Momentum transport and flow shear suppression of turbulence in tokamaks

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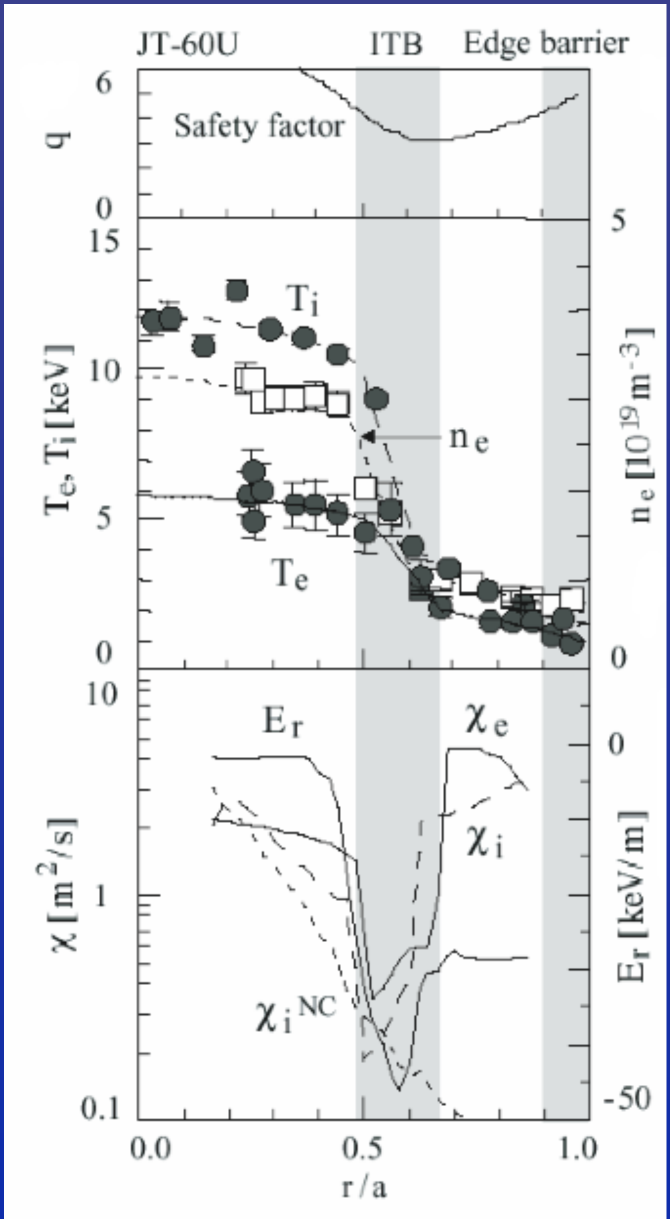


Objective

- Identify mechanism(s) for achieving enhanced confinement
- Internal transport barriers experimentally observed with temperature gradients well above threshold
- Often accompanied by large E x B shear and low or negative magnetic shear
- How do ITBs work, and how can we make them better?

Connor et al. (2004)

Multiple scale problem

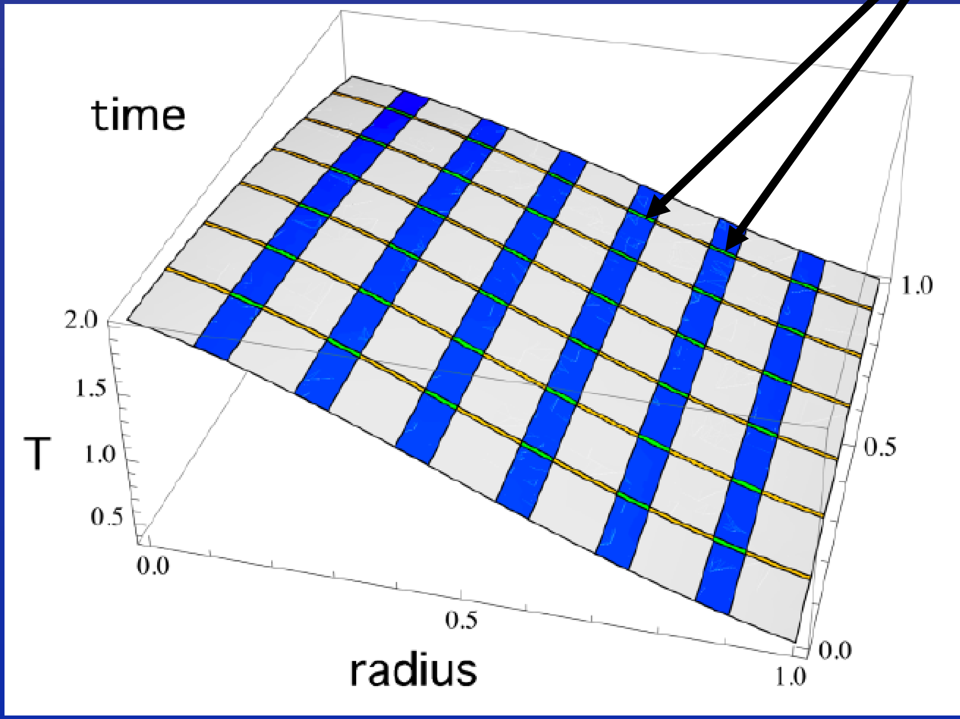
Physics	Perpendicular spatial scale	Temporal scale
Turbulence from ETG modes	$k_\perp^{-1}$ ~ 0.005 – 0.05 cm	ω_* ~ 0.5 - 5.0 MHz
Turbulence from ITG modes	$k_\perp^{-1}$ ~ 0.3 - 3.0 cm	ω_* ~ 10 - 100 kHz
Transport barriers	Measurements suggest width ~ 1 - 10 cm	100 ms or more in core?
Discharge evolution	Profile scales ~ 200 cm	Energy confinement time ~ 2 - 4 s

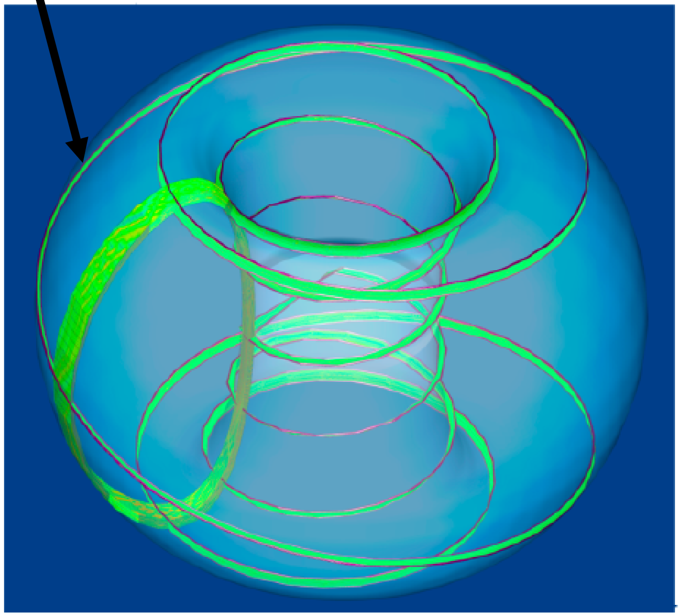
simulation cost: $(L_\parallel/\Delta_\parallel) \times (L_\perp/\Delta_\perp)^2 \times (L_v/\Delta_v)^2 \times (L_t/\Delta t) \sim 10^{21}$

Multiscale simulation (TRINITY)

- Turbulent fluctuations calculated in small regions of fine space-time grid embedded in “coarse” grid (for mean quantities)

Flux tube simulation domain





Overview

- Effect of rotational shear on turbulent transport
- Implications for local gradients (0D)
- Extension to radial profiles (1D)

Model

GK equation with mean flow satisfying $\frac{\rho}{L} \ll M \ll 1$

but : $\nabla u \sim v_{th}/L$

$$\boxed{\begin{aligned} \frac{dh}{dt} \quad &+ \quad \left(\mathbf{v}_{\parallel} + \mathbf{v}_D + \langle \mathbf{v}_E \rangle\right) \cdot \nabla h - \langle C[h] \rangle \\ &= \quad \frac{eF_0}{T} \frac{d\langle \varphi \rangle}{dt} - \langle \mathbf{v}_E \rangle \cdot \nabla \psi \left(\frac{dF_0}{d\psi} + \frac{mv_{\parallel}}{T} \frac{RB_{\phi}}{B} \frac{d\omega}{d\psi} F_0 \right) \end{aligned}}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + R\omega(\psi)\hat{\mathbf{e}}_{\phi} \cdot \nabla$$

$$\boxed{\mathbf{u} = R\omega\hat{\mathbf{e}}_{\phi}}$$

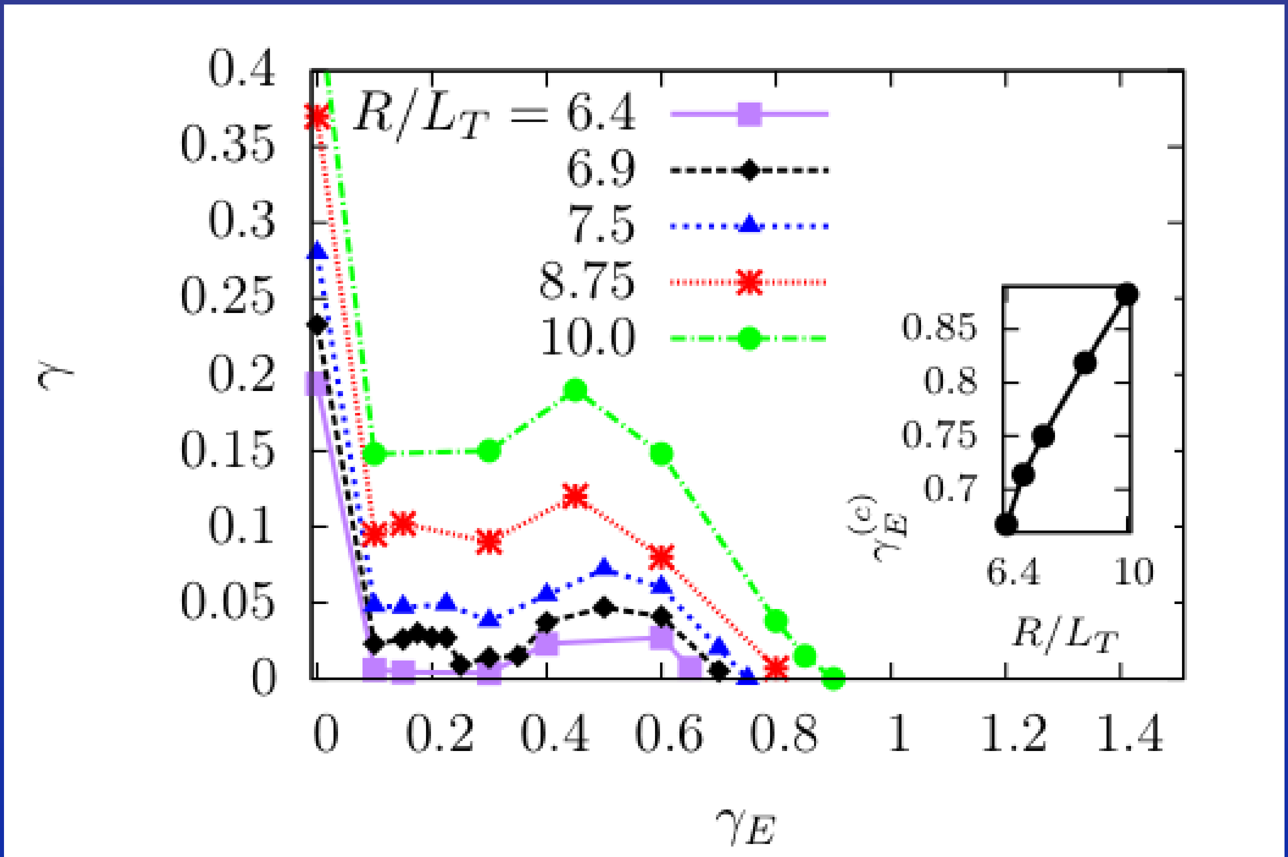
$$\gamma_E \equiv \frac{\psi}{q} \frac{d\omega}{d\psi} \frac{R_0}{v_{th}}$$

Local approximation:

$$\omega(\psi) \approx \omega(\psi_0) + (\psi - \psi_0) \left.\frac{d\omega}{d\psi}\right|_{\psi_0}$$

Linear stability (GS2)

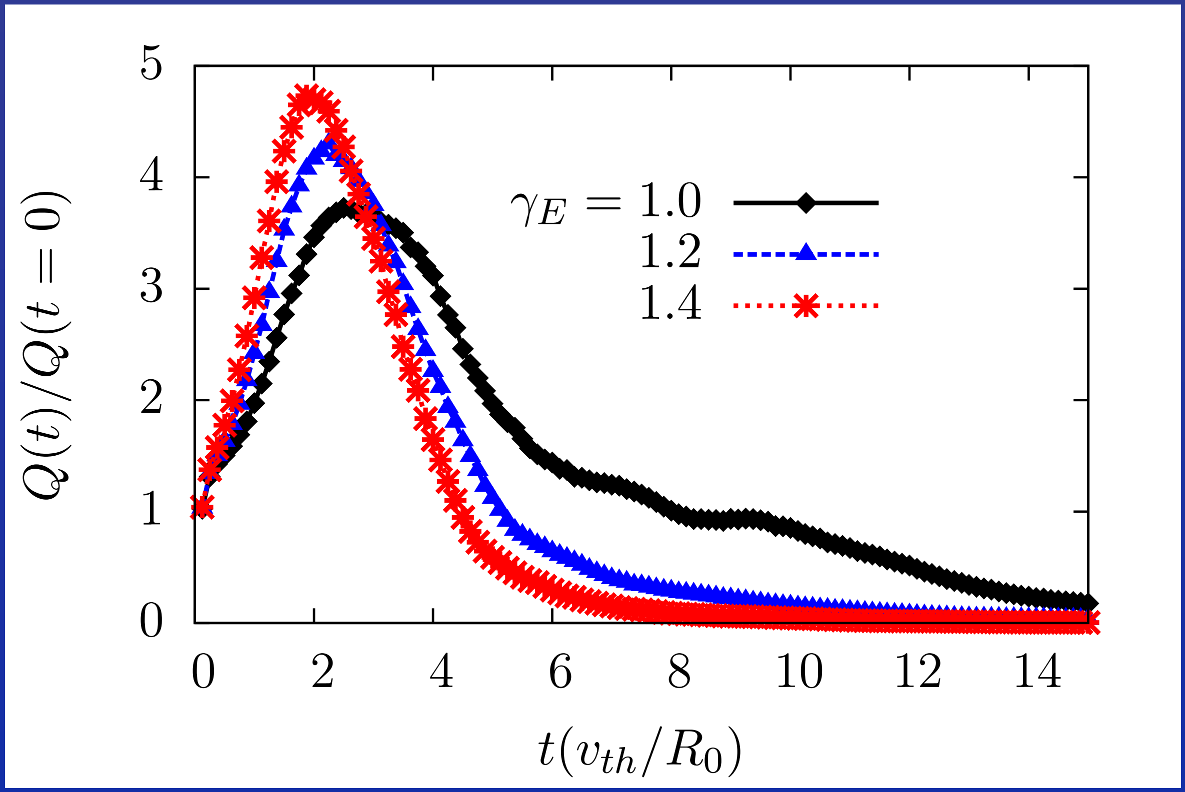
Cyclone base case: $r/R = 0.18 \quad q = 1.4 \quad \hat{s} = 0.8$



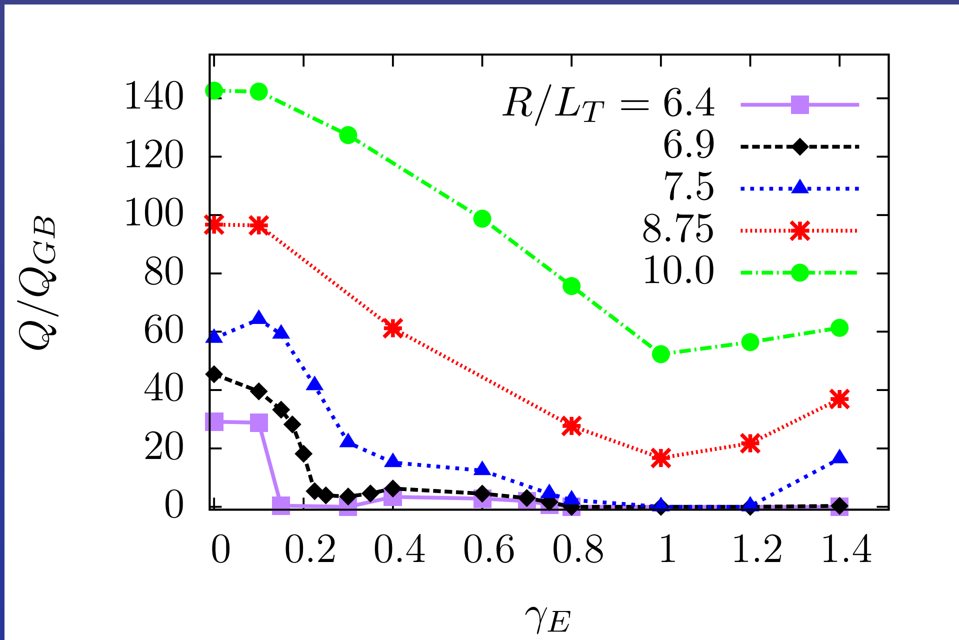
- ITG drive at small shear
- ITG/PVG drive at moderate shear
- Stabilization at large shear
- Roughly linear dependence of critical flow shear on R/LT

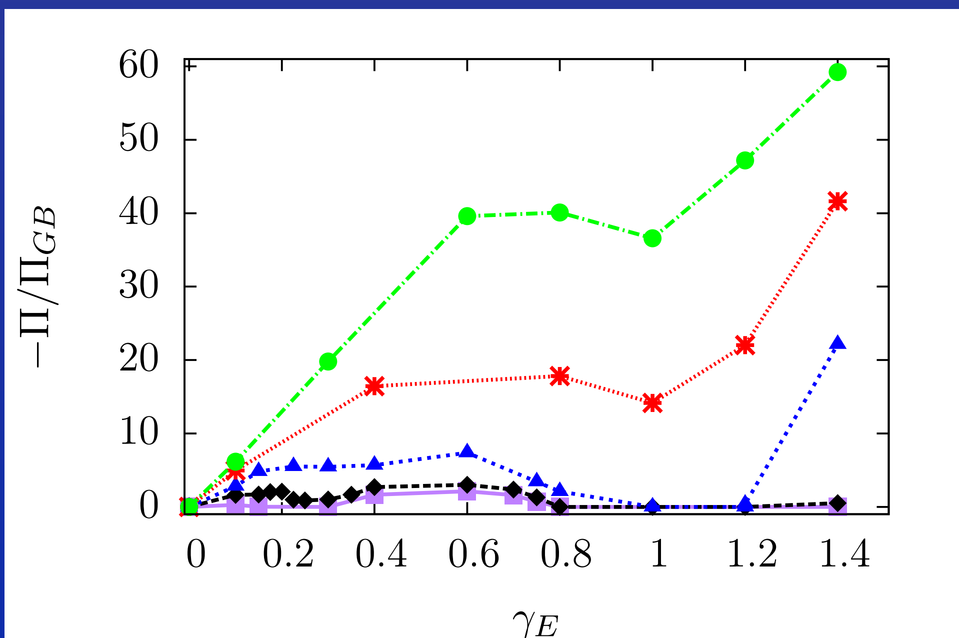
Barnes et al., 2010 (arXiv:1007.3390)

Transient growth



- Beyond critical shear value, transient linear growth
- Amplification of initial amplitude increases with shear
- Cf. Newton et al., 2010 (arXiv: 1007.0040)



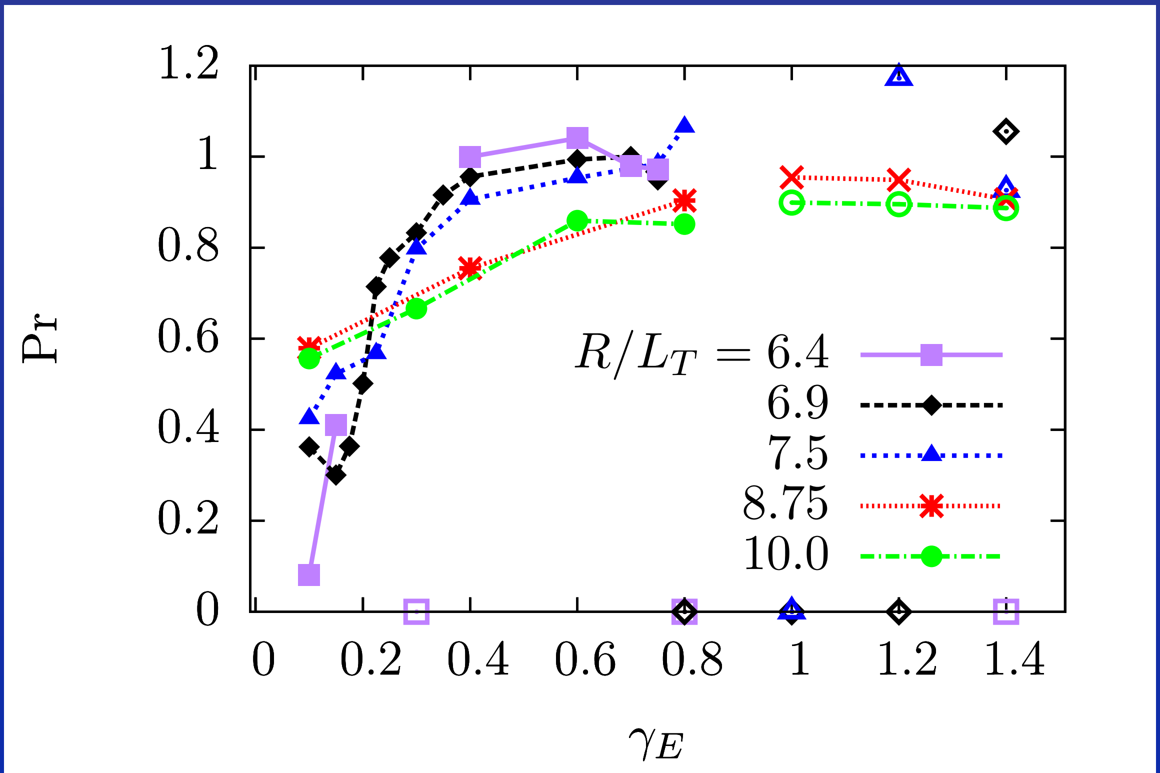


- Fluxes follow linear trends up to linear stabilization point
- Subcritical (linearly stable) turbulence beyond this point
- Optimal flow shear for confinement
- Possible hysteresis
- Maximum in momentum flux => possible bifurcation

Turbulent Prandtl number

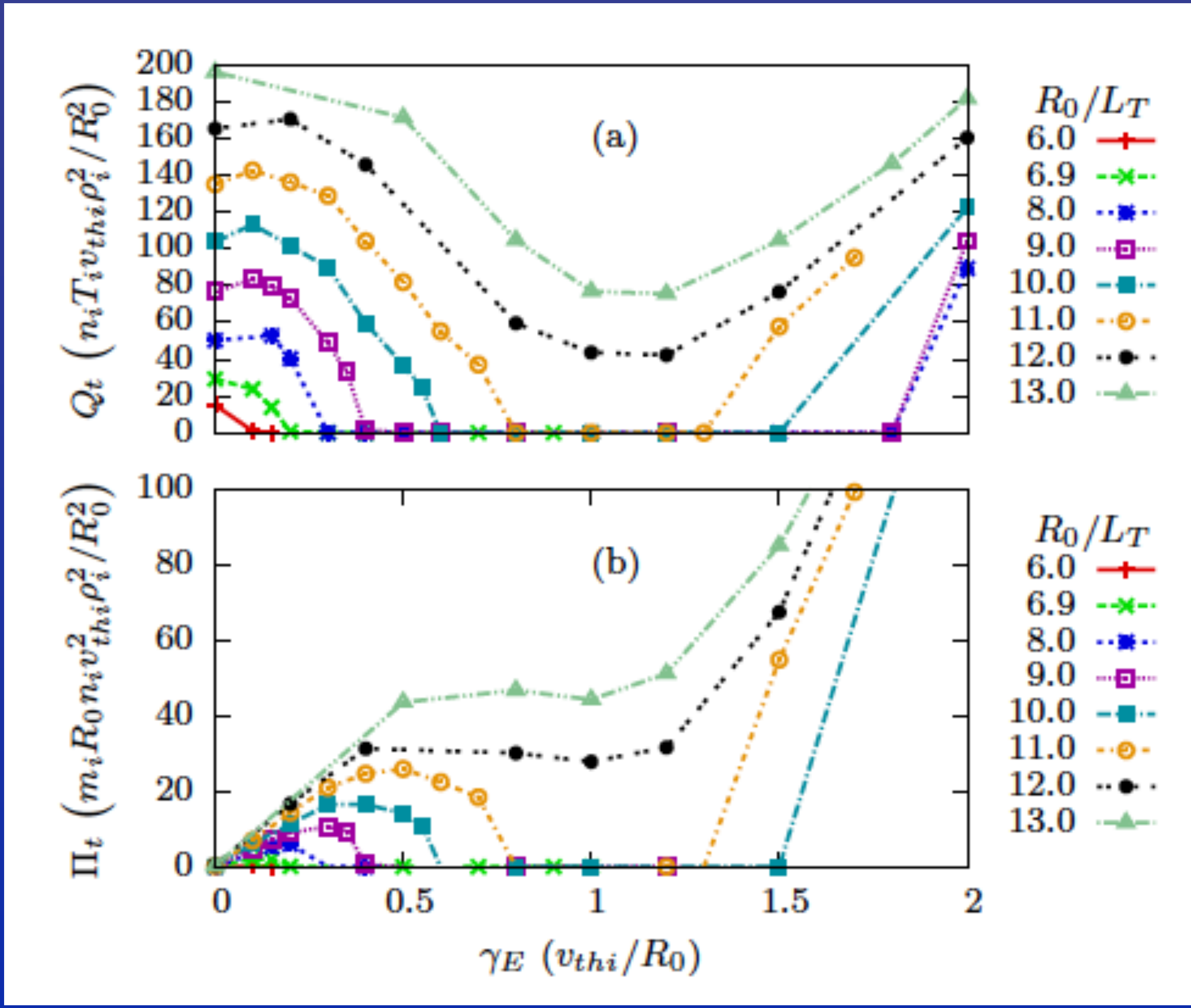
$$\mathrm{Pr} = \frac{\nu_i}{\chi_i} \qquad \Pi_i = -m_i v_{th}(qR_0/r)\nu_i\gamma_E$$

$$Q_i = -\chi_i dT_i/dr$$



- Prandtl number tends to shear- and R/LT-independent value of order unity (in both turbulence regimes)

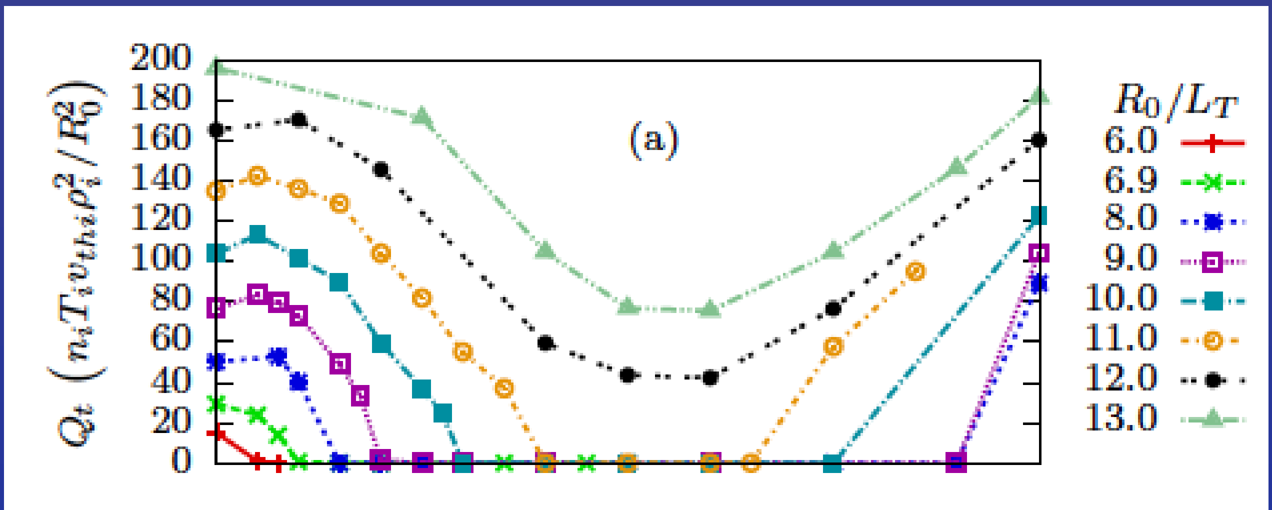
Zero magnetic shear

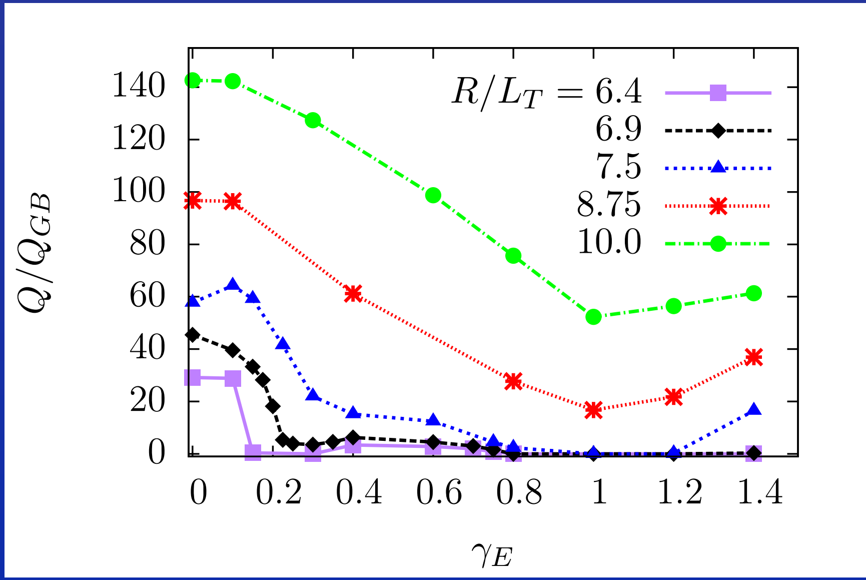


- Similar…sort of
- All turbulence subcritical

Highcock et al., 2010 (arXiv:1008.2305)

Zero magnetic shear





- Similar…sort of
- All turbulence subcritical
- Very different critical flow shear values

Overview

- Effect of rotational shear on turbulent transport
- Implications for local gradients (0D)
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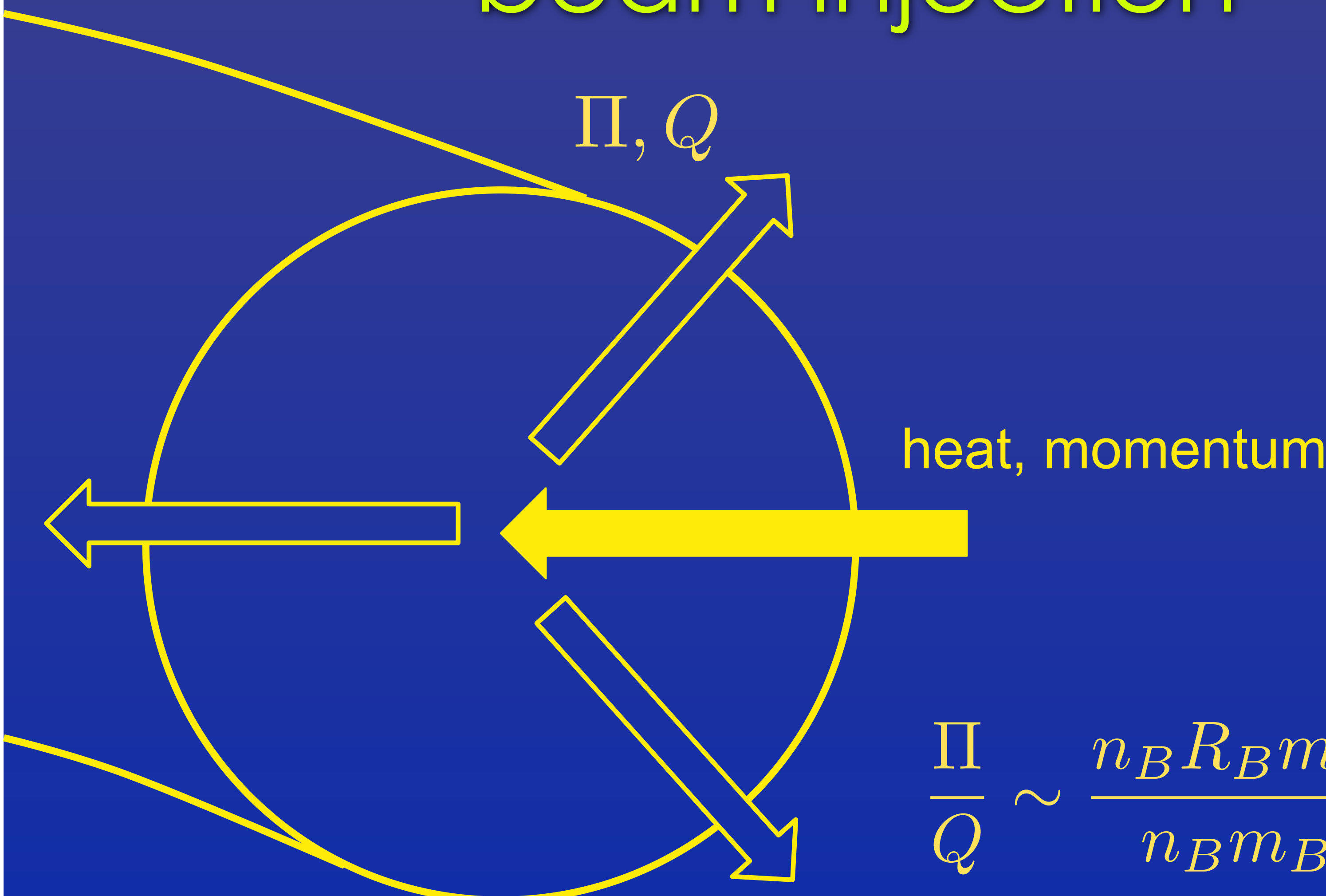
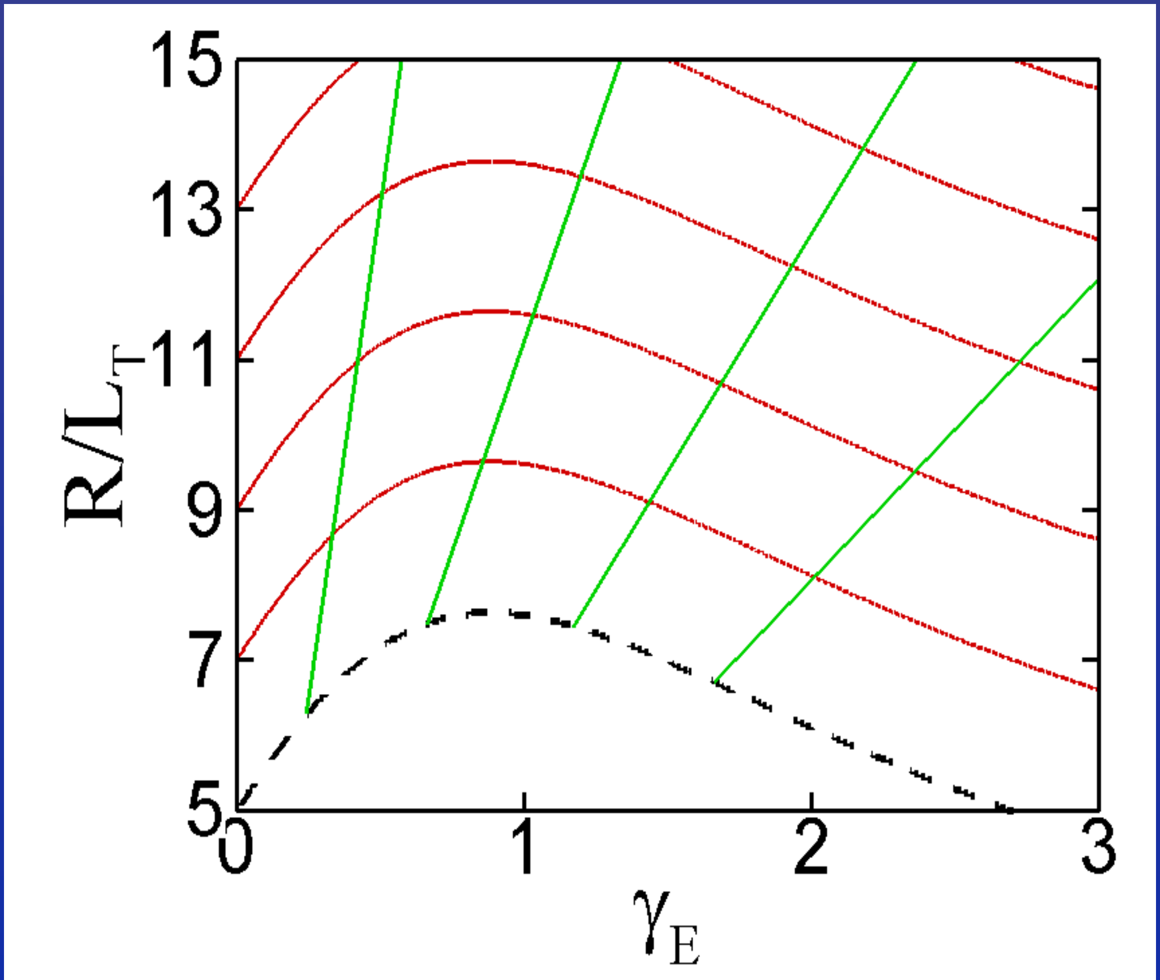
Power/Torque balance for beam injection



$$\frac{\Pi}{Q} \sim \frac{n_B R_B m_B v_B^2}{n_B m_B v_B^3} = \frac{R_B}{v_B}$$

Balance w/o neoclassical



Parra et al., 2010

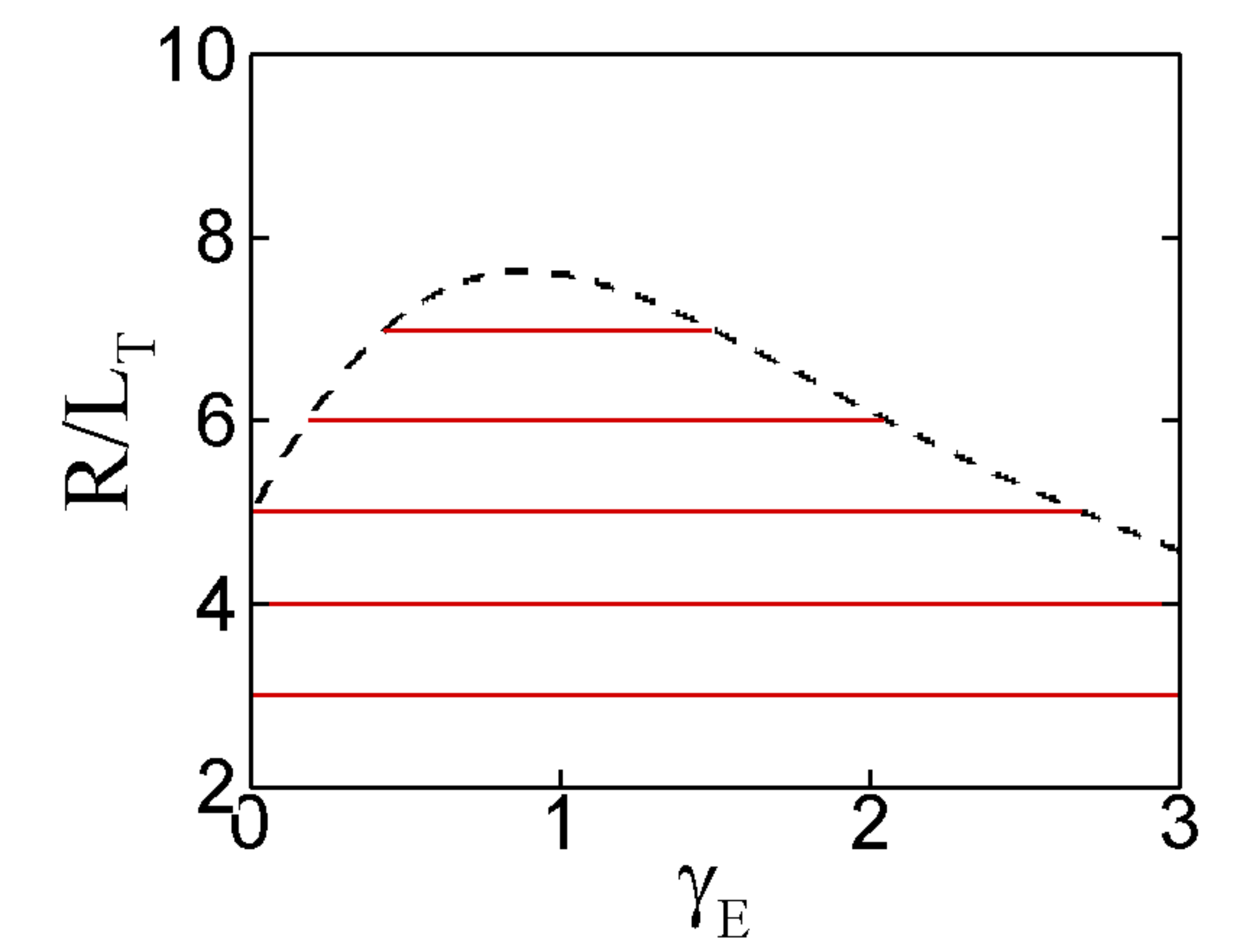
- Q = red lines
- Π/Q = green lines

$$\frac{R}{L_T} = \frac{Pr_t}{\Pi/Q}\gamma_E$$

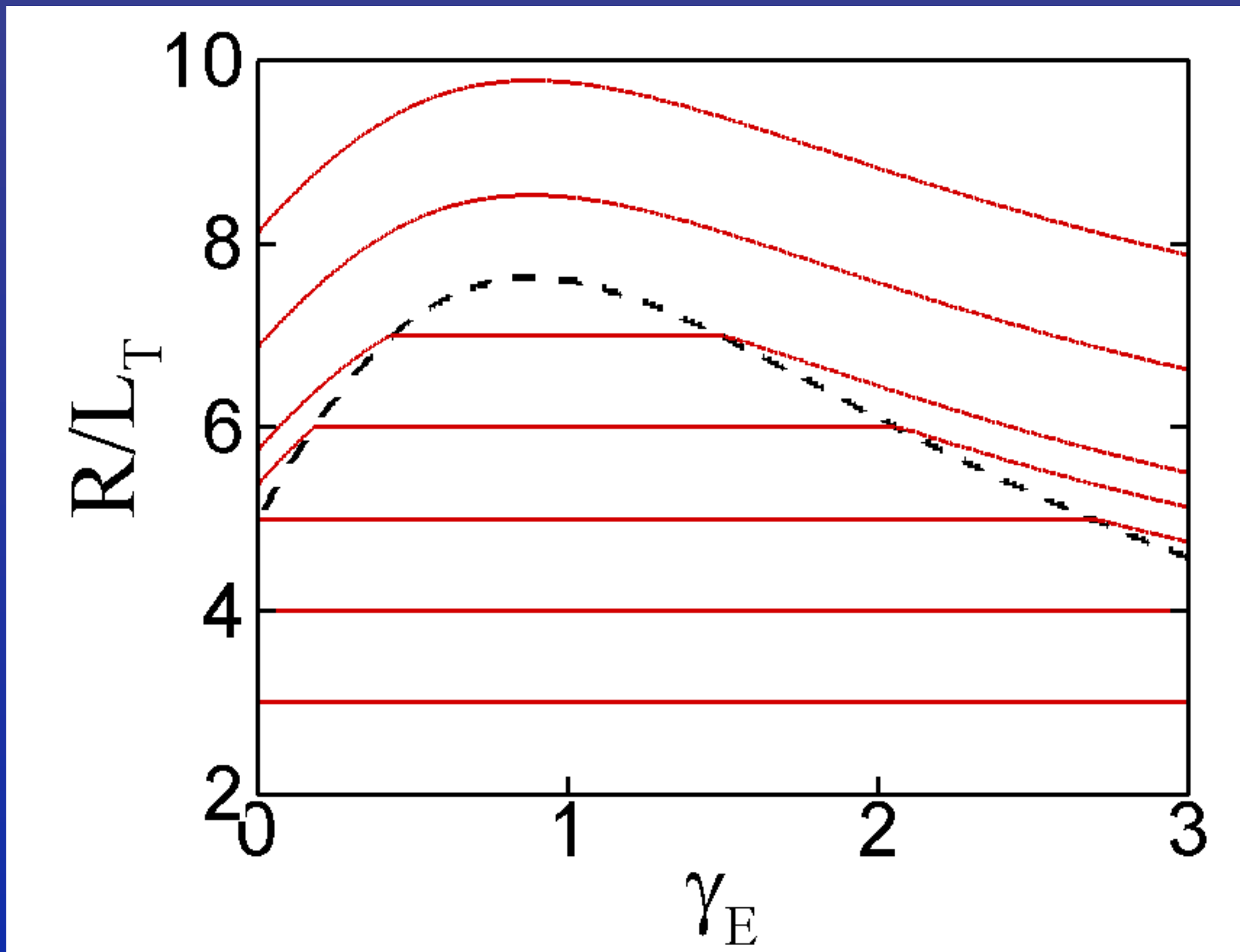
- Critical gradient = dashed line
- For given Π/Q and Q, only one solution

  No bifurcation!

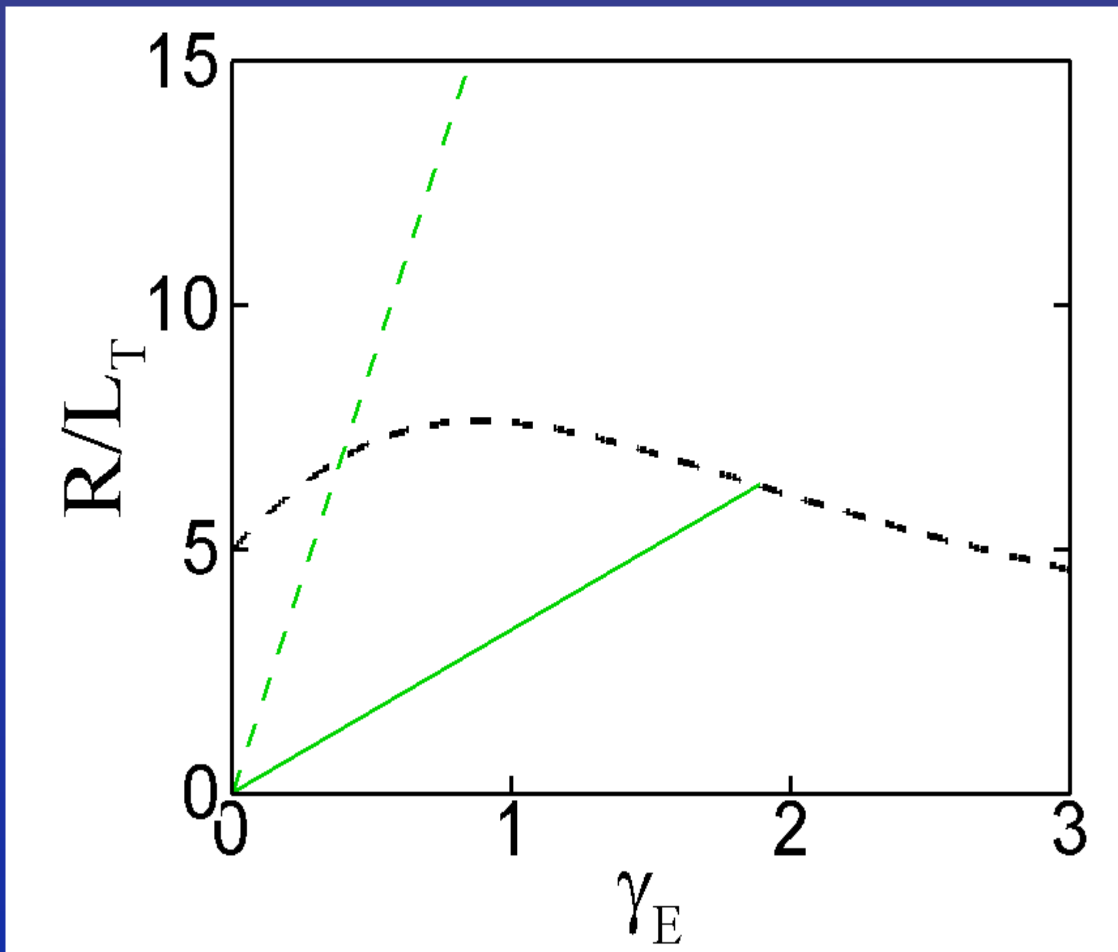
Neoclassical energy flux



Neoclassical energy flux



Curves of constant Π/Q



- Neoclassical

$$\frac{R}{L_T} = \frac{Pr_n}{\Pi/Q}\gamma_E$$

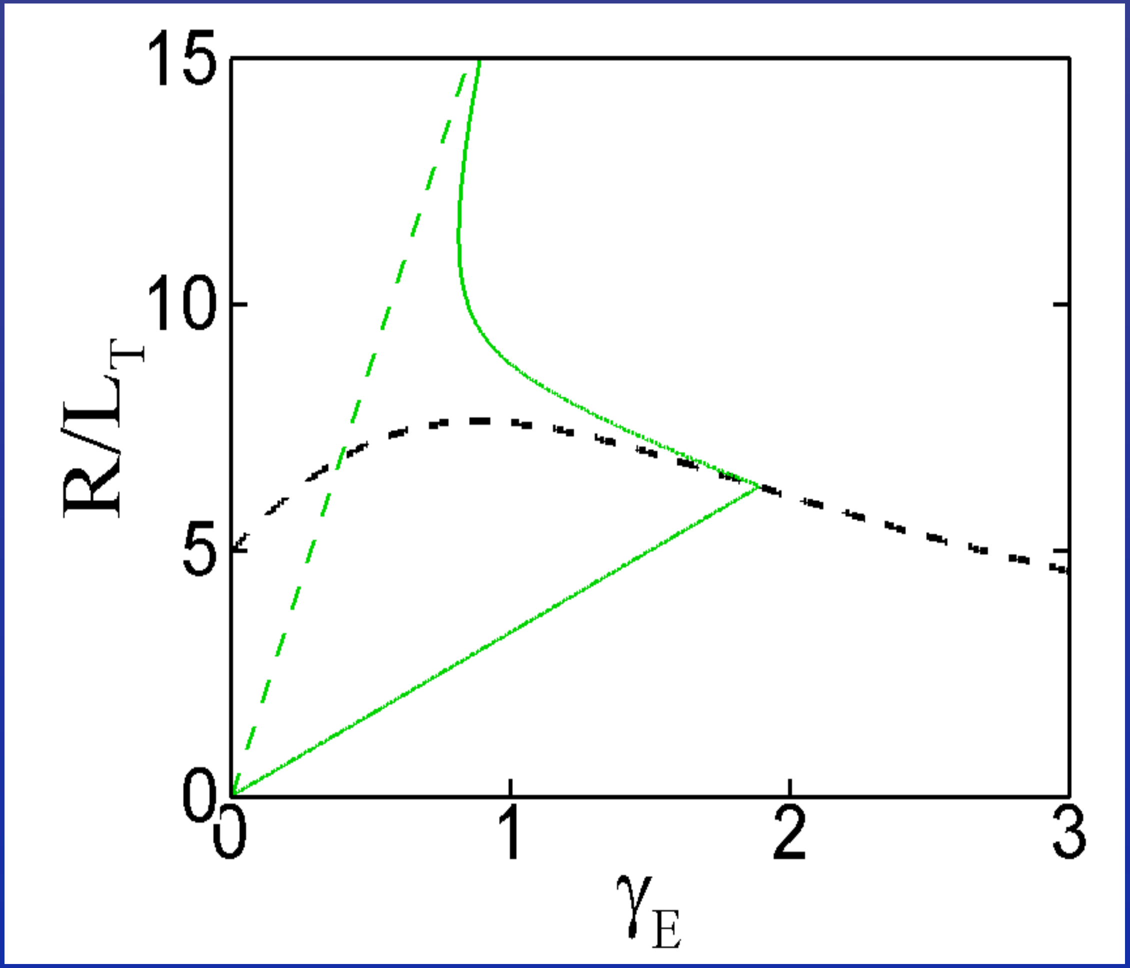
- Turbulent

$$\frac{R}{L_T} = \frac{Pr_t}{\Pi/Q}\gamma_E$$

- Prandtl numbers

$$Pr_n < Pr_t$$

Curves of constant Π/Q



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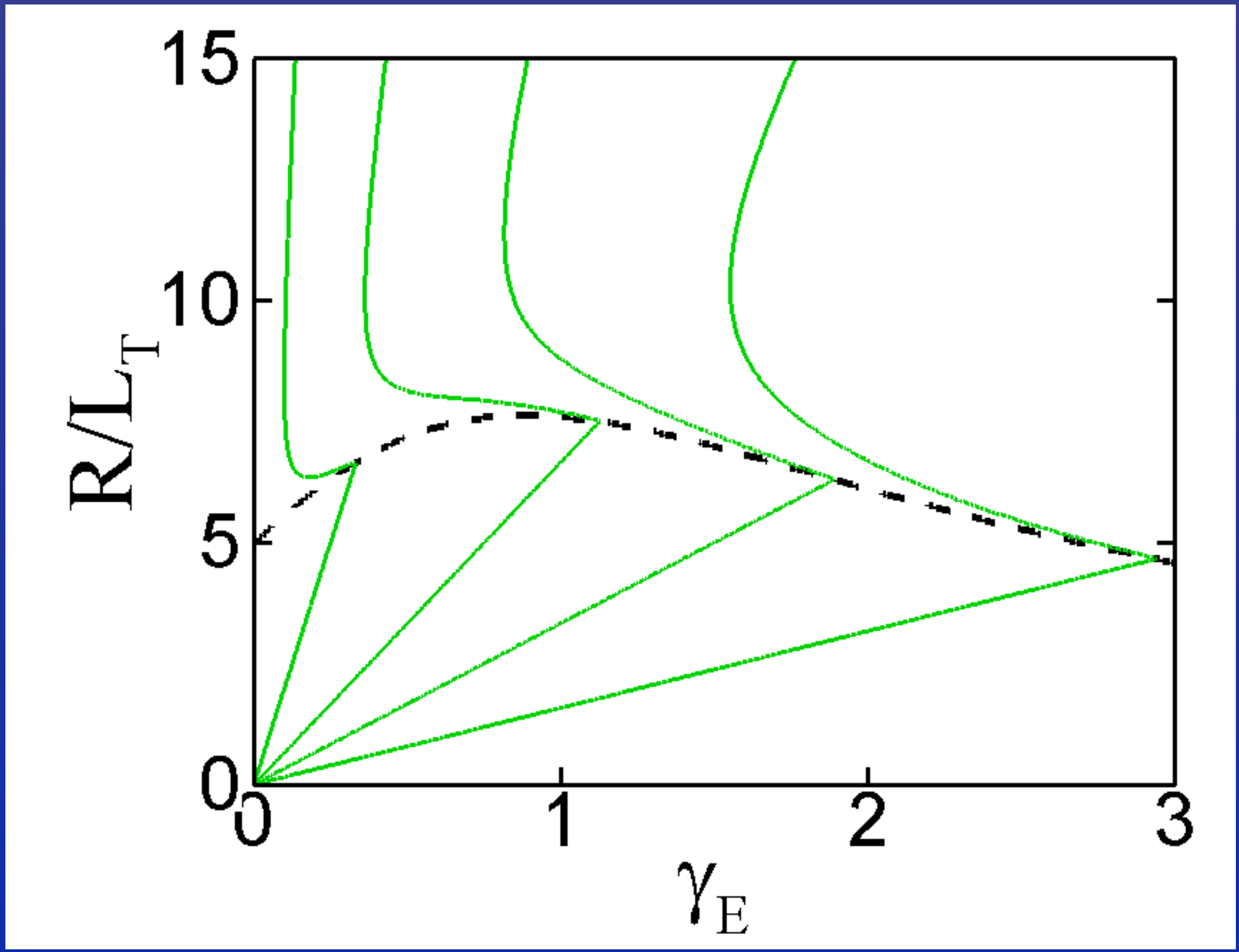
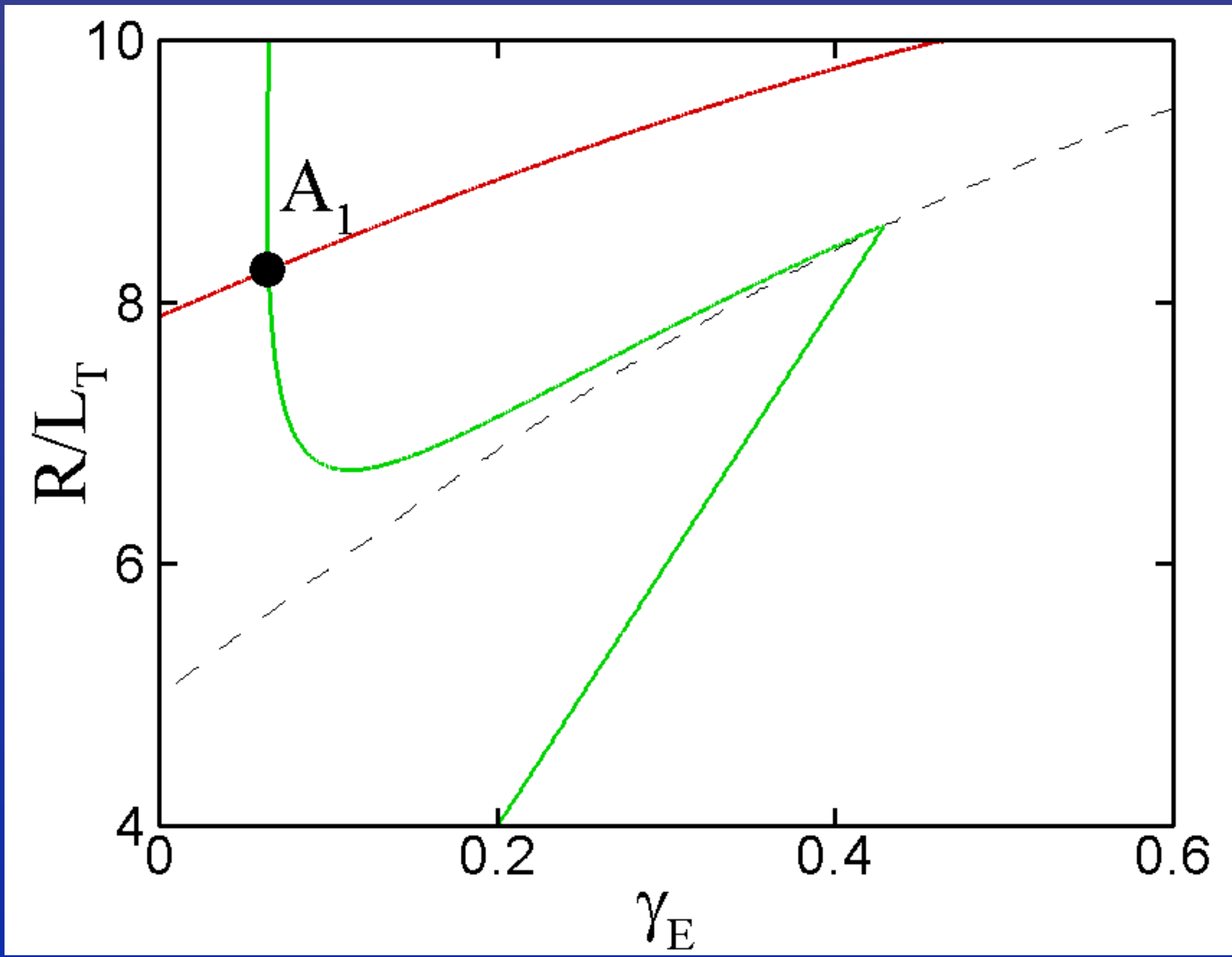


Possible solutions



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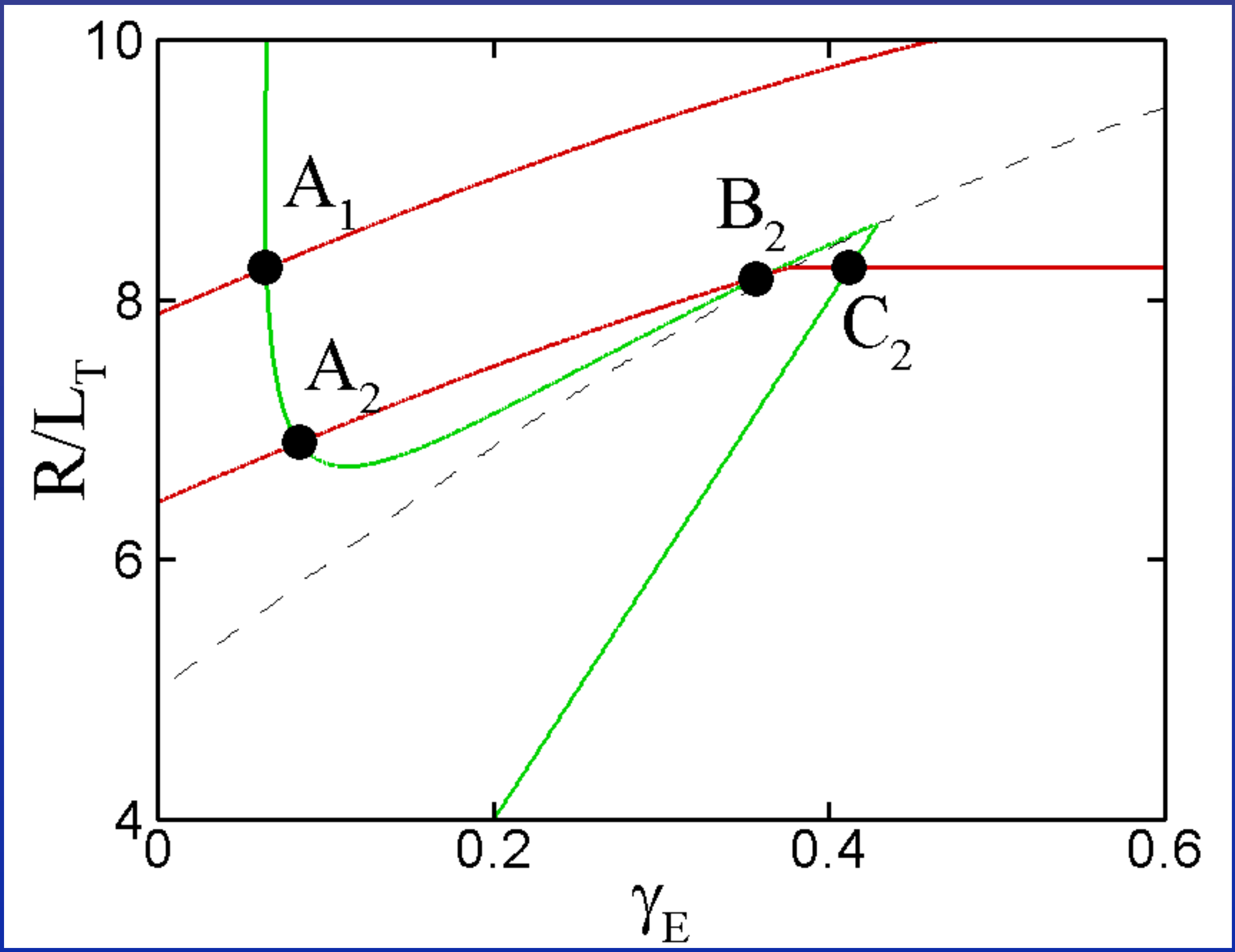

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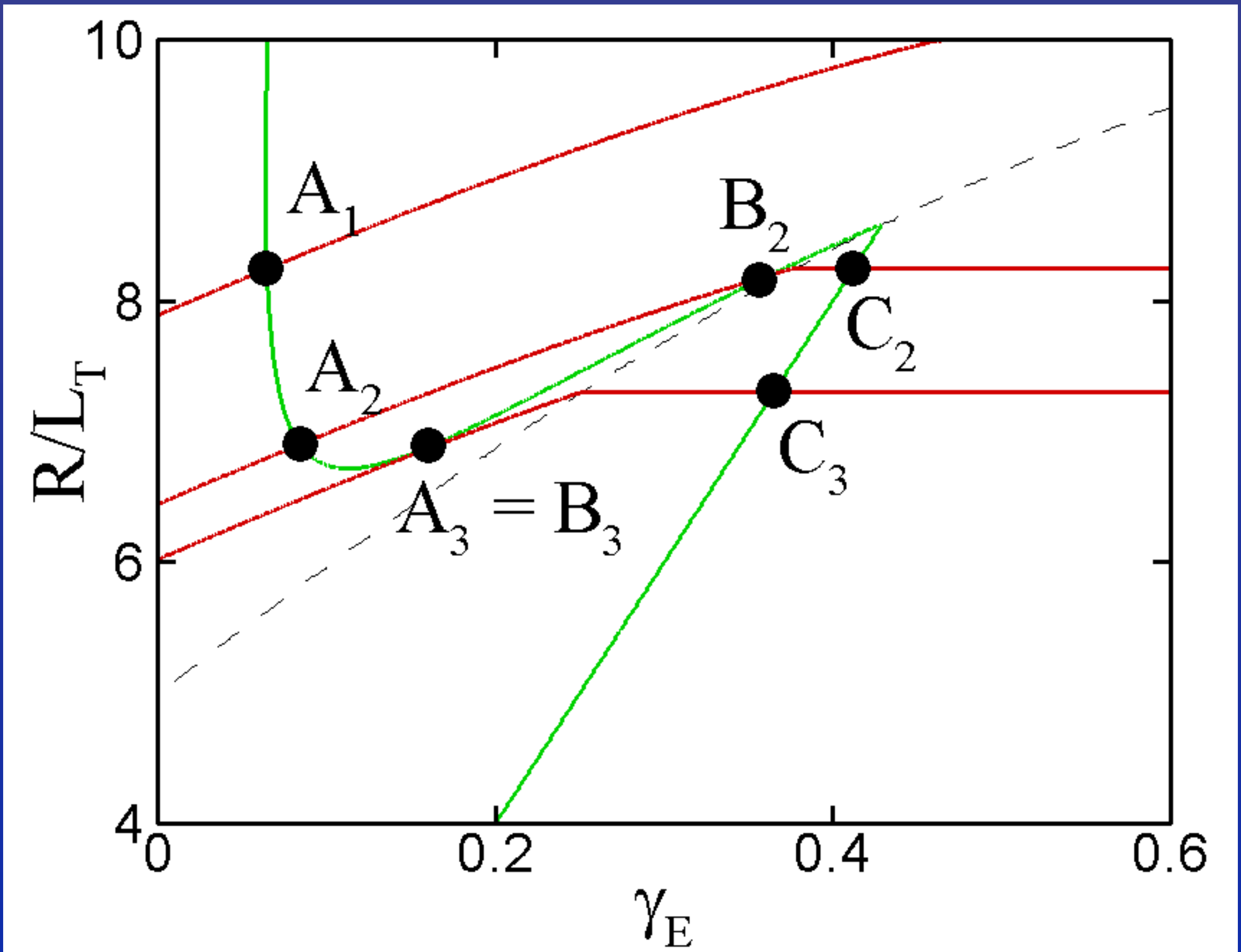
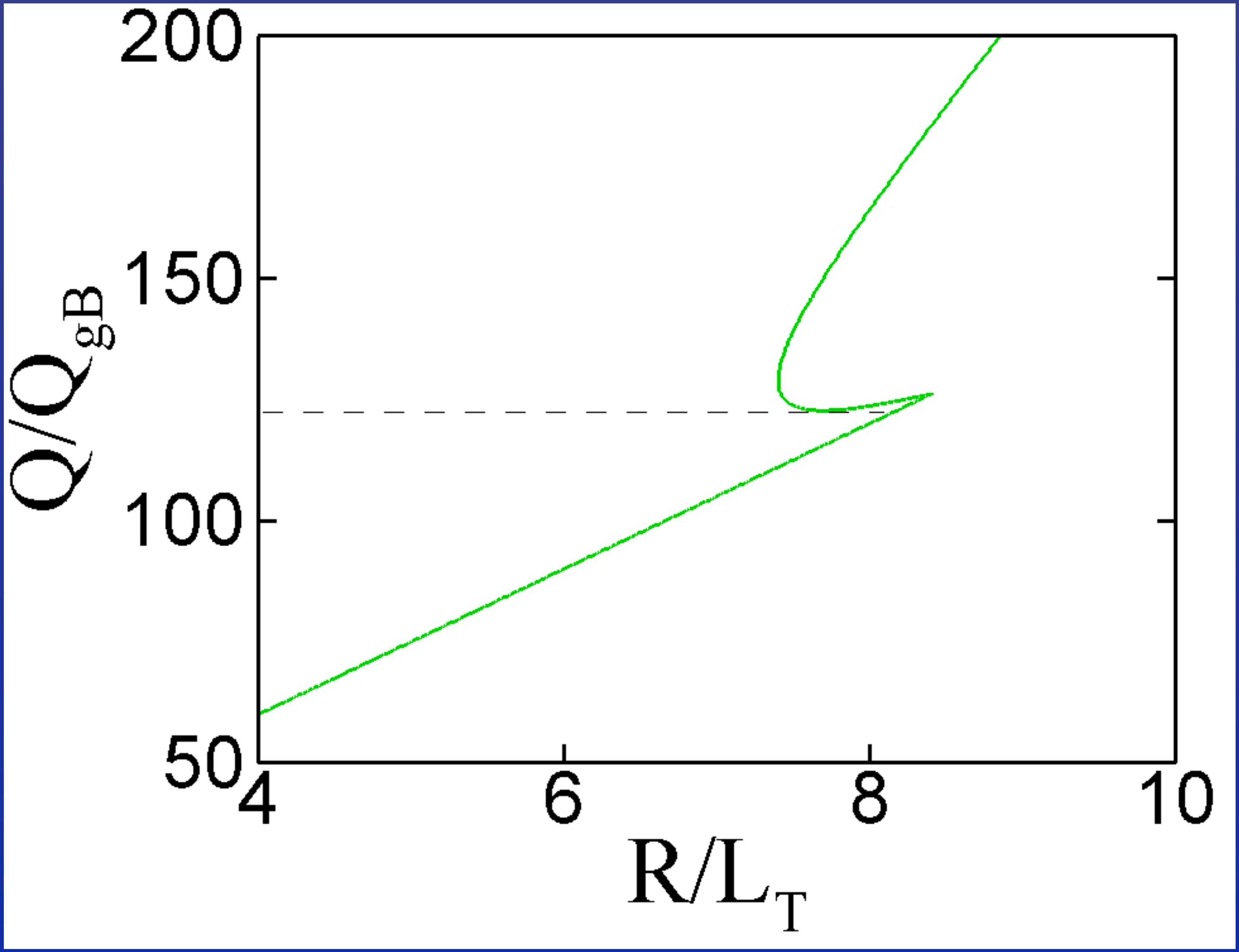
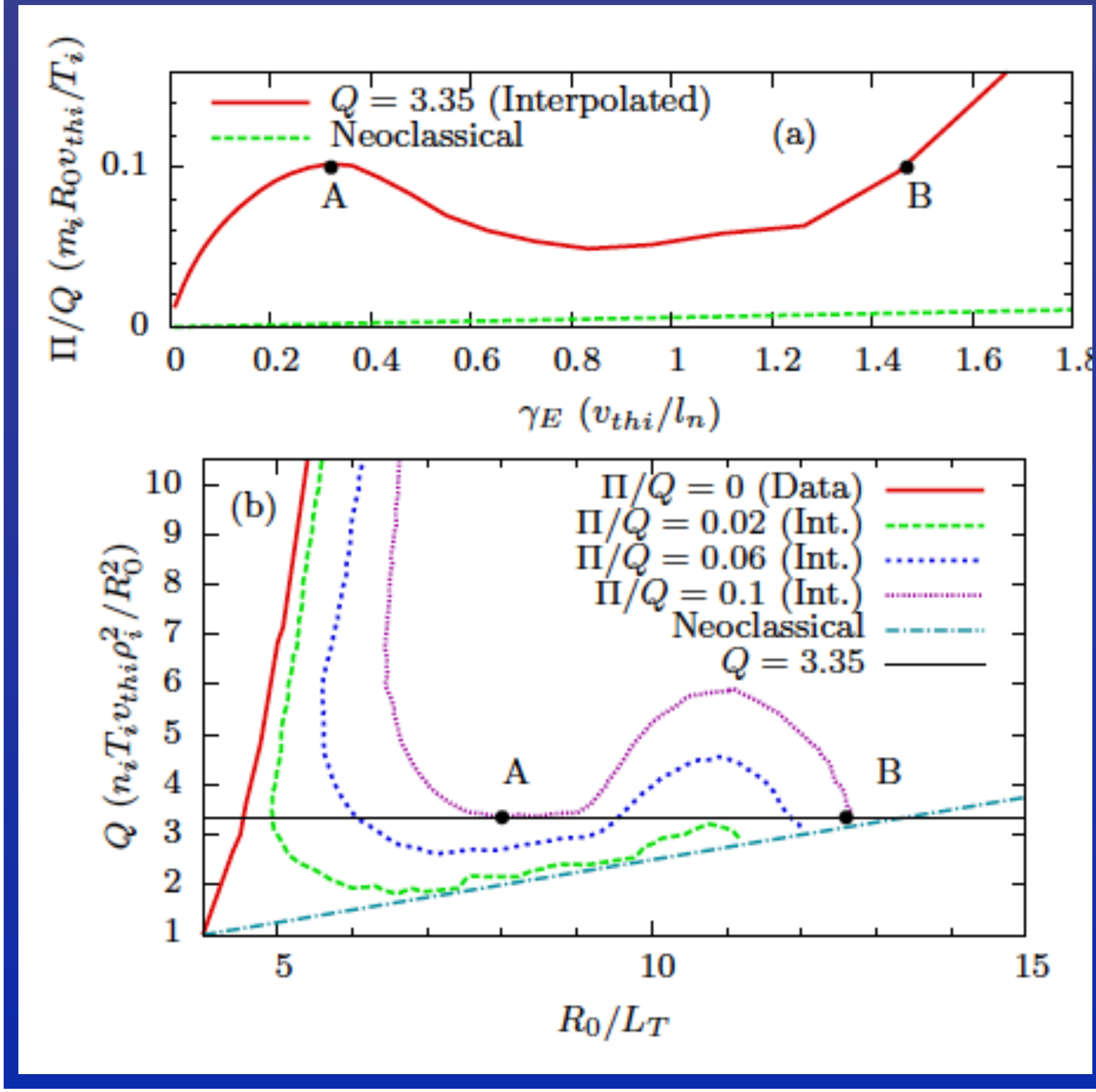


Total energy flux



Bifurcations



- Consider inverse problem: for fixed fluxes, what are gradients?
- With inclusion of neoclassical fluxes, we see bifurcation to much larger flow shear and R/LT

Highcock et al., 2010

Overview

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- Implications for local gradients (0D)
- Extension to radial profiles (1D)

Transport equations in GK

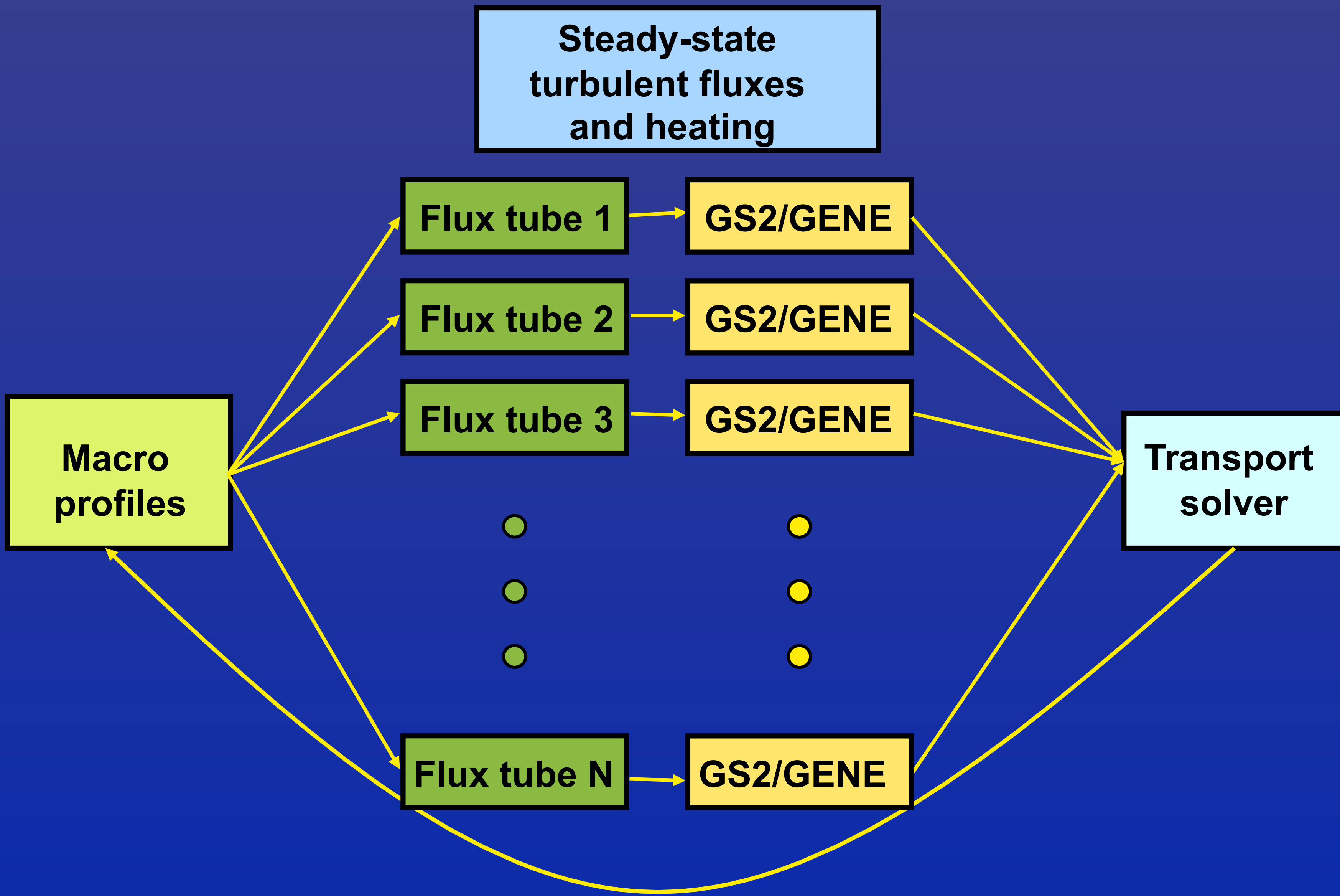
Moment equations for evolution of mean quantities:

$$
\begin{aligned}
\frac{\partial n_s}{\partial t} &= -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V' \left\langle \mathbf{\Gamma}_s \cdot \nabla\psi \right\rangle\right) + S_n \\
\frac{3}{2}\frac{\partial n_s T_s}{\partial t} &= -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V' \left\langle \mathbf{Q}_s \cdot \nabla\psi \right\rangle\right) \\
&+ T_s \left(\frac{\partial \ln n_s}{\partial \psi} - \frac{3}{2}\frac{\partial \ln T_s}{\partial \psi}\right)\left\langle \mathbf{\Gamma}_s \cdot \nabla\psi \right\rangle + \frac{\partial \ln T_s}{\partial \psi}\left\langle \mathbf{Q}_s \cdot \nabla\psi \right\rangle \\
&- \left\langle \int d^3v \frac{h_s T_s}{F_{0s}} \left\langle C[h_s] \right\rangle_{\mathbf{R}} \right\rangle + n_s \nu_\epsilon^{su}\left(T_u - T_s\right) + S_p
\end{aligned}
$$

$$
\frac{\partial L}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V' \sum_s \left\langle \pi_s \right\rangle\right) + S_L
$$

Sugama (1997), Abel (2010)

TRINITY schematic



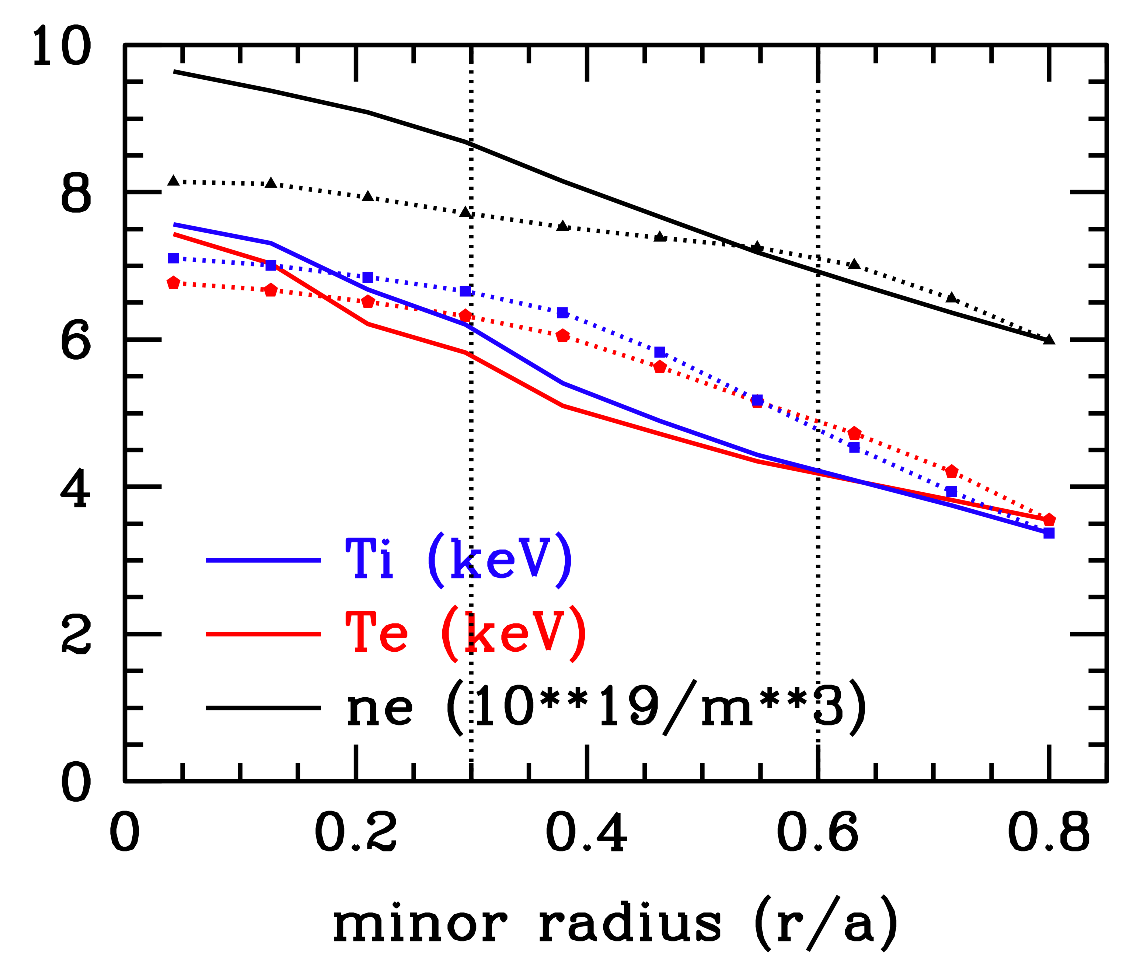
TRINITY transport solver

- Transport equations are stiff, nonlinear PDEs. Implicit treatment via Newton's Method (multi-step BDF, adaptive time step) allows for time steps ~0.1 seconds (vs. turbulence sim time ~0.001 seconds)
- Challenge: requires computation of quantities like

$$\Gamma_j^{m+1} \approx \Gamma_j^m + \left(\mathbf{y}^{m+1} - \mathbf{y}^m\right) \left.\frac{\partial \Gamma_j}{\partial \mathbf{y}}\right|_{\mathbf{y}^m} \qquad \mathbf{y} = \left[\{n_k\}, \{p_{i_k}\}, \{p_{e_k}\}\right]^T$$

- Local approximation: $\dfrac{\partial \Gamma_j}{\partial n_k} = \dfrac{\partial \Gamma_j}{\partial n_j} + \dfrac{\partial \Gamma_j}{\partial (R/L_n)_j} \dfrac{\partial (R/L_n)_j}{\partial n_k}$
- Simplifying assumption: normalized fluxes depend primarily on gradient scale lengths

Evolving density profile



- Costs ~120k CPU hrs (<10 clock hrs)
- Dens and temp profiles agree within ~15% across device
- Energy off by 5%
- Incremental energy off by 15%
- Flow shear absent

Model fluxes

- Simple model for fluxes with parameters chosen to fit zero magnetic shear results from GS2:

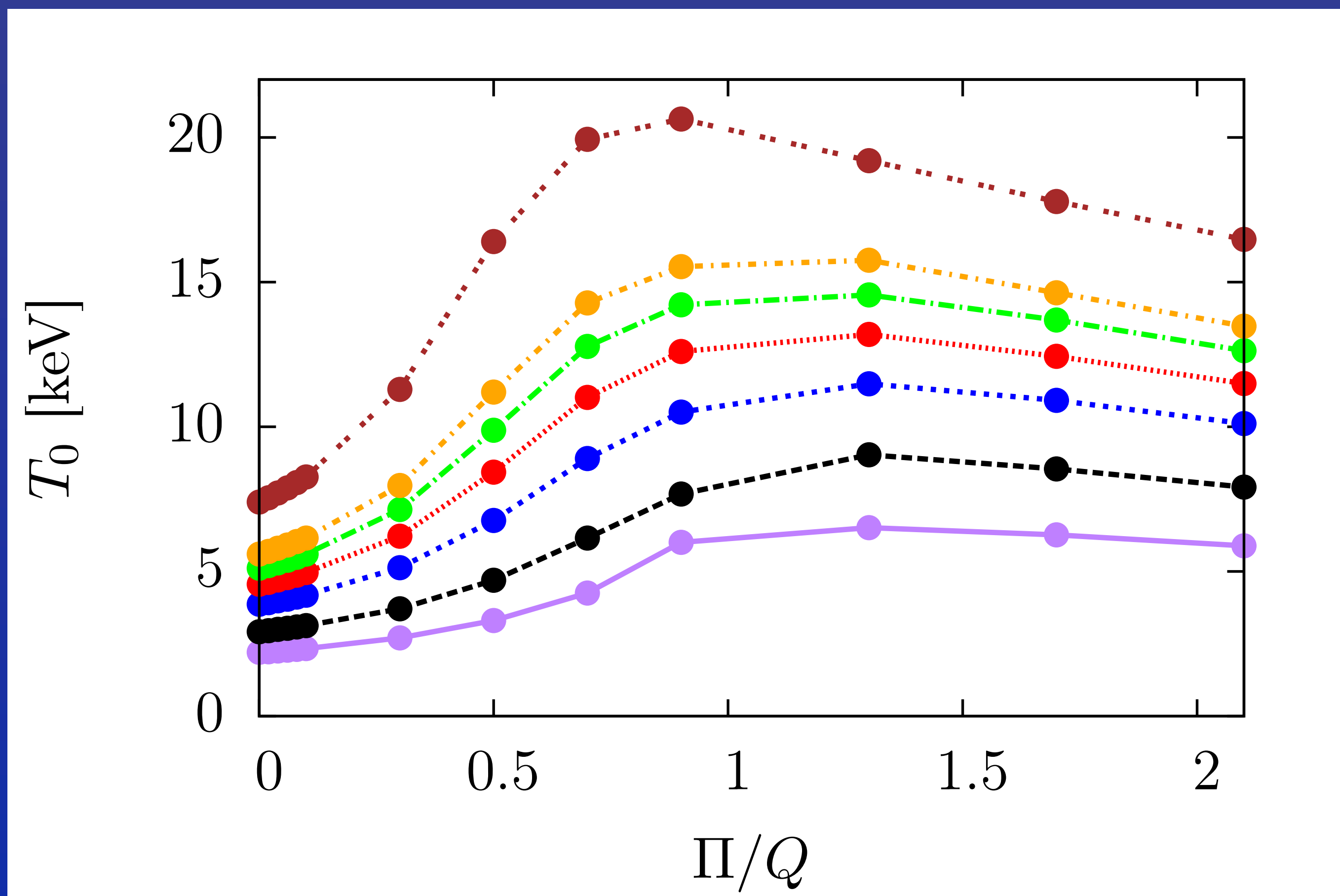
$$Q = Q_t + Q_n \qquad\qquad \Pi = \Pi_t + \Pi_n$$

$$Q_t = \chi_t \left(\frac{R}{L_T} - \left(\frac{R}{L_T} \right)_c [\gamma_E] \right)$$

$$\Pi_t = Q_t \mathrm{Pr}_t \frac{\gamma_E}{R/L_T}$$

$$\left(\frac{R}{L_T} \right)_c [\gamma_e] = \frac{\alpha_1 \gamma_E + (R/L_T)_{c0}}{1 + \alpha_2 \gamma_E^2}$$

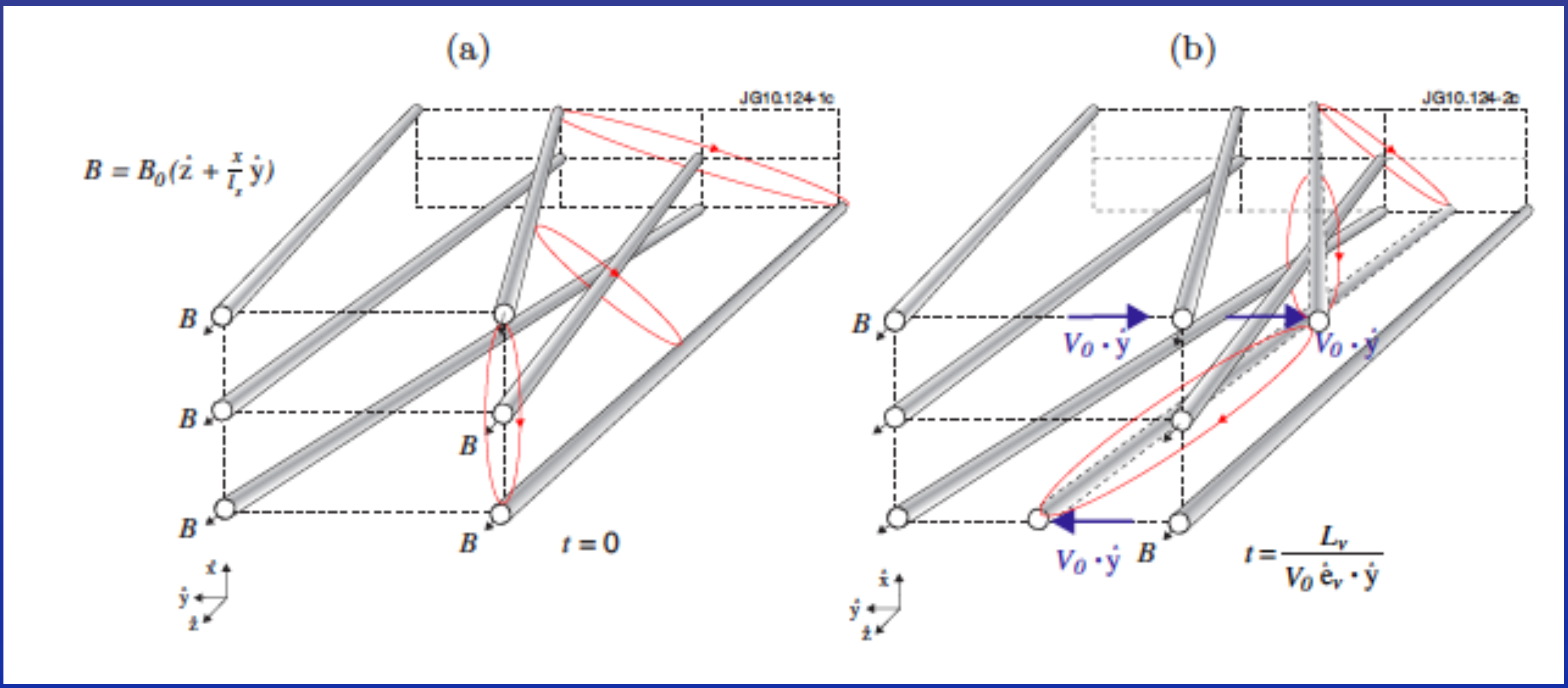
Preliminary results



Conclusions

- Maximum temperature gradient for given heat flux due to flow shear driven subcritical turbulence
- Turbulent Prandtl number is constant of order unity for moderate to large flow shear values.
- Flow shear more effective in stabilizing turbulence as zero magnetic shear
- Bifurcation observed in 0D model (GS2 simulations). Requires neoclassical contribution to fluxes
- Preliminary work on extension to 1D transport simulations shows peak in core temperature at beam energy which varies with power (peak at higher energies for higher powers)

Shear stabilization



Newton et al., 2010 (arXiv:1007.0040)

Bifurcation condition

$$\left.\frac{\partial(R/L_T)}{\partial\gamma_E}\right|_Q \simeq \left(1-\frac{\chi_n}{\overline{\chi}_t}\right)\frac{d(R/L_T|_c)}{d\gamma_E}$$

$$\left.\frac{\partial(R/L_T)}{\partial\gamma_E}\right|_{\Pi/Q} \simeq \frac{d(R/L_T|_c)}{d\gamma_E} - \frac{\overline{\Pi}}{1-\overline{\Pi}/\overline{Q}}\frac{R/L_T|_c}{\gamma_E} + \frac{1}{1-\overline{\Pi}/\overline{Q}}\left[\overline{\Pi}+\frac{\chi_n}{\overline{\chi}_t}\left(\frac{\overline{\Pi}}{\overline{Q}}-\frac{Pr_n}{Pr_t}\right)\right]\frac{d(R/L_T|_c)}{d\gamma_E}.$$

$$\frac{d(R/L_T|_c)}{d\gamma_E} \geq \frac{\overline{\Pi}}{\overline{\Pi}+(\chi_n/\overline{\chi}_t)(1-Pr_n/Pr_t)}\frac{R/L_T|_c}{\gamma_E} > \frac{B_\phi}{B_p}\frac{Pr_n^2}{Pr_t}\frac{1}{\Pi/Q},$$

Parra et al., 2010