Phase mixing in kinetic plasma turbulence

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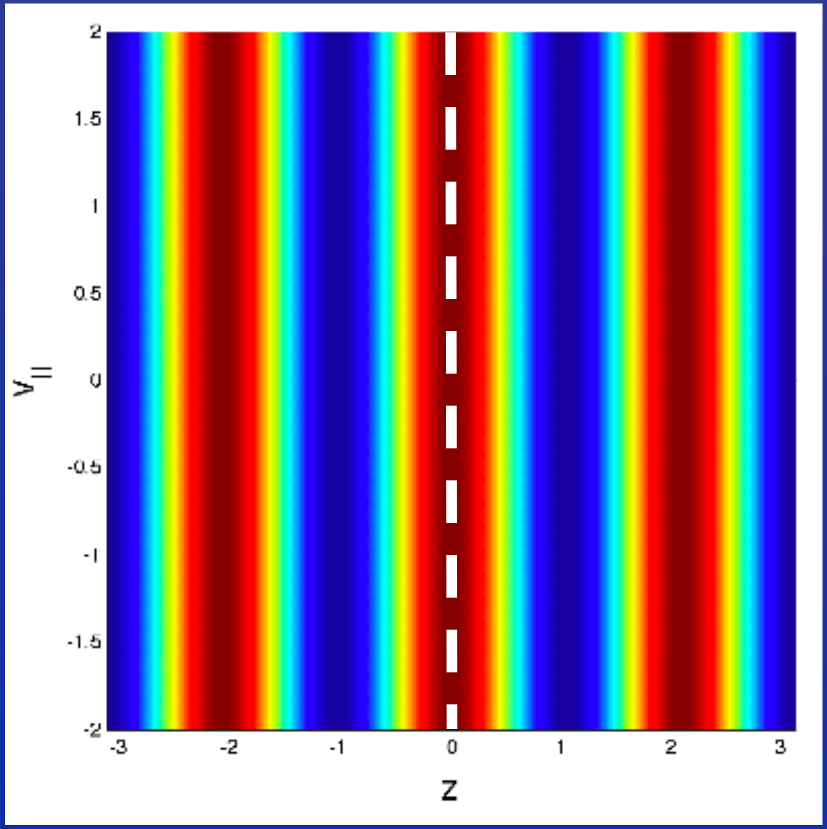
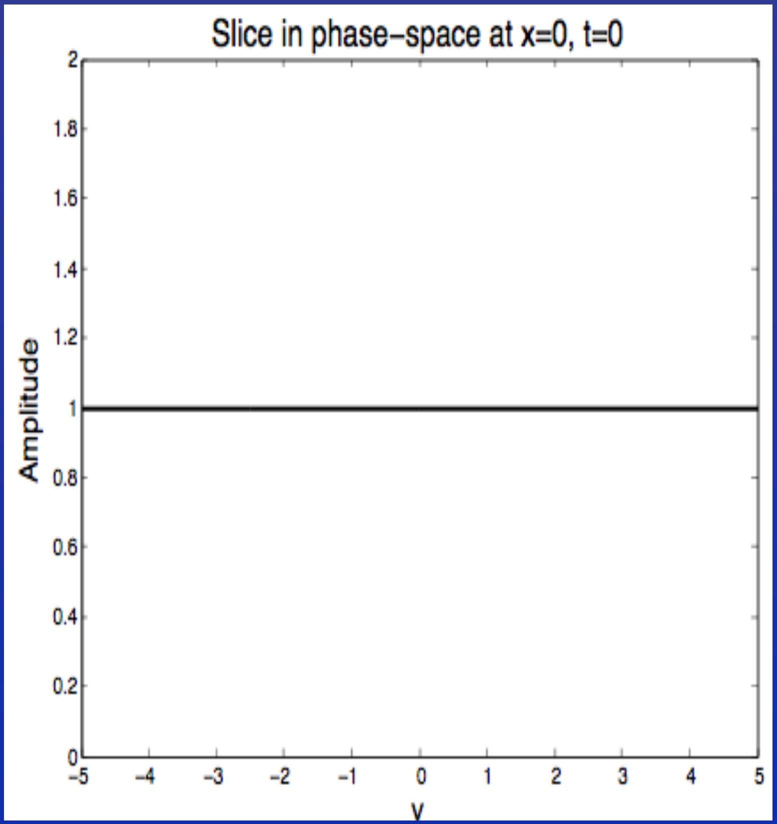
Simple picture of parallel phase-mixing

$$h(z, v_{||}, t_0) \sim \cos(k_{||} z)$$

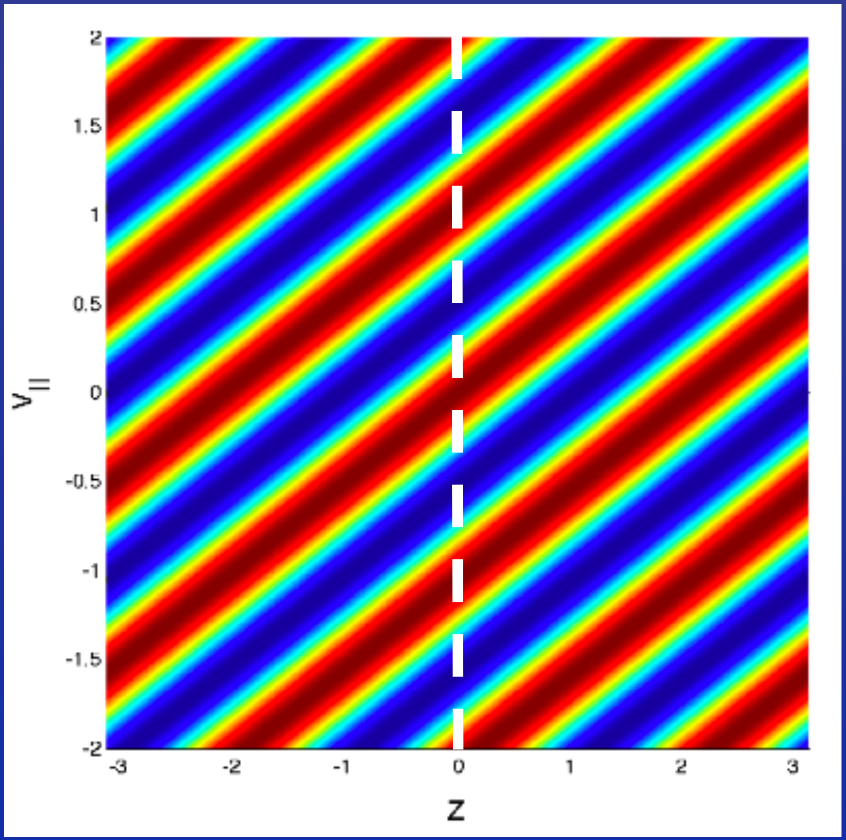


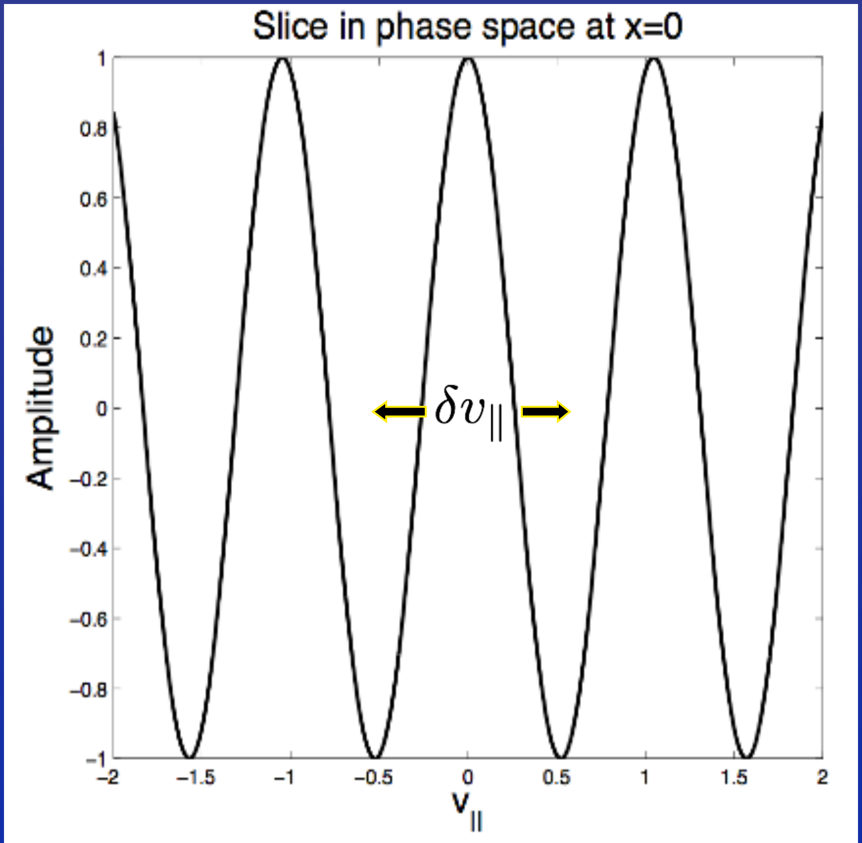


Simple picture of parallel phase-mixing

$$h(z, v_{\|}, t) \sim \cos\left[k_{\|}\left(z - v_{\|} t\right)\right]$$

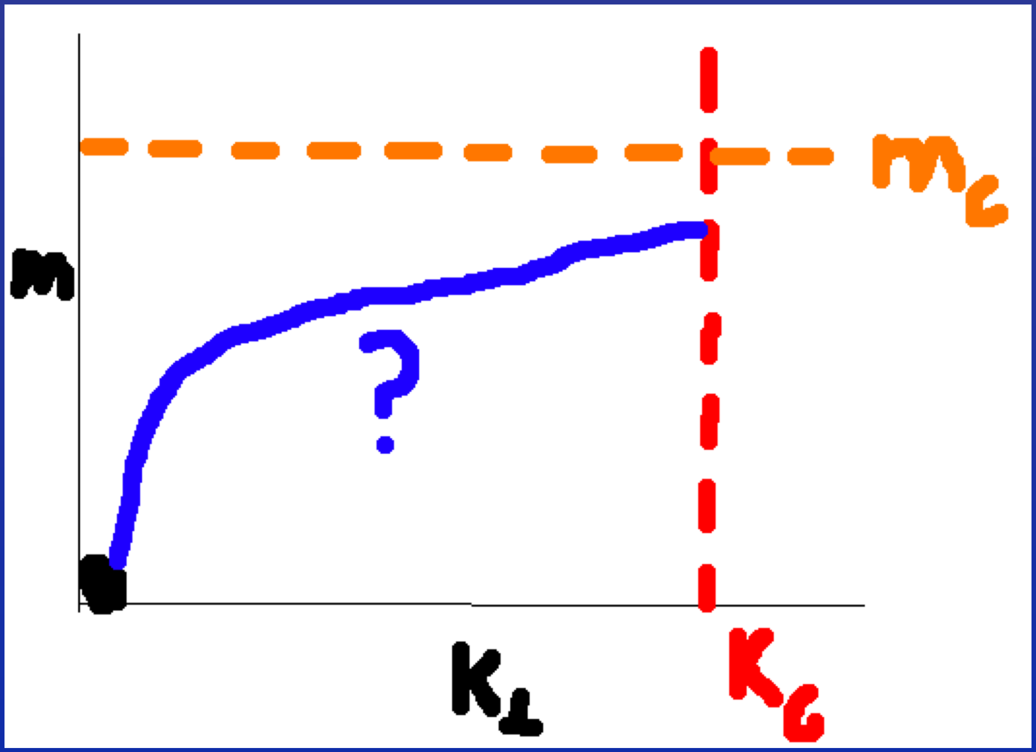
$$\delta v_{\|} \sim \left(k_{\|} t\right)^{-1}$$



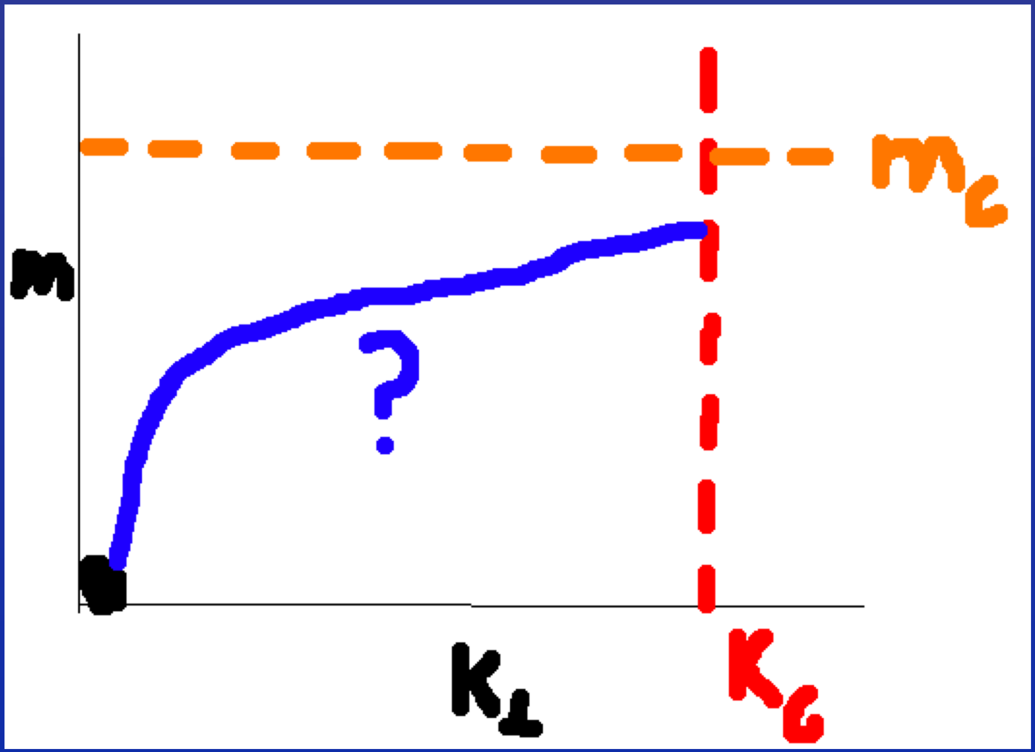


5D cascade path

- Interesting theoretical physics question:
  - What are the general properties of kinetic plasma turbulence, and are they universal?



5D cascade path



- Practical questions:
- Given a collision frequency, what are the smallest scales allowed in each dimension of phase space?
- If scale size restrained in a dimension, does it affect system dynamics in other dimensions?
- Can we use our knowledge of turbulence properties to design sub-grid models for turbulence?

Gyrokinetic-Poisson system

GK equation

$$\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}} \left(\frac{\partial g_{\mathbf{k}}}{\partial t} + ik_{\|} v_{\|} \left(g_{\mathbf{k}} + \frac{q \langle \varphi \rangle_{\mathbf{k}}}{T_i} F_0 \right) + \mathcal{N}_{\mathbf{k}}[g] = C_{\mathbf{k}}[g] - \mathbf{v}_{\varphi} \cdot \nabla F_0 \right)$$

$$g \equiv \langle \delta f \rangle \qquad \mathcal{N}_{\mathbf{k}}[g] \equiv \frac{c}{B} \sum_{\mathbf{k}'} \hat{z} \cdot (\mathbf{k} \times \mathbf{k}') \langle \varphi \rangle_{\mathbf{k}'} g_{\mathbf{k}-\mathbf{k}'}$$

Quasineutrality

$$\frac{q\varphi_{\mathbf{k}}}{T_i} = \Lambda_{\mathbf{k}}^{-1} \int d^3v \; J_0(kv_{\perp}/\Omega) g_{\mathbf{k}}$$

$$\Lambda_{\mathbf{k}} \equiv \left(1 + T_i/T_e - \Gamma_0 \left(k\rho_i\right)\right) n_0$$

Hermite spectra

- Simple Collision operator: $C[g] \equiv \nu \partial_{v_\parallel} \left(\partial_{v_\parallel} + v_\parallel \right) g$
- Hermite transform GK equation:

For m > 2

$$\frac{\partial \hat{g}_m}{\partial t} + ik_\parallel v_{th} \left(\sqrt{\frac{m+1}{2}} \hat{g}_{m+1} + \sqrt{\frac{m}{2}} \hat{g}_{m-1} \right) + \int dv_\perp v_\perp J_0 \left(\frac{k_\perp v_\perp}{\Omega} \right) \mathcal{N}[g_m] = -2\nu m \hat{g}_m$$

Definitions:

$$\hat{g}_m \equiv \int d\vartheta \int dv_\perp v_\perp J_0(k_\perp v_\perp / \Omega) g_m \qquad g = \sum_m g_m(v_\perp) \frac{H_m(v_\parallel)}{\sqrt{\pi 2^m m!}}$$

$$\int dx H_m(x) H_n(x) = \delta_{mn} \pi 2^m m!$$

Scalings and collisional cutoff

$$\frac{\partial \left|\hat{g}_m\right|^2}{\partial t} + \frac{\partial \Gamma_m}{\partial m} + \frac{1}{2}\left(\hat{g}_m^* \mathcal{N}[g_m] + \hat{g}_m \mathcal{N}[g_m^*]\right) = -2\nu m \left|\hat{g}_m\right|^2$$

$$\Gamma_m \equiv \frac{i k_\parallel v_{th}}{2\sqrt{2}} \sqrt{m} \left(\hat{g}_m \hat{g}_{m-1}^* - \hat{g}_m^* \hat{g}_{m-1}\right)$$

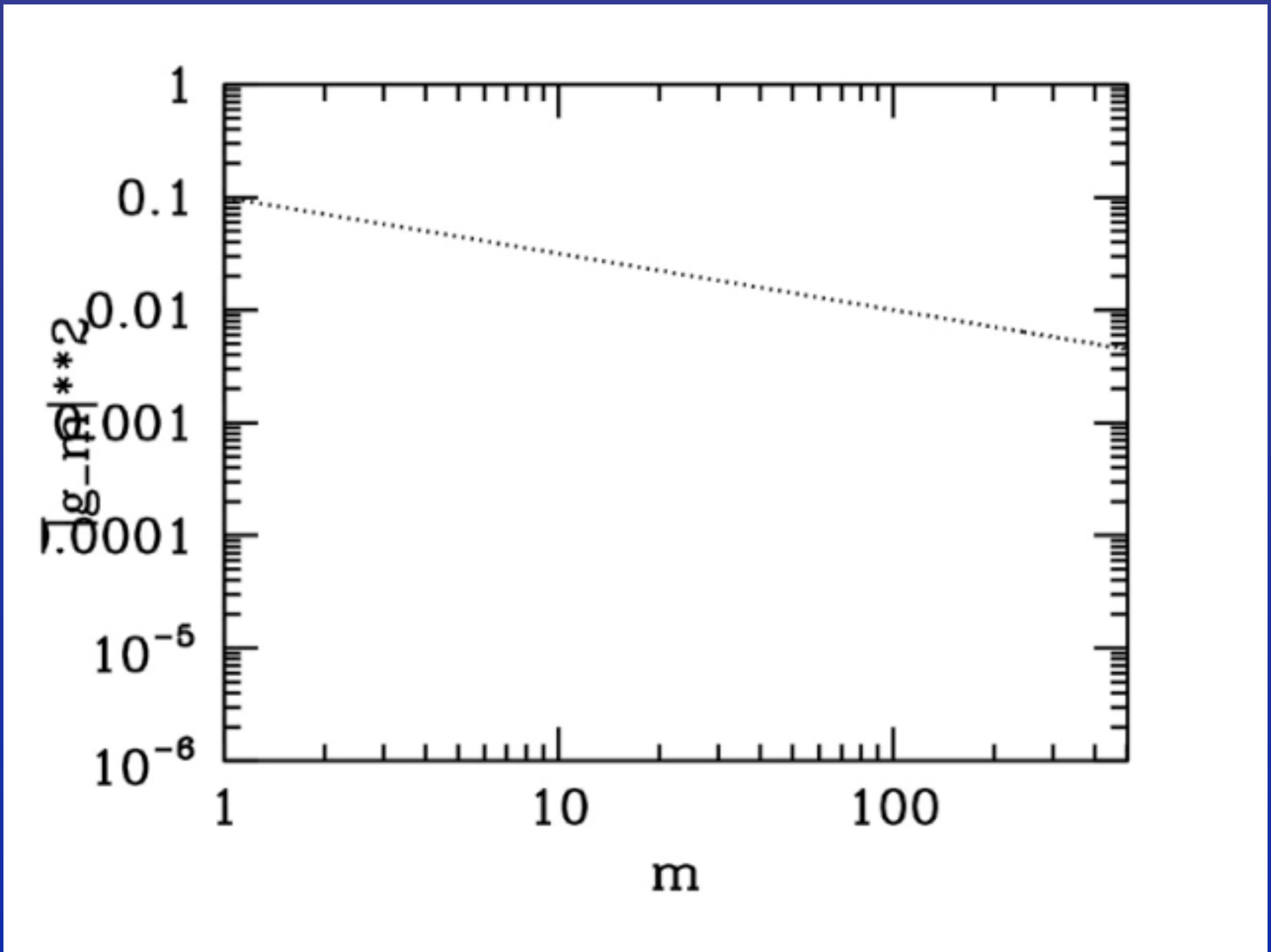
Linear or weakly nonlinear

$$\frac{\partial \Gamma_m}{\partial m} \approx 0 \Rightarrow \left|\hat{g}_m\right|^2 \propto m^{-1/2}$$

Collisional cutoff

$$\int_0^{m_c} dm \left(\frac{\partial \Gamma_m}{\partial m} + 2\nu m \left|g_m\right|^2\right) \approx 0 \Rightarrow m_c \propto \left(\frac{k_\parallel v_{th}}{\nu}\right)^{2/3}$$

Parallel velocity spectrum



Model nonlinearity

Locality and weak k-dependence: $\mathcal{N}_{\mathbf{k}}[g_m] \approx \gamma_k \hat{g}_{m,\mathbf{k}}$

$$\frac{1}{2}\left(\hat{g}_m^* \mathcal{N}[g_m] + \hat{g}_m \mathcal{N}^*[g_m]\right) \approx \gamma_k \left|\hat{g}_m\right|^2$$

Inertial range: $\dfrac{\partial \Gamma_{m,\mathbf{k}}}{\partial m} + \gamma_{\mathbf{k}} \left|g_{m,\mathbf{k}}\right|^2 \approx 0$

$$\Rightarrow \left|\hat{g}_{m,\mathbf{k}}\right|^2 \propto \frac{e^{-2\tilde{\gamma}_{\mathbf{k}}\sqrt{m}}}{\sqrt{m}} \qquad \tilde{\gamma}_{\mathbf{k}} \equiv \frac{\gamma_{\mathbf{k}}}{k_{\parallel} v_{th}}$$

Caveats/Questions

- Summation over k of GK equation gives entropy balance:

$$\frac{\partial \delta S_m}{\partial t} + \frac{\partial \tilde{\Gamma}_m}{\partial m} = -2\nu m \delta S_m \Rightarrow \delta S_m \propto m^{-1/2}$$

- How are spectra in kpar, kperp, m related?
- Next step: 5D simulations in AstroGK