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10/6/2007

Plasma Turbulence:- Lecture #1.

(i) Only the kind in FUSION experiments, particularly ITER.

(ii) I will (in 3 lectures) cover: a) Phenomenology in experiments.
b) Key Instability. "I.T.G."
c) Important nonlinear effects.

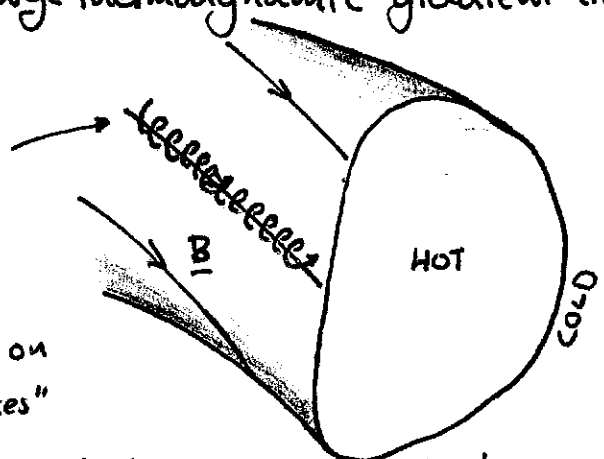
(iii) FUSION PRIMER

$$D + T \rightarrow He^4 + n$$

deuterium, tritium, "alpha" 3.5 MeV, 14 MeV.

Needs $T_{PLASMA} \sim 10 - 20$ keV (Temperature)

(iv) Large thermodynamic gradient in experiment:



∇T = SOURCE OF "FREE ENERGY" FOR INSTABILITIES.

CONTAINED BY $\underline{B}$ FIELD

$\underset{\sim}{B} \sim 5$ Tesla.

- Particles – ions; deuterium and tritium. Larmor (spiral) radius. $\rho_i \simeq 1$ mm
  electrons; Larmor radius. $\rho_e \simeq 1/60$ mm.

(V) TIMESCALES. Energy confinement time $= \tau_E$
(time for energy to leak out –)

$$\boxed{\frac{nT}{\tau_E} = \frac{\text{Heating}}{\text{Volume}}}$$

n = density of particles.

Heating = fusion heating ($\sim n^2T^2$) + external heating.

ITER needs $\tau_E \sim 4$s.

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(vi) • TIME FOR ION TO GO AROUND MACHINE: $\sim L/V_{thi} \sim 3\times10^{-5}\,s$ $\quad L \sim 30\,m$, $V_{thi} \sim 10^{6}\,m s^{-1}$

• TIME FOR ION TO COLLIDE: $\tau_{collision} \sim 1\,ms$, $\lambda_{mean\ free\ path} \sim 1\,km$!

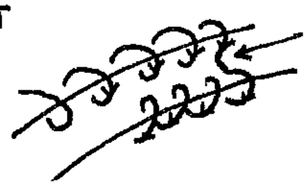
SINCE $\tau_E \gg \tau_{collision}$ TIME TO ESTABLISH LOCAL THERMODYNAMIC EQUILIBRIUM ON EACH SURFACE.

DISTRIBUTION OF PARTICLES $= f \sim F_{MAXWELLIAN}$

(vii) TRANSPORT – the loss of heat & particles across $\underline{B}$.

• CLASSICAL TRANSPORT

Each collision ion jumps ρ_i across the field.



Diffusion Coefficient $\sim \dfrac{\rho_i^2}{\tau_{coll}} \sim 10^{-3}\,m^2 s^{-1}$

TOO SLOW TO EXPLAIN THE OBSERVED LOSS.

• TURBULENT TRANSPORT: Small scale density fluctuations seen in plasma

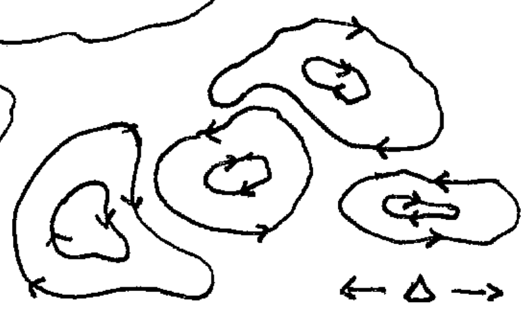
Presumed $\underline{E}$ field fluctuations.

$\uparrow \underline{E}$, $\underline{B}$

PERPENDICULAR VELOCITY OF PARTICLES IN $\underline{E}$ & $\underline{B}$ FIELD $\sim \dfrac{\underline{E}\times\underline{B}}{B^2} = V_{E\times B}$

Eddies

$\cdot\,\underline{B}$



Electrostatic $\underline{E}$ mostly

$\underline{E} = -\nabla\phi$

$\underline{V}_{E\times B} = \dfrac{\underline{b}\times\nabla\phi}{B}$ ($\underline{b}$: unit vector)

$\phi \equiv$ Electrostatic Potential & STREAM FUNCTION OF PERPENDICULAR FLOW

(viii) MIXING LENGTH ESTIMATE We might assume that particles make a random walk of step size $\Delta \sim$ EDDY SIZE and correlation time τ_{corr}.

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(ix) TURBULENT DIFFUSIVITY $\sim \chi \sim \dfrac{\Delta^2}{\tau_{corr.}}$

$\tau_{corr} \sim 3\times10^{-5}s$ (OBSERVED)

$\Delta \gtrsim 1cm$ (OBSERVED)

$\Rightarrow \boxed{\chi \gtrsim \frac{1}{3} m^2 s^{-1}}$ $\qquad \chi_{OBSERVED} \sim 1$

//———//

(X) This is also consistent with the observed density fluctuations:

$\delta n \sim \xi \cdot \nabla n_0$ $\qquad \xi \simeq$ PLASMA DISPLACEMENT $\sim \Delta$?

$\dfrac{\delta n}{n_0} \sim \dfrac{\Delta}{\ell_n} \sim (0.5-1)\%$ $\qquad \ell_n = \dfrac{n_0}{|\nabla n_0|} \simeq (1-2)m.$

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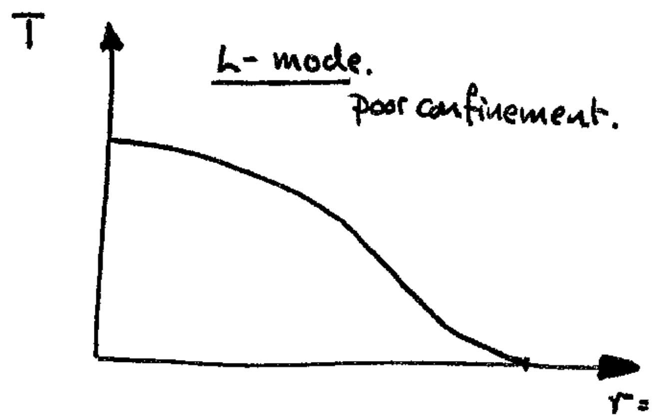
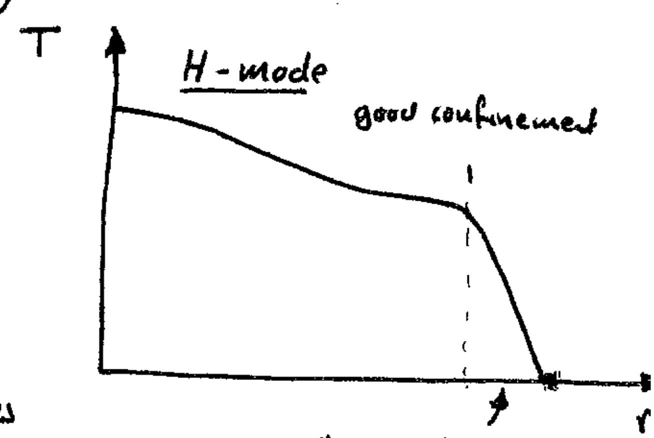
(xi) This is all very encouraging - but hardly a theory.

Experiments show us something else: TURBULENCE FREE REGIONS



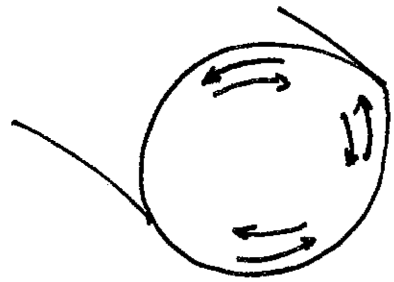


SOMETIMES - when you heat the plasma a lot the edge flips into a a state of high confinement - low turbulence.

(xii) Turbulence free region has a very interesting phenomenum:

$\boxed{\text{SPONTANEOUS SHEAR FLOWS.}}$

(xiii) "ZONAL FLOWS"



Shear flow regions seem to suppress the turbulence.

∇T_i INSTABILITY $\Rightarrow$ TURBULENCE $\Rightarrow$ SECONDARY INSTABILITY $\Rightarrow$ SHEAR FLOW $\Rightarrow$ SUPPRESSES TURBULENCE.

- It is as though the turbulence is self-healing.

(xiv) What do we need to know?

a) What drives the turbulence? How is the plasma unstable?

b) What sets the eddy sizes?

c) How does the turbulence dissipate?

d) What drives the shear flows

e) How do the shear flows saturate?

(xv) I cannot do this in full detail. Because $\tau_{\text{collision}} \gg \tau_{\text{corr}}$ this is fully kinetic/collisionless turbulence. I will make brutal approximations to illuminate these questions.

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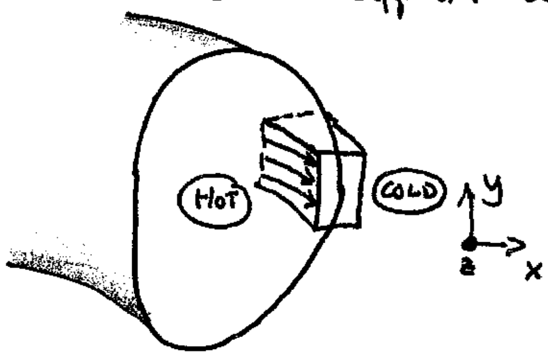
Plasma Turbulence: Lecture #2. Ion Temperature Gradient Instability.

(i) Here we examine the most important instability for fusion turbulence the <u>I</u>on <u>T</u>emperature <u>G</u>radient Mode.

(ii) We extract a "slab" approximation of the plasma,



TAKE $\underline{B} = B_0 \hat{\underline{z}}$ ← CONSTANT.

$$T = T_0 + x\frac{dT}{dx}$$

$$n = n_0 \quad \text{(CONSTANT)}$$

$$\underline{V}_0 = 0$$

(iii) We perturb this situation with an electrostatic field:

$$\delta \underline{E} = -\nabla\phi$$

of the form $\phi = \phi_0(x)\, e^{iky + ik_\parallel z - i\omega t}$

where we look for solutions with $\omega = \omega_R + i\gamma$ ← growth ra

(iv) We also look for solutions with: $\omega \sim k_\parallel V_{thi}$ ← ion thermal velocity $= \sqrt{2}$

This means that electrons move much faster than the instabil

Note $V_{the} \equiv$ Electron Thermal velocity $= \sqrt{\frac{m_i}{m_e}}\, V_{thi} \sim 60\, V_{thi}$ (in deuteriu

Thus key to this instability is the different response of ions and electrons.

(v) Electron Response Two key simplifications:

- Fast moving electrons equalize perturbed electron temperature along $\underset{\sim}{B}$ $\Rightarrow$ $\boxed{\underset{\sim}{B}\cdot\nabla\delta T_e = 0}$

- Electron Inertia is small $\therefore$ FORCE ON ELECTRONS ALONG $\underline{B}$ $= -\nabla_{\parallel}\delta p_e + en_0\nabla_{\parallel}\phi \sim 0$ "$E_{\parallel}$"

$$\Rightarrow \boxed{\frac{\delta n_e}{n_0} = \frac{e\phi}{T_e}}$$ "Boltzmann Response."

(vi) Ion Response We should really use kinetic equations for the ions but for simplicity we use fluid equations.

ION PERPENDICULAR MOTION $= \frac{\underline{E}\times\underline{B}}{B^2} = \frac{\hat{\underline{z}}\times\nabla\phi}{B_0} = \underline{V}_{\perp}$

note $\nabla\cdot\underline{V}_{\perp} = 0$

ION DENSITY RESPONSE
$$\frac{\partial\delta n_i}{\partial t} + \nabla\cdot n_0\underline{V} = 0 \Rightarrow \boxed{\frac{\partial\delta n_i}{\partial t} + n_0\frac{\partial V_{\parallel}}{\partial z} = 0}$$

ION PRESSURE RESPONSE (no heat flow)
$$\frac{\partial\delta p_i}{\partial t} + \underline{V}_{\perp}\cdot\nabla p_0 + \gamma p_0\nabla\cdot\underline{V} = 0$$
γ = ratio of specific heat
$$\Rightarrow \boxed{\frac{\partial\delta p_i}{\partial t} + \underline{V}_{\perp}\cdot\nabla p_0 + \gamma p_0\frac{\partial V_{\parallel}}{\partial z} = 0}$$

ION PARALLEL MOMENTUM RESPONSE
$$\boxed{m_0 n_0\left(\frac{\partial V_{\parallel}}{\partial t}\right) = -\nabla_{\parallel}\delta p_i - en_0\nabla_{\parallel}\phi}$$

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(vii) Quasi-neutrality. To a good approximation the plasma must remain neutral - ϕ adjusts to make it neutral hence.

$$\boxed{e\delta n_e = \frac{e^2\phi}{T_e} n_0 = e\delta n_i}$$

This links ion and electron responses.

(viii) Local Response If $k \gg \frac{1}{T}\frac{dT}{dx}$ we can set $\phi_0 = \bar{\phi}$

DEFINE OMEGA STAR DIAMAGNETIC DRIFT FREQUENCY

$$\boxed{\omega_T^* = \frac{k}{eB}\frac{dT}{dx}} \sim (k\rho_i)\frac{V_{thi}}{l_T}$$

(ix) After a little algebra we obtain the DISPERSION RELATION.

$$\boxed{\omega^2 = k_\parallel^2 V_{thi}^2 \left\{ \left(1 - \frac{\omega_T^*}{\omega}\right)\frac{T_e}{T_i} + \gamma \right\}}$$

STABLE (3 Real Roots) WHEN: $\frac{\omega_T^*}{k_\parallel V_{thi}} < \frac{(1+\gamma)^{2/3}}{(1/2)^{2/3} + (\frac{1}{2})^{1/3}}$

UNSTABLE (1 Real Root, 1 unstable, 1 stable): $\frac{\omega_T^*}{k_\parallel V_{thi}} >$ ..

(X) VERY STABLE LIMIT: $\omega^2 \doteq k_\parallel^2 C_s^2$ $C_s = V_{thi}\sqrt{1+\gamma}$

SOUND WAVES. SOUND SPEED

(Xi) VERY UNSTABLE: $\omega^3 = -\omega_T^* k_\parallel^2 V_{thi}^2$

UNSTABLE ROOT $\equiv$ $\omega = e^{i\pi/3}\left(\omega_T^* k_\parallel^2 V_{thi}^2 \frac{T_e}{T_i}\right)^{1/3} = e^{i\pi/3}\omega_0$

STEVE

SIMPLIFICATION IN UNSTABLE LIMIT:

$$\left.\begin{aligned} &\frac{\partial \delta P_i}{\partial t} + \underline{v}_\perp \cdot \nabla p_0 = 0 \\ &m_i n_0 \frac{\partial V_{\parallel}}{\partial t} = -\frac{\partial \delta P}{\partial z} \\ &\frac{\partial \delta n}{\partial t} = -\frac{\partial V_{\parallel}}{\partial z} \end{aligned}\right\}$$

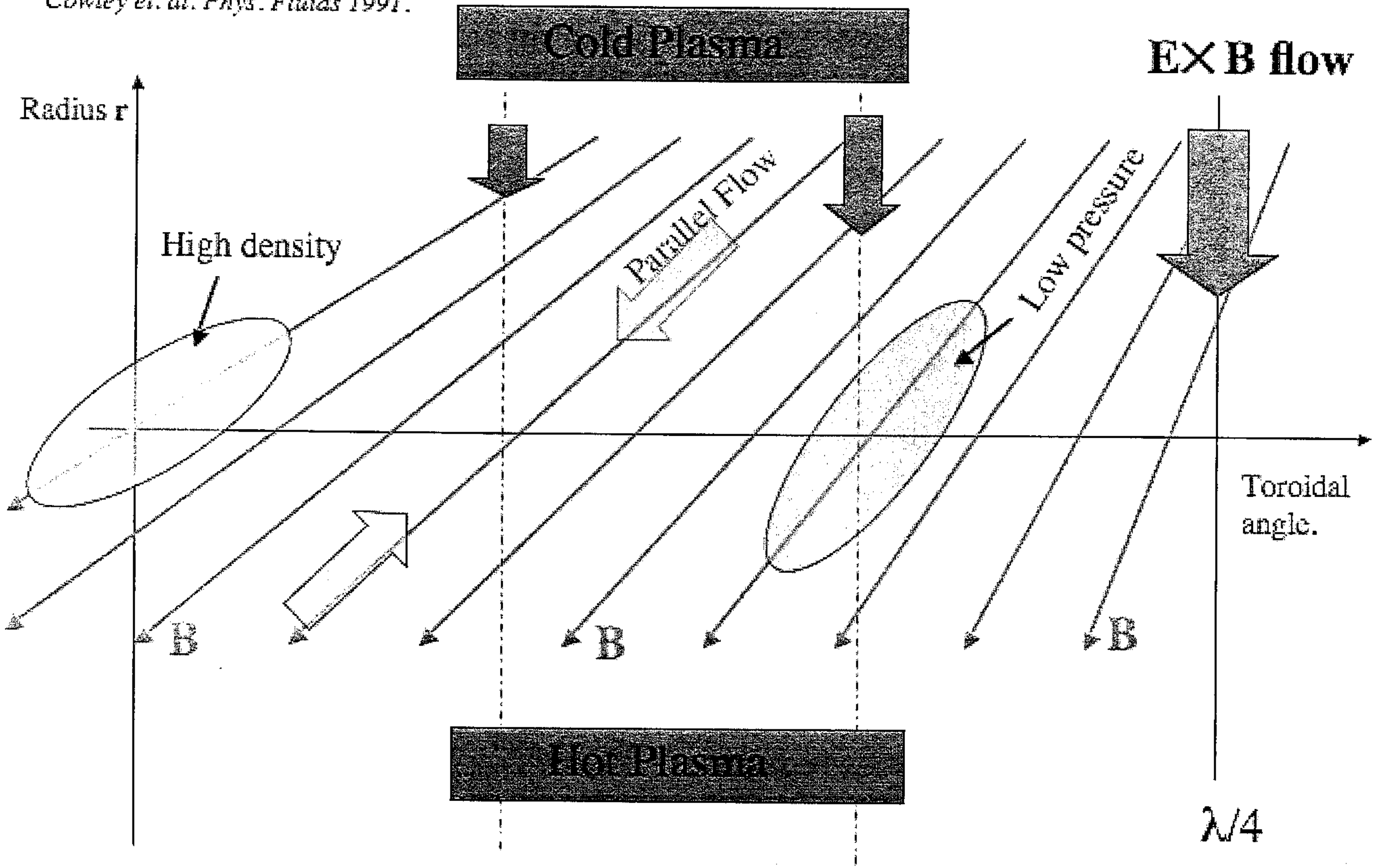
$$\delta n = \frac{e\phi}{T_e}$$

$$\delta P_i = -\frac{\omega_T^*}{\omega_0}\frac{T_e}{T_i}\,\delta\hat{n}\, e^{\gamma t} \cos(ky - \omega_r t \; _{\vec{3}})$$

$$V_{\parallel} = \frac{\omega_0}{k_{\parallel}}\delta\hat{n}\, e^{\gamma t} \cos(ky - \omega_r t - \pi/6) \cos$$

Cartoon of "I.T.G." Instability.

Cowley et. al. Phys. Fluids 1991.



Plasma Turbulence: Lecture #3. ITG Turbulence.

i) Review: ITG instability: (see CARTOON).

sort of sound wave with negative compressibilitl when density goes up pressure goes down. This sucks in more plasma increasing density etc.

(ii) To be unstable: $k_{\parallel} V_{thi} < \omega_T^* \cdot$ constant.

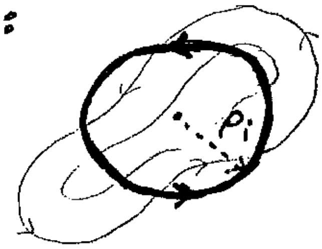
$$\frac{1}{L_T} = \frac{1}{T_i}\frac{dT_i}{dR}$$

BUT $\omega_T^* = (k\rho_i)\frac{V_{thi}}{L_T}$

UNSTABLE: $\boxed{(k_{\parallel}L_T) < (k\rho_i)}$

ρ_i = LARMOR RADIUS $\sim 1mm$
L_T = 1m in ITER.

(iii) FINITE LARMOR RADIUS:



: when ion larmor radius is bigger than "eddy" it averages $\langle \underline{E} \times \underline{B} \rangle$ – to a small effect.
∴ INSTABILITY MUST HAVE $k\rho_i <$

$$\Rightarrow \boxed{k_{\parallel}L_T < k\rho_i < 1}$$

– VERY LONG PARALLEL WAVELENGTH
– VERY SHORT PERPENDICULAR SCALE.

(iv) We have one more constraint to fit into tokamak. Instability cannot stretch all the way around to the "inside" of the surfaces so

$$k_{\parallel} > \frac{1}{L_{CONNECT}}$$

$L_{CONNECT}$ = Distance from outside to inside of surfaces along $\underline{B}$ lines.

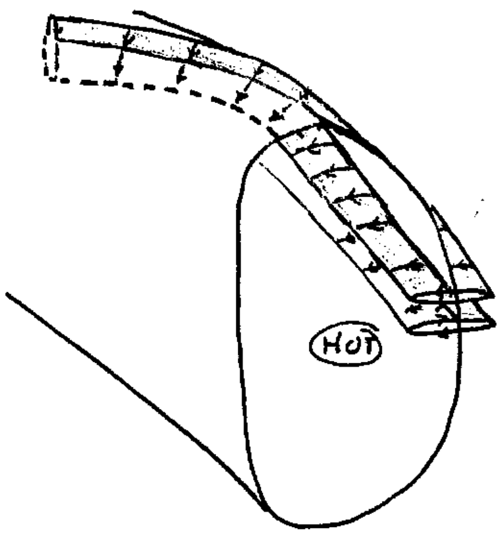
All this handwaving is done precisely by stability codes.

FINALLY

$$\boxed{\left(\frac{L_T}{L_{CONNECT}}\right) < k\rho_i < 1}$$

$$\frac{L_T}{L_{CONNECT}} \lesssim 0.1$$

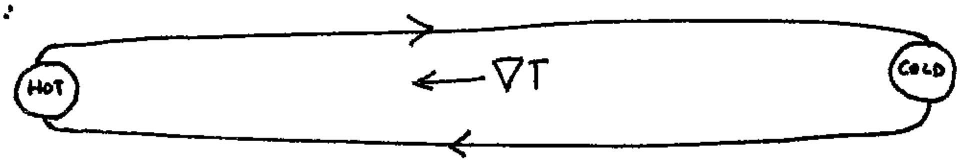
(v)



- TYPICAL EDDY SHAPE FROM EIGENMODE CALCULATION. (LINEAR)
- CONSTANT ALONG FIELD LINES. ($\lambda_{\parallel} \sim L_{conne}$
- ELONGATED IN RADIAL DIRECTION.

(vi) BUT: These eddies are not stable

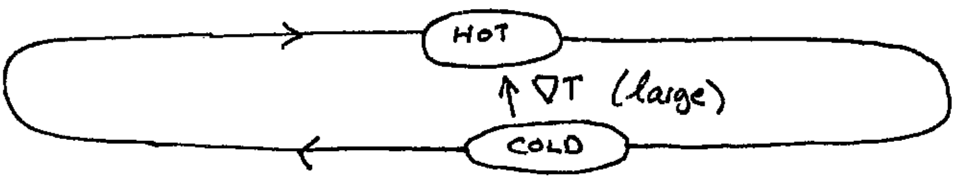
INITIALLY:



$\uparrow \underline{v}_{\perp} = \dfrac{\underline{E} \times \underline{B}}{B^2}$

NOT SHOWN ARE THE ASSOCIATED Parallel velocitie.

LATER:



(vii) Eddies make large gradients in:

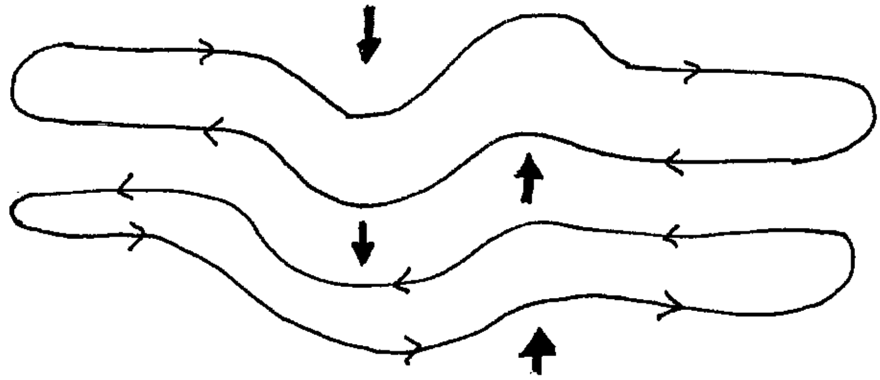
$\nabla \underline{V}_{\perp}$: $\underline{V}_{\perp}$ $\Rightarrow$ DRIVES SECONDARY KELVIN HELMHOLTZ INSTABILITIES.

∇T : T $\Rightarrow$ DRIVES SECONDARY ITG MODES.

$\nabla V_{\parallel}$: $V_{\parallel}$ $\Rightarrow$ DRIVES SECONDARY PARALLEL VELOCITY INSTABILITIES. (KIND OF KELVIN HELMHOLTZ TOO)

- DORLAND THESIS (199
- DORLAND & JENKO
- Lin ~ (2000)
- Like HM. Nazarenko 1990.

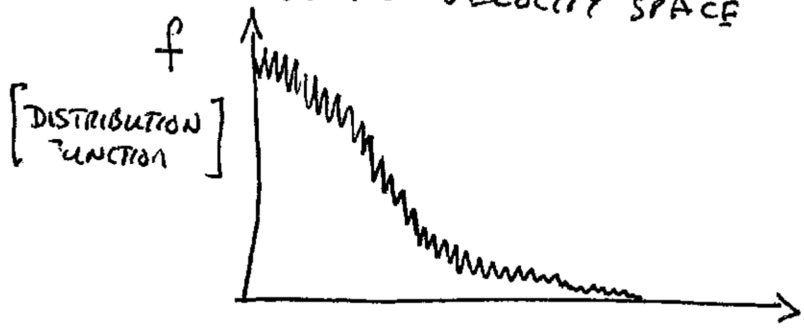
(viii) Often the most unstable secondary mode is the "zonal flow" $\left(\frac{\partial}{\partial y} = 0\right)$ shear flow.



(SEE MOVIES)

(ix) <u>ALL</u> this happens on times short compared to a collision time.

MIXES MAXWELLIANS OF DIFFERENT TEMPERATURES. CREATES SMALL SCALES IN VELOCITY SPACE



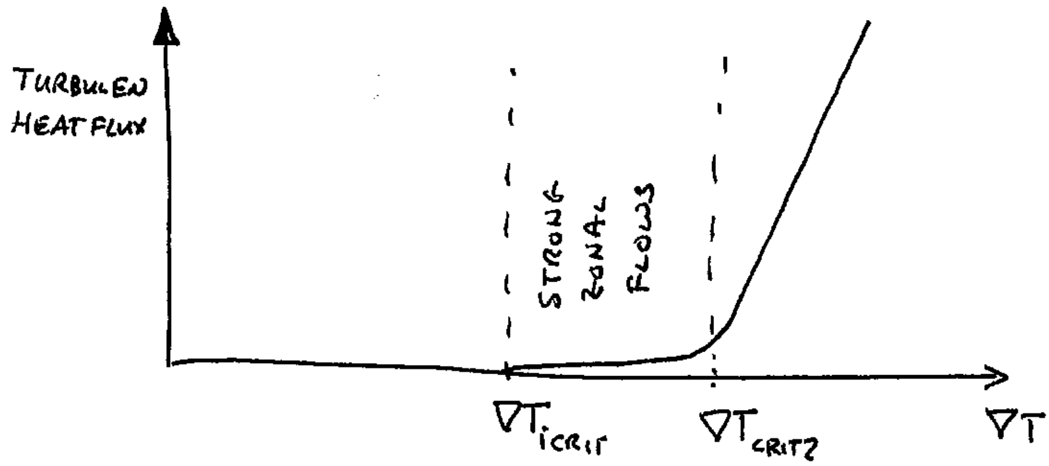
"ENTROPY CASCADE IN VELOCITY SPACE"

- COLLISIONS RAPIDLY ERASE VELOCITY SPACE WIGGLES.

$$\text{ENTROPY PRODUCTION} = \int d^3v \ln f \, C(f)$$

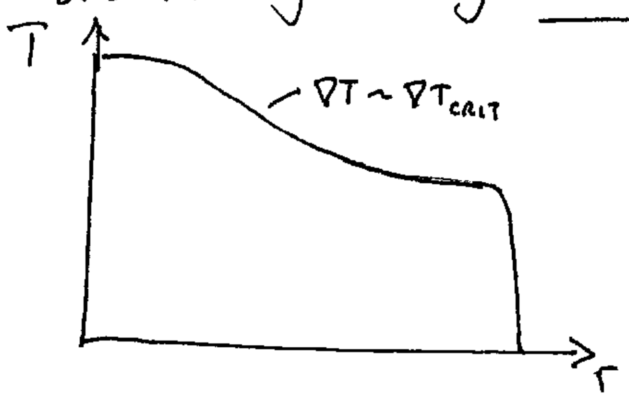
$C \equiv$ COLLISION OPERATOR

(x) ITG has a linear threshold at $\nabla T_i = \nabla T_{i\,crit}$.



When $\nabla T > \nabla T_{crit}$ LINEAR INSTABILITY. BUT WHEN $\nabla T < \nabla T_{crit2}$ WE GET STRONG ZONAL FLOWS THAT SUPPRESS TURBULENT HEAT FLUX SO EFFECTIVELY THRESHOLD IS SHIFTED TO ∇T_{crit2} (Dimits shift)

(xi) Usually we can't get much above ∇T_{crit2} because turbulence becomes <u>too</u> strong. Can roughly model situation by setting $\underline{\nabla T = \nabla T_{crit2}}$.



(xii) CAN WE SWITCH OFF TURBULENCE? IN ITER?

It's a multi-billion dollar question!