# Plasma Physics 222: Homework 1.

*Lecturer: Steve Cowley*

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## Question 1. Plasma Parameters.

This question investigates the regime of several common plasmas. In the following plasmas calculate a typical value of: **a)** The Debye length, **b)** The plasma parameter, **c)** The larmor radius, **d)** The mean free path.

(i) **A Tokamak Fusion Plasma, JET**. With approximate plasma parameters: electron density $n_e \sim 10^{20} m^{-3}$, plasma temperature $T \sim 10keV$ and magnetic field $B \sim 3\, Tesla$.

(ii) **The center of the sun**. With approximate plasma parameters: electron density $n_e \sim 10^{32} m^{-3}$, plasma temperature $T \sim 1.5keV$ and magnetic field $B \sim 0.3\, Tesla$.

(iii) **Laser Plasma Accelerator**. With approximate plasma parameters: electron density $n_e \sim 10^{23} m^{-3}$, plasma temperature $T \sim 1keV$ and magnetic field $B = 0$.

## Question 2. Resistivity and Runaway Electrons.

In this question we *estimate* the resistivity of a plasma and look at the phenomenon of runaway electrons. The arguments are not precise so don't try to be too accurate.

(i) Show that for an electron with velocity $v$ the cross section for collisions with ions (that are almost stationary) is roughly:

$$\sigma \sim A \frac{e^2}{m^2 v^4 \epsilon_0^2}, \tag{1}$$

where $A$ is a constant of order unity.

(ii) Show that the typical time between collisions with ions is roughly

$$\tau_c \sim \frac{1}{n\sigma v}. \tag{2}$$

We assume that at each collision the electron velocity direction is randomized – i.e. the mean velocity of the electron is zero after a collision. Since the electron is very light its speed is not changed by collisions with ions.

(iii) In an constant electric field the electron is accelerated between collsisions. *In most situations the electric field is weak so that* $\mathbf{V} \ll \mathbf{v}$ *and thus the acceleration does not affect the velocity appearing in the cross section appreciably.* Show that the average electron velocity is then roughly:

$$\mathbf{V} \sim -\frac{e}{2m} \mathbf{E} \tau_c \tag{3}$$

Find $\mathbf{V}$ for the average electron, *i.e.* one moving with speed $v_{the} \sim \sqrt{kT/m}$

(iv) The current density is given by $\mathbf{J} = -ne\mathbf{V}$. Find the resistivity $\eta$ of the plasma where:

$$\mathbf{E} = \eta \mathbf{J}. \tag{4}$$

(v) Suppose the Electric field is stronger so that $|\mathbf{V}| > v$. Find the critical field for which $|\mathbf{V}| = v$. What happens to an electron for which the field exceeds this field *Hint. consider the change in the speed $v$ after one collision and one acceleration.* Show that for such electrons their speed increases indefinitely – we call these electrons runaway electrons.

(vi) The Dreicer field, $E_D$, is defined to be the field at which the thermal electrons begin to run away. Give an expression for $E_D$. For $|\mathbf{E}| \ll E_D$ estimate the number of runaways (*Hint, you may assume the distribution of electrons is essentially a maxwellian*).

# Plasma Physics, 222: Homework 2.

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## Question 1. Sheared Magnetic Field.

To model more complicated plasmas simple configurations have been studied extensively. One such configuration is the *sheared slab* here we look at this simple model as an example of the things we discussed in class.

(i) The *sheared slab* field is:

$$\mathbf{B} = B_0(\mathbf{z} + \frac{x}{l_s}\mathbf{y}), \tag{1}$$

where $\mathbf{z}$ and $\mathbf{y}$ are unit vectors in the $z$ and $y$ direction respectively and $B_0$ is a constant. Show that $\nabla \cdot \mathbf{B} = 0$ and calculate the current density. Are the field lines curved?

(ii) Draw the field lines – do your best this is hard. You might try drawing the field lines on different x planes separately.

(iii) Find Clebsch potentials $\alpha$ and $\beta$ so that

$$\mathbf{B} = \nabla\alpha \times \nabla\beta. \tag{2}$$

*Hint. Start by finding a direction that is always perpendicular to the field, this will determine one of the potentials – say $\beta$.*

(iv) A flux tube has a circular cross section $x^2 + y^2 = 1$ in the plane $z = 0$. What shape is the cross section in the plane $z = l_s$. Draw the tube.

# Plasma Physics, 222: Homework 3.

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## Question 1. Relativistic Motion in Fields.

In astrophysical and in laser plasma interaction we encounter relativistic plasmas. One hint about approaching this subject is to try and stick in one frame – do not keep making Lorentz transformations it only leads to confusion

The relativistic equation of motion is:

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{1}$$

where $\mathbf{p} = m\gamma\mathbf{v}$ and $\gamma = (1 - v^2/c^2)^{-1/2}$.

(ii) Calculate the motion of a charged particle in a constant magnetic field $\mathbf{B} = B\mathbf{b}$ with $\mathbf{E} = 0$. *Hint, follow the same steps we took for non-relativistic motion.*

(iii) **Harder and optional.** Solve for the relativistic motion of a a charged particle that starts from rest in a constant electric field that is perpendicular to a constant magnetic field – i.e. $\mathbf{B} \cdot \mathbf{E} = 0$.

# Plasma Physics, 222: Homework 4.

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## Question 1. Magnetic Mirrors and the earth.

The earth's magnetic field is taken in this problem to be a dipole.

(i) Consider a field line that at the equator has a radius 5 earth radii. What is the magnetic field strength on this field line at the equator and at the point it hits the earth. You will need to look up the field in a book or in the plasma formulary.

(ii) Suppose the particles at the equator have an isotropic distribution i.e. they are equally likely to have a velocity at any angle to the field line. Calculate the fraction of these particles that hit the earth.

(iii) Near the equator the magnetic field strength can be written as approximately

$$B \sim B_0(1 + c_1(\Delta\theta)^2) \tag{1}$$

where $c_1$ is a constant and $\Delta\theta = \theta - \pi/2$. Find an expression for $c_1$. Solve for the motion of a trapped particle whose bounce points are at small $\Delta\theta$.

# Plasma Physics, 222: Homework 5.

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## Question 1. Ring Current.

Again the earth's magnetic field is taken in this problem to be a dipole.

(i) The electrons and ions in the magnetosphere are drifting around the earth. Estimate the drift velocity for ions and electrons from the guiding center equations. Find the rough velocity for $1Mev$ protons and $100keV$ electrons. How long does it take one of these drift around the earth at $R = 4R_{earth}$.

(ii) Estimate current in these particles assuming they orbit in a band between $R = 3R_{earth}$ and $R = 4R_{earth}$ and have a density of $n \sim 10^7 m^{-3}$.

(iii) Estimate the change in the field on earth due to this cutrrent

# Plasma Physics, 222: Homework 7.

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## Question 1. Maxwellians and Integrals.

In thermal equilibrium the distribution function is:

$$f(\mathbf{v}) = n(\frac{m}{2\pi kT})^{3/2} \exp{(\frac{m(\mathbf{v}-\mathbf{V})^2}{2kT})} \tag{1}$$

(i) From this distribution calculate the mean kinetic energy of the particles.

(ii) Calculate the current density if these particles have charge $q$.

(iii) Calculate the approximate fraction of deuterium ions in a deuterium plasma with temperature $20kev$ with energy greater than $100kev$. (Roughly the temperature needed to cause deuterium-deuterium fusion.)