\frac{q\phi}{T} + 2\bar{\gamma}k_\alpha^2\rho^2\omega_E'\boldsymbol{\nabla}\psi\cdot\boldsymbol{\nabla}\left(\frac{q\phi}{T}\frac{\chi_z'}{\bar{\gamma}}\right). \quad (14)
$$

The terms within the bracket on the first line are new terms that are due to electron trapping. The final term is required when the mode resides where shear is non-zero. As we have mentioned above, we may need to consider the trapped particle fraction as small. This now becomes clear since the bracketed quantity would be otherwise be large and dominate over the other (drive terms), stabilizing the mode. However, we also note that if the tertiary mode is independent of the field-line-following coordinate, then $\phi = \overline{\phi}$, and the bracketed terms vanish. In this case, the effect of electron trapping appears only on the first term – but it is negligible if the trapped particle fraction is small $f_{tr} \ll 1$. We therefore find it convenient to allow the trapped particle fraction to be finite, to include the case that the bracketed term exhibits significant cancelation.

We can simplify Eqn. 14 a bit more by assuming that the radial derivative of the tertiary mode is small (we argue that the mode envelope should be on the scale of the zonal mode) and so $\boldsymbol{\nabla}_\perp^2\phi \approx -k_\alpha^2|\boldsymbol{\nabla}\alpha|^2\phi$. Let us further assume that $\phi_z'' = 0$ in line with previous observations in tokamak geometries [8, 15]. We can also denote the bracketed quantity on the first line of Eqn. 14 as $-\Delta q\phi/T$, to obtain a more compact dispersion relation:

$$
(1-f_{tr})\tau\bar{\gamma}^2 + \bar{\gamma}\left[2ik_\alpha^3\rho^2|\boldsymbol{\nabla}\alpha|^2\chi_z' + \Delta\right] - 2k_\alpha^2\rho^2|\boldsymbol{\nabla}\psi|^2\chi_z'\phi_z''' = 0. \quad (15)
$$

Examining this equation, magnetic geometry affects several quantities. First, $|\boldsymbol{\nabla}\alpha|$ and $|\boldsymbol{\nabla}\psi|$, which include the effect of flux tube compression and magnetic shear, control the amplitude of the stabilizing and destabilizing terms, respectively. We note that these terms vary in l, in a way that is generally distinct from geometric factors that control linear mode stability (*e.g.* normal curvature), and therefore establish a distinct parallel connection length for the tertiary mode. (This fact was noted by [13], and used to explain the unexpected strength of nonlinear damping of zonal flows in the HSX stellarator.)

The effect of trapped electrons on the tertiary mode can be considered in different limits: First, consider the limit where $\phi = \overline{\phi}$, *e.g.* the mode is constant across magnetic wells, so that $\Delta = 0$. In that case, the dispersion relation is written as $(1-f_{tr})\bar{\gamma}^2 + ib\bar{\gamma} - c = 0$ with b and $c > 0$ real constants. If $4c \leq b^2$ the mode is always stable. Assuming $4c > b^2$, and $0 \leq f \leq 1$, the growth rate is minimized by the choice $f = 0$, *i.e.* electron trapping is destabilizing. Now let us consider $\Delta \neq 0$. if the trapped fraction f is finite, $f \sim \mathcal{O}(1)$, then the term proportional to $\Delta \sim f$ dominates the equation and the mode is stable. However, if we take $\Delta \neq 0$ but further assume $f_{tr} \ll 1$, the situation is less clear. We may neglect f_{tr} may from the quadratic term, and the remaining contribution from trapped electrons is due to Δ, which is purely imaginary and enters the linear term in this quadratic dispersion relation, whose remaining part is also purely imaginary. Overall, the linear term can only be stabilizing. In regions of maximal shear, the trapped electron contribution is the only contribution, so in that case electron trapping is stabilizing. Otherwise, the contribution of the electron trapping term depends on the solution ϕ. In summary, although the effect of electron trapping is not completely clear at low trapped particle fraction, it must be stabilizing when the fraction is sufficiently large. We conclude, that destabilization of the tertiary by kinetic electrons in electrostatic ITG turbulence at low trapped particle fraction could be responsible for the destabilization of turbulence in simulations of both tokamak and stellarator geometries [17], but further investigation is necessary.

3.1 Slab limit

To help make the instability more transparent, we consider the case of uniform magnetic geometry. We change from flux coordinates ψ and α to cartesian coordinates x and y, and apply the usual normalizations $(q\chi/T)L/\rho \to \chi$, $(q\phi/T)L/\rho$, $(x,y)/\rho \to (x,y)$, $tv_{\mathrm{th}}/L \to t$. The electron trapping terms drop out of Eqn. 14, yielding

$$\tau\bar{\gamma}^2\phi + 2\bar{\gamma}ik_y\chi_z'(k_y^2 - \partial_x^2)\phi - 2k_y^2\left[\phi_z'''\chi_z'\phi + \bar{\gamma}\phi_z''\frac{\partial}{\partial x}\left(\frac{\phi\chi_z'}{\bar{\gamma}}\right)\right] = 0. \tag{16}$$

We can consider two simple limits. If the mode is localized about the minimum of $E\times B$ shear we take $\phi_z''=0$ and the final term within the bracket is zero. If we can further take the local limit $\partial_x \ll k_y$, we obtain the solution

$$\bar{\gamma} = -ik_y^3\chi_z'/\tau \pm \sqrt{-k_y^6(\chi_z')^2/\tau^2 + 2k_y^2\phi_z'''\chi_z'/\tau}. \tag{17}$$

If, however, we instead take $\phi_z' = \chi_z' = 0$, as suggested by our simulations, then we obtain

$$\bar{\gamma} = \pm\sqrt{2k_y^2\phi_z''\chi_z''/\tau}. \tag{18}$$

This growth rate has a similar scaling as that of the first limit ($\sim k_x^2 k_y\sqrt{\phi_z\chi_z}$), but it is stronger, since it avoids the stabilizing influence of the real frequency.

4 Numerical investigation

Several questions motivate us to conduct a numerical study of the tertiary mode, and of the nonlinear decay mechanism in general, starting with a simple magnetic geometry: It has been shown that the onset of the tertiary is responsible for transition to turbulence at the nonlinear critical gradient in ITG turbulence simulations [8]. However, what is the role of this mode in the saturated state of the turbulence? One would also like to understand the saturation phase of the zonal flows themselves, beyond the regime of applicability of linearized calculations. How are the flows modified, initially, by the back-reaction of the nonlinear tertiary mode? In the fully developed state, is it possible to observe the signature of the tertiary mode, just as linear eigenmodes are detectable by their characteristic phases, *etc.*, in nonlinear simulations [18, 19]?

A gyrokinetic simulation, in 2D slab geometry, with an initial condition dominated by a zonal mode, demonstrates the evolution through three phases, as shown in Fig. 1: (1) growth of tertiary modes, (2) saturation phase exhibiting finite energy transfer, between and (3) steady state with an energetic balance. The corresponding growth rate spectrum for the growth phase is plotted in Fig. 2. Investigation of all three of these phases will continue.

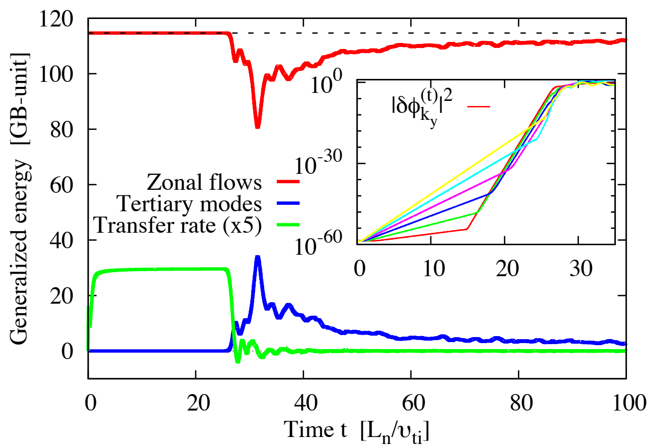

Figure 1: Free energy evolution for simple 2D gyorkinetic system.

Finally, we demonstrate the localization to maxima of ϕ by plotting the potential in the x-y plane during the growth phase of the tertiary. In Fig. 3, the initial condition and final condition are shown, including a version of ϕ with the zonal part removed, revealing localized mode structure of the tertiary, around positive peaks of ϕ. The fact that only one sign of ϕ is allowed is reflected in the growth rate formula for this branch of the tertiary, Eqn. 18.

References

[1] V. B. Lebedev, P. N. Yushmanov, P. H. Diamond, S. V. Novakovskii, and A. I. Smolyakov. Plateau regime dynamics of the relaxation of poloidal rotation in tokamak plasmas. *Physics of Plasmas*, 3(8):3023–3031, 1996.

[2] S. V. Novakovskii, C. S. Liu, R. Z. Sagdeev, and M. N. Rosenbluth. The radial electric field dynamics in the neoclassical plasmas. *Physics of Plasmas*, 4(12):4272–4282, 1997.

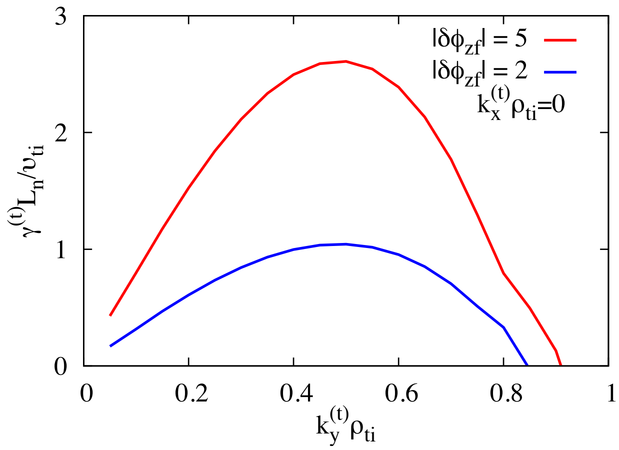

Figure 2: Growth rate spectrum of the tertiary mode (as measured for the $k_x = 0$ component of each mode).

[3] H. Sugama and T.-H. Watanabe. Dynamics of zonal flows in helical systems. *Phys. Rev. Lett.*, 94:115001, Mar 2005.

[4] H. Sugama and T.-H. Watanabe. Collisionless damping of zonal flows in helical systems. *Phys. Plasmas*, 13(1):–, 2006.

[5] A. Mishchenko, P. Helander, and A. Könies. Collisionless dynamics of zonal flows in stellarator geometry. *Phys. Plasmas*, 15(7):–, 2008.

[6] P. Helander, A. Mishchenko, R. Kleiber, and P. Xanthopoulos. Oscillations of zonal flows in stellarators. *Plasma Phys Contr F*, 53(5):054006, 2011.

[7] M. N. Rosenbluth and F. L. Hinton. Poloidal flow driven by ion-temperature-gradient turbulence in tokamaks. *Phys. Rev. Lett.*, 80:724–727, Jan 1998.

[8] B. N. Rogers, W. Dorland, and M. Kotschenreuther. Generation and stability of zonal flows in ion-temperature-gradient mode turbulence. *Phys. Rev. Lett.*, 85(25):5336–5339, Dec 2000.

[9] Liu Chen, Zhihong Lin, and Roscoe White. Excitation of zonal flow by drift waves in toroidal plasmas. *Phys. Plasmas*, 7(8):3129–3132, 2000.

[10] D. Strintzi and F. Jenko. On the relation between secondary and modulational instabilities. *Phys. Plasmas*, 14(4), 2007.

[11] G G Plunk and A Ban Navarro. Nonlinear growth of zonal flows by secondary instability in general magnetic geometry. *New Journal of Physics*, 19(2):025009, 2017.

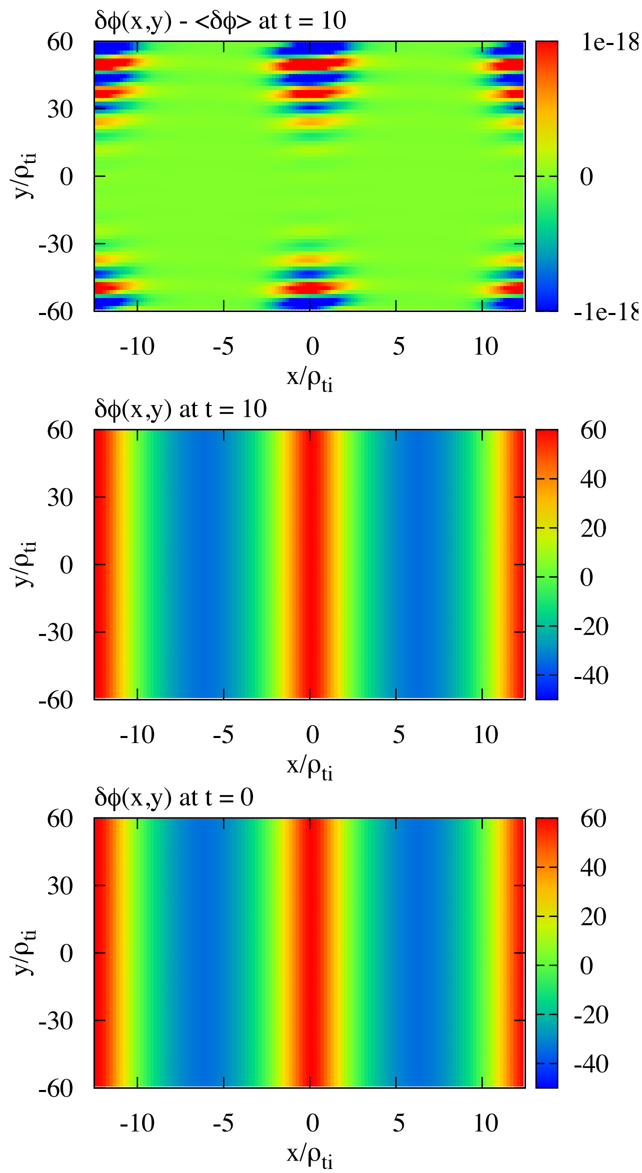

Figure 3: Electrostatic potential during growth phase of tertiary. The first panel shows only the non-zonal component of ϕ. Unfiltered field (second panel) is indistinguishable from initial condition (third panel) since the tertiary mode is still at very low amplitude.

[12] R. E. Waltz and C. Holland. Numerical experiments on the drift wave–zonal flow paradigm for nonlinear saturation. *POP*, 15(12):122503, 2008.

[13] G. G. Plunk, P. Xanthopoulos, and P. Helander. Distinct turbulence saturation regimes in stellarators. *Phys. Rev. Lett.*, 118:105002, Mar 2017.

[14] G G Plunk, T Tatsuno, and W Dorland. Considering fluctuation energy as a measure of gyrokinetic turbulence. *New Journal of Physics*, 14(10):103030, 2012.

[15] Maurice Maurer. *On the Limitation of the Dimits Shift in Plasma Microturbulence.* PhD thesis, Free University of Berlin, 2011.

[16] Matt Landreman, Gabriel G. Plunk, and William Dorland. Generalized universal instability: transient linear amplification and subcritical turbulence. *Journal of Plasma Physics*, 81(5), 10 2015.

[17] Josefine Proll. personal communication.

[18] Tilman Dannert and Frank Jenko. Gyrokinetic simulation of collisionless trapped-electron mode turbulence. *Phys. Plasmas*, 12(7):–, 2005.

[19] T. Görler and F. Jenko. Multiscale features of density and frequency spectra from nonlinear gyrokinetics. *Phys. Plasmas*, 15(10):–, 2008.