Direct multi-scale coupling of a transport code to gyrokinetic turbulence codes

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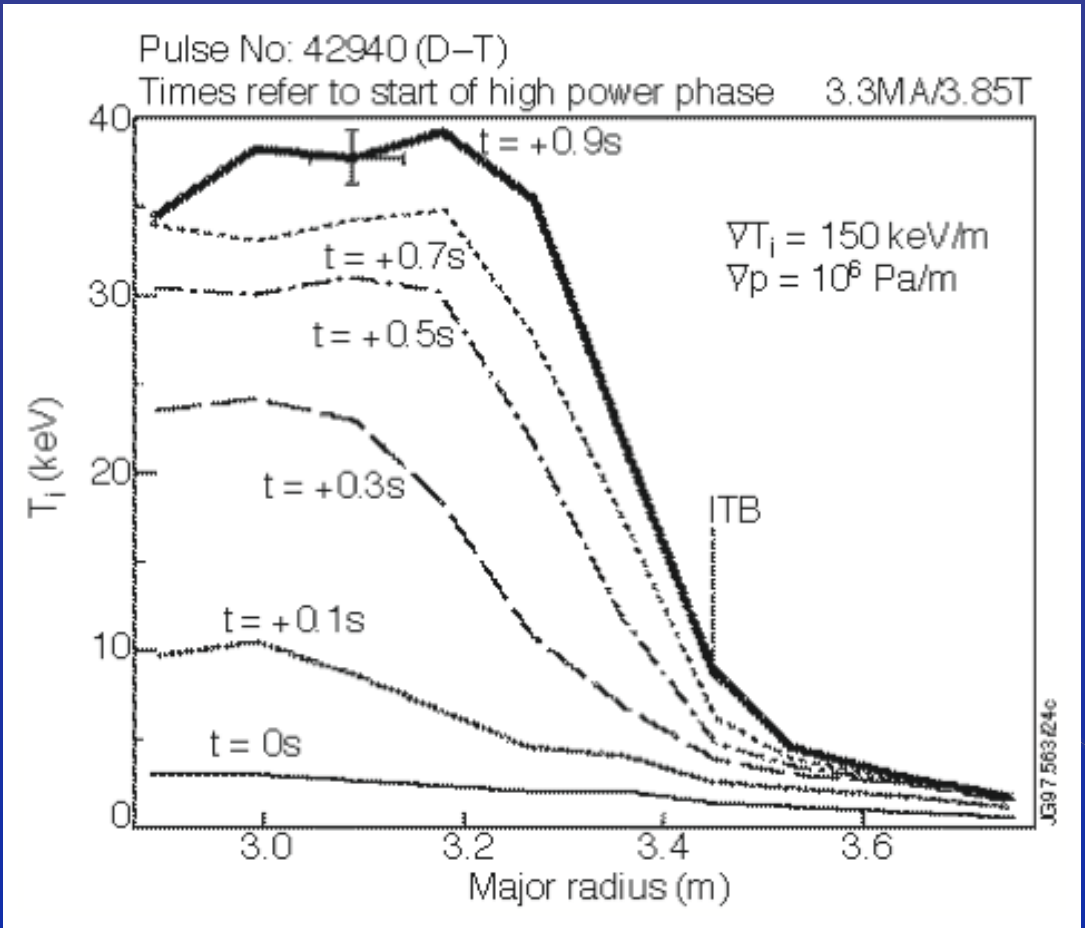
Overview

- Motivation
- Multi-scale model
- Trinity simulation results
- Conclusions and future work

Objective



Connor et al. (2004)

Core:
multi-physics,
multi-scale

Edge:
multi-physics,
multi-scale

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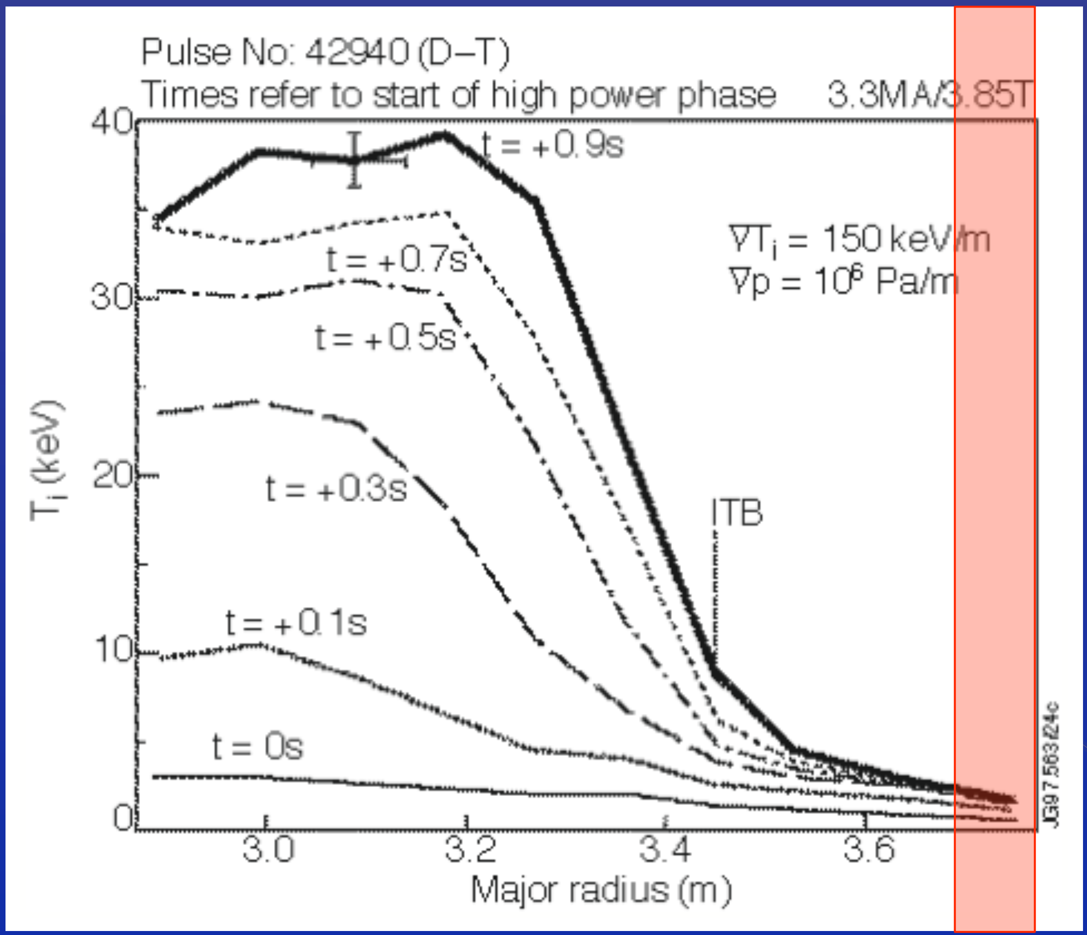

Connor et al. (2004)

Core:
multi-physics,
multi-scale

- **kinetic turbulence**
- neoclassical
- sources
- magnetic equilibrium
- MHD

Multiple scale problem

Physics	Perpendicular spatial scale	Temporal scale
Turbulence from ETG modes	$k_\perp^{-1}$ ~ 0.005 – 0.05 cm	ω_* ~ 0.5 - 5.0 MHz
Turbulence from ITG modes	$k_\perp^{-1}$ ~ 0.3 - 3.0 cm	ω_* ~ 10 - 100 kHz
Transport barriers	Measurements suggest width ~ 1 - 10 cm	100 ms or more in core?
Discharge evolution	Profile scales ~ 200 cm	Energy confinement time ~ 2 - 4 s

$$(L_{\|}/\Delta_{\|}) \times (L_\perp/\Delta_\perp)^2 \times (L_v/\Delta_v)^2 \times (L_t/\Delta t) \sim 10^{21}$$

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Gyrokinetic multiscale assumptions

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial f}{\partial \mathbf{R}} + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} + \frac{dE}{dt}\frac{\partial f}{\partial E} + \frac{d\vartheta}{dt}\frac{\partial f}{\partial \vartheta} = C[f]$$

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$$f = F + \delta f \qquad\qquad \delta f \sim \epsilon f$$

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- Separation of space scales:

$$\nabla F \sim F/L, \;\; \nabla_{||}\delta f \sim \delta f/L, \;\; \nabla_{\perp}\delta f \sim \delta f/\rho$$

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- “Smooth” velocity space:

$$\epsilon \lesssim \nu/\omega \lesssim 1 \Rightarrow \sqrt{\epsilon} \lesssim \delta v/v_{th} \lesssim 1$$

Transport equations in GK

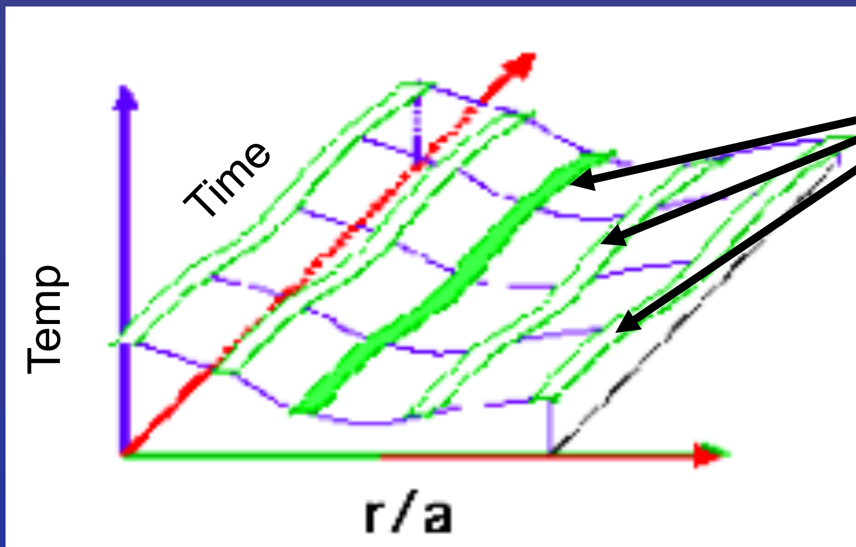
Moment equations for equilibrium evolution:

$$
\begin{aligned}
\frac{\partial n_s}{\partial t} &= -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \boldsymbol{\Gamma}_s \cdot \nabla\psi\right\rangle\right) + S_n \\
\frac{3}{2}\frac{\partial n_s T_s}{\partial t} &= -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \mathbf{Q}_s \cdot \nabla\psi\right\rangle\right) \\
&+ T_s\left(\frac{\partial \ln n_s}{\partial \psi} - \frac{3}{2}\frac{\partial \ln T_s}{\partial \psi}\right)\left\langle \boldsymbol{\Gamma}_s \cdot \nabla\psi\right\rangle + \frac{\partial \ln T_s}{\partial \psi}\left\langle \mathbf{Q}_s \cdot \nabla\psi\right\rangle \\
&- \left\langle \int d^3v \frac{h_s T_s}{F_{0s}}\left\langle C[h_s]\right\rangle_{\mathbf{R}}\right\rangle + n_s \nu_\epsilon^{su}\left(T_u - T_s\right) + S_p
\end{aligned}
$$

$$
\frac{\partial L}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\sum_s \left\langle \pi_s\right\rangle\right) + S_L
$$

Sugama (1997)

Multiscale grid

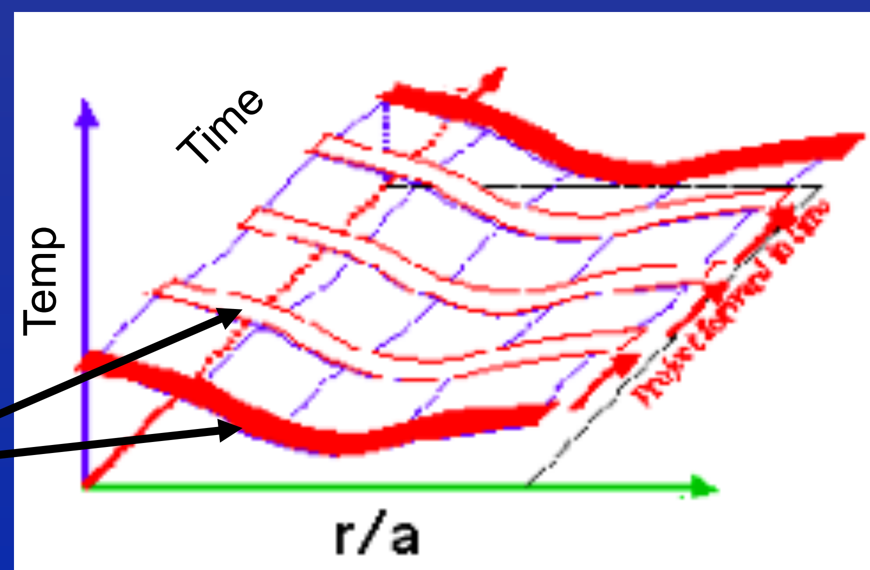


Flux tube simulation domain

- Turbulent fluxes calculated in small regions of fine grid embedded in “coarse” radial grid (for equilibrium)

- Steady-state (time-averaged) turbulent fluxes calculated in small regions of fine grid embedded in “coarse” time grid (for equilibrium)

Flux tube simulation domain

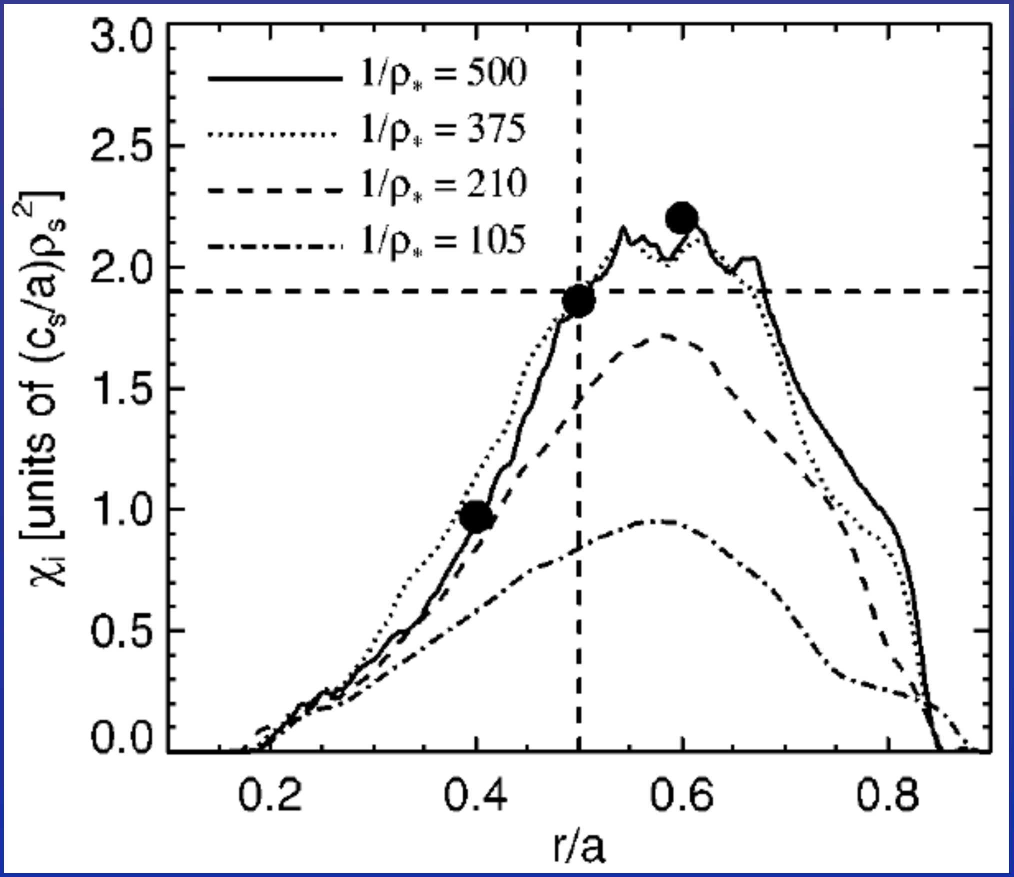


Flux tube assumptions

- Macroscopic quantities (density, flow, temperature, etc. constant across simulation domain)
- Gradient scale lengths of macroscopic quantities constant across simulation domain
  - Total gradient NOT constant (corrugations possible due to fluctuation + equilibrium gradients)
- In addition to delta-f assumption that equilibrium quantities constant in time over simulation
- => No important meso-scale physics

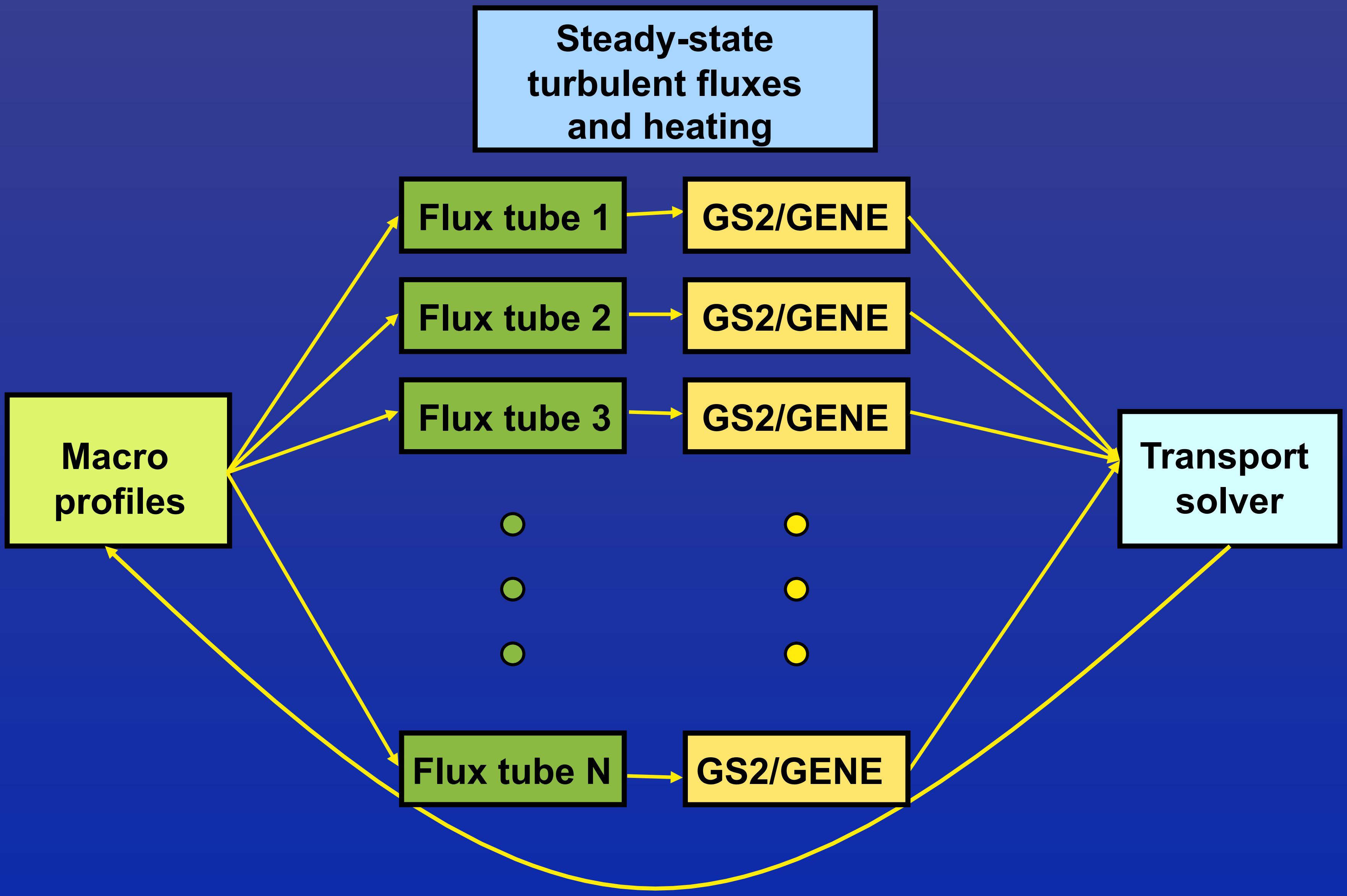
Validity of flux tube approximation

- Lines represent global simulations from GYRO
- Dots represent local (flux tube) simulations from GS2
- Excellent agreement for $\rho_* \ll 1$

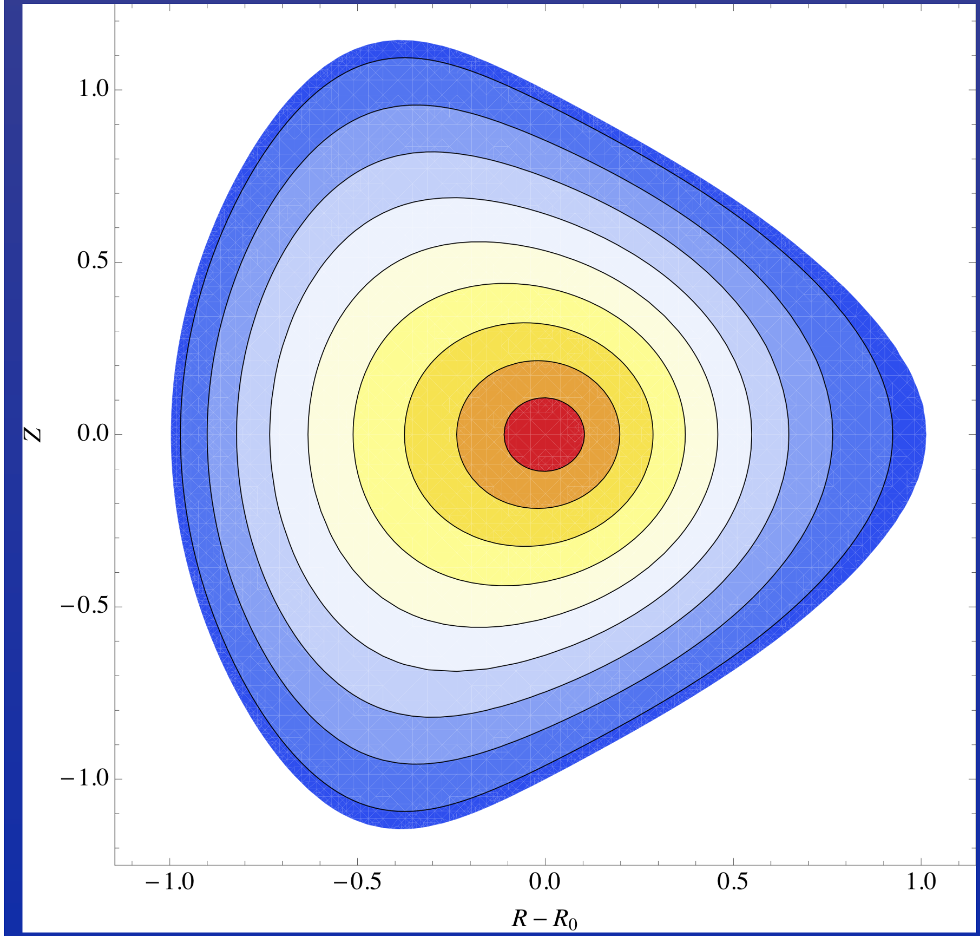


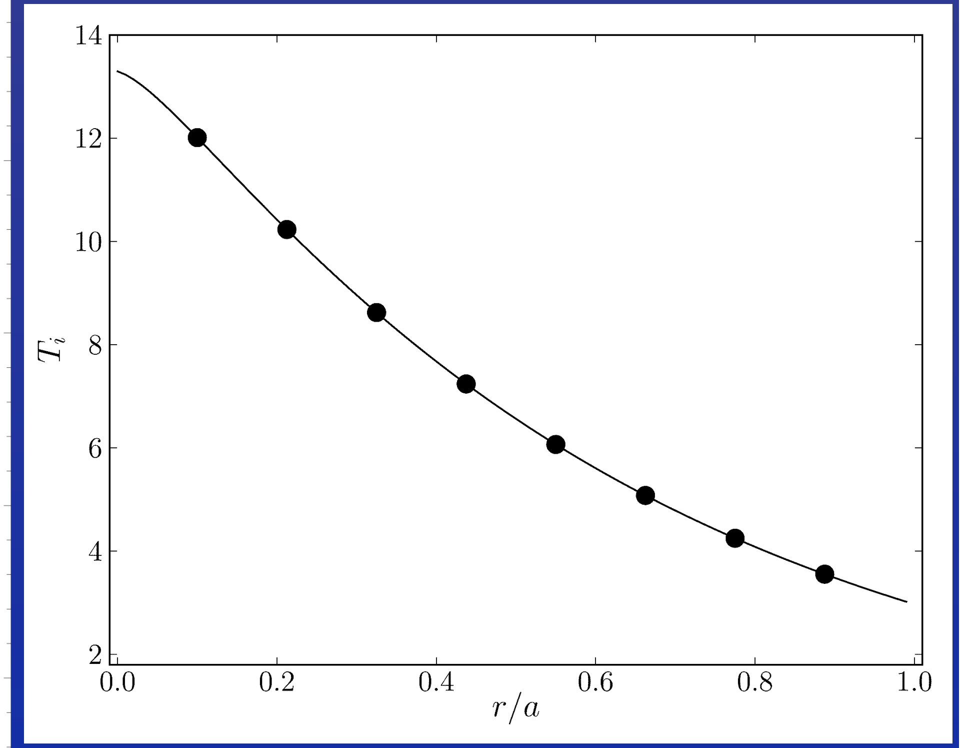
Candy et al (2004)

Trinity schematic



Sampling profile with flux tubes





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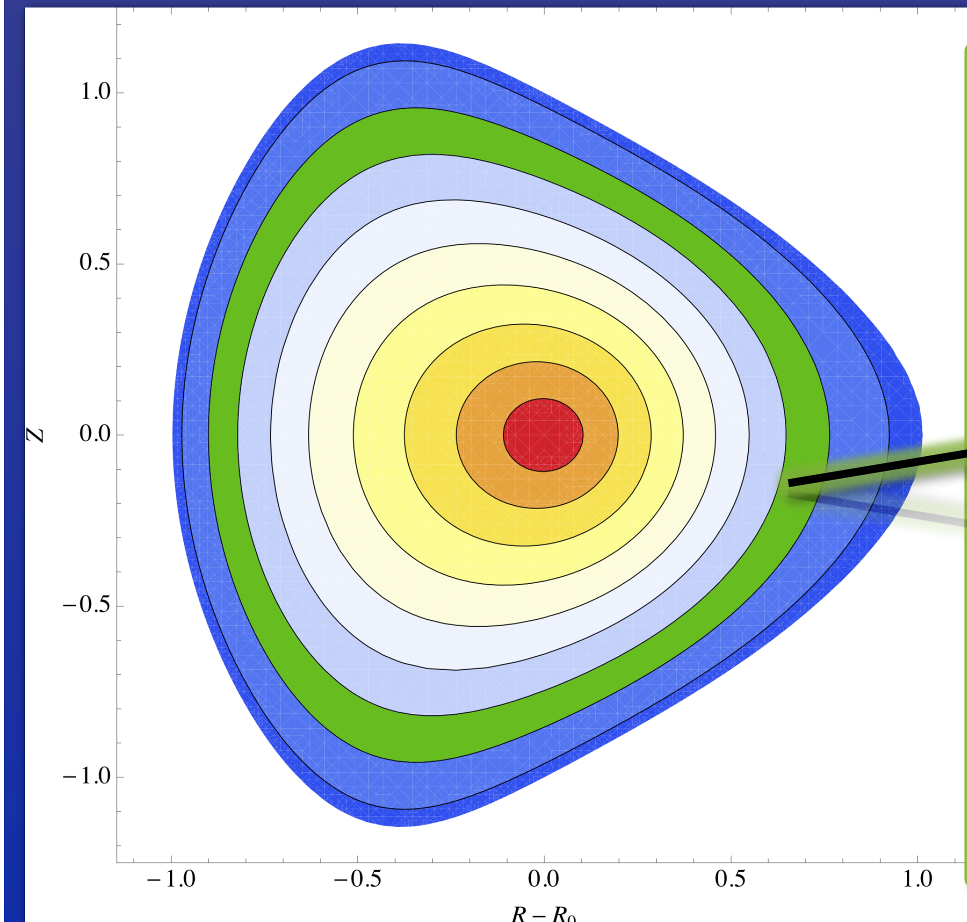

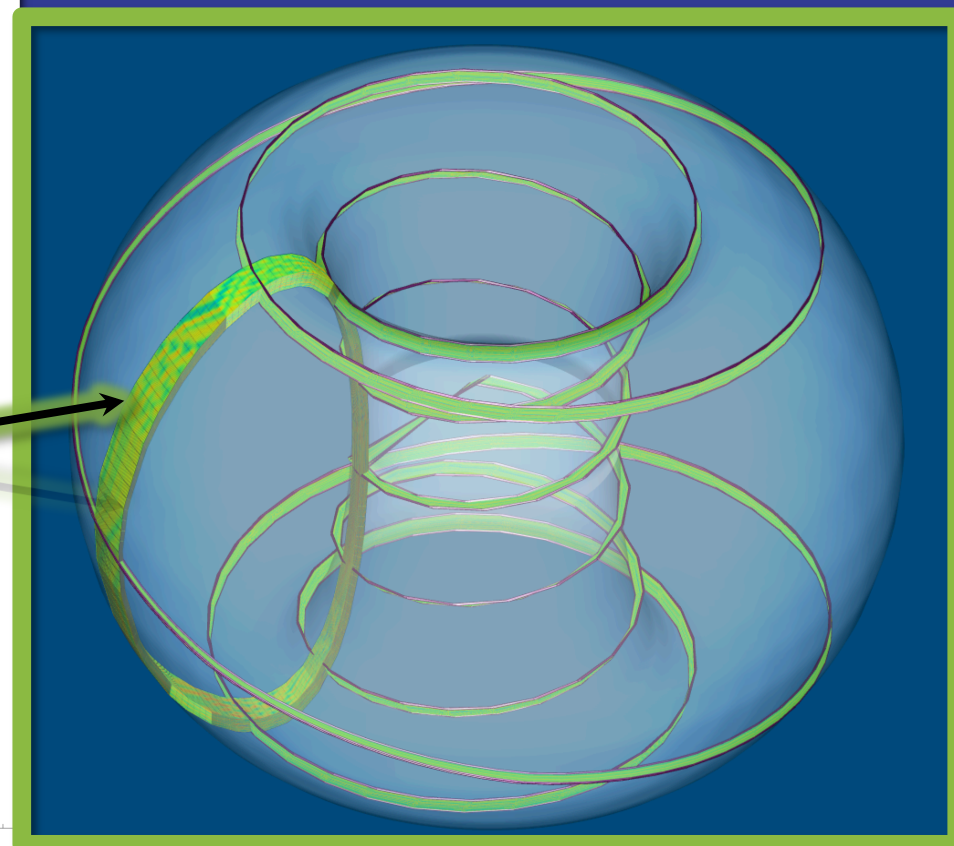

Simulation volume reduced by factor of ~100

Trinity transport solver

- Transport equations are stiff, nonlinear PDEs. Implicit treatment via Newton's Method (multi-step BDF, adaptive time step) allows for time steps ~0.1 seconds (vs. turbulence sim time ~0.001 seconds)
- Challenge: requires computation of quantities like

$$\Gamma_j^{m+1} \approx \Gamma_j^m + \left(\mathbf{y}^{m+1} - \mathbf{y}^m\right) \left.\frac{\partial \Gamma_j}{\partial \mathbf{y}}\right|_{\mathbf{y}^m} \qquad \mathbf{y} = \left[\{n_k\}, \{p_{i_k}\}, \{p_{e_k}\}\right]^T$$

- Local approximation: $\dfrac{\partial \Gamma_j}{\partial n_k} = \dfrac{\partial \Gamma_j}{\partial n_j} + \dfrac{\partial \Gamma_j}{\partial (R/L_n)_j} \dfrac{\partial (R/L_n)_j}{\partial n_k}$
- Simplifying assumption: normalized fluxes depend primarily on gradient scale lengths

Trinity transport solver

- Calculating flux derivative approximations:
  - at every radial grid point, simultaneously calculate $\Gamma_j[(R/L_n)_j^m]$ and $\Gamma_j[(R/L_n)_j^m + \delta]$ using 2 different flux tubes
  - Possible because flux tubes independent (do not communicate during calculation)
  - Perfect parallelization
  - use 2-point finite differences:

$$\frac{\partial \Gamma_j}{\partial (R/L_n)_j} \approx \frac{\Gamma_j[(R/L_n)_j^m] - \Gamma_j[(R/L_n)_j^m + \delta]}{\delta}$$

Trinity scaling

- Example calculation with 10 radial grid points:
  - evolve density, toroidal angular momentum, and electron/ion pressures
  - simultaneously calculate fluxes for equilibrium profile and for 4 separate profiles (one for each perturbed gradient scale length)
  - total of 50 flux tube simulations running simultaneously
  - ~2000-4000 processors per flux tube => scaling to over 100,000 processors with >85% efficiency

Multi-scale simulation savings

- Statistical periodicity in toroidal direction takes advantage of $k_\perp^{-1} \ll L_\theta$: volume savings factor of ~100
- Exploitation of scale separation between turbulence and equilibrium evolution: time savings factor of ~100
- Extreme parallelizability: savings factor of ~10
- Total saving of ~10^5: simulation possible on current machines

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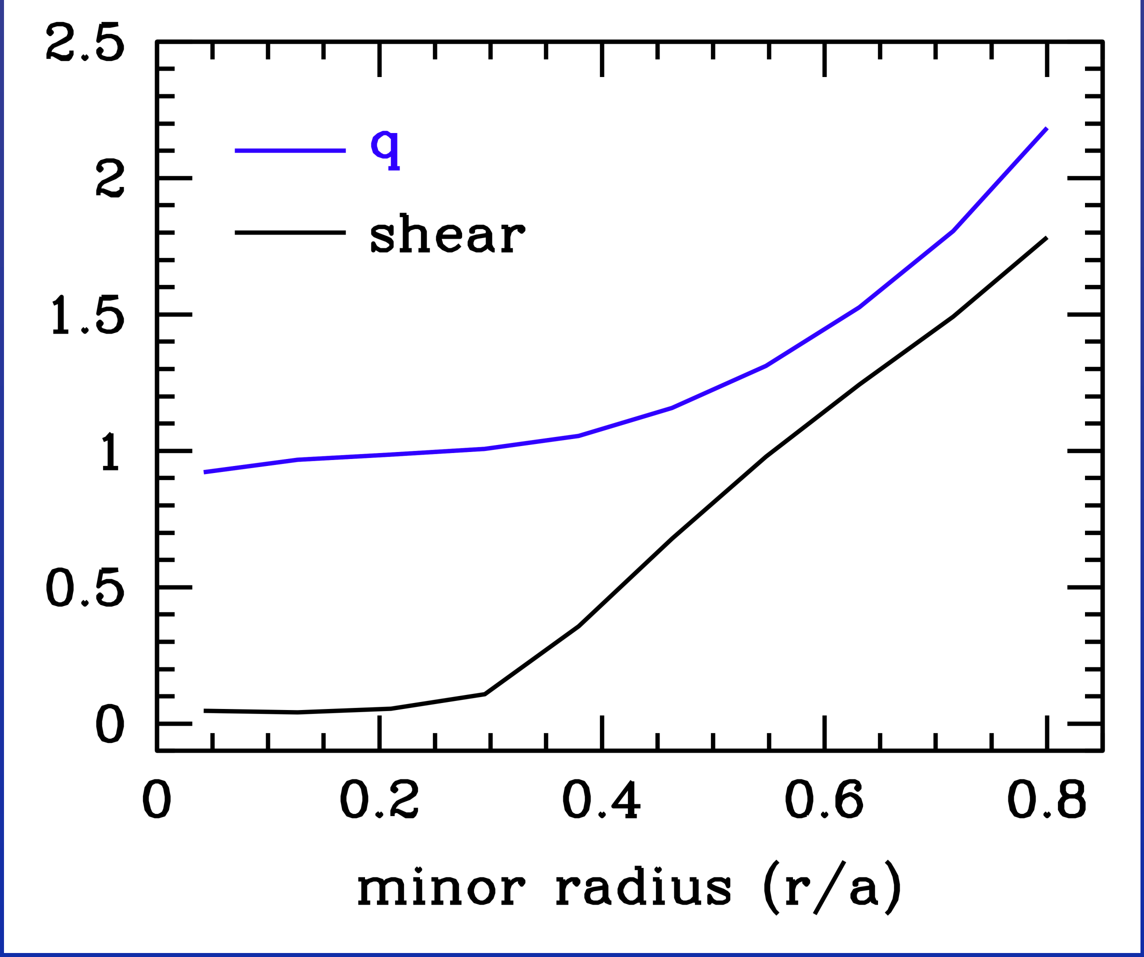
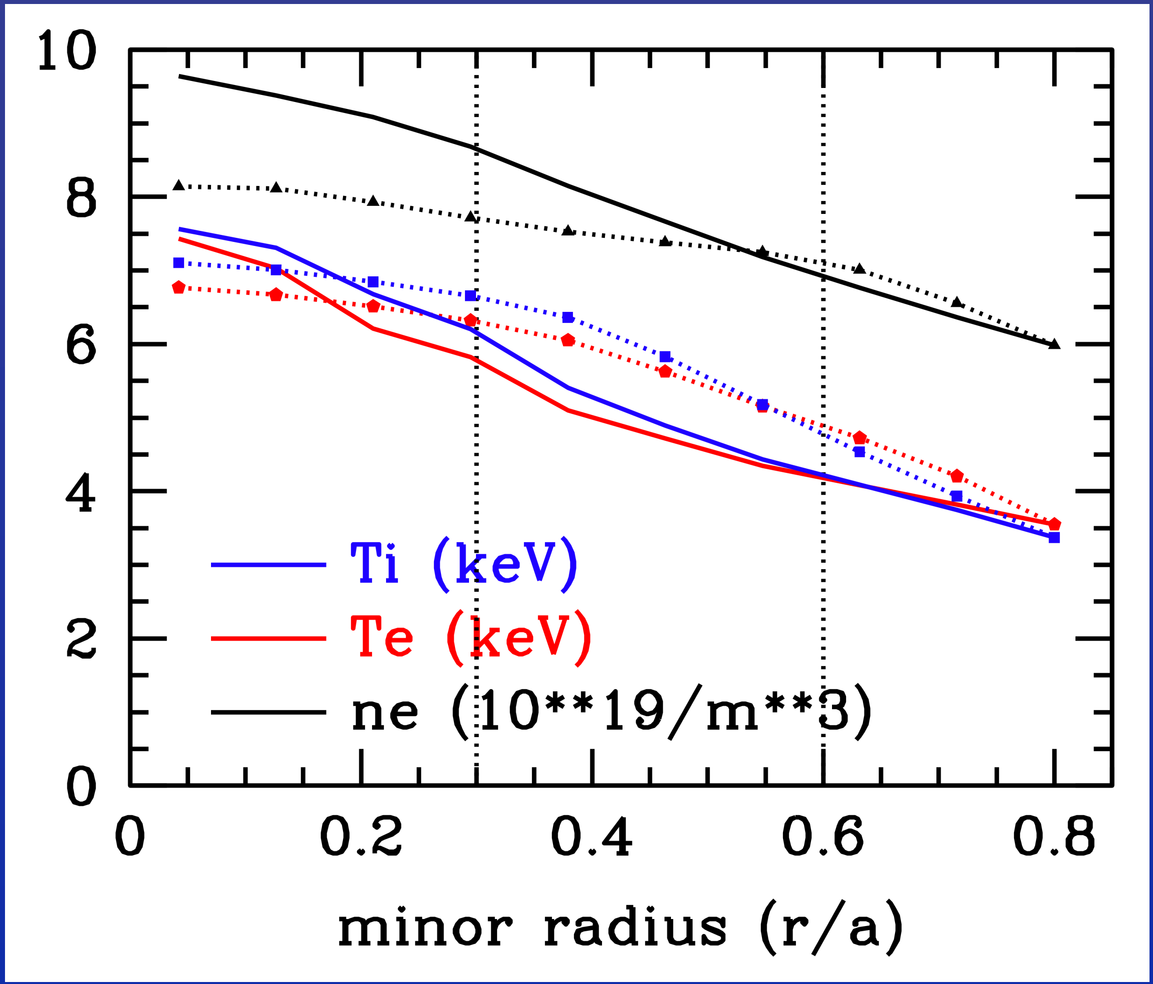
JET shot #42982



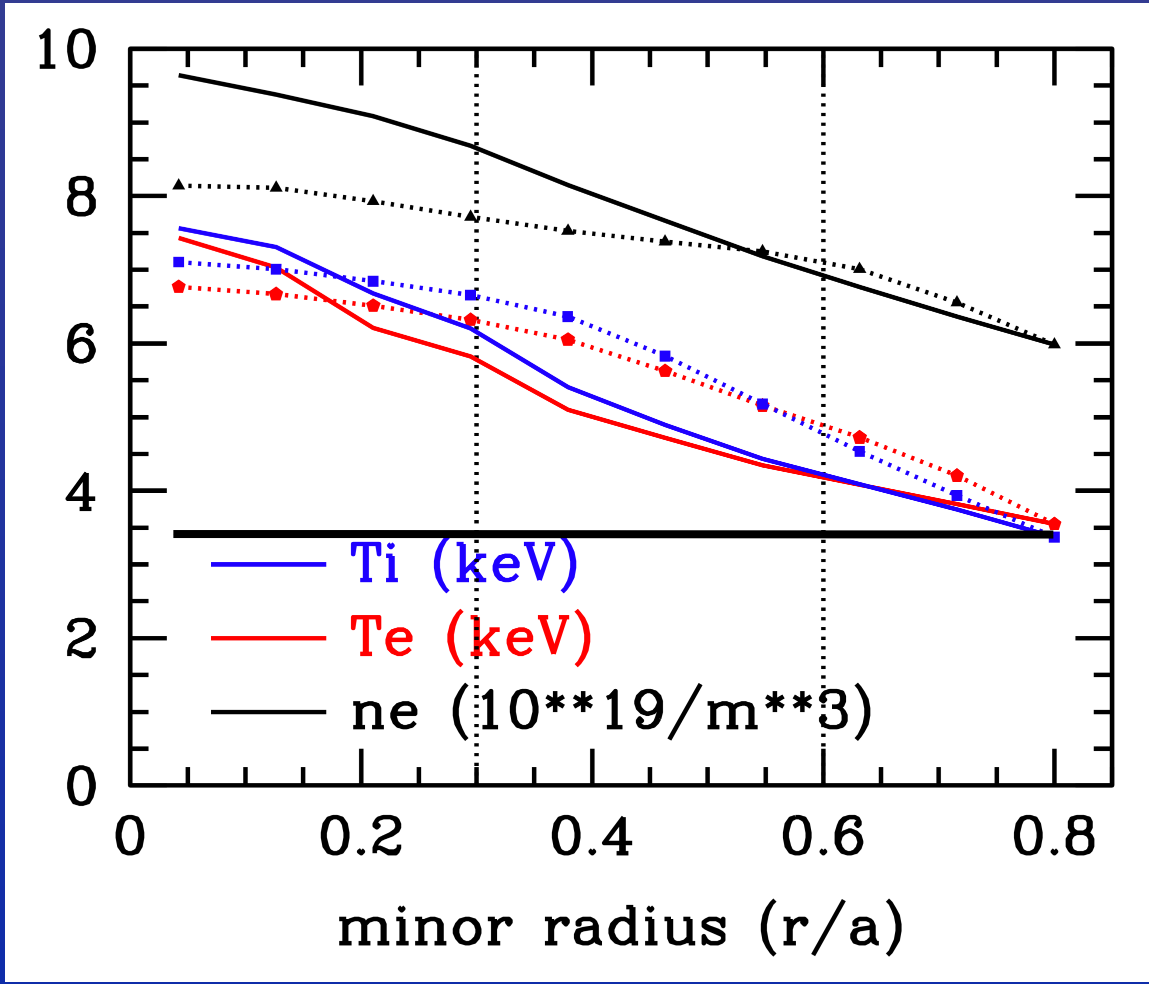
- ITER demo discharge
- H-mode D-T plasma, record fusion energy yield
- Miller local equilibrium model: q, shear, shaping
- B = 3.9 T on axis
- TRANSP fits to experimental data taken from ITER profile database

Evolving density profile



- 10 radial grid points
- Costs ~120k CPU hrs (<10 clock hrs)
- Dens and temp profiles agree within ~15% across device
- Energy off by 5%
- Incremental energy off by 15%
- Sources of discrepancy:
  - Large error bars
  - Flow shear absent

Evolving density profile



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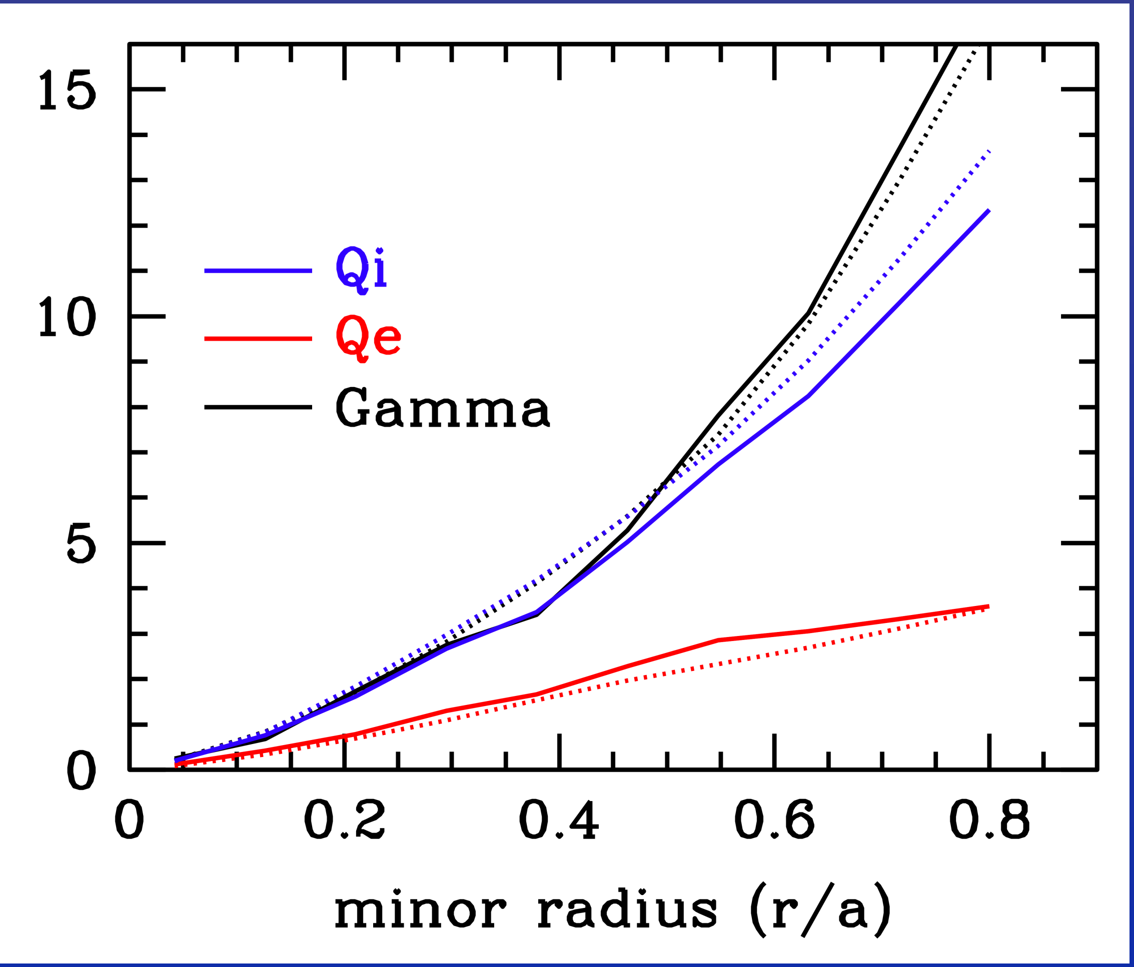
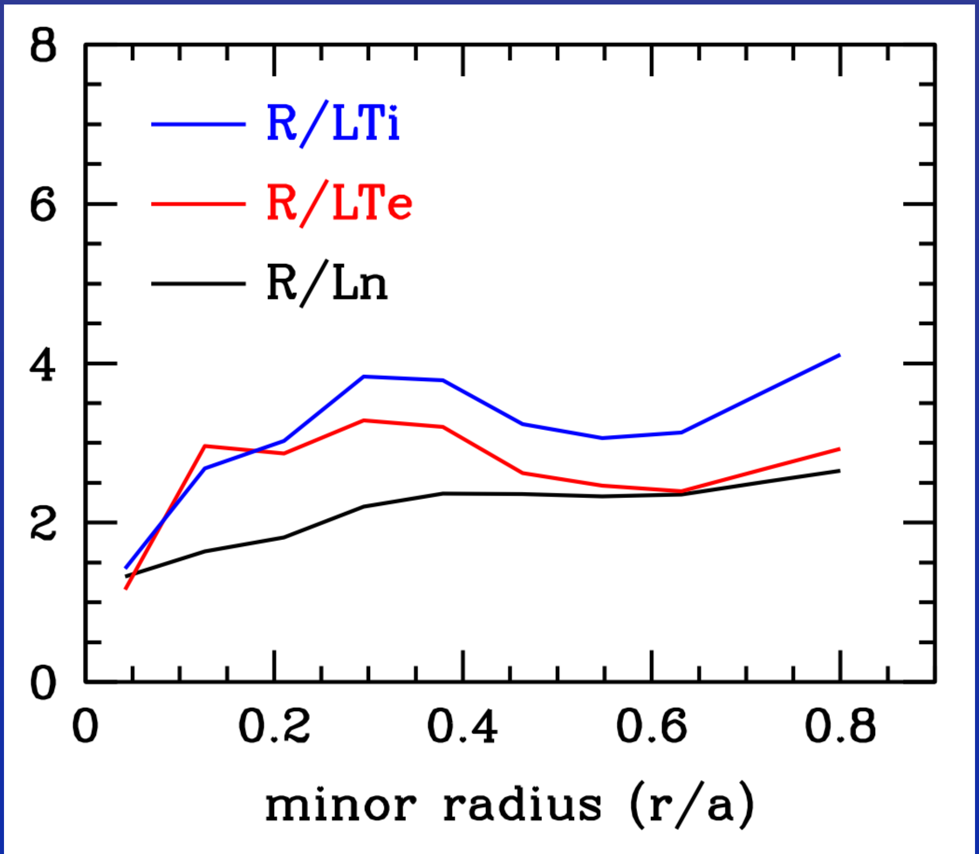
Power balance

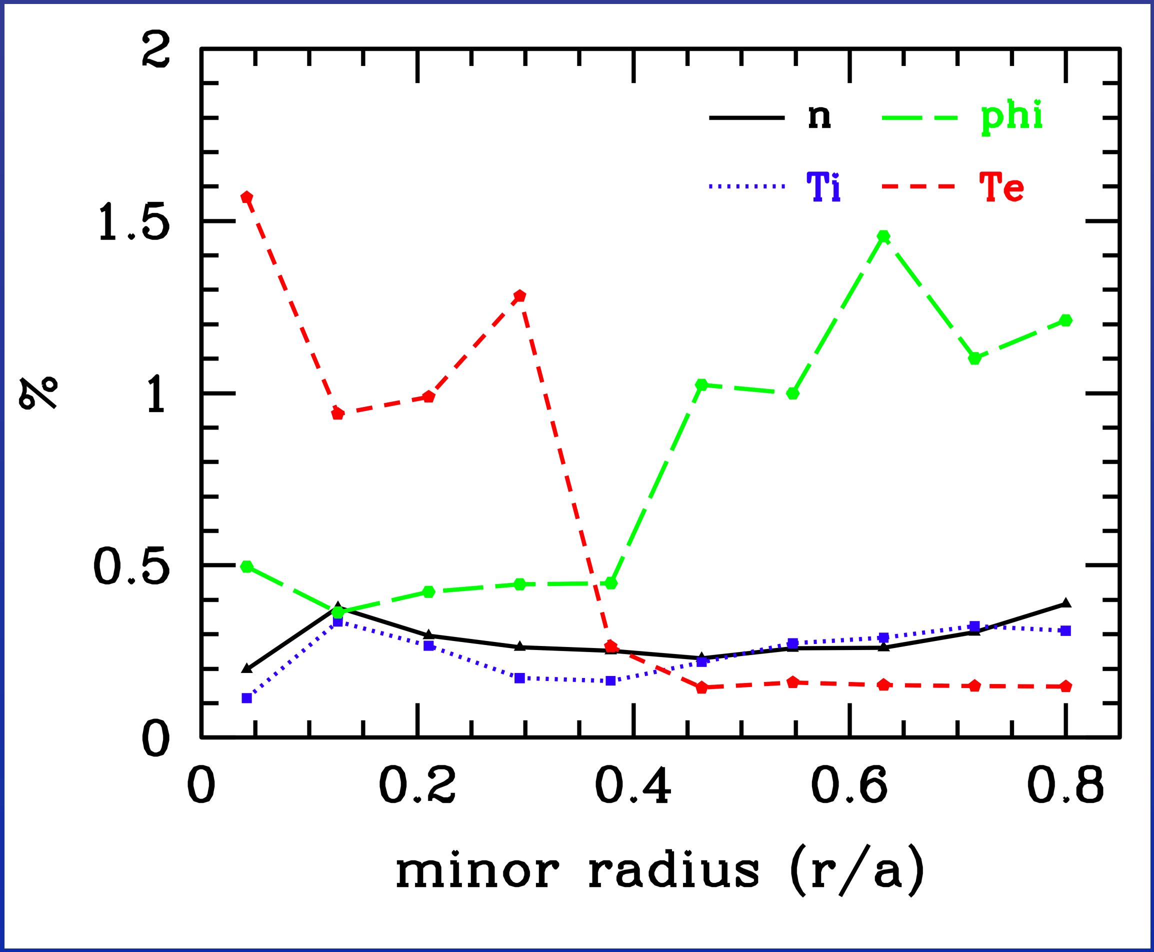


Profile stiffness

- ~ flat grad scale lengths indicative of stiffness (near critical gradient across most of minor radius)



Fluctuations



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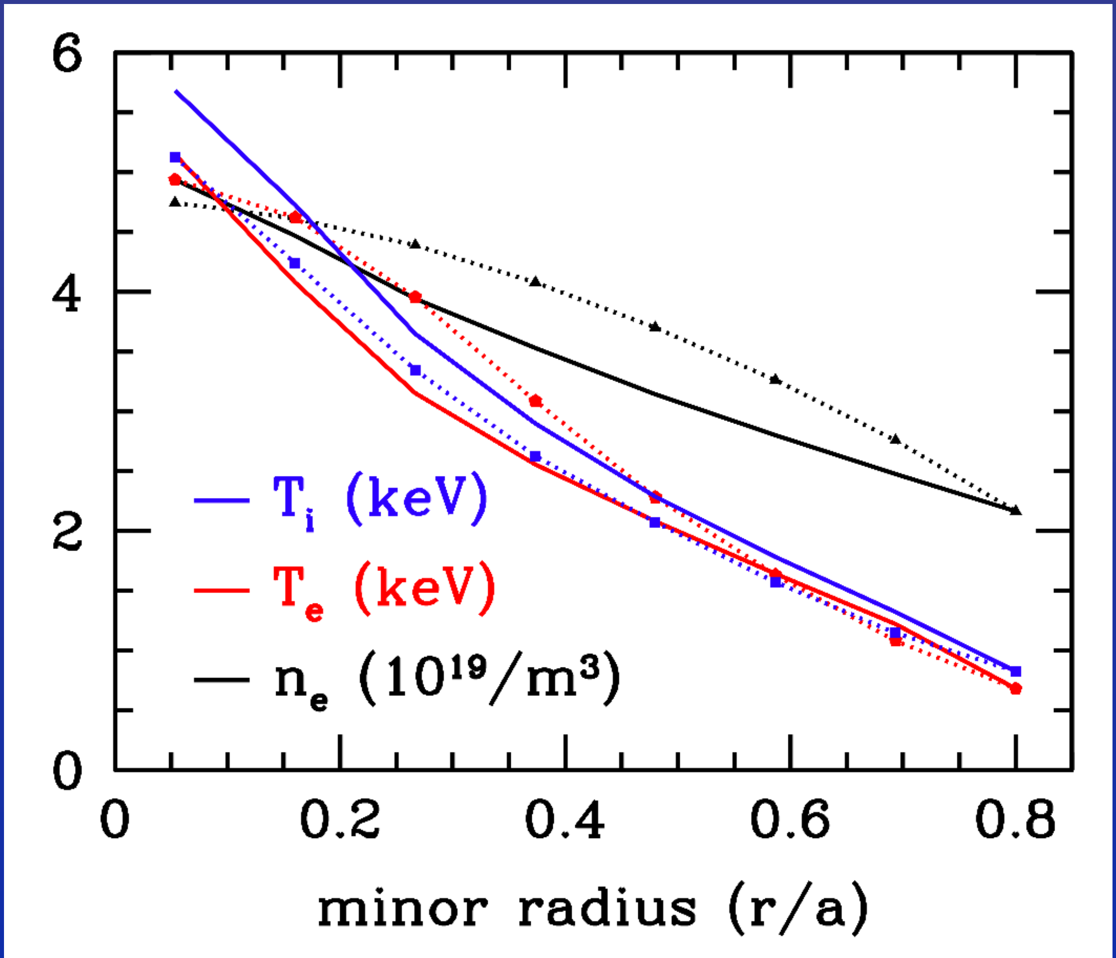
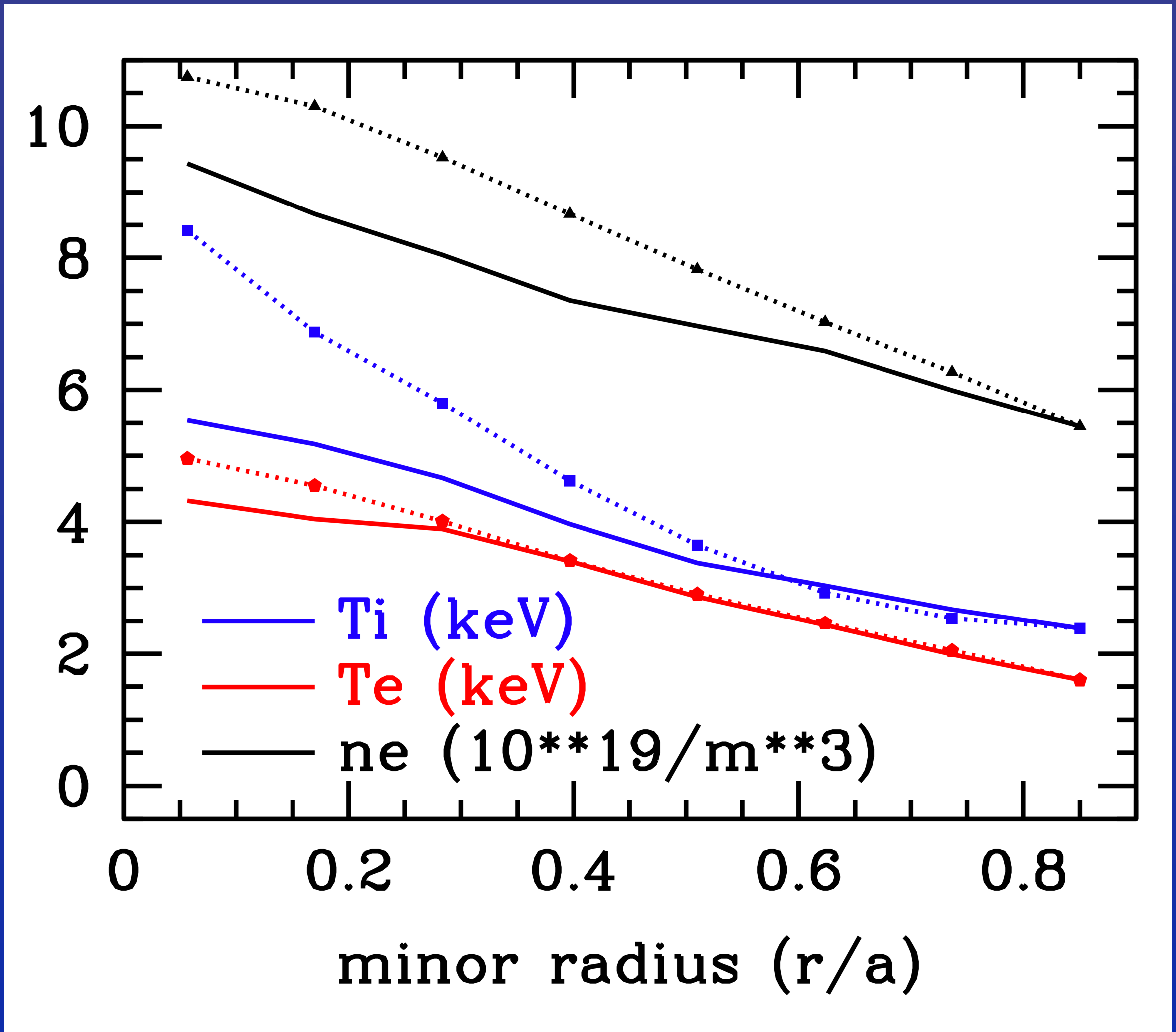


- L-mode discharge
- 8 radial grid points
- Costs ~25k CPU hrs (4 clock hrs)
- Flow shear absent

AUG shot #13151



- Fluxes calculated with GENE
- 8 radial grid points
- Costs ~400k CPU hrs (<24 clock hrs)
- Dens and electron temp profiles agree within ~10% across device
- Flow shear absent

Conclusions and future work

- Multi-scale approach provides savings of $\sim 10^5$
- Routine first-principles simulations of self-consistent interaction between turbulence and equilibrium possible
- Future work:
  - Further comparisons with experimental measurements
  - Momentum transport simulations
  - Magnetic equilibrium evolution
  - MHD stability
  - Improved neoclassical model
  - Coupling to global flux solver