Reduced transport regions in rotating tokamak plasmas

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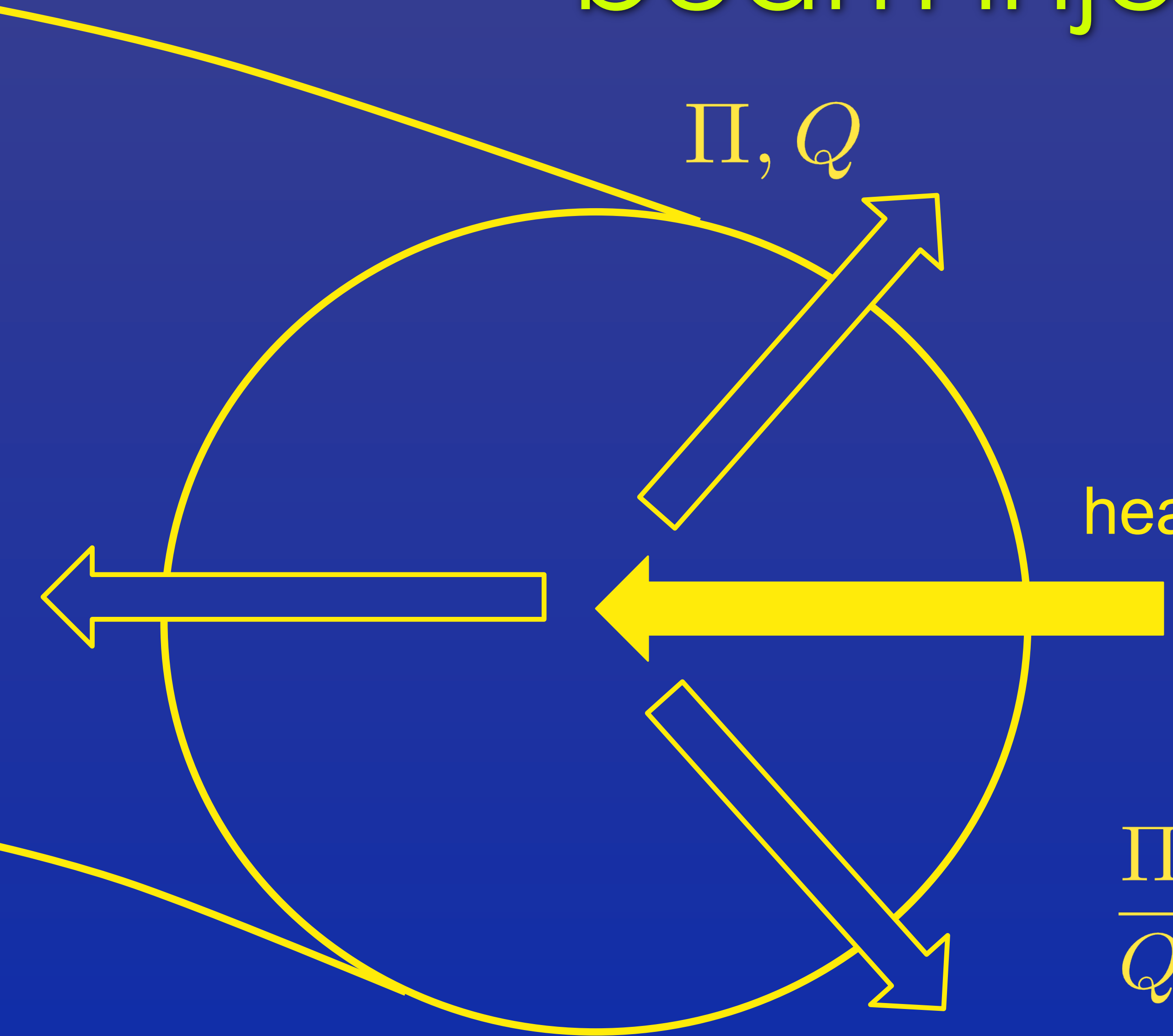
Power/Torque balance for beam injection



$$\frac{\Pi}{Q} \sim \frac{n_B R_B m_B v_B^2}{n_B m_B v_B^3} = \frac{R_B}{v_B}$$

Model fluxes

- Simple model for fluxes with parameters chosen to fit zero magnetic shear results from GS2:

$$Q = Q_t + Q_n$$

$$\Pi = \Pi_t + \Pi_n$$

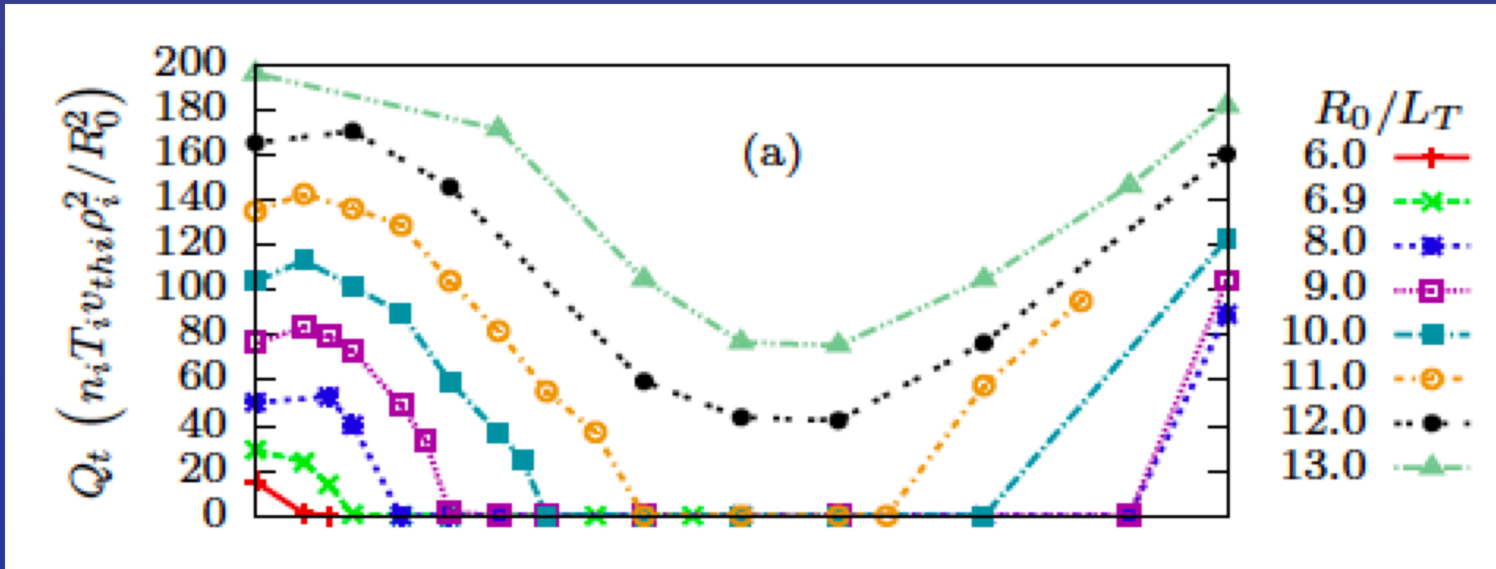
$$\overline{Q}_t \equiv \frac{Q_t}{nTv_{th}} \left(\frac{R}{\rho}\right)^2 \equiv \chi_t \left[\frac{R}{L_T} - \left(\frac{R}{L_T}\right)_c\right]$$

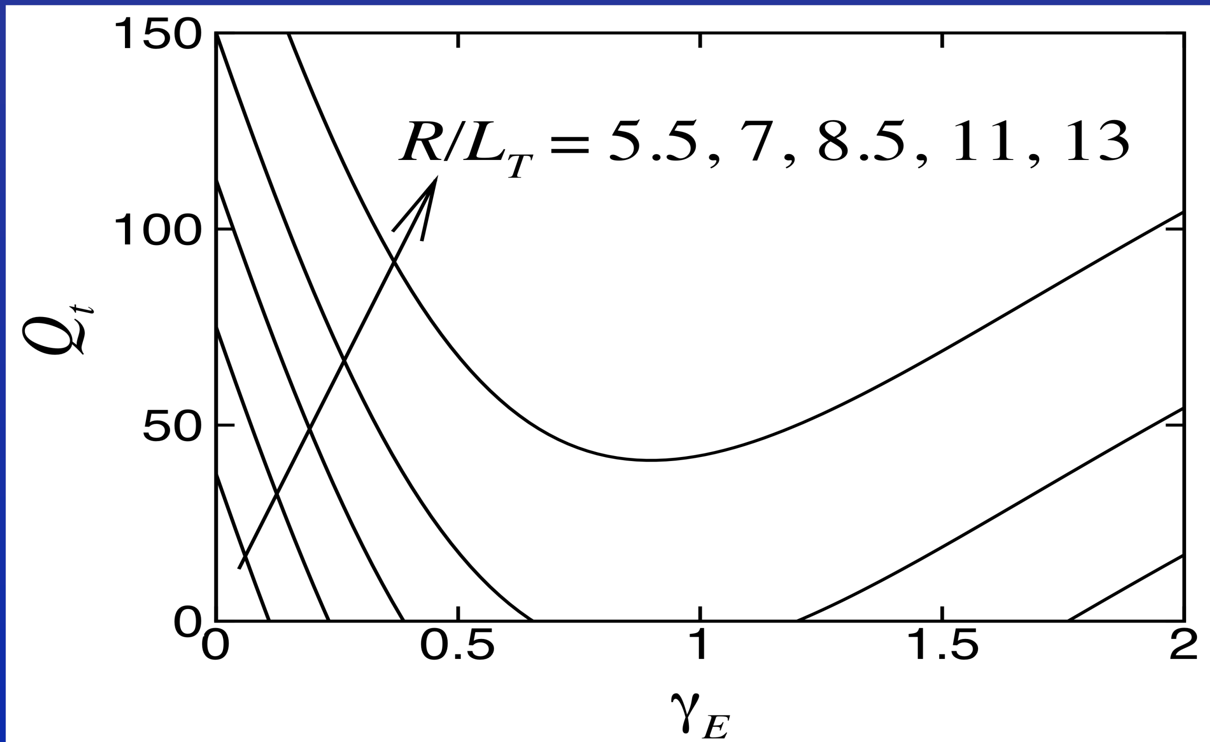
$$\overline{Q}_n \equiv \frac{Q_n}{nTv_{th}} \left(\frac{R}{\rho}\right)^2 \equiv \frac{\chi_n}{T^2} \frac{R}{L_T}$$

$$\left(\frac{R}{L_T}\right)_c \equiv \frac{\alpha_1 \gamma_E + (R/L_T)_{c0}}{1 + \alpha_2 \gamma_E^2}$$

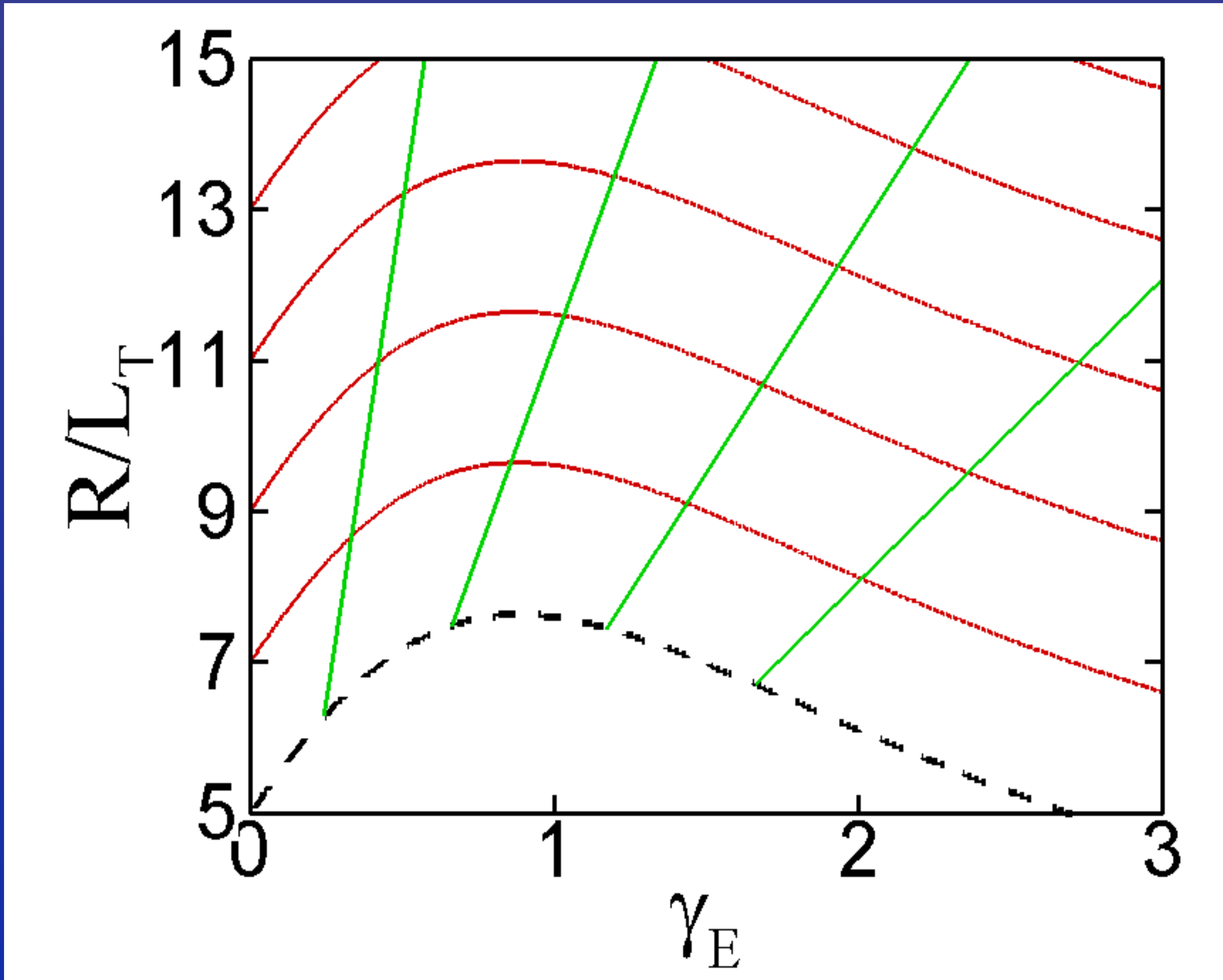
$$\overline{\Pi}_{t,n} \equiv \frac{\Pi_{t,n}}{mnRv_{th}^2} \left(\frac{R}{\rho}\right)^2 = \overline{Q}_{t,n} \mathrm{Pr}_{t,n} \frac{\gamma_E}{R/L_T}$$

Model fluxes





Balance w/o neoclassical

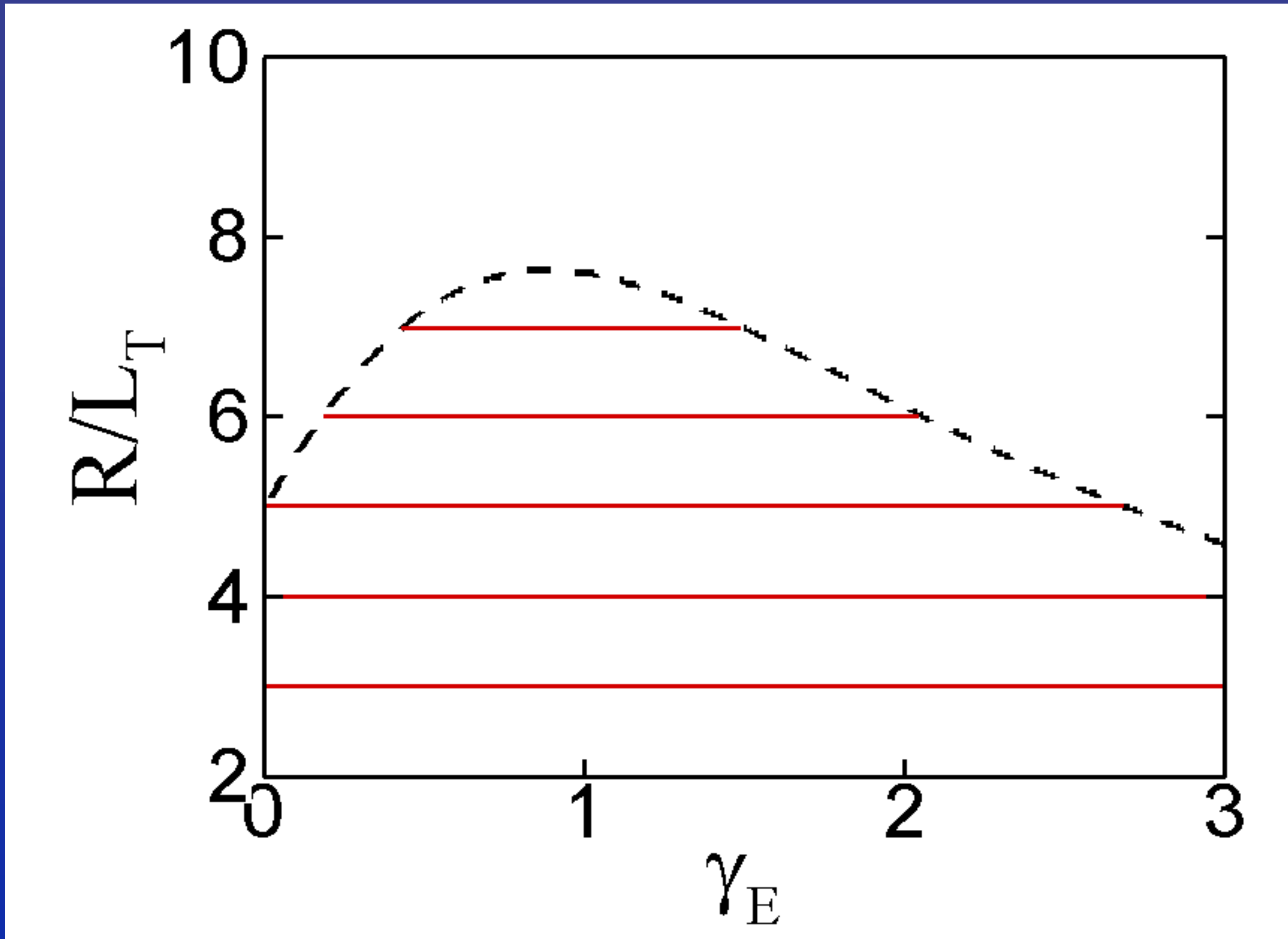


- $\overline{Q}$ = red lines
- $\overline{\Pi}/\overline{Q}$ = green lines

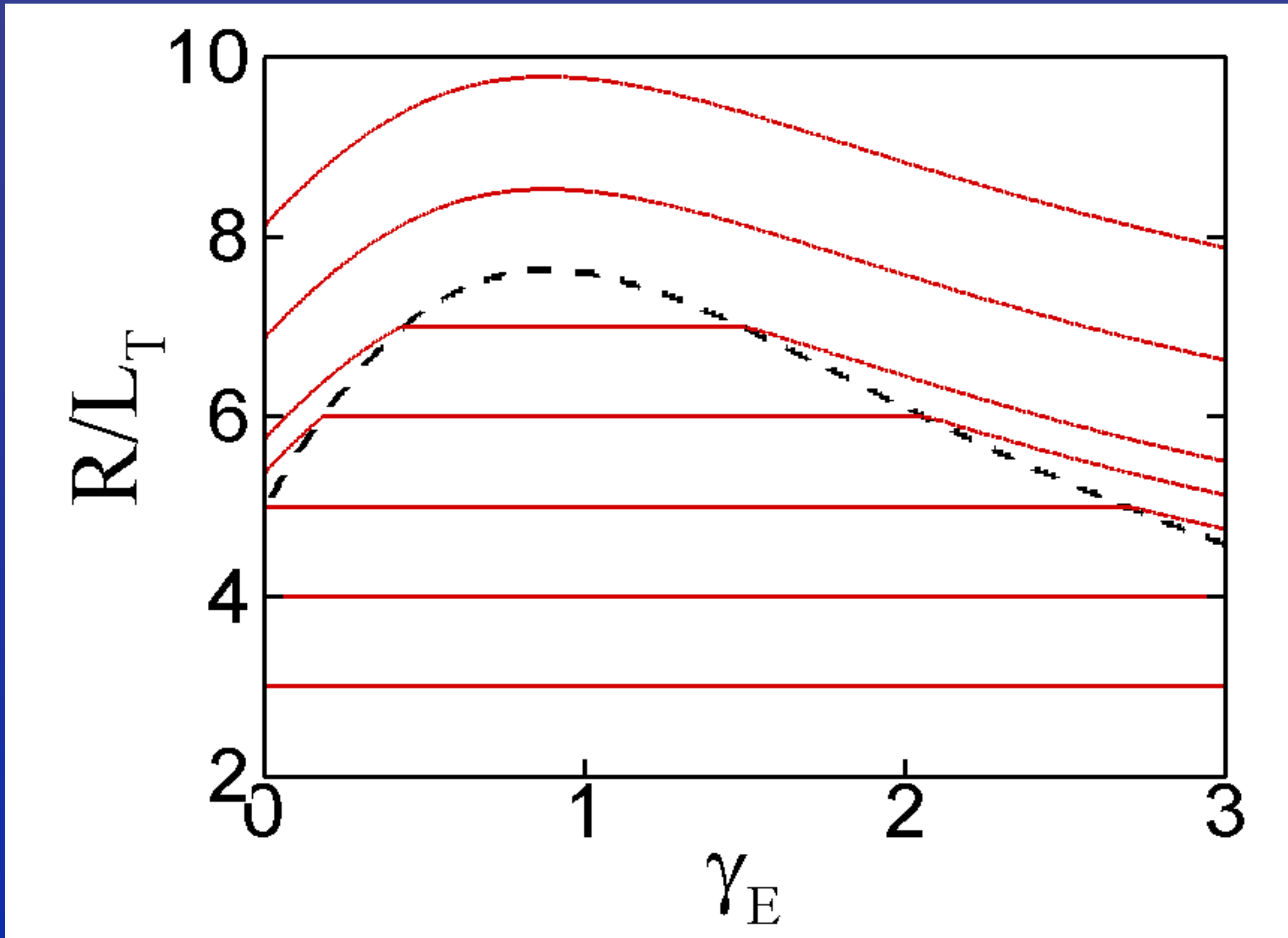
$$\frac{R}{L_t} = \frac{\mathrm{Pr}_t}{\overline{\Pi}/\overline{Q}}\gamma_E$$

- Critical gradient = dashed line
- For given $\overline{\Pi}/\overline{Q}$ and $\overline{Q}$, only one solution

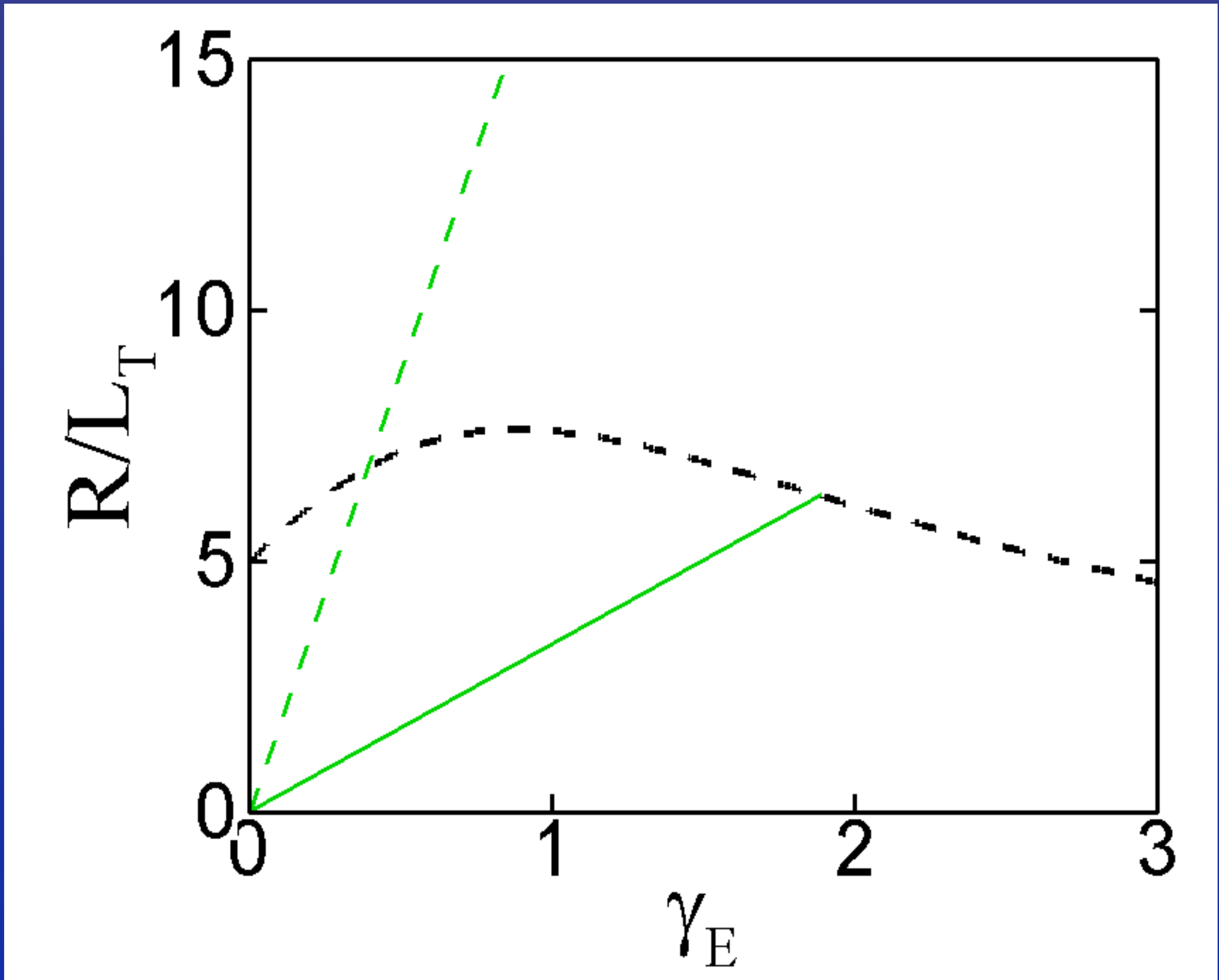
Neoclassical energy flux



Balance with neoclassical



Curves of constant $\overline{\Pi}/\overline{Q}$



- Neoclassical

$$\frac{R}{L_t} = \frac{\mathrm{Pr}_n}{\overline{\Pi}/\overline{Q}}\gamma_E$$

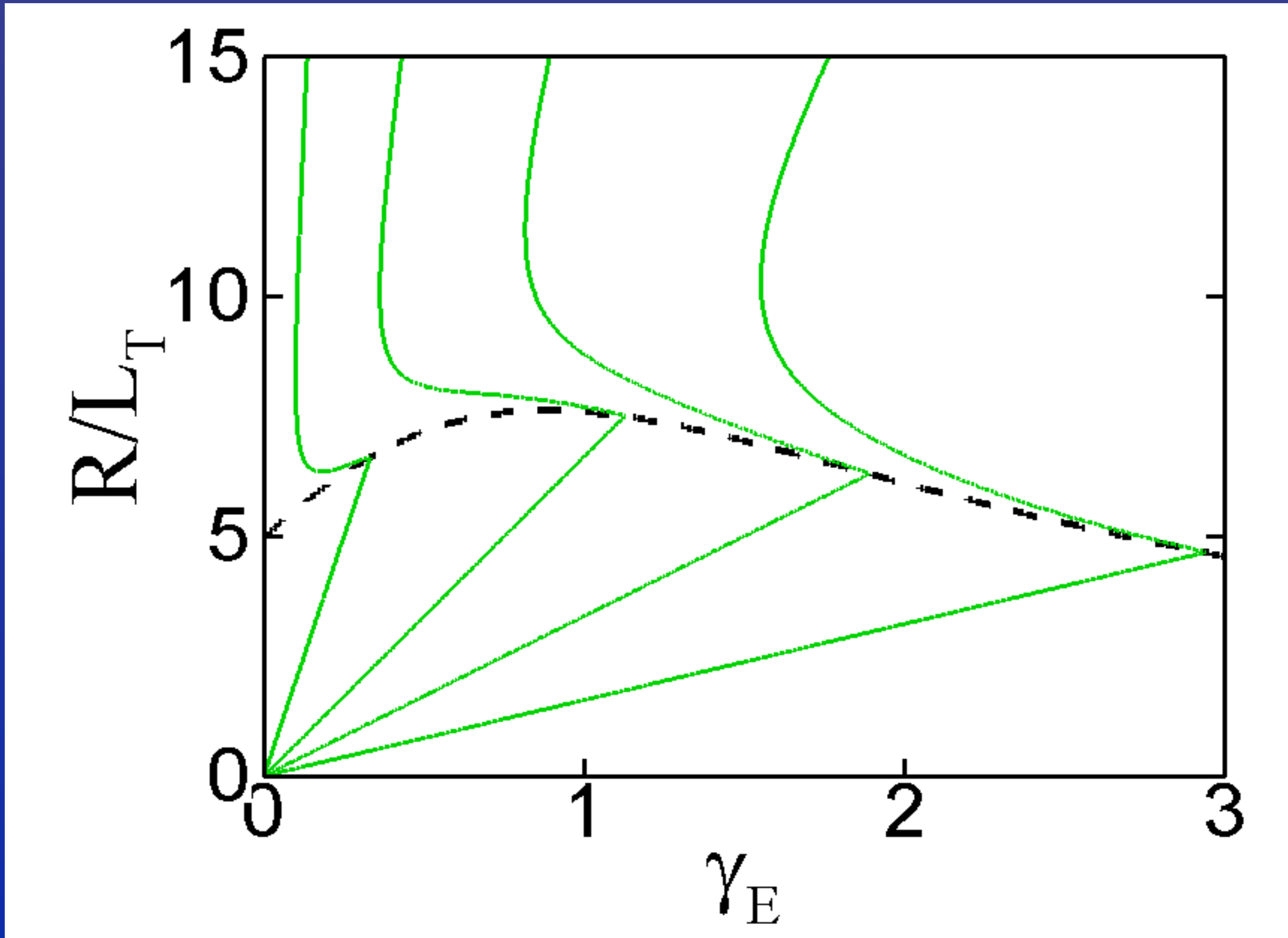
- Turbulent

$$\frac{R}{L_t} = \frac{\mathrm{Pr}_t}{\overline{\Pi}/\overline{Q}}\gamma_E$$

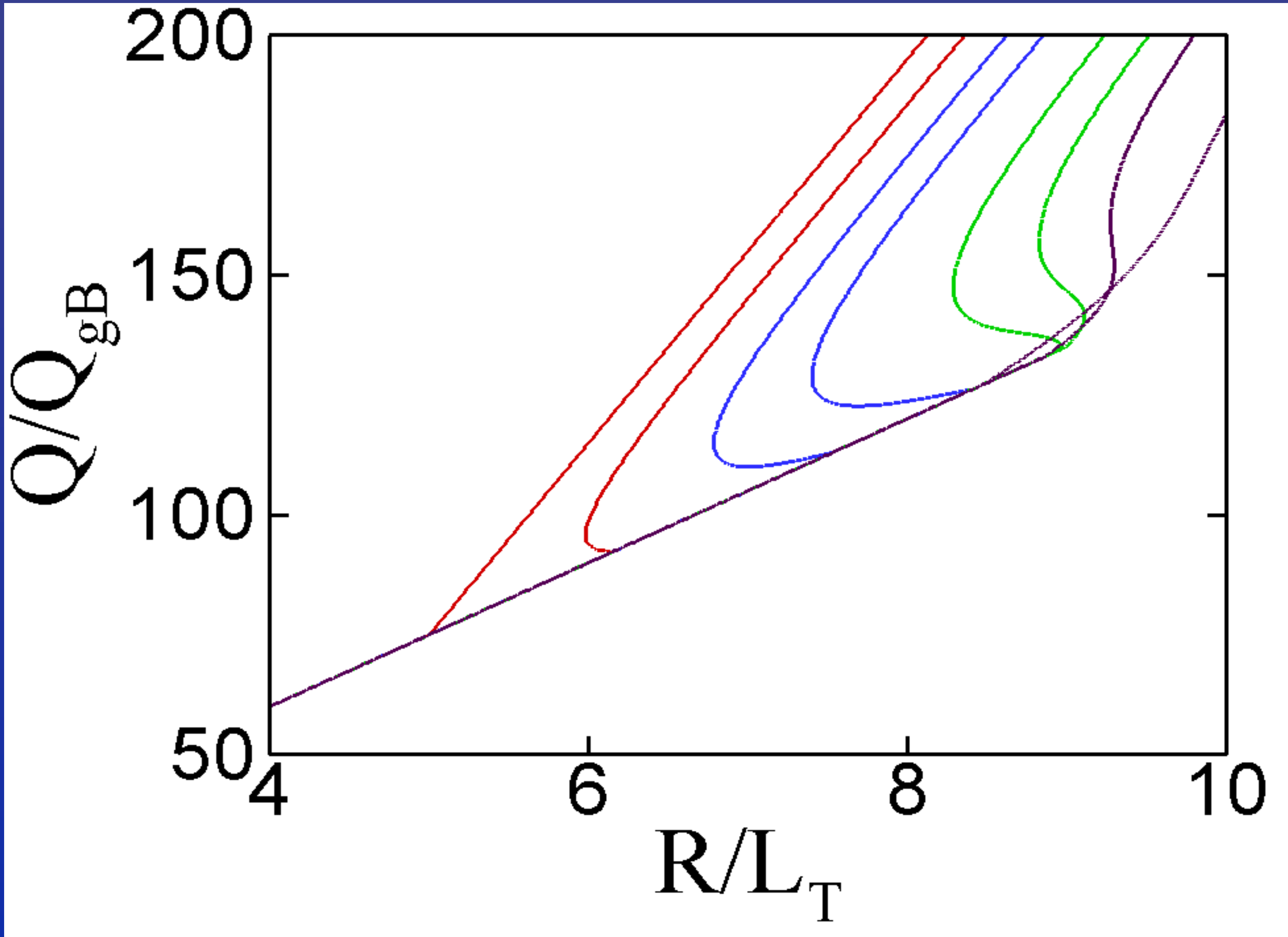
- Prandtl numbers

$$\mathrm{Pr}_n \ll \mathrm{Pr}_t$$

Curves of constant $\overline{\Pi}/\overline{Q}$



Total energy flux



Temperature dependence

- Have been using $\overline{Q}$ and $\overline{\Pi}/\overline{Q}$ for local analysis, but these quantities depend on local temperature:

$$\overline{Q} \sim \frac{Q}{T^{5/2}} \qquad \frac{\overline{\Pi}}{\overline{Q}} \sim \frac{\Pi}{Q} T^{1/2}$$

- Consequently, $\overline{Q}$ is a label for radius, and the contours of constant $\overline{\Pi}/\overline{Q}$ vary from radius to radius
- Better to consider $\overline{\Pi}/\overline{Q}^{4/5}$ or $\hat{\Pi}/\hat{Q}$, which are independent of temperature

$$\hat{Q} = \frac{Q}{T_0^{5/2}} \qquad \hat{\Pi} = \frac{\Pi}{T_0^2}$$

Solving for radial profiles

- Expressions for fluxes:

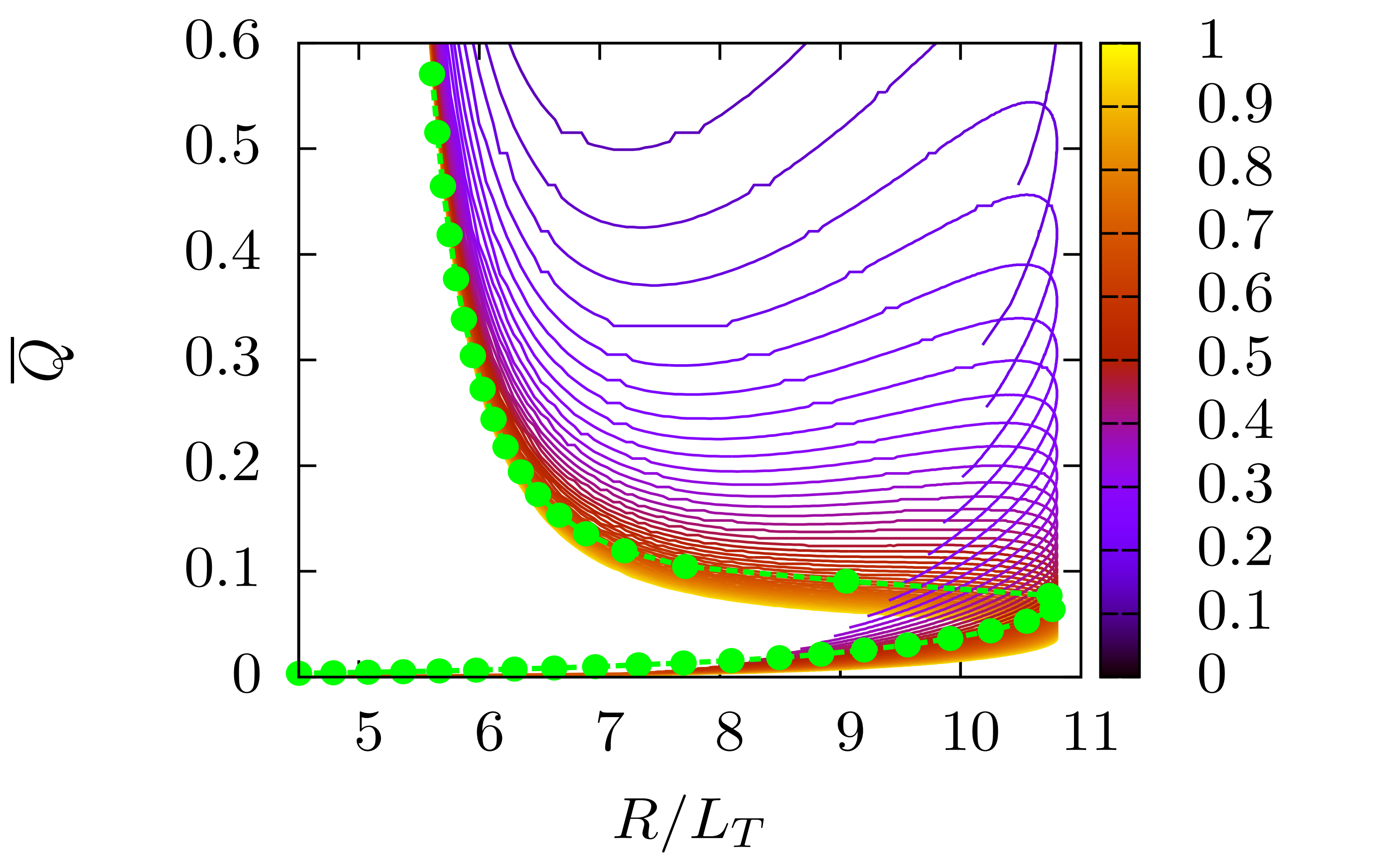
$$\hat{Q}(\kappa, \gamma_E, T) = \hat{T}^{5/2}\left(\hat{\chi}_t\left(\kappa - \kappa_c\right) + \frac{\hat{\chi}_n}{\hat{T}^2}\kappa\right)$$

$$\hat{\Pi}(\kappa, \gamma_E, T) = \gamma_E\left(\hat{\chi}_t\left(1 - \frac{\kappa_c}{\kappa}\right)\mathrm{Pr}_t\hat{T}^2 + \hat{\chi}_n\mathrm{Pr}_n\right)$$

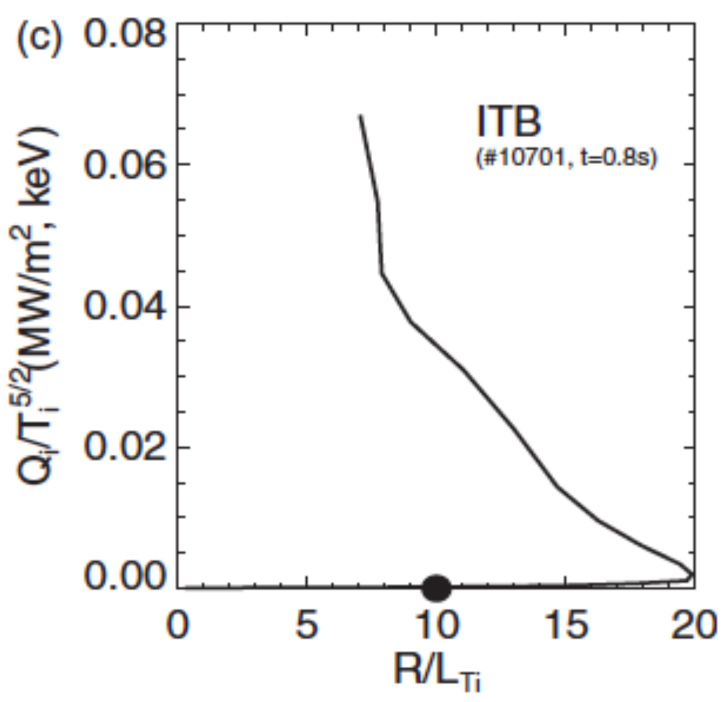
- Radial profiles of $\hat{Q}$ and $\hat{\Pi}$ are inputs. Given $\hat{T}$ at one radius, we can solve for γ_E and κ at that radius. With $\hat{T}$ and κ, we can obtain $\hat{T}$ at nearby radii. Repeat process to construct radial profiles.

Numerical results

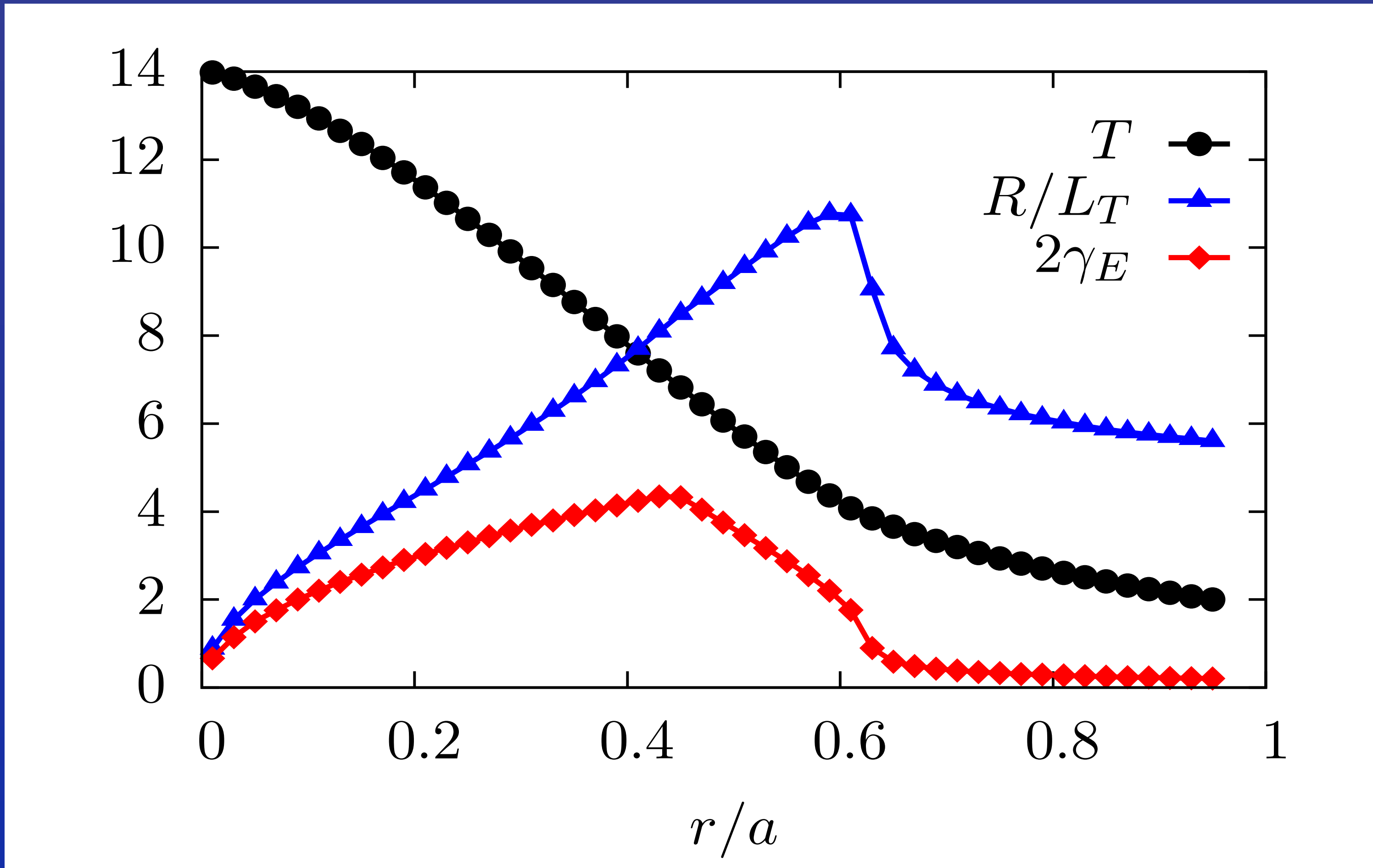
- Here, Q~sqrt(r/a), Pi/Q=0.1, Edge T=2 keV

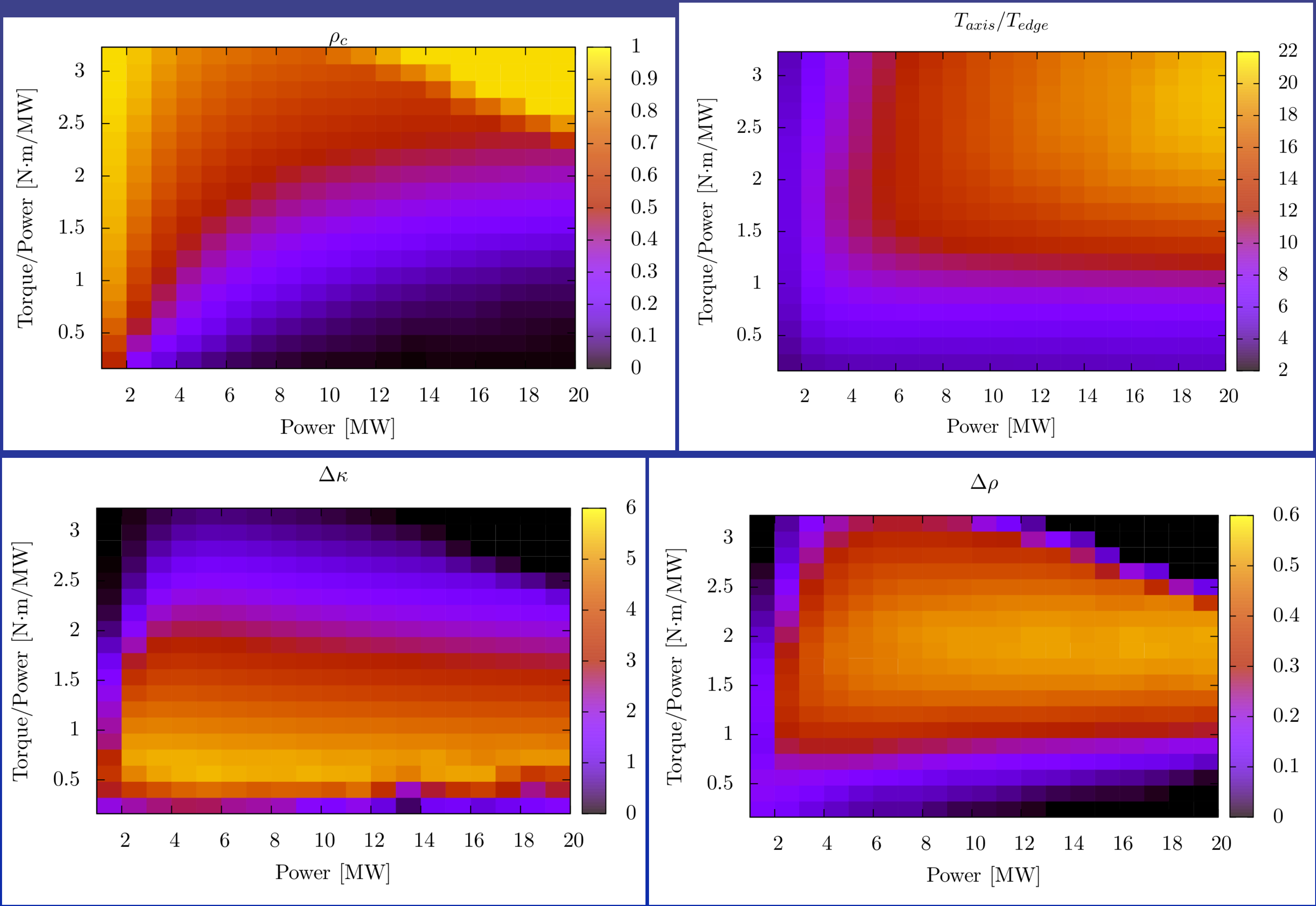


Wolf, PPCF **45** (2003)



Extension to 1D (radial)





ρ_c
Torque/Power [N·m/MW]
Power [MW]
2 4 6 8 10 12 14 16 18 20
0.5 1 1.5 2 2.5 3
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
T_{axis}/T_{edge}
Torque/Power [N·m/MW]
Power [MW]
2 4 6 8 10 12 14 16 18 20
0.5 1 1.5 2 2.5 3
2 4 6 8 10 12 14 16 18 20 22
$\Delta\kappa$
Torque/Power [N·m/MW]
Power [MW]
2 4 6 8 10 12 14 16 18 20
0.5 1 1.5 2 2.5 3
0 1 2 3 4 5 6
$\Delta\rho$
Torque/Power [N·m/MW]
Power [MW]
2 4 6 8 10 12 14 16 18 20
0.5 1 1.5 2 2.5 3
0 0.1 0.2 0.3 0.4 0.5 0.6

What have we learned?

- Significant enhancement of temperature gradient obtained solely through flow shear and magnetic shear
- Enhancement for simple geometry not sufficient to account for strong ITBs
  - Shafranov shift
  - Plasma shaping
- Multiple solutions not necessary to obtain localized enhancement of temperature gradient
- Understanding transition to enhanced gradient regime is work in progress

The future: multiscale simulation

- In TRINITY [Barnes *et al.*, PoP **17**, 056109 (2010)], turbulent fluctuations calculated in small regions of fine space-time grid embedded in "coarse" grid (for mean quantities)

Flux tube simulation domain