Numerical implementation of a local, δf-gyrokinetic model for intrinsic rotation

Michael Barnes and Felix Parra

University of Oxford

Culham Centre for Fusion Energy

Motivation

- Significant ‘intrinsic’ rotation observed in experiments with no obvious momentum injection
- Rotation profiles depend on heating mechanism and can reverse sign – can’t be explained solely by ‘pinch’ from edge of plasma (see Parra poster)
- Lowest-order GK equation gives no momentum flux for up-down symmetric plasma without flow
- Must include higher-order terms to calculate flux generating intrinsic rotation

Outline

- First derive how fluctuations scale with B/B_p so we can simplify higher-order GK equation
- Identify new terms to add to lowest-order GK equation implemented in GS2
- Details of implementation
- Preliminary results for ‘intrinsic’ momentum flux profiles
- Conclusions
- Bonus material: Fluctuation scalings for inertial and dissipation ranges

Fluctuation scalings: critical balance

- Conjecture: parallel streaming/wave propagation time and nonlinear decorrelation time comparable at all scales
- Physical idea: two points along field can be correlated only if information propagates between the points before turbulence is decorrelated in perpendicular plane

$$\frac{v_{\mathrm{th}}}{\ell_{\|}} \sim \tau_{nl}^{-1} \sim \frac{v_{\mathrm{th}}}{R} \frac{\rho_i^2}{\ell_x \ell_y} \Phi_\ell$$

$$\Phi_\ell \equiv \frac{Ze\varphi_\ell}{T} \frac{R}{\rho_i} \equiv \Phi(\mathbf{r} + \ell) - \Phi(\mathbf{r})$$

- For details of turbulence scalings, see Ref. [1]

Characteristic perpendicular scale

- Define outer scale as scale where injection rate is comparable to nonlinear decorrelation rate

$$\omega_* \sim \frac{\rho_i v_{\rm th}}{\ell_y^o L_T} \sim \tau_{nl}^{-1} \sim \frac{v_{\rm th}}{\ell_\parallel^o}$$

- Conjecture: parallel length scale of turbulence at driving scale is the connection length
- Physical idea: modes cannot extend much beyond connection length due to good curvature

$$\ell_\parallel^o \sim qR \Rightarrow \boxed{\frac{\ell_y^o}{\rho_i} \sim q\frac{R}{L_T}} \quad (1)$$

Fluctuation amplitudes

- Conjecture: Scale lengths in the two dimensions of the perpendicular plane are comparable
- Physical idea: linear drive creates $\ell_x^{-1} \lesssim \ell_y^{-1}$. Smaller ℓ_x set nonlinearly through magnetic and zonal flow shear:

$$\ell_x^{-1} \sim \ell_y^{-1} \left(S_{\mathrm{ZF}} \tau_{nl} + \hat{s}\theta \right) \sim \ell_y^{-1}$$

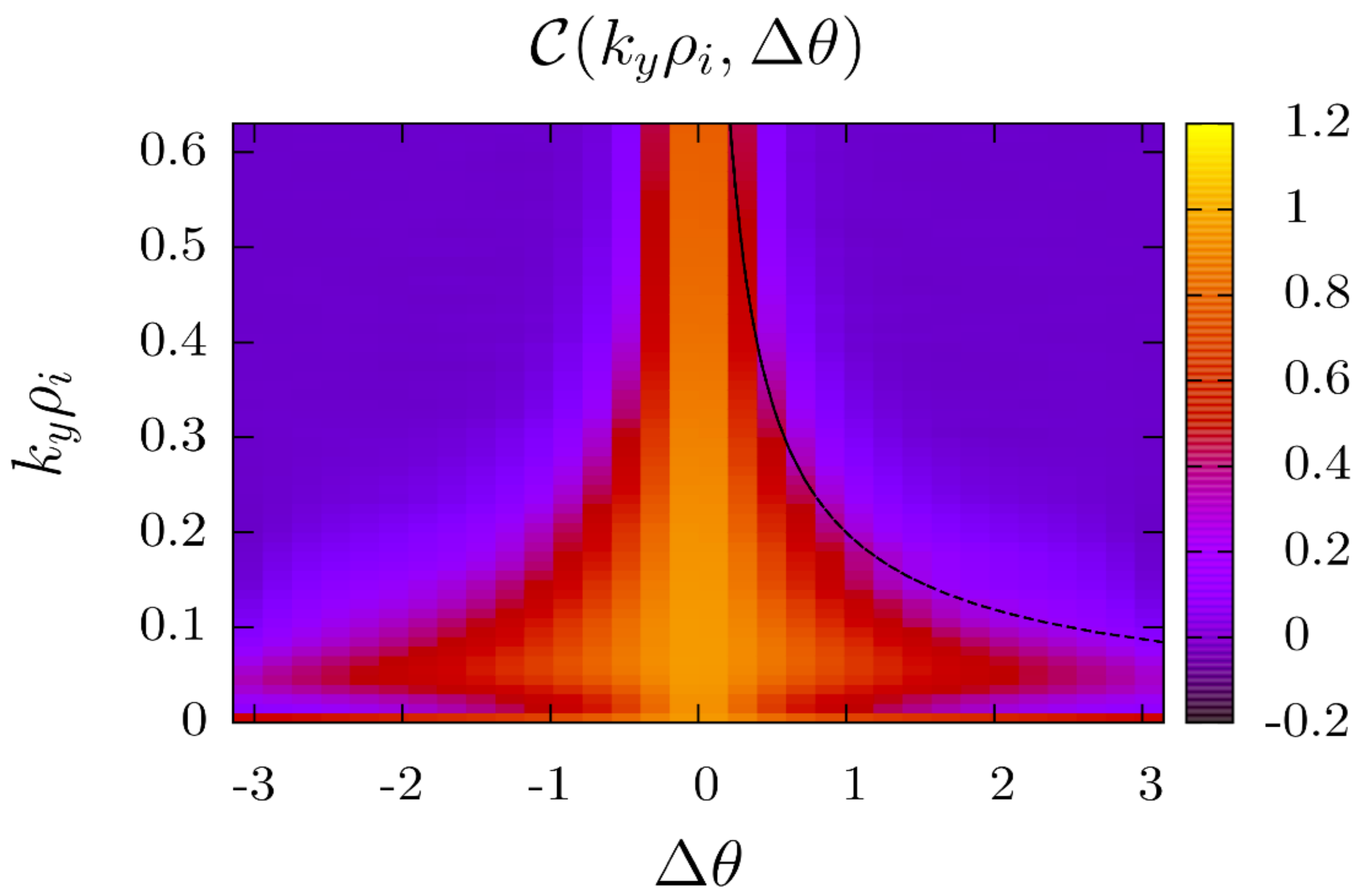
$$\ell_x^o \sim \ell_y^o \Rightarrow \boxed{\Phi_o \sim q \left(\frac{R}{L_T} \right)^2} \quad (2)$$

Heat flux: $\Rightarrow \boxed{\dfrac{Q_i}{n_i T_i v_{\mathrm{th}}} \left(\dfrac{R}{\rho_i} \right)^2 \sim q \left(\dfrac{R}{L_T} \right)^3}$

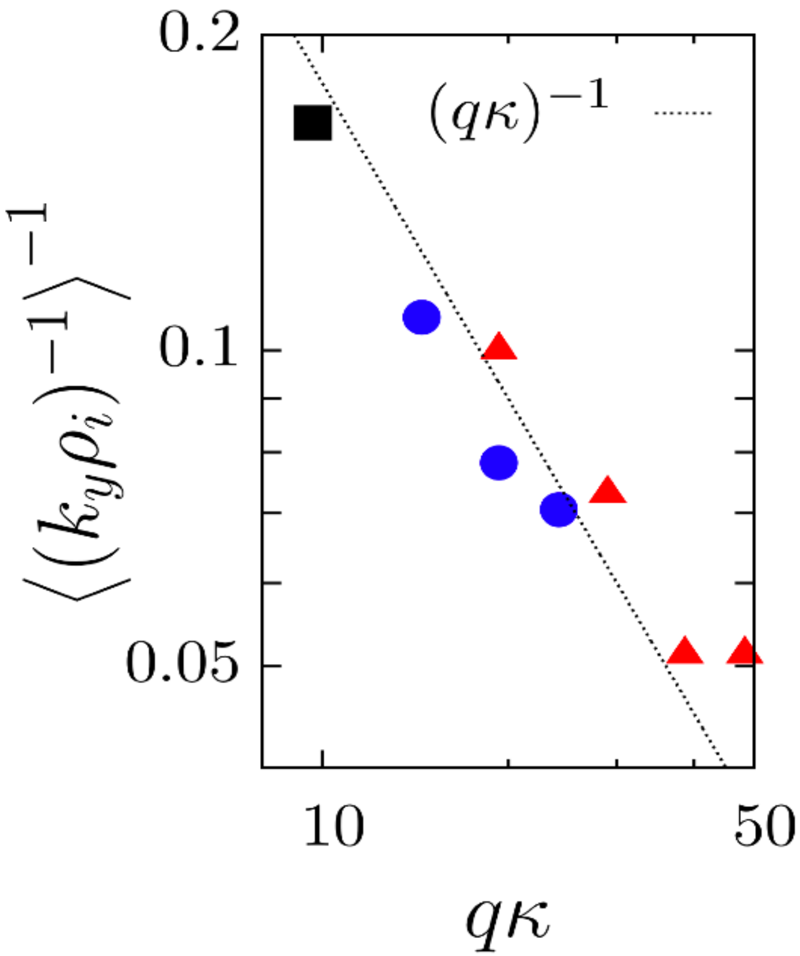
Critical balance test

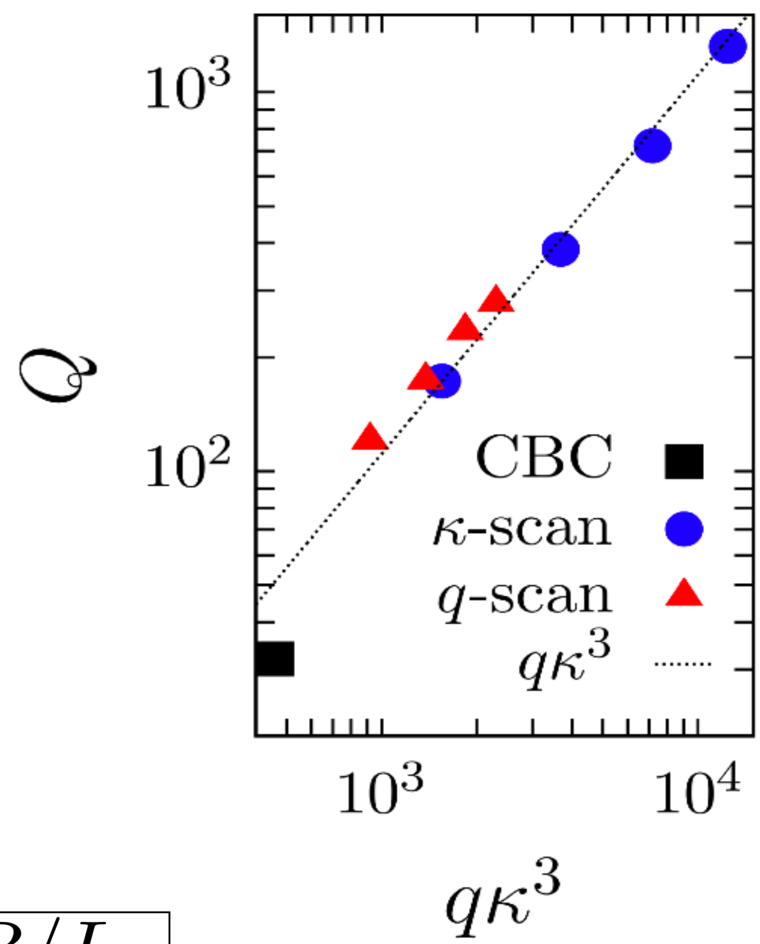
Correlation function:

$$\mathcal{C}(k_y, \Delta\theta) = \frac{\sum_{k_x} \Phi(\theta = 0)\Phi^*(\theta = \Delta\theta)}{\sum_{k_x} |\Phi(\theta = 0)|^2}$$



Turbulence scaling tests





$\kappa = R/L_T$

Note that Q at large R/L_T much larger than found in previous studies (box size used here for $R/L_T \approx 20$ was $\approx 1000\rho_i$)

Higher-order GK equation

- Using scalings (1) & (2), B_p/B<<1, and $\nu\tau_{nl}$<<1 [2]:

$$\frac{dg_s}{dt} + \mathbf{v}_\parallel \cdot \nabla \left(g_s - Z_s e \langle \varphi \rangle \frac{\partial F_{0s}}{\partial E} \right) + \langle \mathbf{v}_E^\perp \rangle \cdot \nabla F_{0s}$$

$$+ \left(\mathbf{v}_{Cs} + \mathbf{v}_{Ms} + \langle \mathbf{v}_E^\perp \rangle \right) \cdot \nabla_\perp \left(g_s - Z_s e \langle \varphi \rangle \frac{\partial F_{0s}}{\partial E} \right)$$

$$= - \mathbf{v}_{Ms} \cdot \nabla\theta \frac{\partial}{\partial\theta} \left(g_s - Z_s e \langle \Phi \rangle \frac{\partial F_{0s}}{\partial E} \right) - \left\langle \mathbf{v}_E^\parallel \right\rangle \cdot \nabla_\perp g_s$$

$$- \langle \mathbf{v}_E^\perp \rangle \cdot \nabla\theta \frac{\partial g_s}{\partial\theta} - \left\langle \mathbf{v}_E^\parallel \right\rangle \cdot \nabla F_{0s} - \langle \mathbf{v}_E^\perp \rangle \cdot \nabla F_{1s}$$

$$+ Z_s e \left(\mathbf{v}_\parallel \cdot \nabla \langle \Phi \rangle + \mathbf{v}_{Ms} \cdot \nabla_\perp \langle \Phi \rangle \right) \left(\frac{\partial g_s}{\partial E} + \frac{\partial F_{1s}}{\partial E} \right)$$

$$+ \psi\text{-profile variation}$$

(R,E,μ) variables

Higher-order GK equation

- LHS is identical to lowest-order high-flow GK Eq.
- New terms on RHS (except ψ-profile variation) now implemented in local GK code GS2
- Blue terms are corrections to lowest-order turbulence gradients accounting for slow variation along B-field
- Red terms are corrections to lowest-order equilibrium distribution function (neoclassical)
- Green term is parallel nonlinearity
- Note: $\mathbf{v}_E^{\|} \equiv \frac{c}{B}\hat{\mathbf{b}} \times \nabla\theta \frac{\partial\Phi}{\partial\theta}, \quad \mathbf{v}_E^{\perp} = \frac{c}{B}\hat{\mathbf{b}} \times \nabla_{\perp}\Phi$

Neoclassical terms

- Neoclassical correction to equilibrium distribution function (F_1) obtained from NEO [3] at multiple radii; finite differences give ∇F_1
- NEO uses E and $\xi = v_{||}/v$ as v-space variables, so we must take care with derivatives of F_1:

$$\left.\frac{\partial}{\partial E}\right|_{\mu} = \left.\frac{\partial}{\partial E}\right|_{\xi} + \frac{1-\xi^2}{2\xi E}\left.\frac{\partial}{\partial \xi}\right|_{E}$$

$$\left.\nabla\right|_{\mu} = \left.\nabla\right|_{\xi} + \frac{\xi^2-1}{2\xi}\frac{\nabla B}{B}\frac{\partial}{\partial \xi}$$

- Both terms have singularities at $\xi=0$

Removal of singularities

- Remove ξ-singularity in ∇F_1 term and $\nabla B \partial_E F_1$ term by combining:

$$\langle \mathbf{v}_E^{\perp} \rangle \cdot \nabla F_1 - Z_s e \, \mathbf{v}_{\nabla B} \cdot \nabla_{\perp} \langle \Phi \rangle \frac{\partial F_1}{\partial E} = \langle \mathbf{v}_E^{\perp} \rangle \cdot \left(\nabla + \mu \nabla B \frac{\partial}{\partial E} \right) F_1$$

$$\nabla + \mu \nabla B \frac{\partial}{\partial E} = \nabla \Big|_{\xi} + \left(1 - \xi^2\right) \frac{\nabla B}{B} \left(E \frac{\partial}{\partial E}\Big|_{\xi} - \frac{\xi}{2} \frac{\partial}{\partial \xi} \right)$$

- Singularity in other $\partial_E F_1$ terms eliminated because they are multiplied by $v_{||}$ or $v_{||}^2$

Momentum flux

- Neglecting neoclassical contributions, momentum flux given by $\Pi = \Pi_{-1}^{tb} + \Pi_0^{tb}$

$$\Pi_{-1}^{tb} = \frac{1}{\langle|\nabla\psi|\rangle_\psi}\left\langle \int d^3v\ m_s R^2 \mathbf{v}\cdot\nabla\phi\left(\mathbf{v}_E^\perp\cdot\nabla\psi\right)\delta f(\mathbf{R})\right\rangle_\psi$$

$$\Pi_0^{tb} = \frac{I(\psi)}{\langle|\nabla\psi|\rangle_\psi}\frac{m_s^2c^2}{2Z_se}\frac{1}{V'}\frac{\partial}{\partial\psi}\left(V'\left\langle\frac{\hat{\mathbf{b}}}{B^3}\times\nabla_\perp\varphi\cdot\nabla\psi\int d^3v\ v_\parallel^2\delta f\right\rangle_\psi\right)$$

$$-\frac{1}{\langle|\nabla\psi|\rangle_\psi}\frac{m_s^2c}{2Z_se}\left\langle\frac{I^2}{B^2}\int d^3v\ C_{ii}^l[v_\parallel^2]F_2^{tb}\right\rangle_\psi + \frac{m_sc}{2Z_se}\langle R^2\rangle_\psi\frac{\partial p}{\partial t}$$

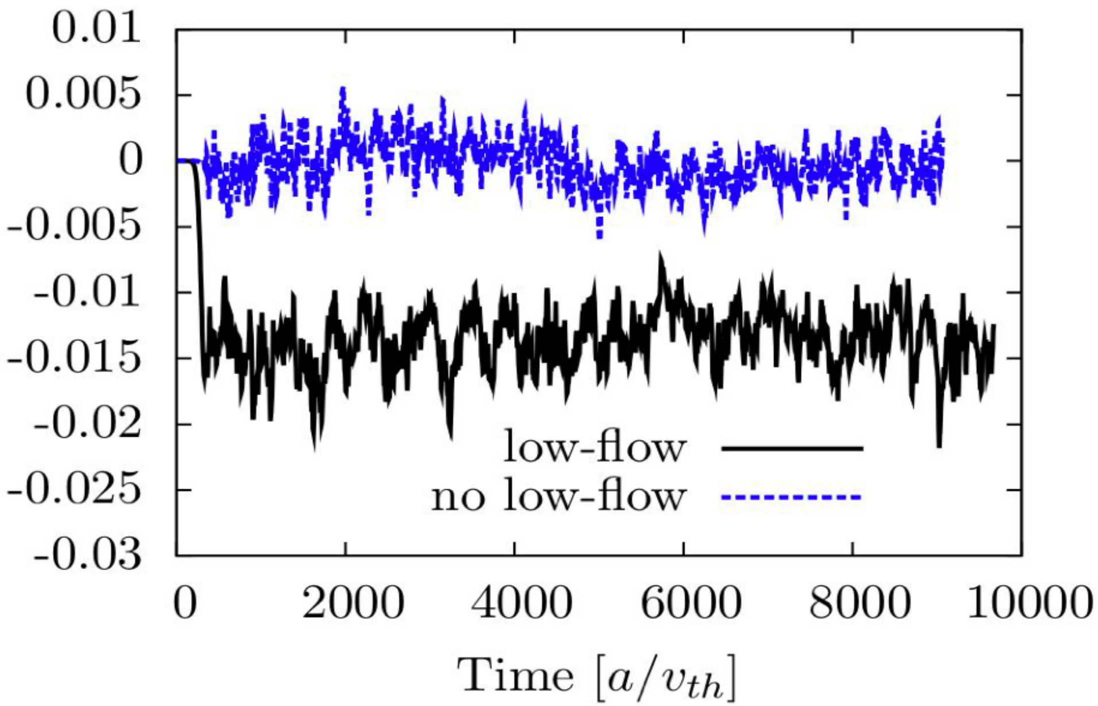
- Black terms currently diagnosed

Preliminary results

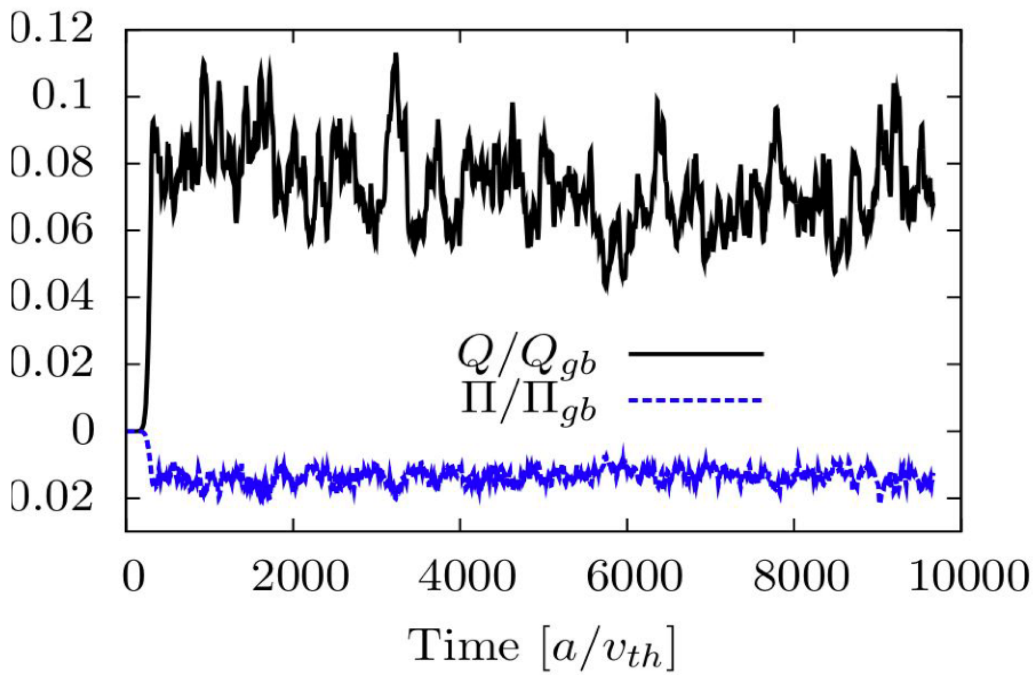
JET shot 19649 (L-mode) at ρ=0.16, no equilibrium flow

Small momentum flux generated by including neoclassical correction to F_0 in GK equation:

Momentum flux (Π^{tb}_{-1})



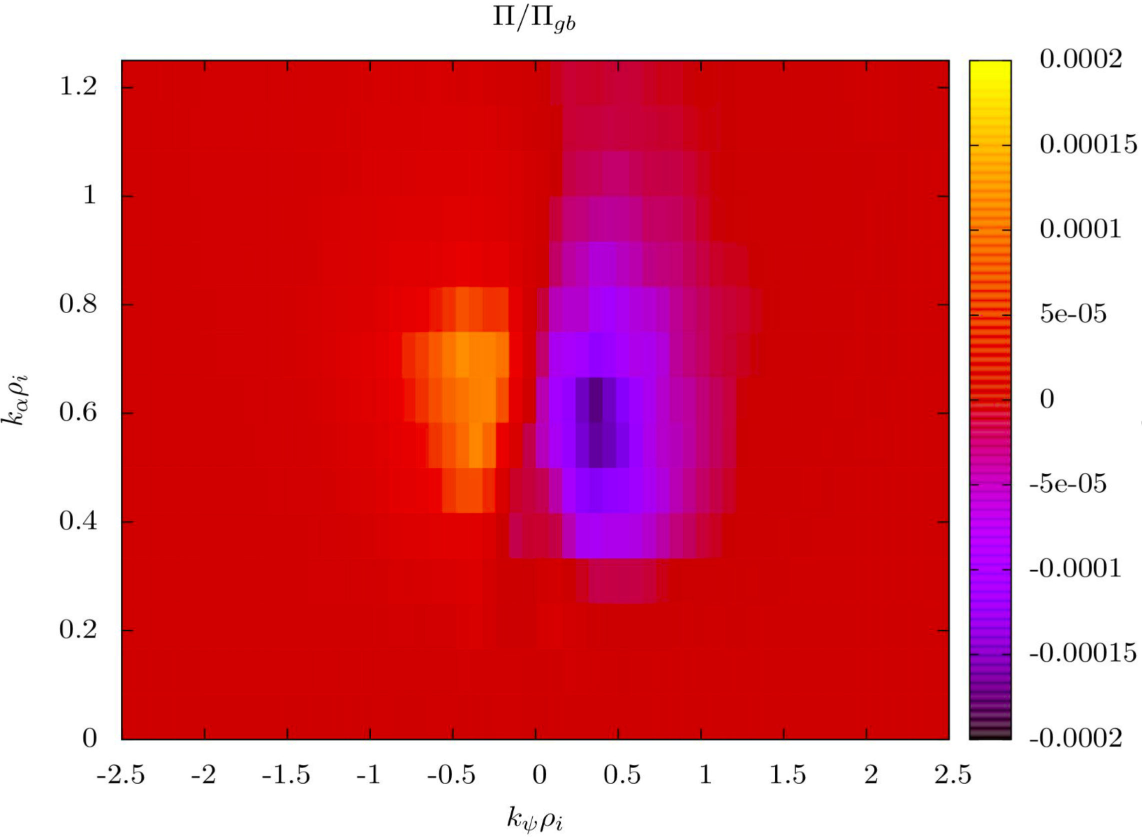
Fluxes



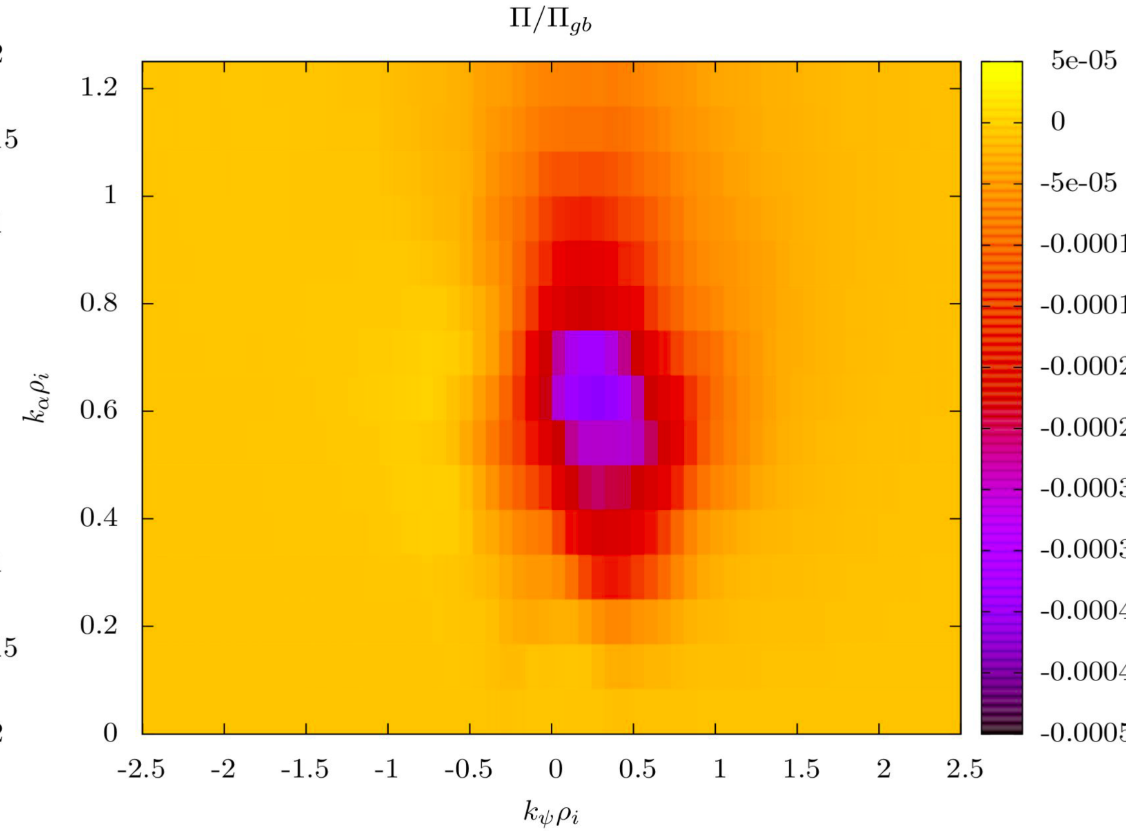
Symmetry breaking

JET shot 19649 (L-mode) at ρ=0.16, no equilibrium flow

k_ψ symmetry broken (see Parra poster and [4])

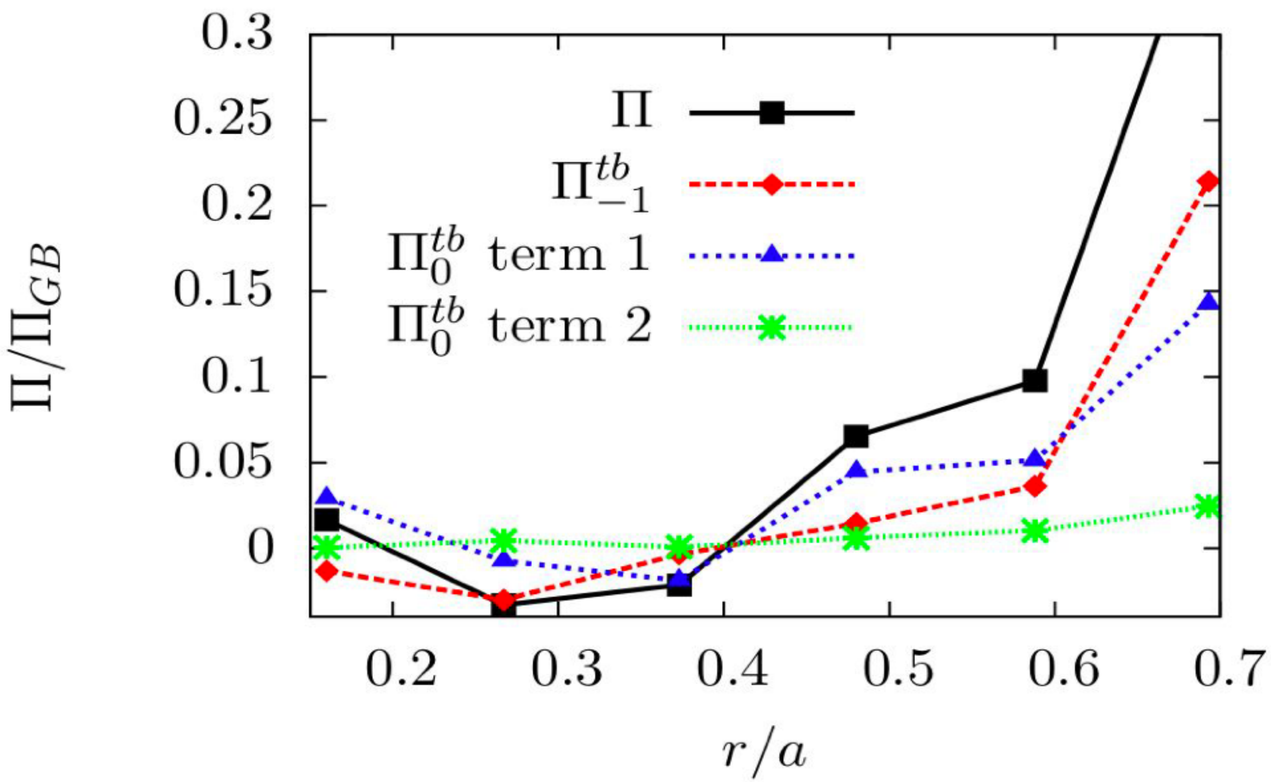


No low-flow



low-flow

Radial momentum flux profiles



JET shot 19649

- Momentum flux sign reversal allows for both co- and counter-rotating regions of plasma
- Different flux contributions of same order and possibly different sign

Conclusions

- Simple scalings for turbulence spatial scales and amplitudes derived and numerically confirmed – plasma turbulence satisfies critical balance
- Higher-order flux-tube GK Eq. implemented in GS2, giving momentum flux in absence of flow or flow-shear – can be used to obtain intrinsic rotation profiles
- Momentum fluxes determined by many effects, not just ψ-profile variation
- So-called $k_{||}$ symmetry breaking not necessary to generate momentum flux – true symmetry of nonlinear GK Eq. is in (θ, $v_{||}$, k_ψ)

Inertial range scalings

- Free energy, W, is nonlinear invariant [5]:

$$W = \sum_s \int d^3r \int d^3v \left(\frac{T_s \delta f_s^2}{2F_{0s}} \right)$$

- <u>Conjecture</u>: There is no significant dissipation or driving between outer scale and Larmor radius
- In this 'inertial' range, flux of free energy, W_ℓ / τ_{nl}, must be independent of $\ell_\perp$:

$$W_\ell / \tau_{nl} \sim \left(\frac{\rho_i}{R} \right)^2 \frac{v_{\rm th}}{R} \frac{\rho_i^2}{\ell_x \ell_y} \Phi_\ell^3 \sim \text{constant}$$

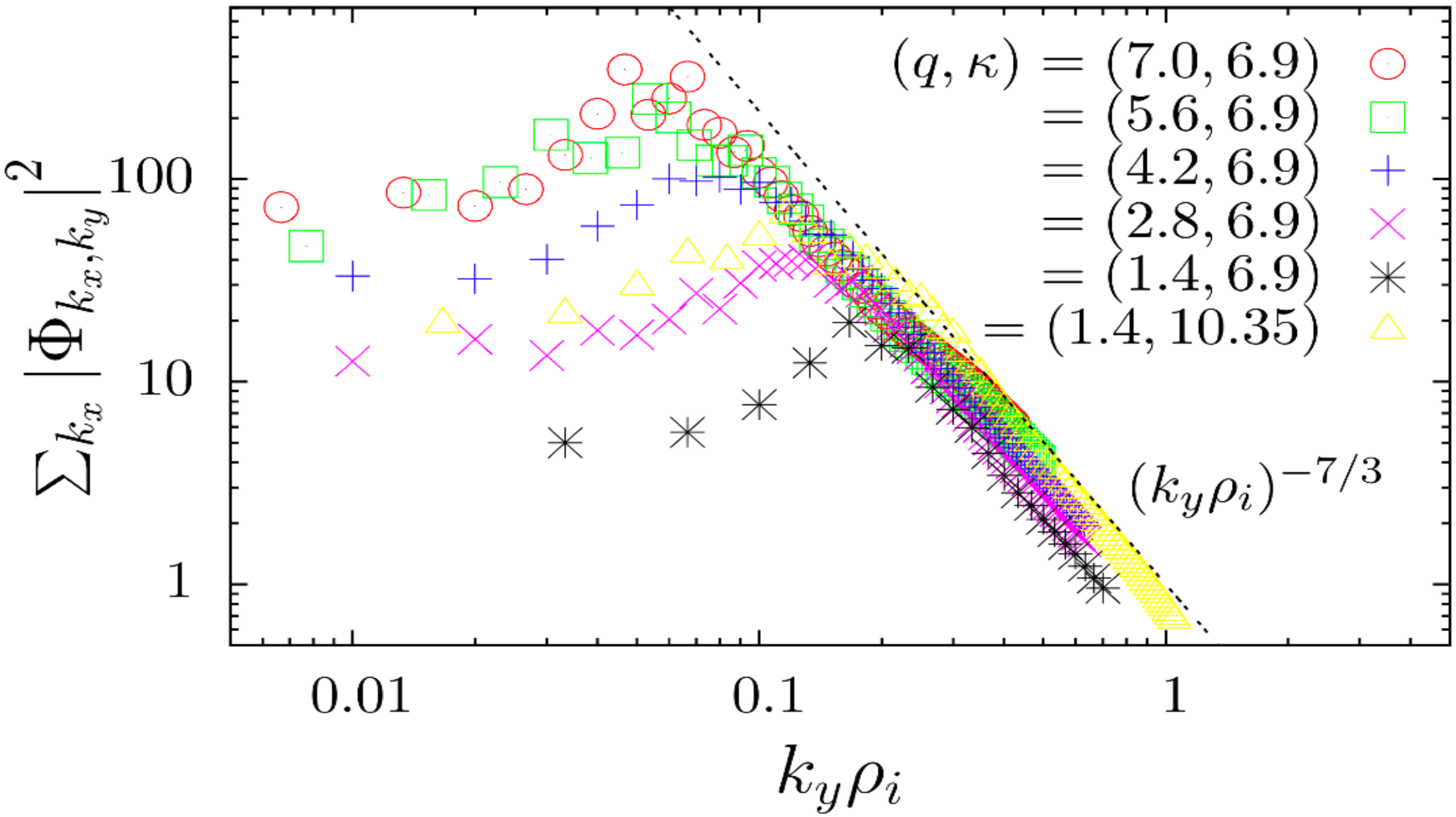
- Solving for Φ_ℓ and using (1) & (2):

$$\Phi_\ell \sim \Phi_o \left(\frac{\ell_\perp}{\ell_y^o} \right)^{2/3} \sim q^{1/3} \left(\frac{R}{L_T} \right)^{4/3} \left(\frac{\ell_\perp}{\rho_i} \right)^{2/3} \quad (3)$$

Inertial range spectra

- Parseval's theorem is used to give 1D spectrum:

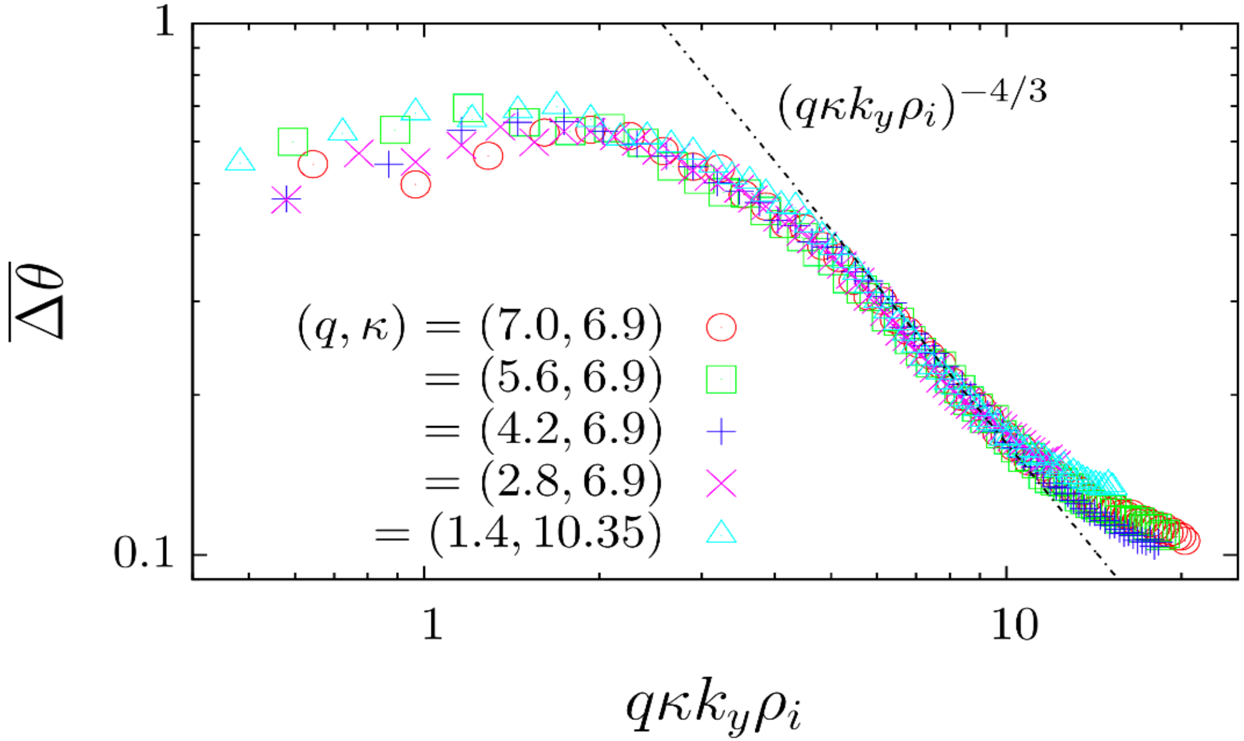
$$k_\perp \rho_i \left|\Phi_{\mathbf{k}}\right|^2 \sim q^{2/3} \left(\frac{R}{L_T}\right)^{8/3} (k_\perp \rho_i)^{-7/3}$$



Inertial range critical balance

- Using (3) and applying critical balance:

$$\frac{\ell_\parallel}{qR} \sim \left(\frac{\ell_\perp}{\rho_i}\frac{L_T}{qR}\right)^{4/3} \quad \& \quad \Phi_\ell^2 \sim q\left(\frac{R}{L_T}\right)^4 \frac{\ell_\parallel}{R}$$



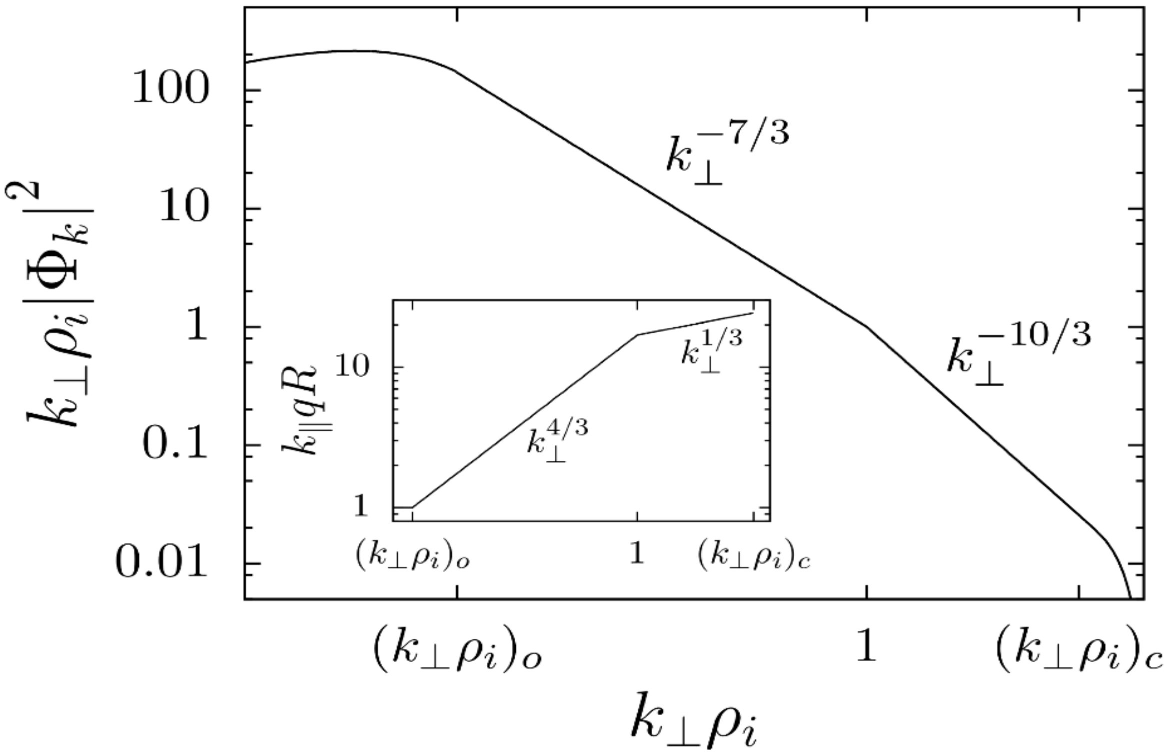
$$\overline{\Delta\theta}(k_y) = \int d(\Delta\theta)\mathcal{C}(k_y, \Delta\theta) \qquad \kappa = R/L_T$$

Sub-Larmor scales

- Below ρ_i, spectrum and wavenumber cutoff are [5,6]

$$k_\perp |\Phi_{\mathbf{k}}|^2 \sim k_\perp^{-10/3} \quad \& \quad (k_\perp \rho_i)_c \sim \mathrm{Do}^{3/5} \equiv (\tau_{\rho_i} \nu_{ii})^{-3/5}$$

- Combining with (1)-(3): $(k_\perp \rho_i)_c \sim q^{1/5} \left(\frac{R}{L_T}\right)^{4/5} \left(\frac{v_{\rm th}}{\nu_{ii} R}\right)^{3/5}$



References

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