Opportunities and priorities for stellarator theory and computation

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There is an art to identifying good research problems. Generally, one seeks research problems that are consequential and scientifically interesting, where effect sizes are large, and where there is reasonable probability of obtaining a conclusive and meaningful answer. Here we attempt to identify some research areas in stellarator theory and computation that meet these criteria, areas that should be priorities for exploration in the next few years.

We take the point of view that the best research problems lie in the intersections of the following sets:

1. Which problems are potential 'show-stoppers' for the stellarator as a fusion energy concept? What kind of theoretical/computational advances are most likely to impact the feasibility of fusion energy?
2. In which areas is theory and computation most trustworthy and predictive? In which areas are the theoretical approximations best satisfied, and neglected effects and terms least likely to matter?
3. What tools, methods, ideas, and techniques have become available (perhaps in other fields) which may enable new advances?
4. Which areas are of personal scientific interest and most aligned with one's expertise?

The last of these topics is different for everyone, so we will instead focus on the first three issues.

I. CRITICAL PROBLEMS

Let us consider the first category from the list above, problems that absolutely must be addressed if the stellarator concept is to lead to commercial fusion energy. While there are many such critical issues, there are ways forward on all of them. In most cases there are ways forward that are computational, which means significant progress can be made even without the time and money required for experiments.

- A stellarator reactor must be designed to have acceptable cost, both cost of the power plant and cost per unit electric power. The latter measure must be low enough to compete with conventional technologies (considering possible government subsidies and taxes.) The cost of the fusion power plant is not as critical as the cost per unit electric power, since one can imagine building a small number of enormous fusion reactors, but the plant cost remains an important issue. Fusion energy will need to compete with fission, an already-available technology for low-carbon baseload electricity, where a primary obstacle to new power plants today is the high up-front capital cost of a plant. There are several drivers for the cost of a stellarator fusion reactor:
  - One driver of both measures of cost is the extremely tight tolerances to which stellarators have typically been designed. It costs far more to build and assemble components to tight tolerances than to generous tolerances. The demise of NCSX can be attributed to the tight tolerances that were thought to be required. However, the plasma performance surely has different sensitivity to different patterns of error fields, and so one way forward is to identify and provide control over these important field components.
  - Stellarator reactors, particularly designs extrapolated from W7-X and LHD, have extremely large size, owing in part to the high aspect ratio. A solution may be to use quasi-axisymmetric designs, which exist at much lower aspect ratio.
  - The large size of stellarator reactors also follows from the need to fit a neutron stopping distance (1-1.5 m) between the plasma and coils, while keeping the coils sufficiently close to the plasma to produce the needed shaping of the field. Typically these competing aims are reconciled by scaling a plasma and coil

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configuration up uniformly until the plasma-coil separation is sufficient. This consideration determined the size of ARIES-CS, for example[1]. A solution may be to optimize the plasma shape for compatibility with more distant coils [2].

- Even with extensive optimization, and even neglecting the transport of fast particles by instabilities, stellarators still lose a few percent of fusion alpha particles before they thermalize. For example, for ARIES-CS, the lost fraction was $\sim 5\%$ [1]. Even this small loss of high-energy particles is unacceptable, exceeding material limits on the plasma facing components. There is reason to believe the alpha confinement can be improved: the guiding-center alpha loss can in principle be made arbitrarily small in a quasi-helically-symmetric (QHS) or quasi-axisymmetric (QAS) stellarator, since the collisionless guiding-center confinement becomes perfect in the limit of perfect symmetry, which can be approached by increasing the aspect ratio in the QHS case or approaching true axisymmetry in the QAS case.

- There are two related issues associated with particle transport:
  - A strategy must be found for extracting helium ash from the core to prevent poisoning of the fusion reaction. To extract ash, a short particle confinement time is desirable, but this conflicts with the desire for the energy confinement time to be long.
  - At the high densities that are optimal for fusion performance, stellarators are observed (except in the HDH mode of W7-AS [4]) to show strong accumulation of impurities, resulting in radiative collapse. This behavior likely results from the fact that in the absence of symmetry and when all species have low collisionality, neoclassical theory predicts inward impurity accumulation for the typical ion root regime. Transport from microturbulence may provide a mechanism for flushing impurities. Or, breaking of magnetic surfaces in a controlled manner, in an annulus that is swept across the plasma, could perhaps remove impurities in a manner analogous to ELMs.

- There is little knowledge about whether all the requirements of a divertor (acceptable heat flux on solid surfaces, sufficient neutral pressure for pumping out helium ash, sufficient reduction of impurity influx to the core, and robustness to varying plasma conditions) can simultaneously be met in a reactor-relevant stellarator. Several divertor concepts are being considered, including the helical divertor in LHD, the island divertor in W7-X, and a general 'non-resonant' (non-island) divertor [3].

- It remains to be demonstrated that materials exist for the plasma facing components which can withstand the harsh reactor environment for a sufficiently long time. Possible solutions include liquid metal walls, moving to a non-DT fuel cycle (though other requirements then become significantly more demanding), or a lithium vapor-box divertor [5].

- For the high duty cycle required of a power plant, it will be necessary to regularly replace the blanket, moving blanket modules in and out between the coils. Given the very limited access between coils of a stellarator, it is not clear this blanket maintenance can be carried out sufficiently quickly. One solution may be to include chamber access requirements in the coil shape optimization [6]. Another solution may be to build the coils with demountable joints, so the coils can be disassembled to provide access [7]. A third solution may be to provide at least part of the magnetic field shaping with saddle coils or tiles ('monoliths') of superconducting material [8].

II. PREDICTIVE POWER

We next consider the second set of concerns. Theory and computation are most useful when the underlying equations are robust. For example, theory that relies principally on Maxwell's equations will be more robust than theory which relies on models of the plasma response in some asymptotic regime or on atomic physics [9]. Here we attempt to characterize the reliability of theory and computation in various areas of stellarator physics, roughly ordering from the highest to lowest predictive power, though the precise ordering is not meant to be taken too seriously.

- <u>Prediction of the vacuum field from a given coil configuration.</u> In this case, calculations rely only on the Biot-Savart law, and so we can have such high confidence in the theory that there is in fact no need to build experiments for validation.

- <u>Magnetic equilibrium.</u> In cases where good flux surfaces exist, the equilibrium equation $\boldsymbol{j} \times \boldsymbol{B} = \nabla p$ should be robust. The equilibrium depends on $j_{bootstrap}$ from neoclassical theory, which itself is relatively reliable as discussed below. One main limitation in predicting the equilibrium is that it depends on the pressure profile (both directly and via $j_{bootstrap}$), which depends on turbulent transport and so is hard to know with confidence. The other main limitation of MHD equilibrium prediction is that physics beyond ideal MHD becomes important in the presence of islands. All the items below in this list depend on the magnetic equilibrium, and so they necessarily cannot be predicted more reliably than the equilibrium itself.

- RF heating The propagation and absorption of RF waves is generally well understood theoretically. The weak point in this area is probably coupling of RF power through the edge, where it is known for example that the experimental coupling efficiency of lower hybrid power is often far lower than predicted. Since stellarators do not require current drive, RF physics has relatively low impact on the viability of a stellarator reactor.

- Neoclassical transport and $j_{bootstrap}$. Neoclassical transport is largely a function of the guiding-center particle trajectories, which can be computed with high confidence to the extent that the magnetic equilibrium is known. The weakness of neoclassical theory is that it depends on profiles of density and temperature, which are determined in part by turbulence. The bootstrap current is expected to be relatively unaffected by turbulence, except insofar as turbulence sets the density and temperature profiles.

- Fast particle confinement. In the absence of transport by instabilities, fast particle confinement depends on guiding-center trajectories, the equations for which are robust. Particles of sufficiently high energy are not significantly affected by microturbulence, although turbulence starts to matter once the particles have slowed considerably. The transport of fast particles by Alfvenic modes however has significantly greater uncertainty, depending for instance on the saturation level of the instabilities.

- Core turbulent transport. In the core, where gyrokinetic orderings are well satisfied, the gyrokinetic equation that governs turbulence should be robust. Linear stability properties (e.g. linear critical temperature gradients) should be trustworthy, although the relationship between linear properties and the real nonlinear system is unclear due to the Dimits shift and subcritical turbulence. Numerical solution of the gyrokinetic system nonlinearly presents additional challenges. Ideally, turbulence calculations in a stellarator would include an entire flux surface, include multiple kinetic species, and include wavenumbers extending from ion to electron gyroradius scales, but such a computation is presently infeasible.

- MHD stability. MHD stability in stellarators is puzzling because data from multiple experiments suggests that MHD stability limits can be violated. So while solving the linearized MHD equations is a more straightforward numerical task than e.g. solving nonlinear equations for microturbulence, the relevance of linear MHD calculations to real experiments is unclear. Nonlinear calculations and/or including non-ideal effects may provide some resolution, but this remains to be shown.

- The edge and divertor. In this area, theory has high predictive power in one way and low predictive ability in others. The rough positions along which heat and particles strike the plasma facing components can largely be determined by following magnetic field lines, and hence can be determined with high confidence. However, other properties, such as the width of the edge heat flux layer or divertor detachment, depend on much more complicated physical mechanisms. In the edge, the large perturbations to density and temperature and large flows violate standard gyrokinetic orderings, and hence predictions of edge turbulence are difficult. Neutrals and atomic physics are also likely to be important for many edge phenomena.

Several conclusions can be drawn from this list. MHD equilibrium calculation is crucial to all items except the first, and even in the case of the first item there is a connection, since MHD equilibrium is typically an integral component of coil design. Also, a 'weak link' for many of these areas is predictive capability for the pressure profile that enters the MHD equilibrium calculation.

However, sensitivity to the profiles of pressure and plasma current is far weaker in a stellarator (where the finite-β finite-plasma-current equilibrium will be similar to the vacuum field that can be calculated with certainty from the Biot-Savart law) than in a tokamak, where equilibrium does not exist without a plasma current. The stronger self-consistency requirements for the plasma state in a tokamak make numerical optimization and extrapolations to next-step facilities less reliable than in a stellarator. Thus, very large experiments (i.e. time and money) are more important for the tokamak path to fusion energy than for the stellarator path.

III. TOOLS

The third set of considerations is the set of tools, methods, ideas, and techniques that have become available recently. We separately consider advances in non-fusion-specific areas, advances in tokamaks that have bearing on stellarators, and advances specific to stellarators. In each area, there are far more developments than can be reviewed here, so we only attempt to give a few examples.

A. Developments that are not specific to stellarators or tokamaks but which may have bearing on stellarator research

- Computing power continues to increase.
- High-temperature superconducting materials have developed. For magnetic fusion, these materials would reduce the coolant power required for magnetic field coils, reduce the volume of the coils themselves, could enable operation at higher B, and make it easier to include demountable joints [7].
- Optimization, which has been central to stellarator design, has continued to develop in other engineering fields. There are many areas of advance in optimization that may be useful for stellarators.
  - Adjoint methods, which give rapid evaluation of the gradient of the objective function with no need for finite differencing, and which yield sensitivity information about the solution, have yet to be exploited for stellarator optimization and tolerance calculation.
  - Multi-objective optimization algorithms [10], which can yield solutions that are inaccessible to weighted-sum combinations of objective functions, have yet to be exploited for stellarator optimization.
  - Optimization is central to machine learning, a subject which has advanced greatly in recent years. In machine learning, an algorithm's training phase is typically an optimization problem, e.g. optimizing the weights of a neural network to match a training dataset.
  - In other engineering fields, frameworks for combining different physical models and codes into one optimization, known as 'multdisciplinary design optimization,' have reached a high level of sophistication [11]. Algorithms from the field of 'robust optimization' yield optima that are insensitive to uncertainties, implying generous tolerances [12]. Both of these sub-fields of optimization have yet to be explored for stellarators despite their apparent relevance.
- Magnetic field design plays a role in other engineering fields, and there may be ideas which can be appropriated for stellarators. As one example, the optimization of gradient field coils for magnetic resonance imaging (MRI) [13] is similar to the problem of designing stellarator coils.
- Additive manufacturing (3D printing) has advanced significantly.
- Uncertainty quantification, the understanding of sensitivities in the outputs of numerical calculations, has been a popular area of applied mathematics. Uncertainty quantification is noteworthy given the critical issue of tolerances in section I.

B. Developments in tokamak research that may have bearing on stellarators

- There has been significant research on nonaxisymmetric perturbations to tokamaks. Some examples include the study of magnetic perturbations on ELMs [14], effects of symmetry-breaking perturbations on rotation [15], and relating perturbed fields to stable MHD modes [16].
- As a particular case of the previous item, there has been significant theoretical and experimental work on neoclassical toroidal viscosity (NTV) in tokamaks, which has direct bearing on quasisymmetric stellarators.
- Nonlinear extended MHD codes, such as NIMROD and M3D-C1, have continued to develop.
- New ideas for divertors have been developed, such as the snowflake and super-X divertor.
- Transport codes that iteratively call gyrokinetic codes, namely Trinity and TGYRO, have become available for first-principles prediction of temperature and density profiles in tokamaks.

C. Developments in stellarator research

- W7-X has come on-line.
- Gyrokinetic codes have become available for stellarators.
- A number of recent theoretical and computational studies have indicated it may be possible to achieve outward neoclassical impurity transport in stellarators [17–19].
- There has been progress in understanding [2] and reducing [6, 20] the complexity of stellarator coil shapes.
- Progress has been made in computing equilibria with islands [21–23].
- Optimization of stellarator geometry for reduced turbulence has been demonstrated numerically [24, 25].
- A small number of new optimized stellarator equilibria have been found numerically, QIPC [26] and ESTELLE [27]
- Some figures of merit for fast particle confinement have been suggested [28, 29] which have not been thoroughly exploited in stellarator optimization.
- The LHD deuterium campaign is beginning, which enables study of isotope effects.

IV. INTERSECTIONS

We now list several research topics which lie in the intersection of the previous three sections. This list is not intended to be exhaustive, and is rather meant to be a starting point for discussion.

- Perhaps the clearest priority area is coil optimization. Improvements in coil optimization can directly affect the cost of a reactor (through the reactor tolerances and size), and affect the feasibility of reactor maintenance (hence the duty cycle and overall plant economics) [6]. Coils are also the area in which theory is most predictive, due to the dominant importance of the Biot-Savart law. Work on coil design can also take advantage of advances in optimization in other fields. The space of possible coil shapes to explore (including various topologies, multiple layers of coils, supercondicting tiles, etc.) is enormous, and much can be done numerically without the cost and time associated with experiments.
- It is critical to better understand the proper tolerances on coil shapes and other components, and how they can be eased. Tolerances are of utmost important because they drive cost, not only for reactors but also for any nearer-term experiments. New theoretical tools are available for understanding tolerances - uncertainty quantification and adjoint methods - which have not yet been explored for stellarators. It should be possible to develop theoretical and computational understanding of the patterns of magnetic field to which the plasma is sensitive, and to design experiments in which there is good control of these magnetic field patterns, enabling designs with generous tolerances. This understanding of plasma sensitivity could perhaps be done using the framework of the reluctance matrix [16, 30].
- The implications for stellarators of high temperature superconductors should be examined. The physics advantages of operating at $B>$5T should be assessed. Also, even if B remains at the $\sim$5T level of previous reactor studies, coils can be built with smaller cross-section and smaller radius of curvature, and the feasibility of including joints in the coils is greater.
- Optimization of plasma shapes must be continued. (This topic is closely related to coil optimization.) For a stellarator reactor to become feasible, it is critical that plasma optima be found with excellent fast-alpha confinement, compatible with distant coils and with a divertor. As with coil optimization, there is great potential for advance both because of vast parameter space that can be explored, and also due to the possibility of exploiting advances in optimization from other fields. Plasma shape optimization can also take advantage of advances in other areas of stellarator computation, such as MHD equilibria with islands, new metrics for coil complexity, microinstability and microturbulence calculations, and divertor properties.

- More work is required to improve alpha particle confinement. Alpha confinement is a potential show-stopper for the stellarator, and at least the guiding-center confinement of alphas can be predicted with high confidence. Developments in optimization generally and in figures of merit for fast-particle confinement (section III C) can be exploited. Increases in computing power also make it more feasible then ever before to directly simulate guiding-center motion of alpha particles for long times within an optimization iteration.

- Coordinated activity should be taken to evaluate the different approaches to computing 3D equilibria with islands. As already noted, equilibrium has great importance since almost all other calculations depend on it. Many codes have been developed to compute equilibria with islands, but their formulations are quite different, and it is not clear that they give the same answers. Some work has been done to compare codes [31], but not all of the presently available codes were included, and much more can be done. A systematic comparison of the existing codes should include a comparison of their numerical properties, such as whether and how rapidly the output converges with numerical resolution. The availability of nonlinear extended MHD codes (NIMROD, M3D-C1, etc) means it is now possible to examine the approach to the ideal static MHD limit from a more general non-ideal time-dependent model, to evaluate whether this limit coincides with the results of the various static ideal codes.

- It would be very valuable to accelerate the calculation of the bootstrap current with finite-collisionality corrections. Previous stellarator optimization has use crude current models, either ad-hoc (e.g. taking a tokamak current profile for the NCSX optimization), or using the low-collisionality asymptote which is inaccurate and full of resonances that must be smoothed out in an ad-hoc manner. The impact this work would be high, because the bootstrap current affects the equilibrium, which in turn affects all other properties of the plasma. The main limitation to predictive power is the effect of turbulence on the pressure profile, but for a given pressure, we can be confident in the predicted current. New tools for accurate calculation of the stellarator bootstrap current have become available, including recent analytic work [32] and codes [33, 34]; the former is limited by the low-collisionality approximation and the latter is limited by computational cost. A fast and accurate bootstrap current module has yet to be integrated into optimization frameworks such as STELLOPT.

- Two of the critical issues in section I, ash extraction and impurity control, could potentially be addressed by *increasing* the turbulent particle transport for a given energy transport. Perhaps this could be done by controlling the relative phase between the fluctuating density and potential, which depends on trapped particles and therefore on the magnetic geometry. A relatively large fraction of the stellarator community is presently working on microinstabilities and turbulence in stellarators, taking advantage of the new gyrokinetic tools, but little interest has been paid to ash extraction and impurity control. In light of the importance of this topic, there will be a special session at the 2017 International Stellarator and Heliotron Workshop on decoupling energy and particle transport.

It is clear from this list that many opportunities exist for high-impact theory and computation, work that could significantly improve the viability of the stellarator concept for fusion energy.

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