Transport scalings for ITG turbulence

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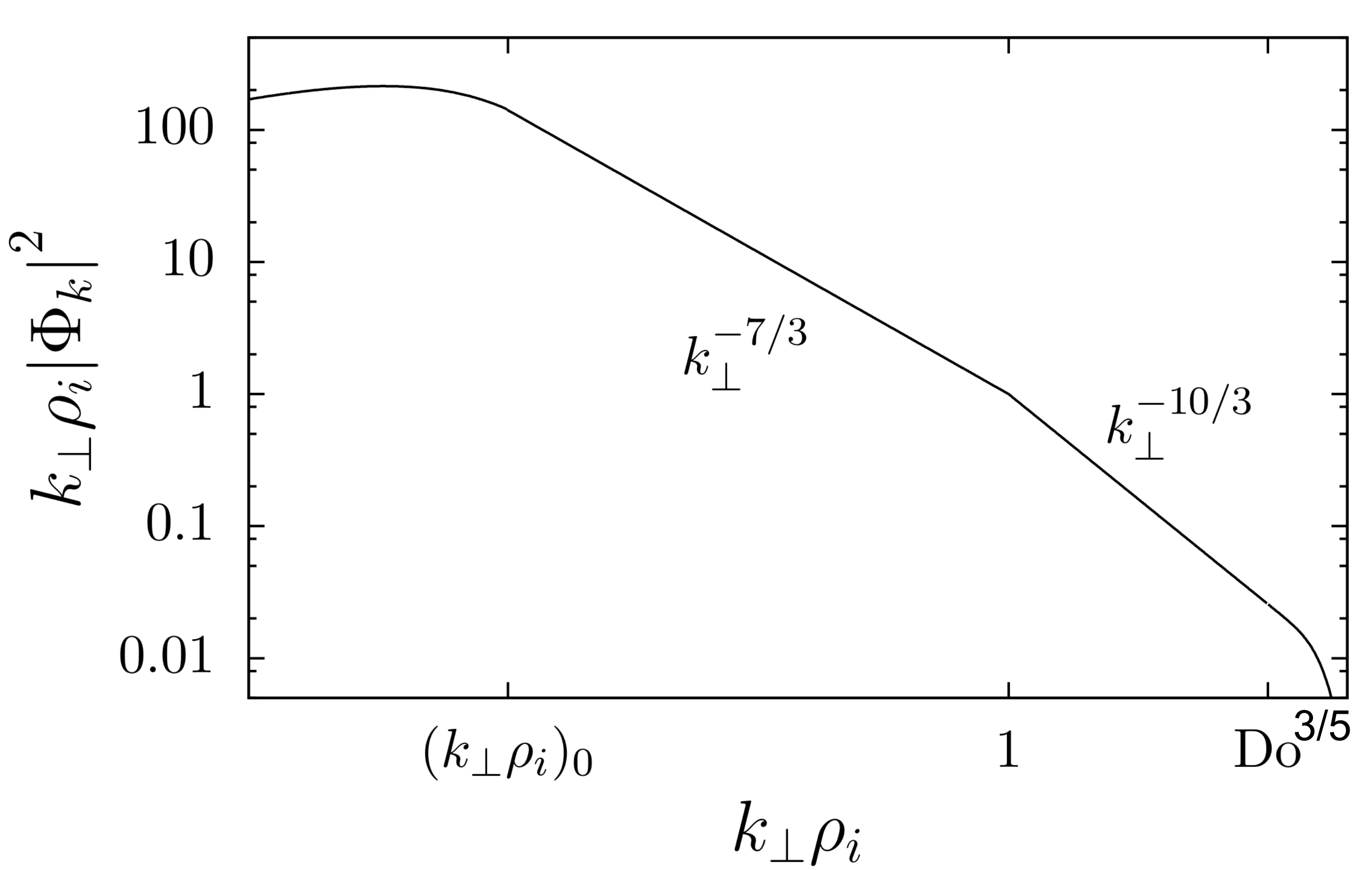
Why do we care?

- Interesting physics problem that might tell us something about properties of turbulence
- Plasma confinement properties are known to depend strongly on quantities such as mean plasma current and temperature gradient
- Analytical results for turbulence are rare, and direct numerical simulations are costly
- Scaling laws useful indicators of gross plasma performance and provide guidance for numerical simulations
- Improved turbulence fluctuation measurements allow for detailed comparisons with experiment

Cartoon of ITG turbulence



Model = gyrokinetics

Gyrokinetic variables: $\mathbf{R},\ E = \frac{mv^2}{2},\ \mu = \frac{mv_\perp^2}{2B}$

$$\frac{\partial}{\partial t}\left(h_s - \frac{Z_s e \langle\Phi\rangle_{\mathbf{R}}}{T_s} F_{M,s}\right) + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_{M,s}\right)\cdot\nabla h_s + \frac{c}{B}\left\{\langle\Phi\rangle_{\mathbf{R}}, h_s\right\}$$

$$= \langle C[h_s]\rangle_{\mathbf{R}} - \langle \mathbf{v}_E\rangle_{\mathbf{R}}\cdot\nabla F_{M,s}$$

Quasineutrality: $\sum_s Z_s \left(\int d^3\mathbf{v}\ h_s - \frac{Z_s e\Phi}{T_s} n_s\right) = 0$

Assume $\int d^3\mathbf{v}\ h \sim v_{th}^3 h$ => $\frac{h}{F_M} \sim \frac{Ze\Phi}{T}$

Conjectures

- The parallel streaming time and nonlinear turnover time are comparable at all scales
- The characteristic parallel length of the turbulence at the outer scale is the connection length
- The time scale associated with the linear drive is comparable to the nonlinear decorrelation time at the outer scale
- Fluctuation scale lengths in the two dimensions of the perpendicular plane are comparable
- There is no significant dissipation or driving between the outer and Larmor scales

Critical balance

- **Conjecture 1: The parallel streaming time and nonlinear turnover time are comparable at all scales**
- Physical idea: two points along field line can be correlated only if information propagates between the points before turbulence is decorrelated in perpendicular plane

$$\frac{v_{th}}{\ell_{\|}} \sim \frac{v_{th}}{R} \frac{\rho_i^2}{\ell_x \ell_y} \tilde{\Phi}_{\ell_\perp} \qquad \tilde{\Phi}_{\ell_\perp} \equiv \frac{Ze\Phi_{\ell_\perp}}{T} \frac{R}{\rho_i}$$

Outer scale

- **Conjecture 2: The characteristic parallel length of the turbulence at the outer scale is the connection length**
- Physical idea: Modes cannot extend much beyond the connection length due to stabilizing effect of good curvature

$$\ell_\parallel^o \sim qR \quad => \quad \frac{v_{th}}{qR} \sim \frac{v_{th}}{R} \frac{\rho_i^2}{\ell_x^o \ell_y^o} \tilde{\Phi}_{\ell_\perp^o}$$

- **Conjecture 3: The time scale associated with the linear drive is comparable to the nonlinear decorrelation time at the outer scale**

$$\frac{v_{th}}{qR} \sim \omega_* \sim \frac{\rho_i v_{th}}{\ell_y^o L_T} \Rightarrow \boxed{\ell_y^o \sim \frac{qR}{L_T}\rho_i}$$

Outer scale

- **Conjecture 4: Fluctuation scale lengths in the two dimensions of the perpendicular plane are comparable**
- Physical idea: Linear drive favors structures with $\ell_x \gtrsim \ell_y$. Smaller ℓ_x formed through magnetic shear and zonal flow shear: $\ell_x^{-1} \sim \left(S_{\mathrm{zf}}\tau_{\mathrm{nl}} + \hat{s}\theta\right)\ell_y^{-1} \sim \ell_y^{-1}$

$$\frac{v_{th}}{qR} \sim \frac{v_{th}}{R}\frac{\rho_i^2}{\ell_x^o \ell_y^o}\tilde{\Phi}_{\ell_\perp^o} \quad + \quad \ell_y^o \sim \frac{qR}{L_T}\rho_i \quad => \quad \boxed{\tilde{\Phi}_{\ell_\perp^o} \sim q\left(\frac{R}{L_T}\right)^2}$$

$$\frac{Q_i}{n_i T_i v_{th}}\left(\frac{R}{\rho_i}\right)^2 \equiv \boxed{\tilde{Q}_i \sim \frac{\rho_i}{\ell_y^o}\tilde{\Phi}^2_{\ell_\perp^o} \sim q\left(\frac{R}{L_T}\right)^3}$$

Entropy balance

- Entropy is conserved quantity in gyrokinetics:

$$S = -\sum_s \int d^3\mathbf{r} \int d^3\mathbf{v} \left(\frac{\delta f_s^2}{2F_{0,s}} \right)$$

- Multiply GK equation by h_ℓ, integrate over phase space, and sum over species to get scale-by-scale entropy balance:

$$\frac{\partial S_\ell}{\partial t} + \mathcal{T}_\ell = \mathcal{Q}_\ell + \mathcal{C}_\ell \qquad S_\ell \equiv \sum_s \int d^3\mathbf{r} \int d^3\mathbf{v} \left(\frac{\delta f_{\ell,s}^2}{2F_{0,s}} \right)$$

$$\mathcal{T}_\ell \equiv \frac{c}{B} \sum_s \int d^3\mathbf{r} \int d^3\mathbf{v} \, \frac{h_{\ell,s}}{F_{0,s}} \left\{ \langle \Phi \rangle_{\mathbf{R}} , h_s \right\}$$

$$\mathcal{Q}_\ell \equiv -\sum_s \int d^3\mathbf{r} \int d^3\mathbf{v} \, \frac{h_{\ell,s}}{F_{0,s}} \mathbf{v}_E \cdot \nabla F_{0,s} \sim Q \frac{R}{L_T}$$

Inertial range

- **Conjecture 5: There is no significant dissipation or driving between the outer and Larmor scales**

i.e. $\mathcal{C}_\ell \approx 0, \quad \mathcal{T}_\ell \sim \mathcal{Q}_\ell \approx \mathcal{Q}_{\ell^o}$

$$\frac{v_{th}}{R}\frac{\rho_i^2}{\ell_\perp^2}\tilde{\Phi}^3_{\ell_\perp}\left(\frac{\rho_i}{R}\right)^2 \sim \frac{v_{th}}{L_T}\frac{\rho_i}{\ell^o_\perp}\tilde{\Phi}^2_{\ell^o_\perp}\left(\frac{\rho_i}{R}\right)^2$$

$$\Rightarrow \boxed{\tilde{\Phi}^2_{\ell_\perp} \sim q^{2/3}\left(\frac{R}{L_T}\right)^{8/3}\left(\frac{\ell_\perp}{\rho_i}\right)^{4/3}}$$

$$\frac{v_{th}}{\ell_\parallel} \sim \frac{v_{th}}{R}\frac{\rho_i^2}{\ell_\perp^2}\tilde{\Phi}_{\ell_\perp} \Rightarrow \boxed{\frac{\ell_\parallel}{qR} \sim \left(\frac{\ell_\perp}{\rho_i}\frac{L_T}{qR}\right)^{4/3}}$$

Sub-Larmor cutoff

- Define number analogous to Reynolds number to measure separation of driving and dissipation scales:

$$\mathrm{Do} \equiv (\nu\tau_\rho)^{-1} \sim \frac{v_{th}}{\nu R}\tilde{\Phi}_{\rho_i} \quad \text{with} \quad \tilde{\Phi}_{\rho_i} = \tilde{\Phi}_{\ell_\perp^o}\left(\frac{\rho_i}{\ell_\perp^o}\right)^{2/3}$$

For outer scale: $\tilde{\Phi}_{\ell_\perp^o} \sim q\left(\frac{R}{L_T}\right)^2$ & $\ell_\perp^o \sim \frac{qR}{L_T}\rho_i$

$$\Rightarrow \quad \mathrm{Do} \sim q^{1/3}\left(\frac{R}{L_T}\right)^{4/3}\frac{v_{th}}{\nu R}$$

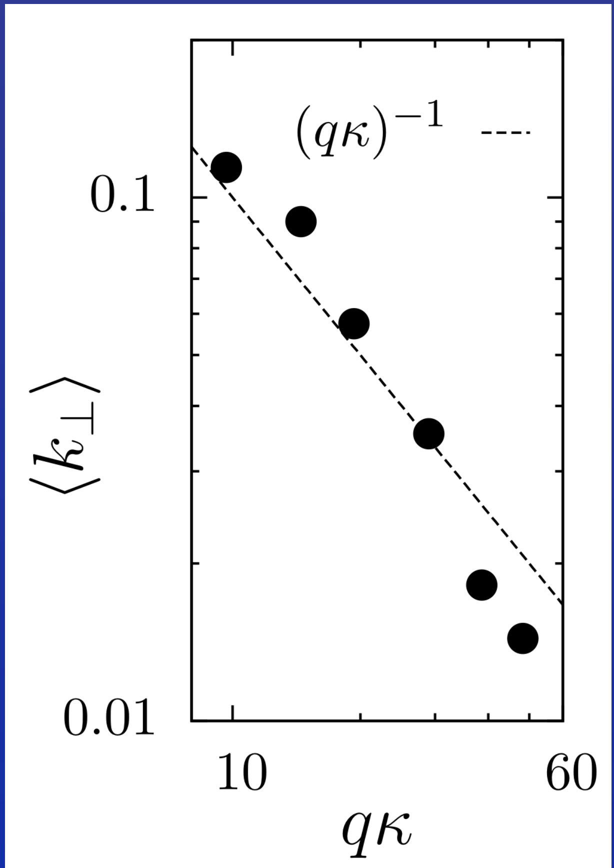
Tatsuno PRL '09 $\left(\frac{\delta v_\perp}{v_{th}}\right)_c \sim \mathrm{Do}^{-3/5} \sim q^{-1/5}\left(\frac{R}{L_T}\right)^{-4/5}\left(\frac{\nu R}{v_{th}}\right)^{3/5}$

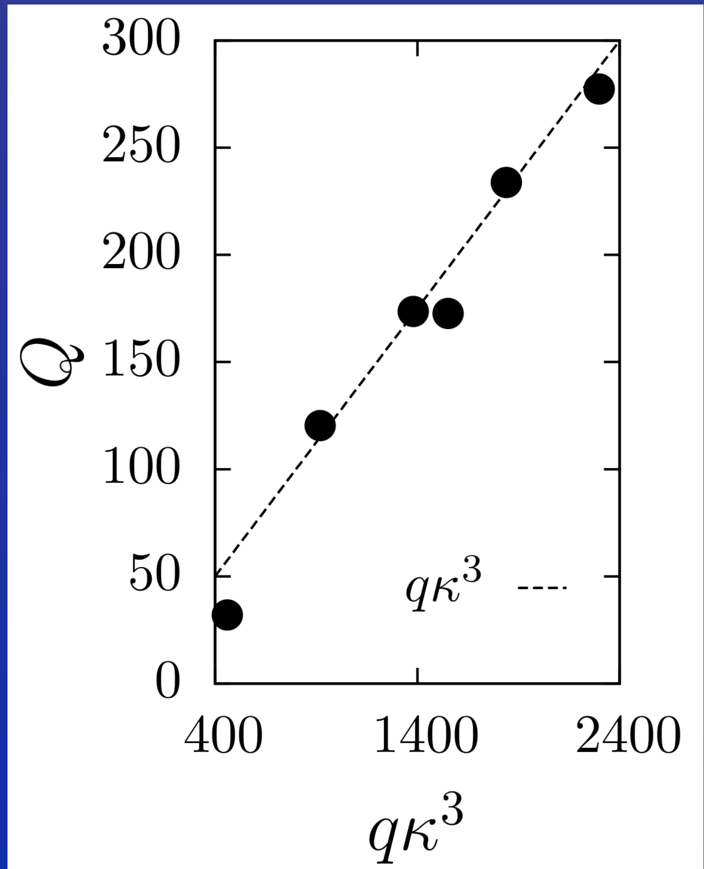
Summary of key results

Outer scale	$\ell_y^o \sim \frac{qR}{L_T}\rho_i$ & $\tilde{\Phi}_{\ell_\perp^o} \sim q\left(\frac{R}{L_T}\right)^2$ $\tilde{Q}_i \sim q\left(\frac{R}{L_T}\right)^3$
Inertial scale	$\tilde{\Phi}^2_{\ell_\perp} \sim q^{2/3}\left(\frac{R}{L_T}\right)^{8/3}\left(\frac{\ell_\perp}{\rho_i}\right)^{4/3}$ $\frac{\ell_\parallel}{qR} \sim \left(\frac{\ell_\perp}{\rho_i}\frac{L_T}{qR}\right)^{4/3}$
Sub-Larmor	$\mathrm{Do} \sim q^{1/3}\left(\frac{R}{L_T}\right)^{4/3}\frac{v_{th}}{\nu R}$

Outer range scalings

- Use GS2 to test scaling predictions
- Cyclone Base Case with varying q and $\kappa = R/L_T$





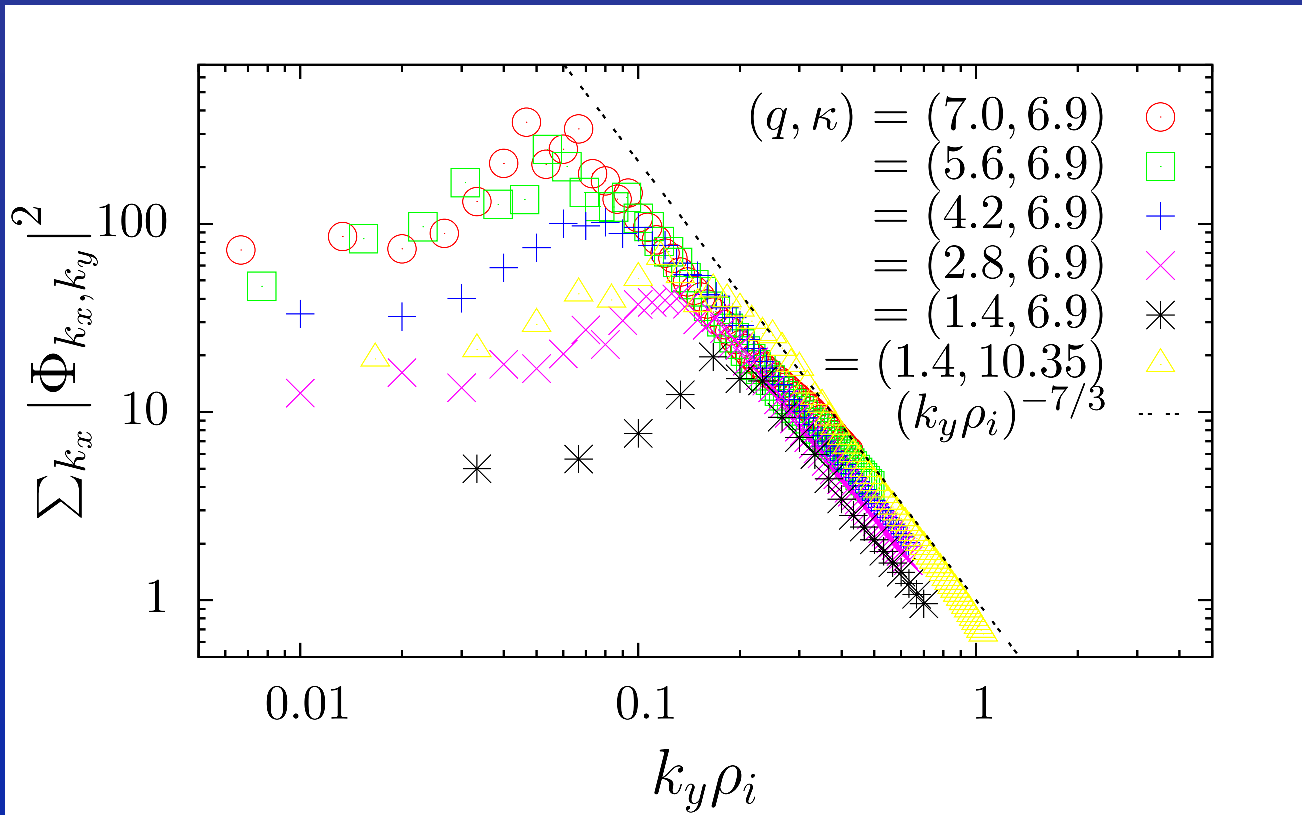
$$k_y \rho_i \sim \frac{L_T}{qR}$$

$$\tilde{Q}_i \sim q \left(\frac{R}{L_T} \right)^3$$

Inertial range spectra

Parseval's theorem: $$\int d^2\mathbf{r}\ \Phi_\ell^2 = \sum_{\mathbf{k}\le\mathbf{k}_\ell} |\Phi_\mathbf{k}|^2 \Rightarrow \Phi_\ell^2 \sim k_\ell^2\, |\Phi_{\mathbf{k}_\ell}|^2$$

1D spectrum: $$k\,|\Phi_k|^2 \sim k^{-7/3}$$



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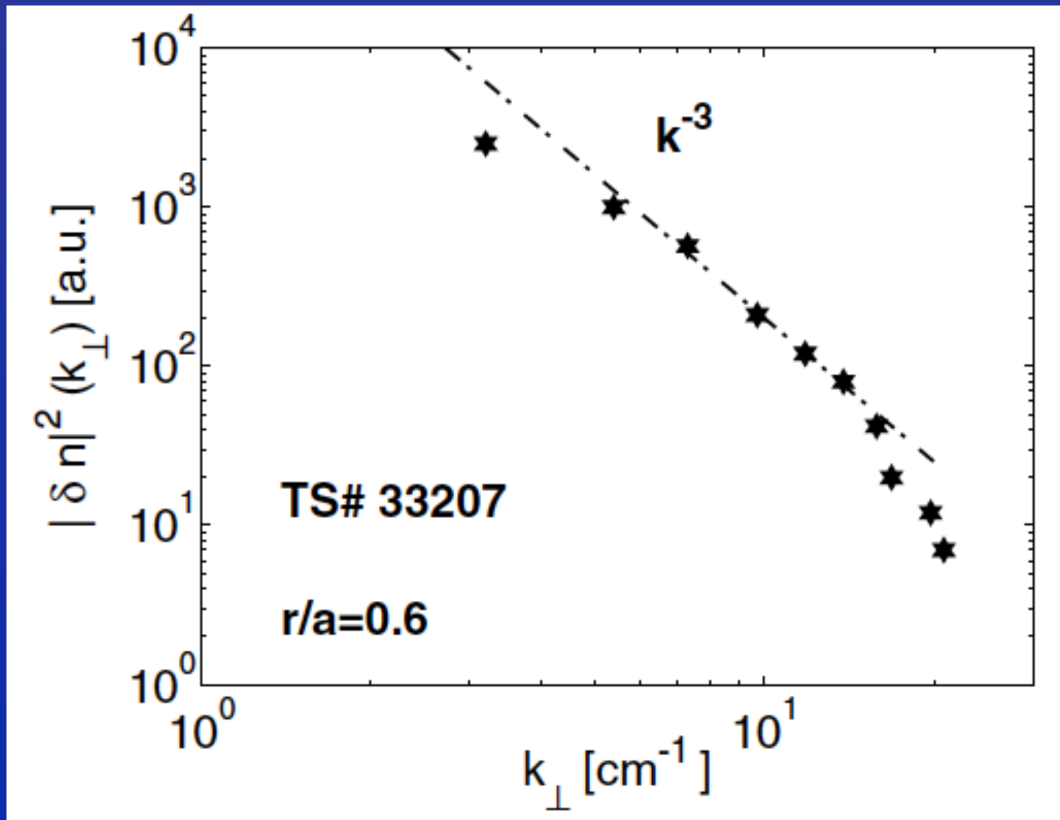
1D spectrum: $k\,|\Phi_k|^2 \sim k^{-7/3}$ => $|\Phi_k|^2 \sim k^{-10/3}$

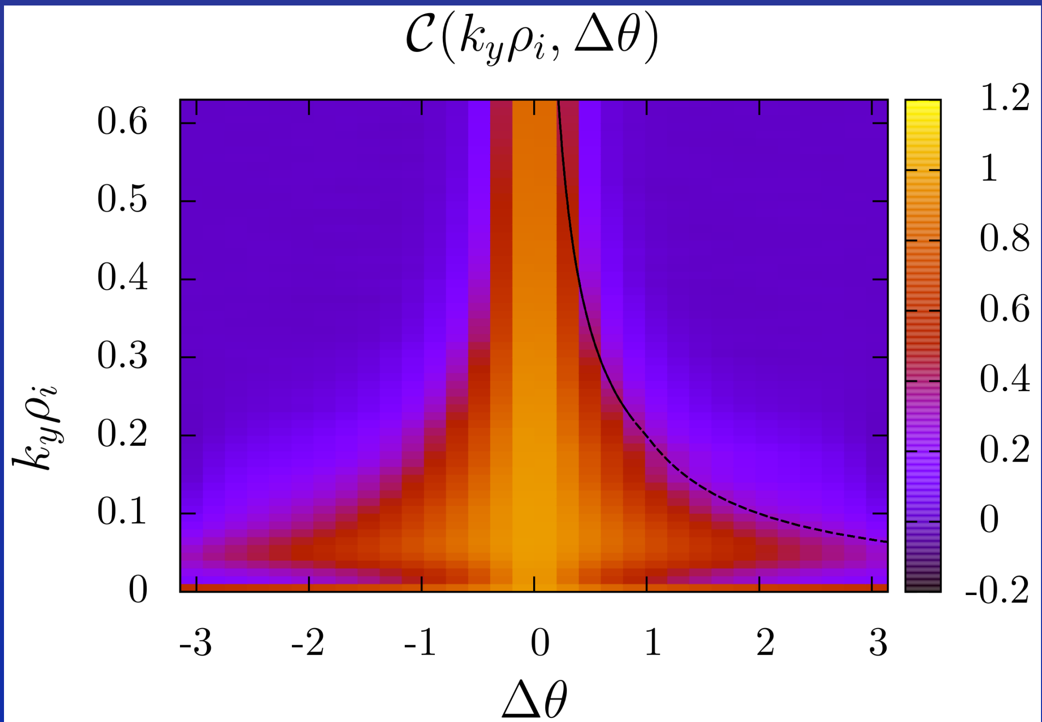
Tore Supra fluctuation spectrum

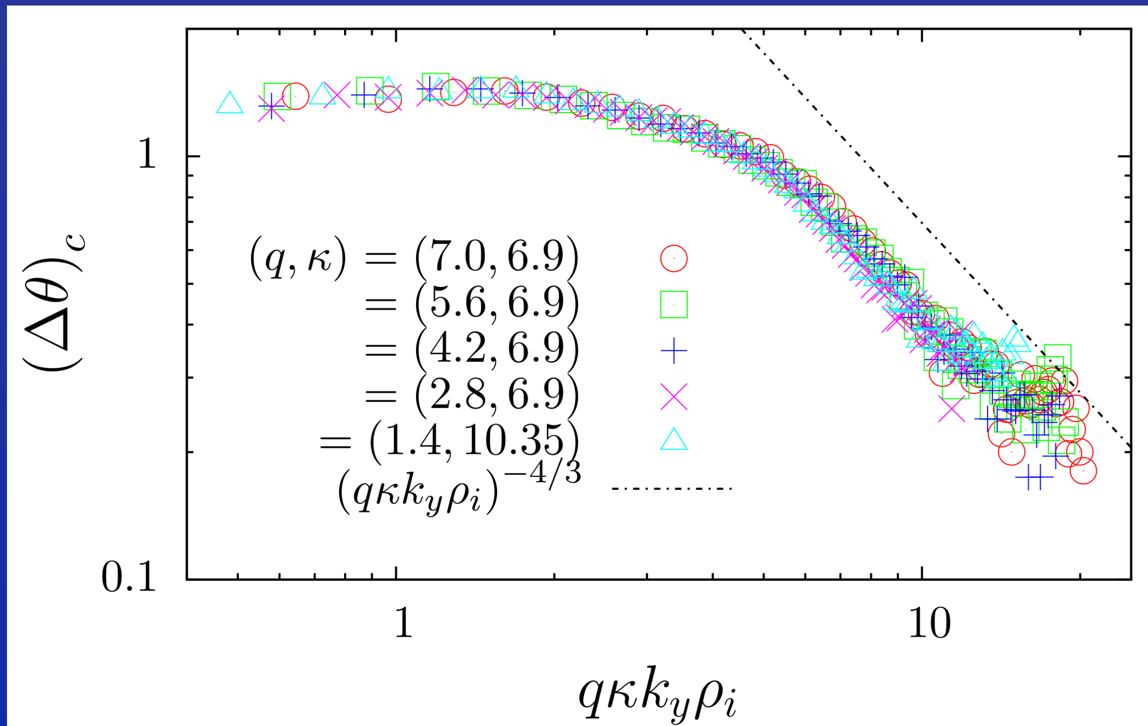


Critical balance test

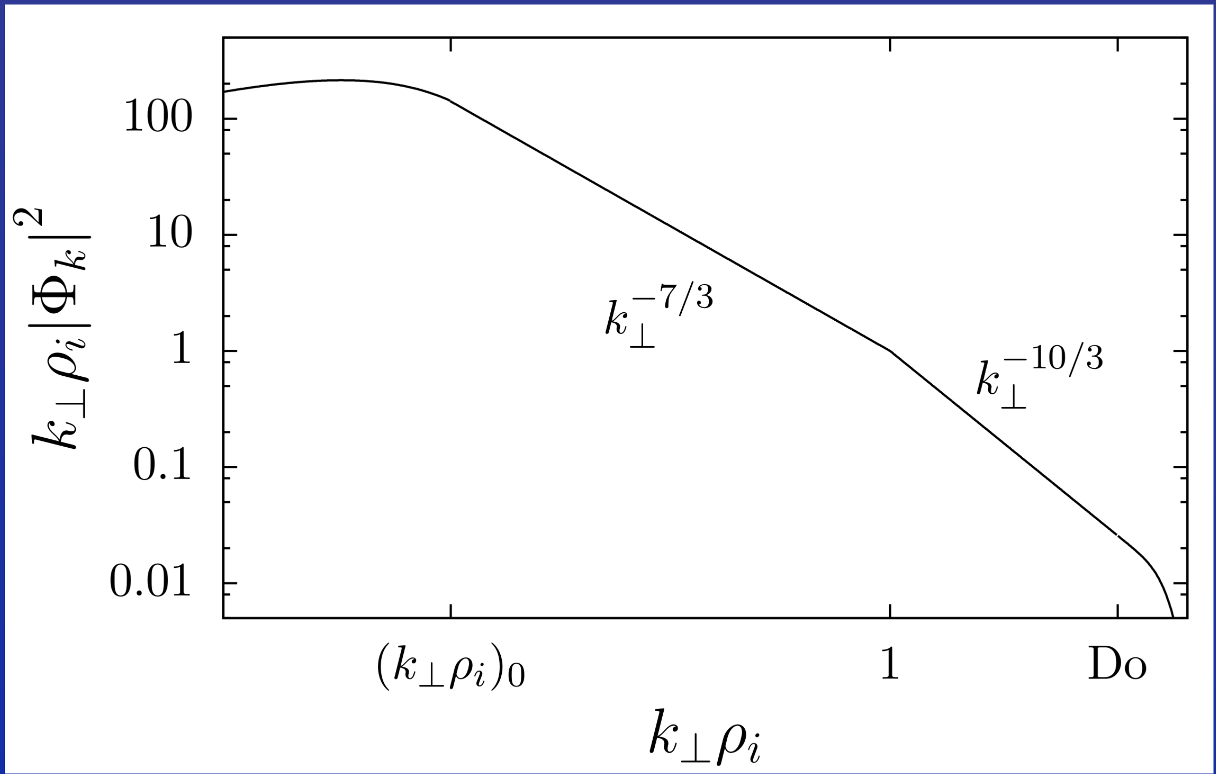
$$\mathcal{C}(k_y, \Delta\theta) = \frac{\sum_{k_x} \Phi(\theta = 0)\Phi^*(\theta = \Delta\theta)}{\sum_{k_x} |\Phi(\theta = 0)|^2}$$

$$\frac{\ell_\parallel}{qR} \sim \left(k_\perp \rho_i \frac{qR}{L_T} \right)^{-4/3}$$





Discussion



- Critical balance is satisfied for system we've considered
- Scalings from simple arguments give remarkably good agreement with simulations
- Modifications for different regions in parameter space