Transitions to reduced transport regimes in rotating tokamak plasmas

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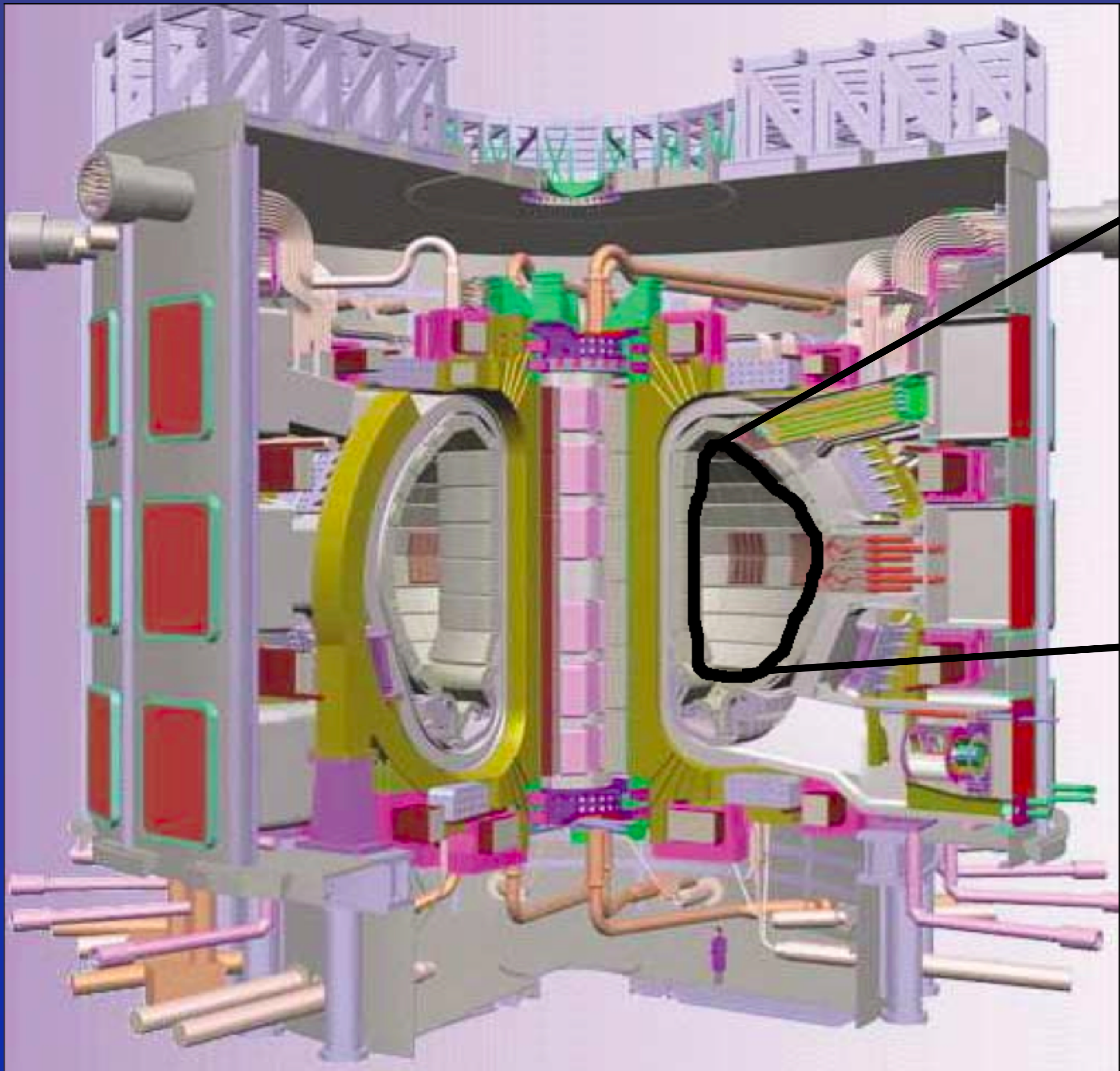
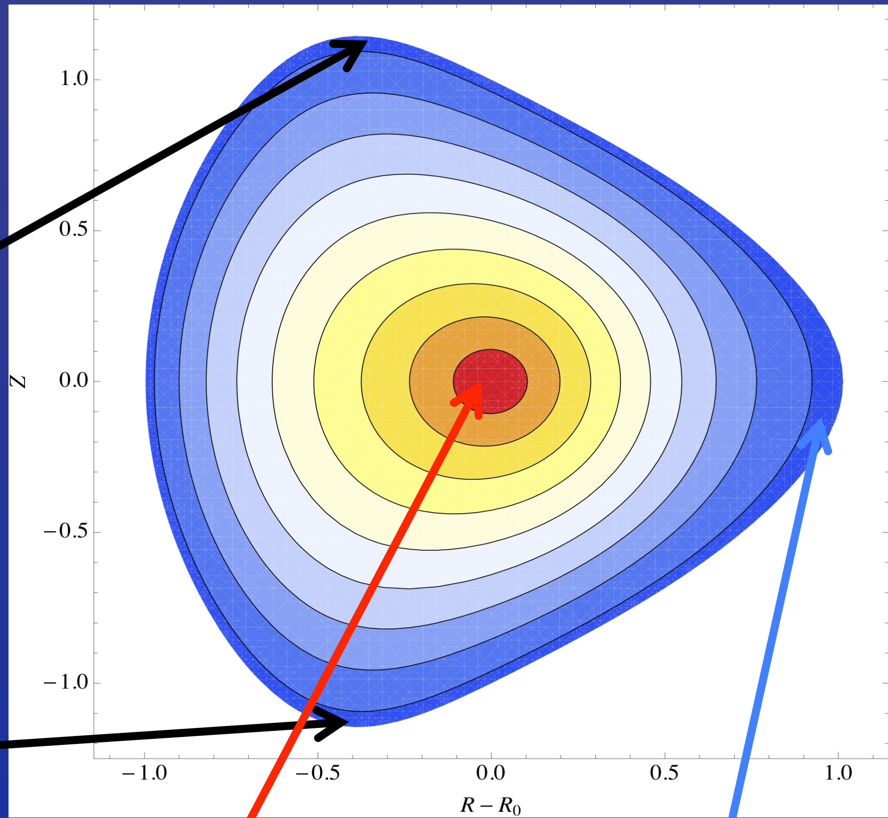
Magnetic confinement fusion



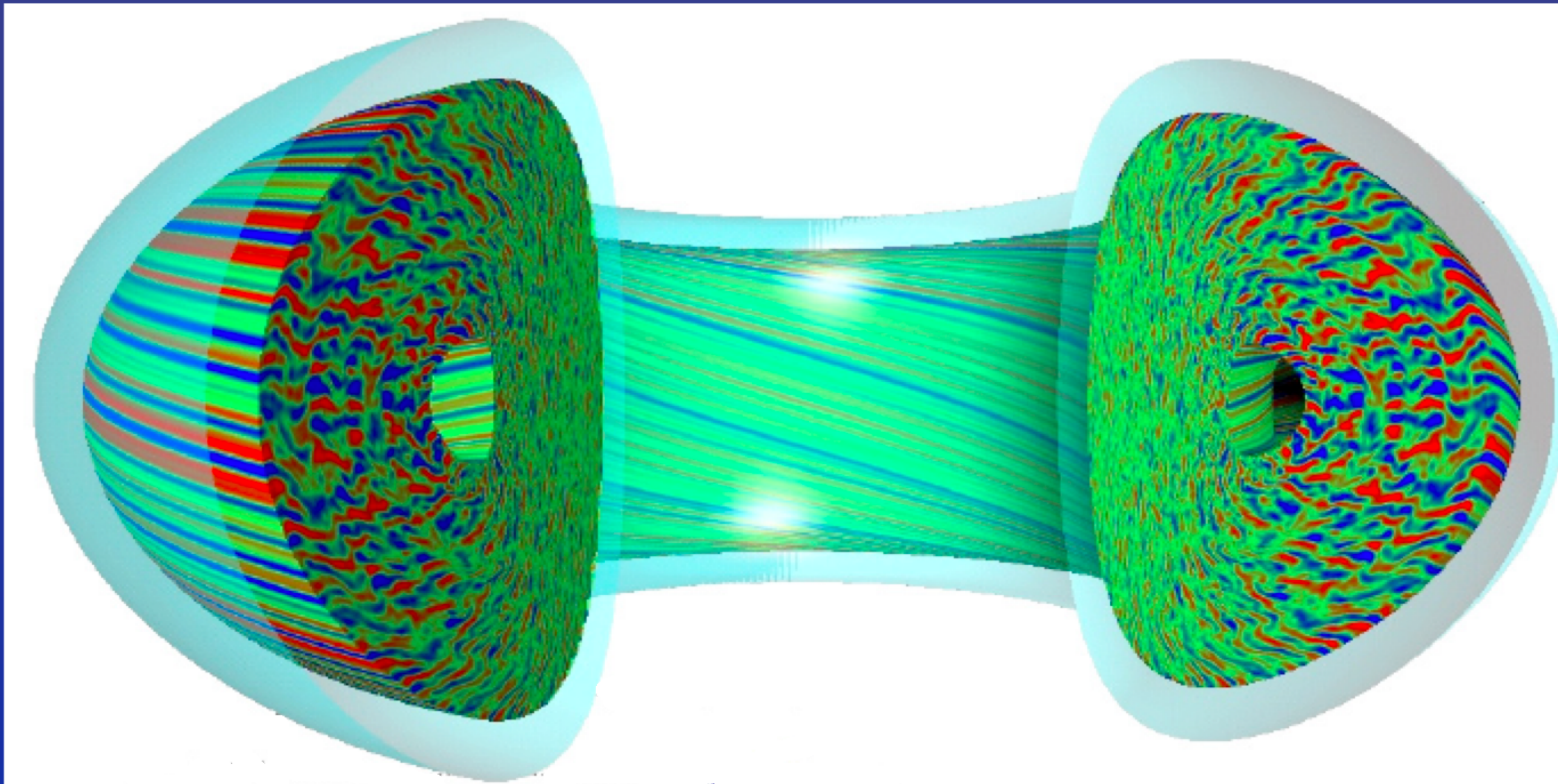
ITER



Hot, dense

Cold, dilute

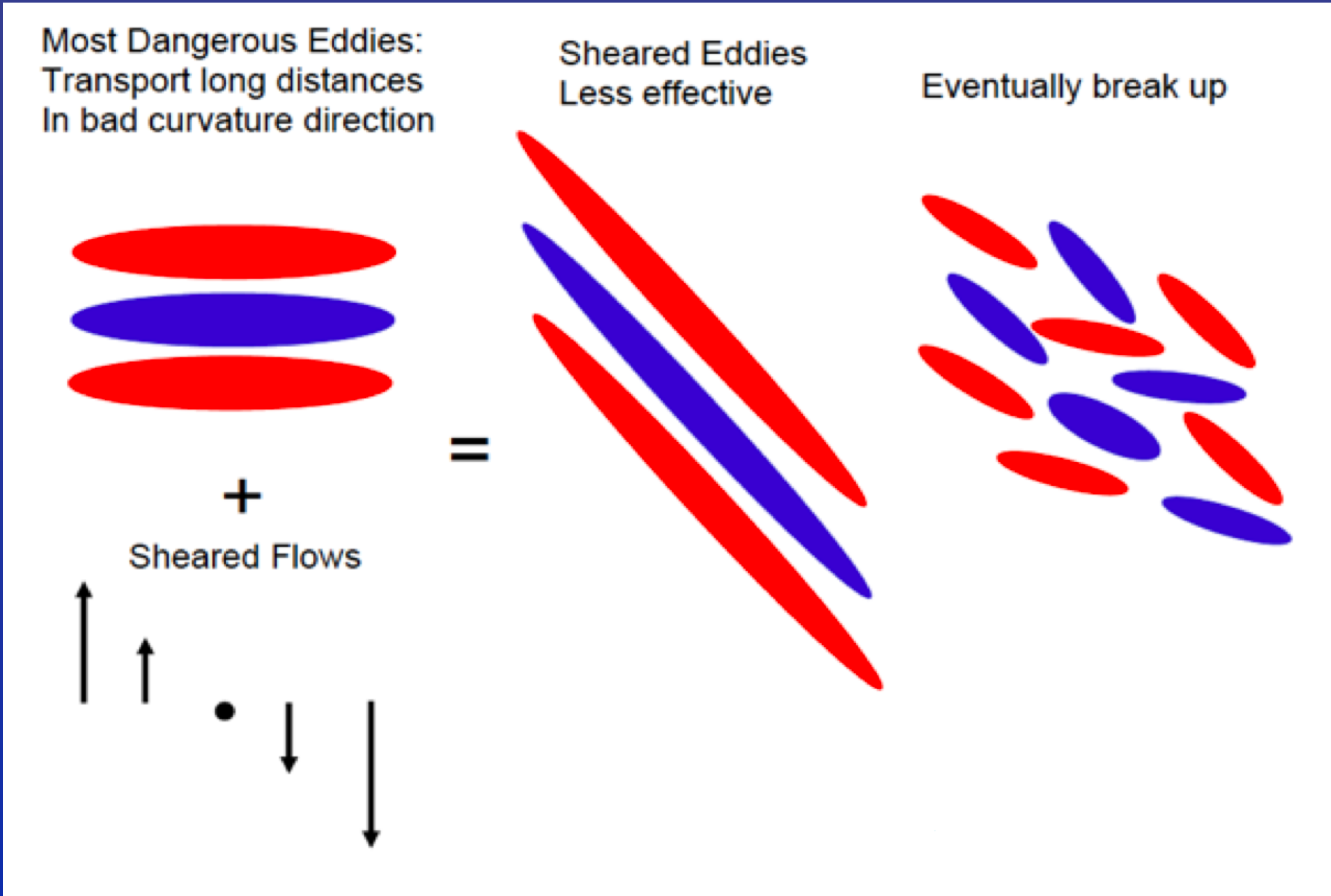
Resultant turbulence



GYRO simulation

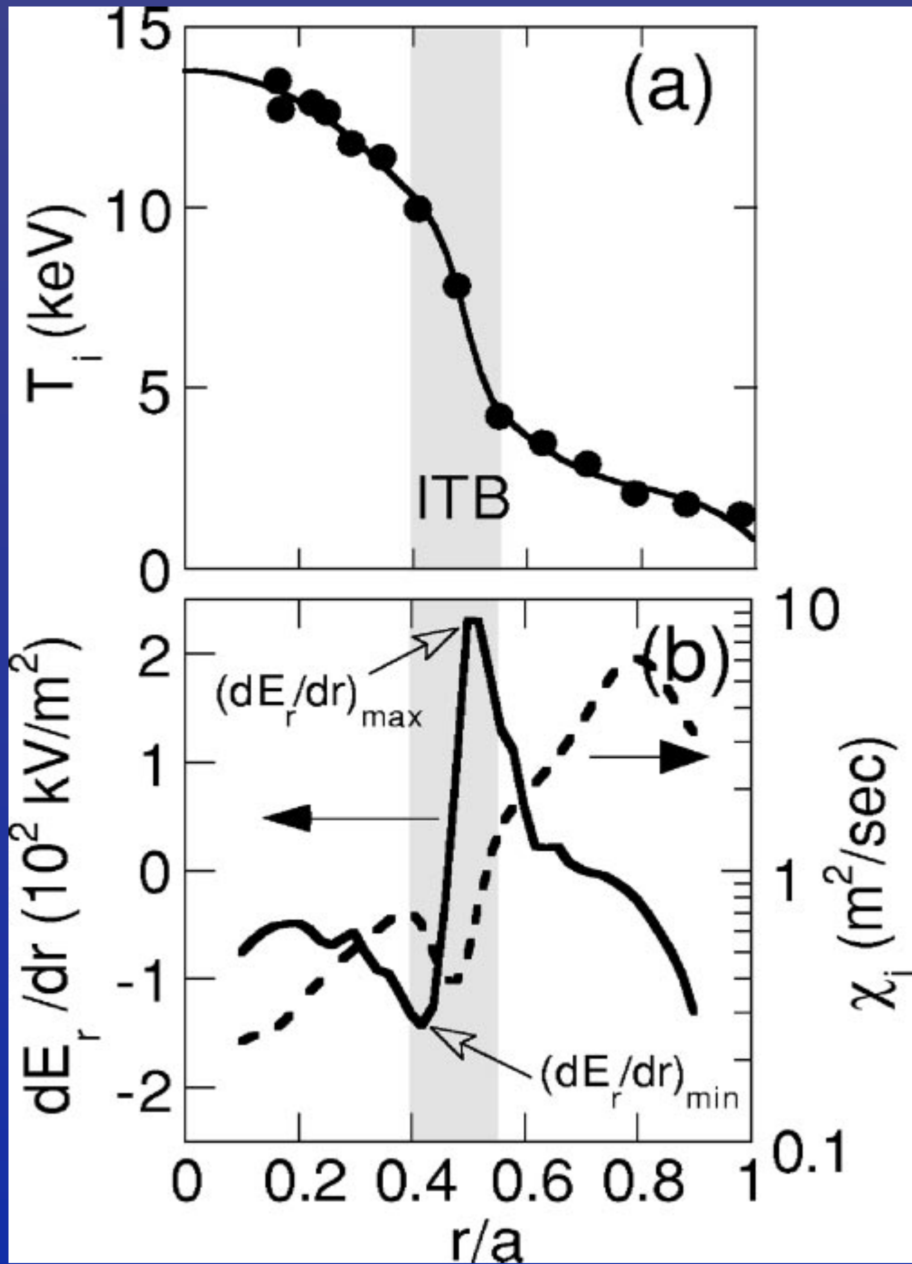
Turbulence-driven heat fluxes limit core plasma temperature

Suppression via sheared flow



Experimental observations

- “Internal Transport Barriers” (ITBs) observed in wide range of fusion devices
- Often accompanied by strong velocity shear and weak or negative magnetic shear
- How do ITBs work, and how can we make them better?



JT-60U data. Y. Miura, *et al.*, 10:1809:2003

Overview

- Theoretical and numerical model
- Effect of rotational shear on turbulent transport
- Implications for local gradients (0D)
- Extension to radial profiles (1D)

Multiple scale problem

- Solve for density, temperature, and flow, which depend on particle, heat, and momentum fluxes
- Fluxes depend on gradients of density, temperature, and flow, so problems are coupled

Physics	Perpendicular spatial scale	Temporal scale
Turbulence from ETG modes	$k_\perp^{-1}$ ~ 0.005 – 0.05 cm	ω_* ~ 0.5 - 5.0 MHz
Turbulence from ITG modes	$k_\perp^{-1}$ ~ 0.3 - 3.0 cm	ω_* ~ 10 - 100 kHz
Transport barriers	Measurements suggest width ~ 1 - 10 cm	100 ms or more in core?
Discharge evolution	Profile scales ~ 200 cm	Energy confinement time ~ 2 - 4 s

Gyrokinetic multiscale assumptions

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial f}{\partial \mathbf{R}} + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} + \frac{dE}{dt}\frac{\partial f}{\partial E} + \frac{d\vartheta}{dt}\frac{\partial f}{\partial \vartheta} = C[f]$$

- Turbulent fluctuations are low amplitude:

$$f = F + \delta f \qquad \delta f \sim \epsilon f$$

- Separation of time scales:

$$\frac{\partial_t \delta f}{\delta f} \sim \omega \sim \epsilon \Omega \qquad \frac{\partial_t F}{F} \sim \tau^{-1} \sim \epsilon^2 \omega$$

- Separation of space scales:

$$\nabla F \sim F/L, \quad \nabla_{||} \delta f \sim \delta f / L, \quad \nabla_{\perp} \delta f \sim \delta f / \rho$$

- "Smooth" velocity space:

$$\epsilon \lesssim \nu/\omega \lesssim 1 \Rightarrow \sqrt{\epsilon} \lesssim \delta v / v_{th} \lesssim 1$$

- Sub-sonic drifts: $v_D \sim \epsilon v_{th}$

Transport equations in GK

Moment equations for evolution of mean quantities:

$$\frac{\partial n_s}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \mathbf{\Gamma}_s \cdot \nabla\psi \right\rangle\right) + S_n$$

$$\begin{aligned}\frac{3}{2}\frac{\partial n_s T_s}{\partial t} &= -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \mathbf{Q}_s \cdot \nabla\psi \right\rangle\right) \\ &+ T_s\left(\frac{\partial \ln n_s}{\partial \psi} - \frac{3}{2}\frac{\partial \ln T_s}{\partial \psi}\right)\left\langle \mathbf{\Gamma}_s \cdot \nabla\psi \right\rangle + \frac{\partial \ln T_s}{\partial \psi}\left\langle \mathbf{Q}_s \cdot \nabla\psi \right\rangle \\ &- \left\langle \int d^3v \frac{h_s T_s}{F_{0s}} \left\langle C[h_s]\right\rangle_{\mathbf{R}} \right\rangle + n_s \nu_\epsilon^{su}\left(T_u - T_s\right) + S_p\end{aligned}$$

$$\frac{\partial L}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\sum_s \left\langle \pi_s \right\rangle\right) + S_L$$

Sugama (1998)

Gyrokinetic equation

GK equation with mean flow satisfying $\frac{\rho}{L} \ll M \ll 1$

but : $\nabla u \sim v_{th}/L$

$$\frac{dh}{dt} + \left(\mathbf{v}_{\parallel} + \mathbf{v}_D + \langle \mathbf{v}_E \rangle\right) \cdot \nabla h - \langle C[h] \rangle = \frac{eF_0}{T}\frac{d\langle\varphi\rangle}{dt} - \langle \mathbf{v}_E \rangle \cdot \nabla\psi \left(\frac{dF_0}{d\psi} + \frac{mv_{\parallel}}{T}\frac{RB_{\phi}}{B}\frac{d\omega}{d\psi}F_0\right)$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + R\omega(\psi)\hat{\mathbf{e}}_{\phi} \cdot \nabla$$

$$\mathbf{u} = R\omega\hat{\mathbf{e}}_{\phi}$$

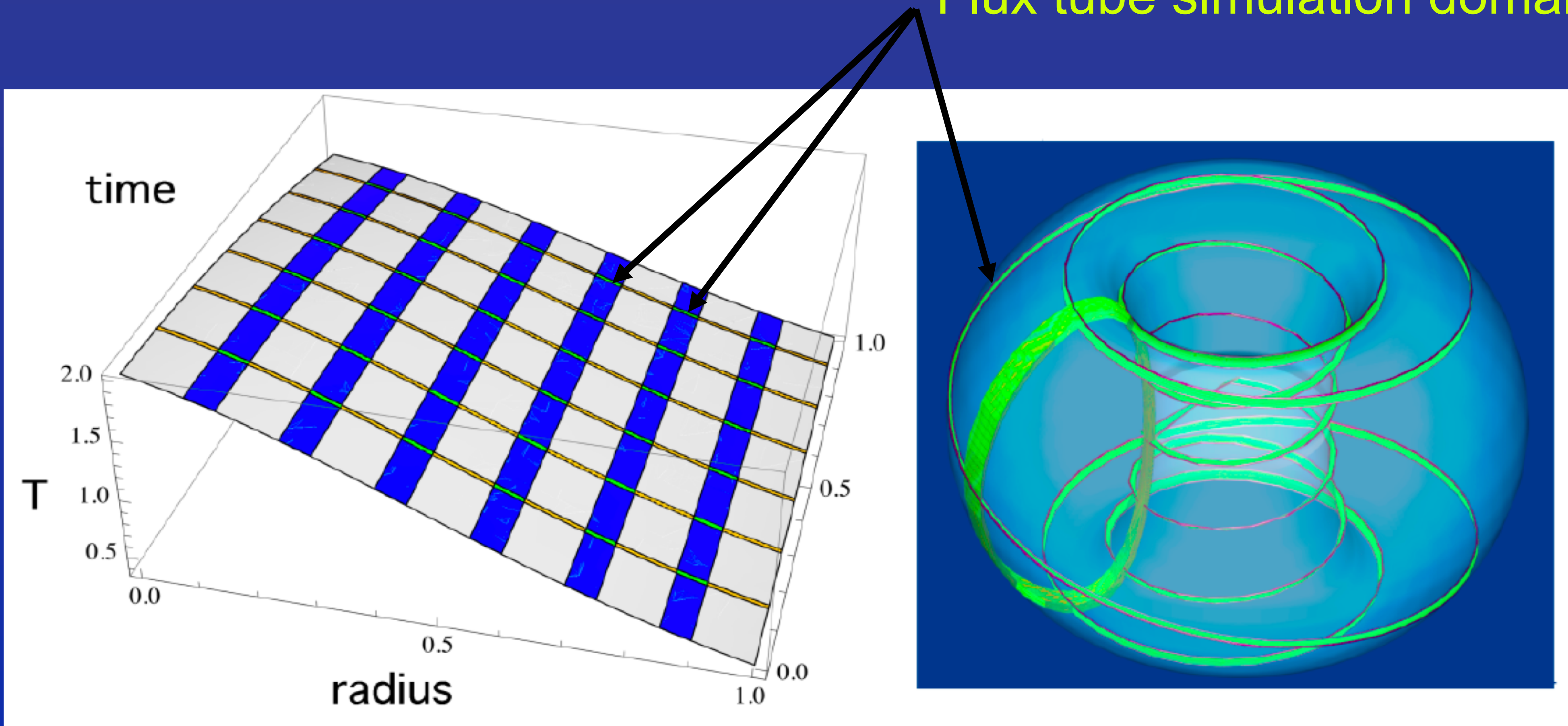
$$\gamma_E \equiv \frac{\psi}{q}\frac{d\omega}{d\psi}\frac{R_0}{v_{th}}$$

Local approximation:

$$\omega(\psi) \approx \omega(\psi_0) + (\psi - \psi_0)\left.\frac{d\omega}{d\psi}\right|_{\psi_0}$$

Multiscale approach

- In TRINITY [Barnes *et al.*, PoP **17**, 056109 (2010)], turbulent fluctuations calculated in small regions of fine space-time grid embedded in "coarse" grid (for mean quantities)



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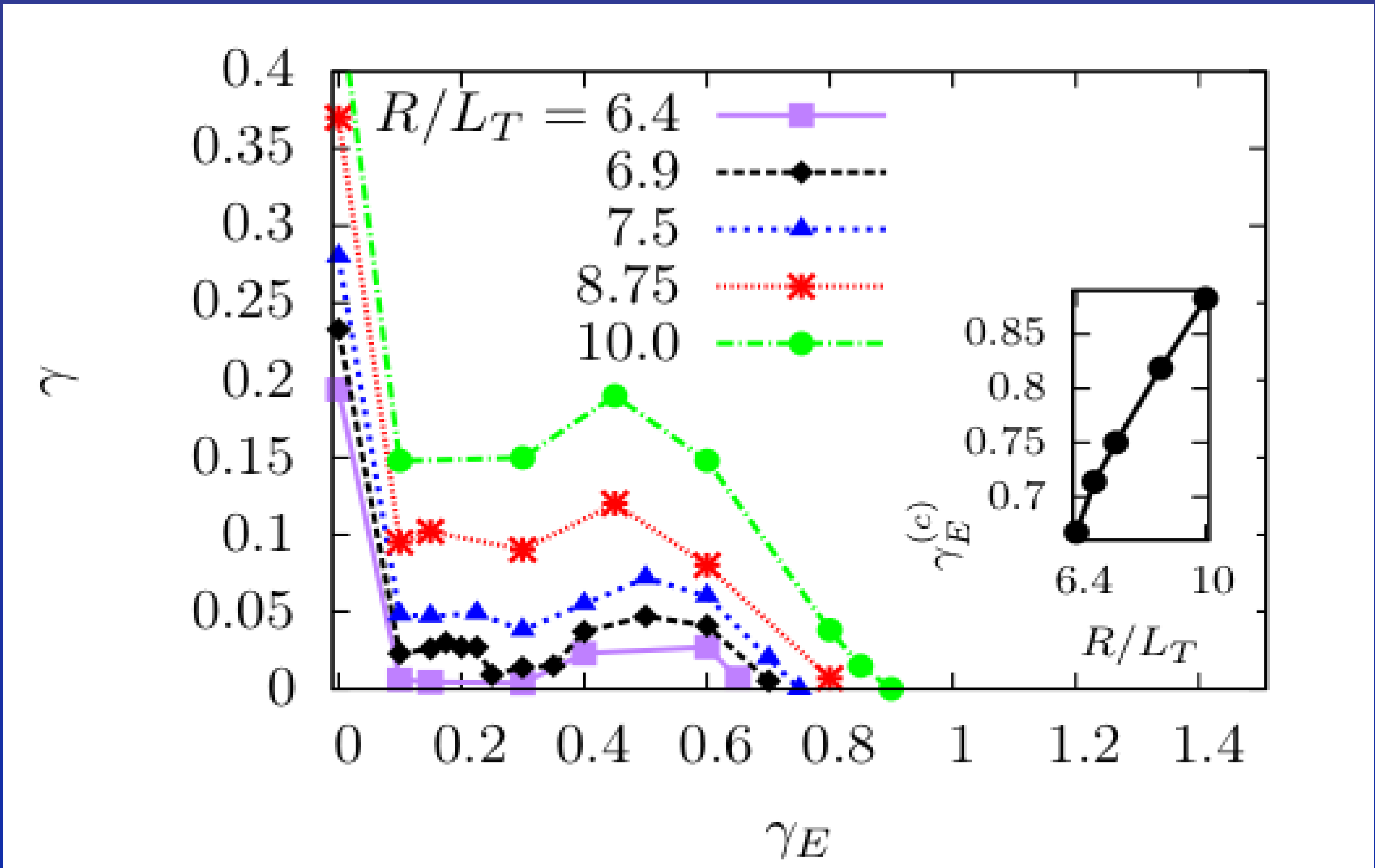
Linear stability (GS2)

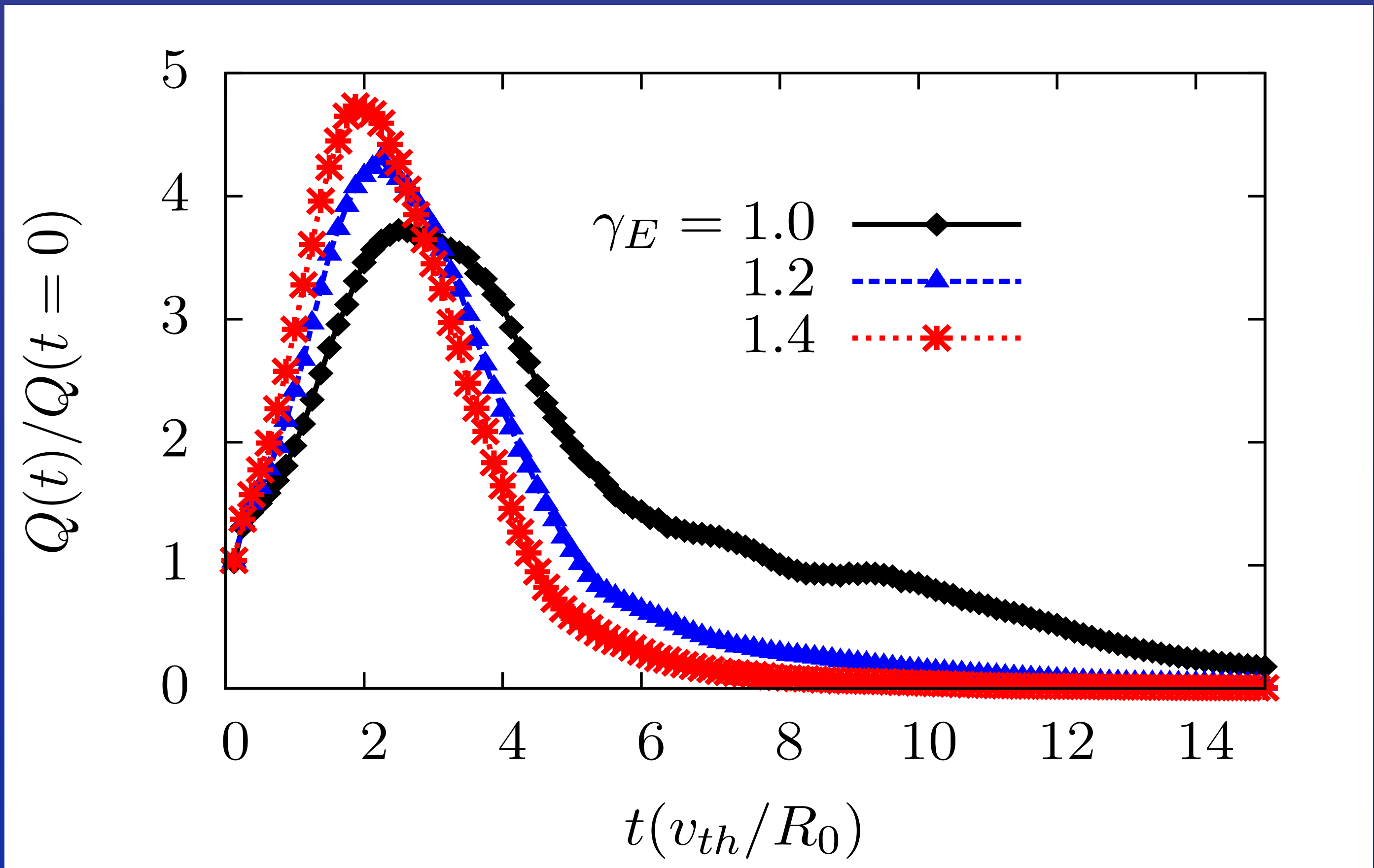
Cyclone base case: $r/R = 0.18 \quad q = 1.4 \quad \hat{s} = 0.8$



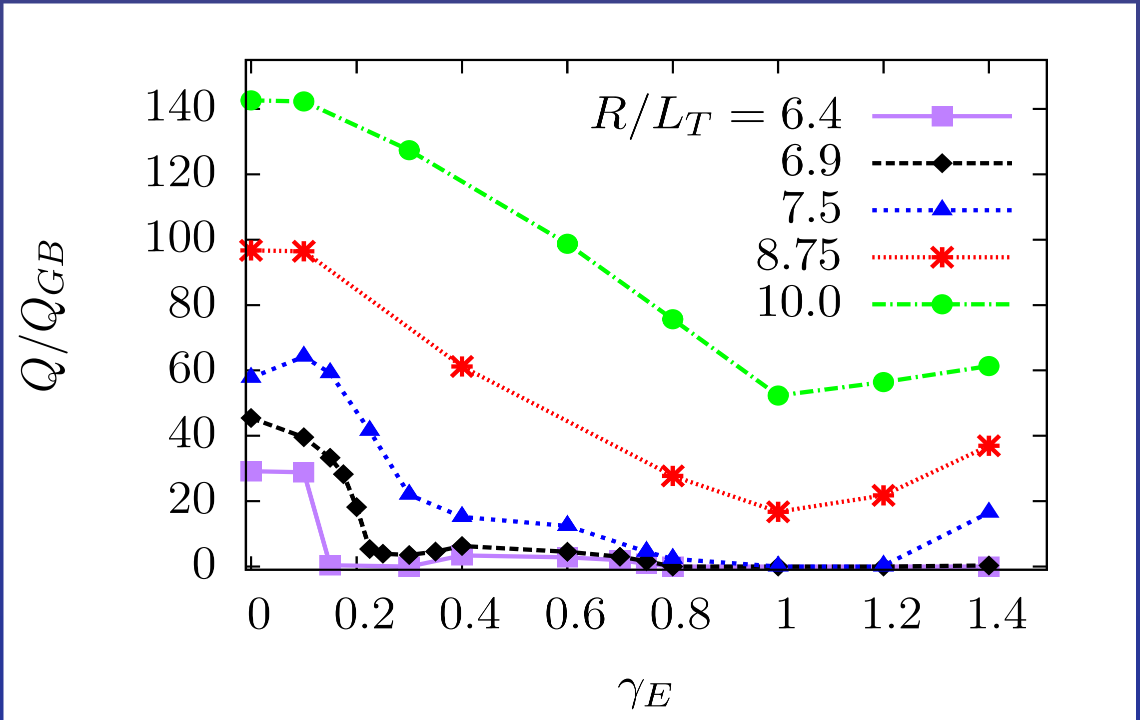
- ITG drive at small shear
- ITG/PVG drive at moderate shear
- Stabilization at large shear
- Roughly linear dependence of critical flow shear on R/LT

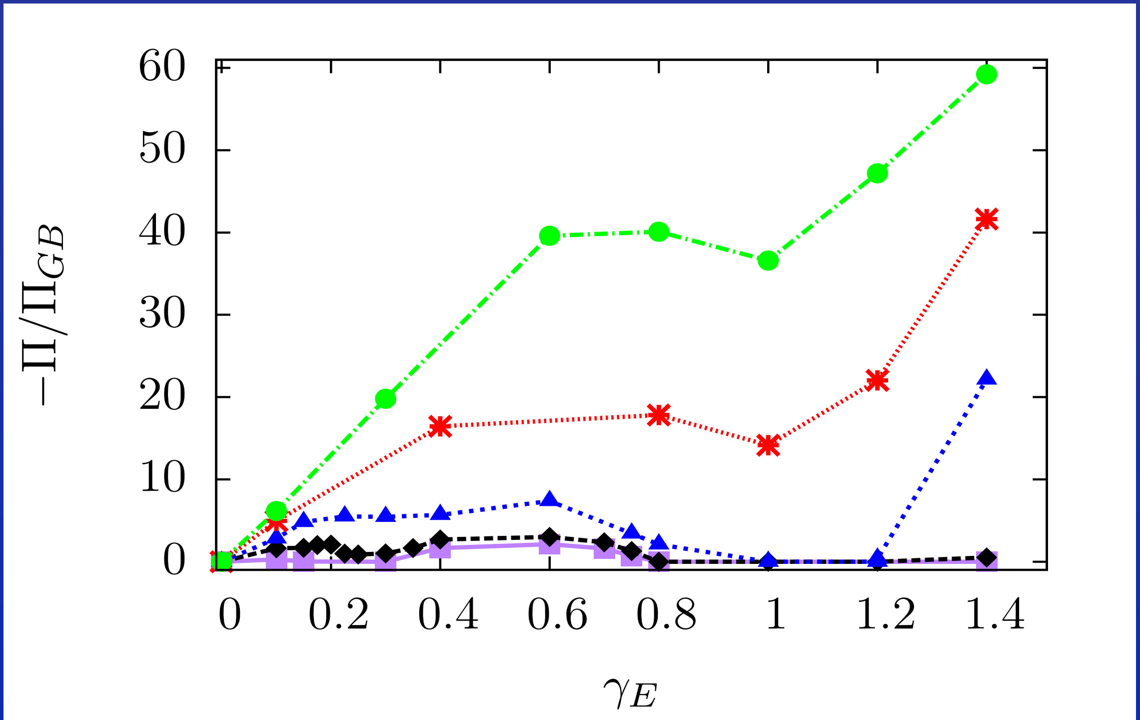
Barnes et al., 2010 (arXiv:1007.3390)

Transient growth



- Beyond critical shear value, transient linear growth
- Amplification of initial amplitude increases with shear
- Cf. Newton et al., 2010 (arXiv: 1007.0040)



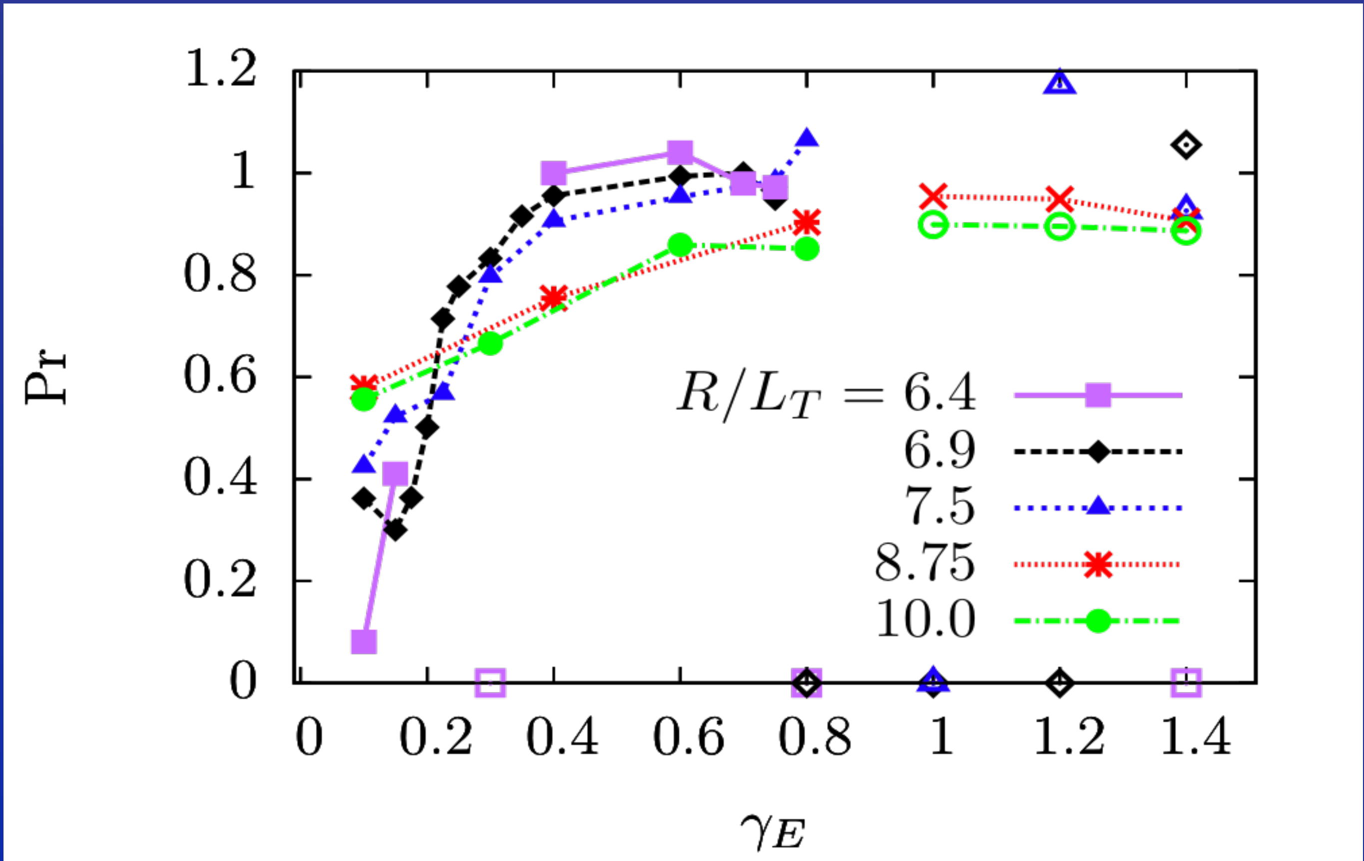


- Fluxes follow linear trends up to linear stabilization point
- Subcritical (linearly stable) turbulence beyond this point
- Optimal flow shear for confinement
- Possible hysteresis
- Maximum in momentum flux => possible bifurcation

Turbulent Prandtl number

$$\mathrm{Pr} = \frac{\nu_i}{\chi_i} \qquad \Pi_i = -m_i v_{th}(qR_0/r)\nu_i\gamma_E$$

$$Q_i = -\chi_i dT_i/dr$$

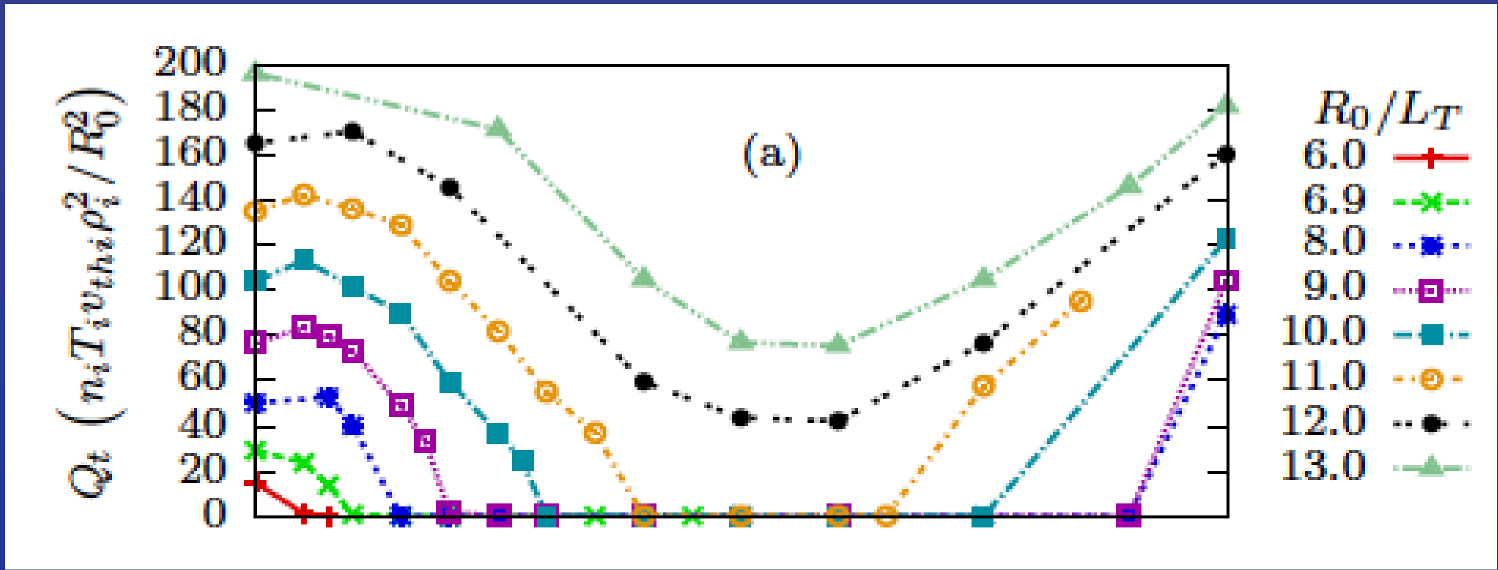


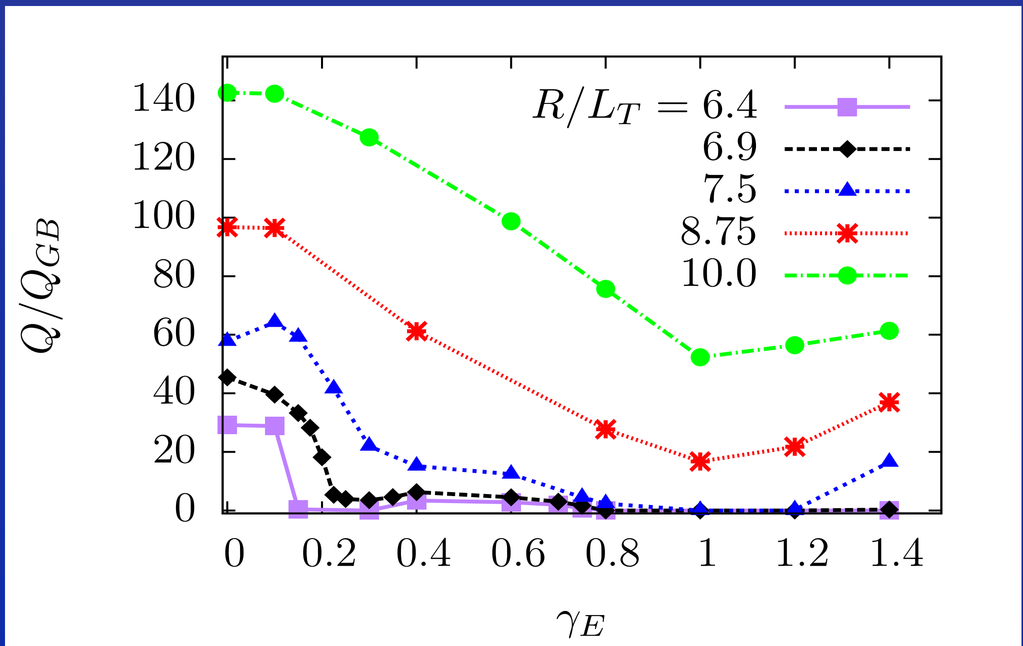
$\hat{s} = 0.8$

- Prandtl number tends to shear- and R/LT-independent value of order unity (in both turbulence regimes)

Barnes *et al.*, PRL submitted (2010).

Zero magnetic shear





- Similar…sort of
- All turbulence subcritical
- Very different critical flow shear values

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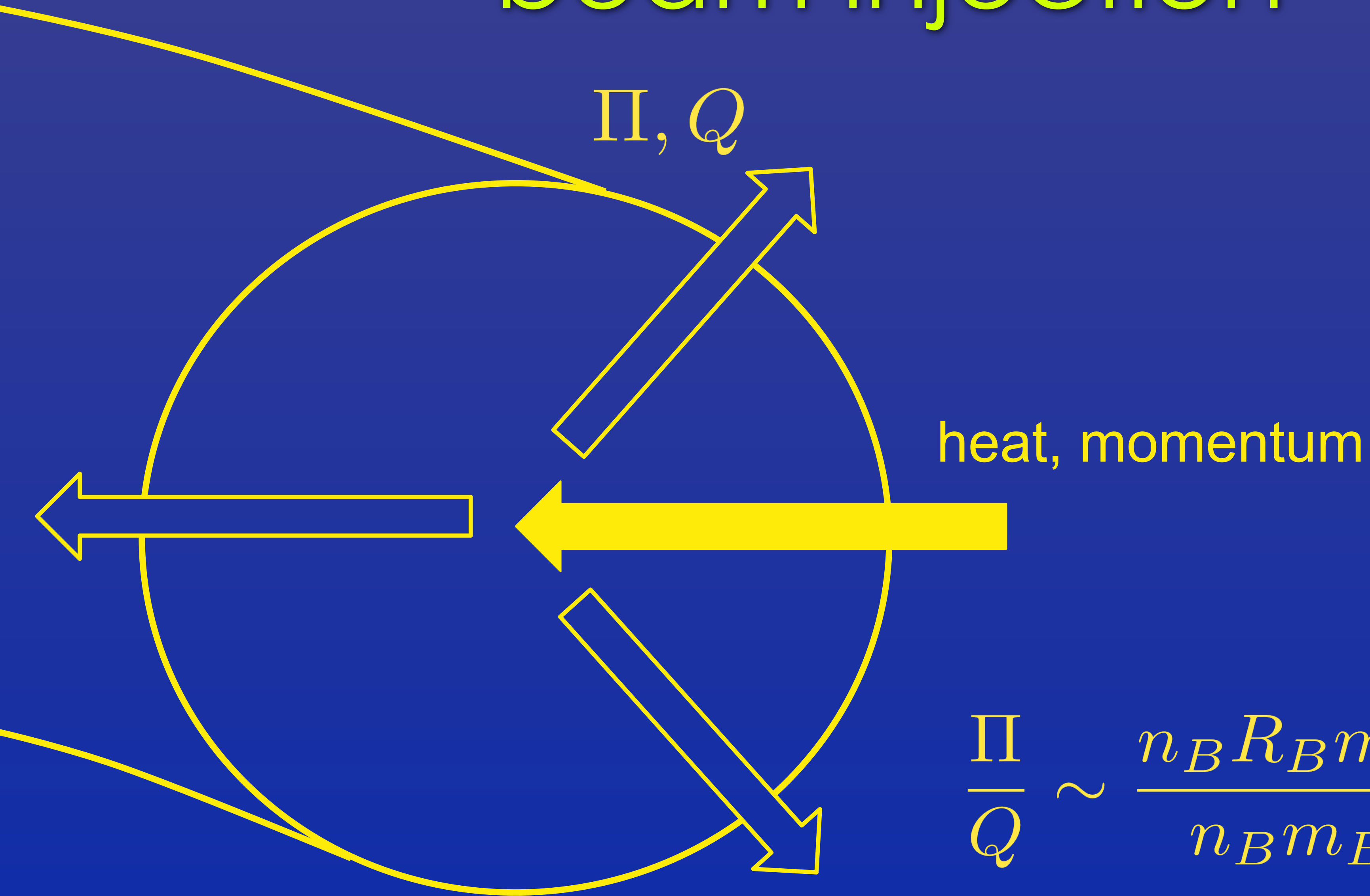
Power/Torque balance for beam injection



$$\frac{\Pi}{Q} \sim \frac{n_B R_B m_B v_B^2}{n_B m_B v_B^3} = \frac{R_B}{v_B}$$

Model fluxes

- Simple model for fluxes with parameters chosen to fit zero magnetic shear results from GS2:

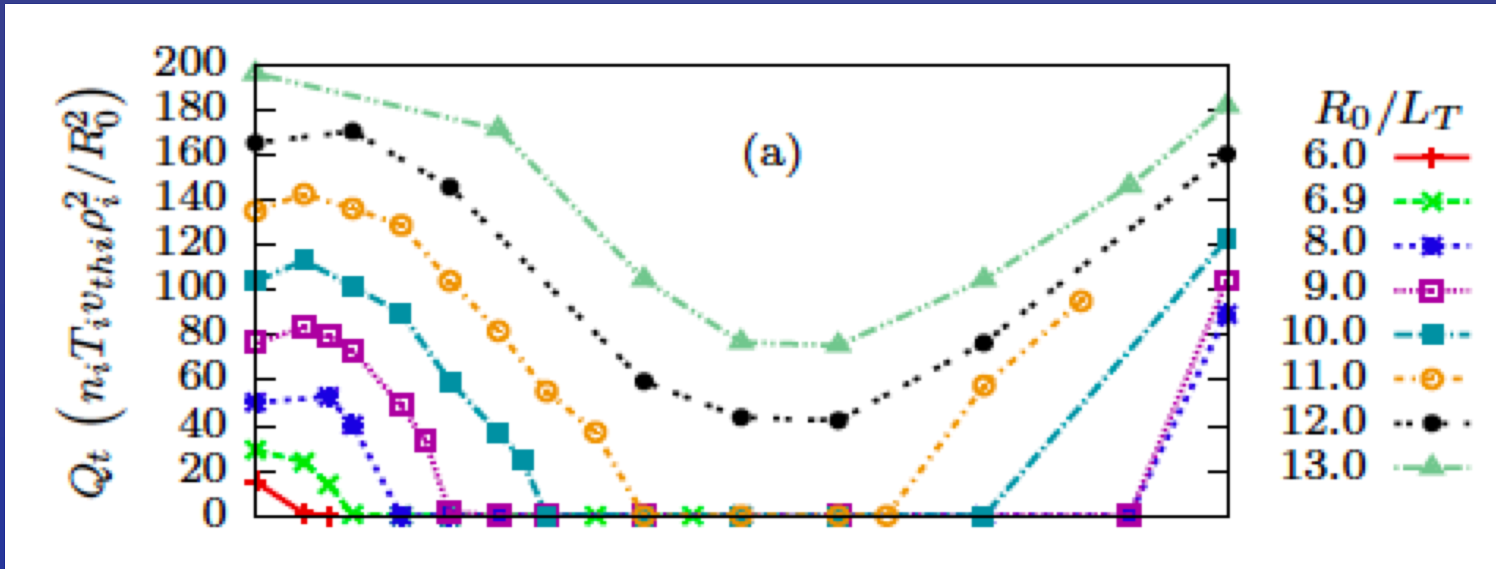
$$Q = Q_t + Q_n \qquad \Pi = \Pi_t + \Pi_n$$

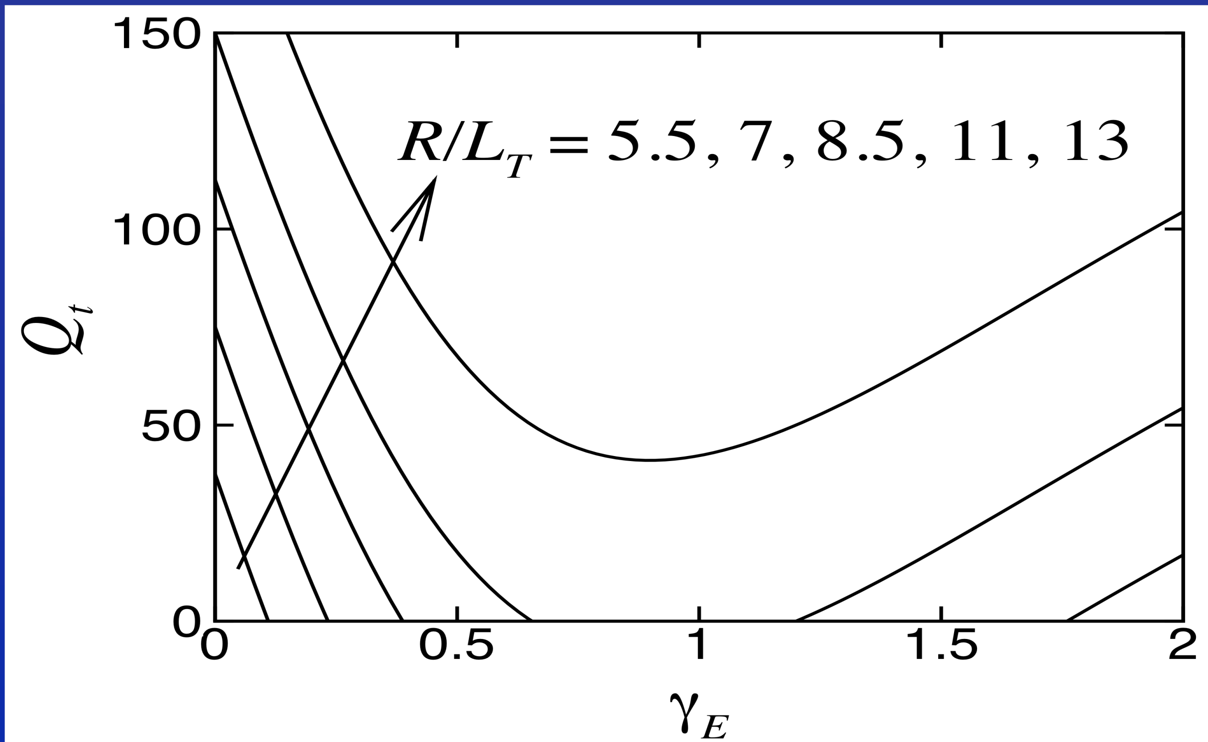
$$\overline{Q}_t \equiv \frac{Q_t}{nTv_{th}}\left(\frac{R}{\rho}\right)^2 \equiv \chi_t\left[\frac{R}{L_T} - \left(\frac{R}{L_T}\right)_c\right]$$

$$\overline{Q}_n \equiv \frac{Q_n}{nTv_{th}}\left(\frac{R}{\rho}\right)^2 \equiv \frac{\chi_n}{T^2}\frac{R}{L_T} \qquad \left(\frac{R}{L_T}\right)_c \equiv \frac{\alpha_1\gamma_E + (R/L_T)_{c0}}{1+\alpha_2\gamma_E^2}$$

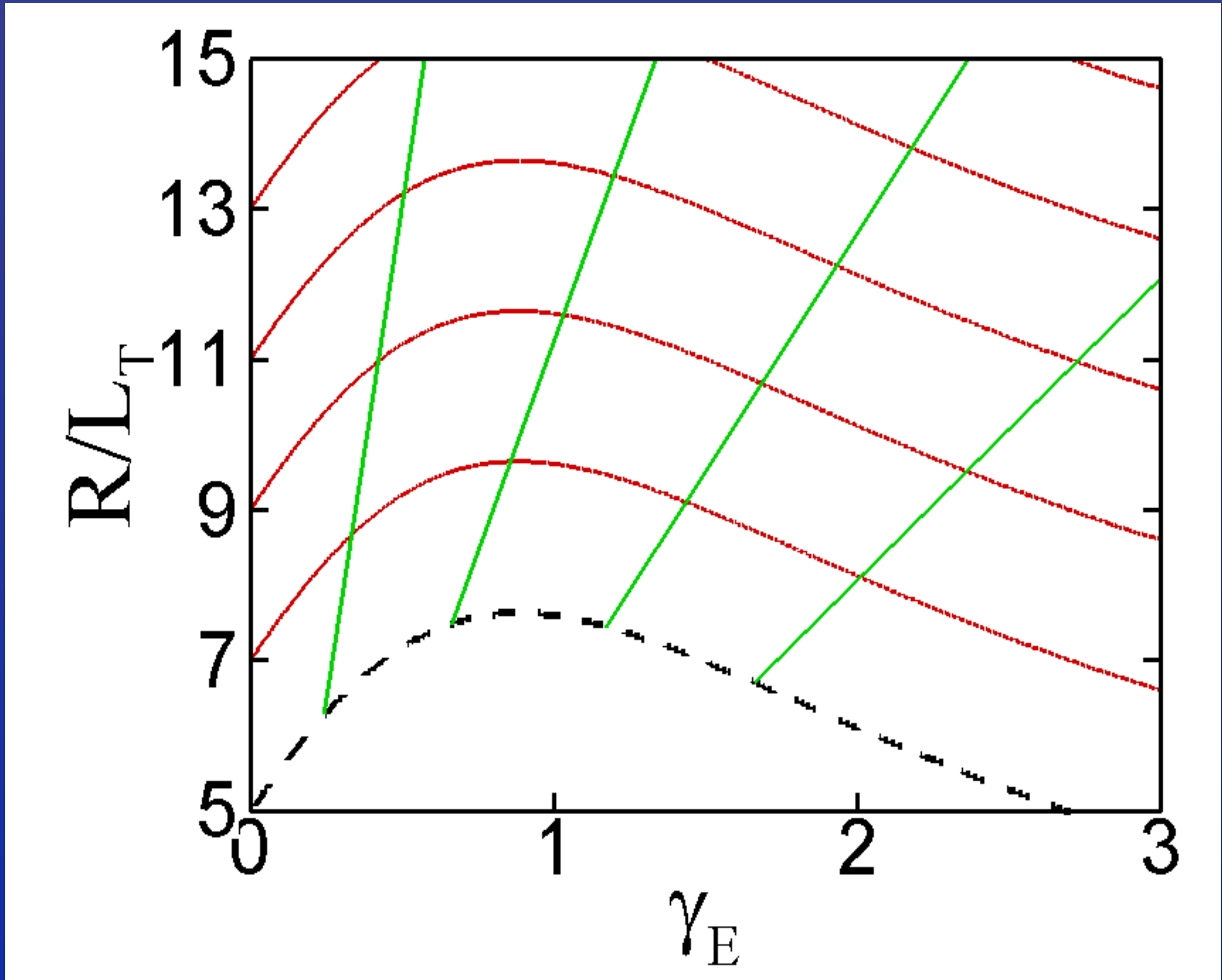
$$\overline{\Pi}_{t,n} \equiv \frac{\Pi_{t,n}}{mnRv_{th}^2}\left(\frac{R}{\rho}\right)^2 = \overline{Q}_{t,n}\mathrm{Pr}_{t,n}\frac{\gamma_E}{R/L_T}$$

Model fluxes





Balance w/o neoclassical



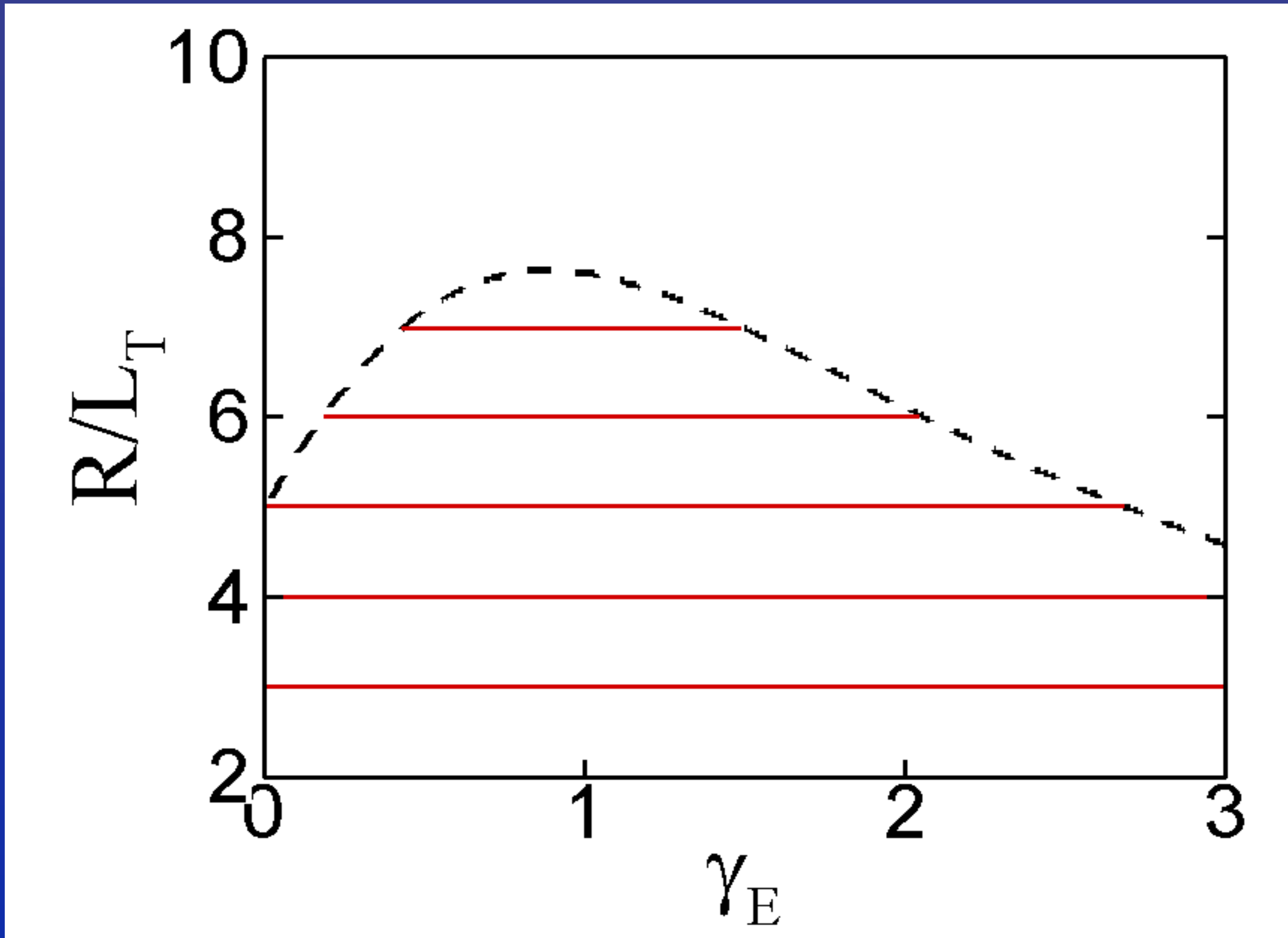
- $\overline{Q}$ = red lines
- $\overline{\Pi}/\overline{Q}$ = green lines

$$\frac{R}{L_t} = \frac{\mathrm{Pr}_t}{\overline{\Pi}/\overline{Q}}\gamma_E$$

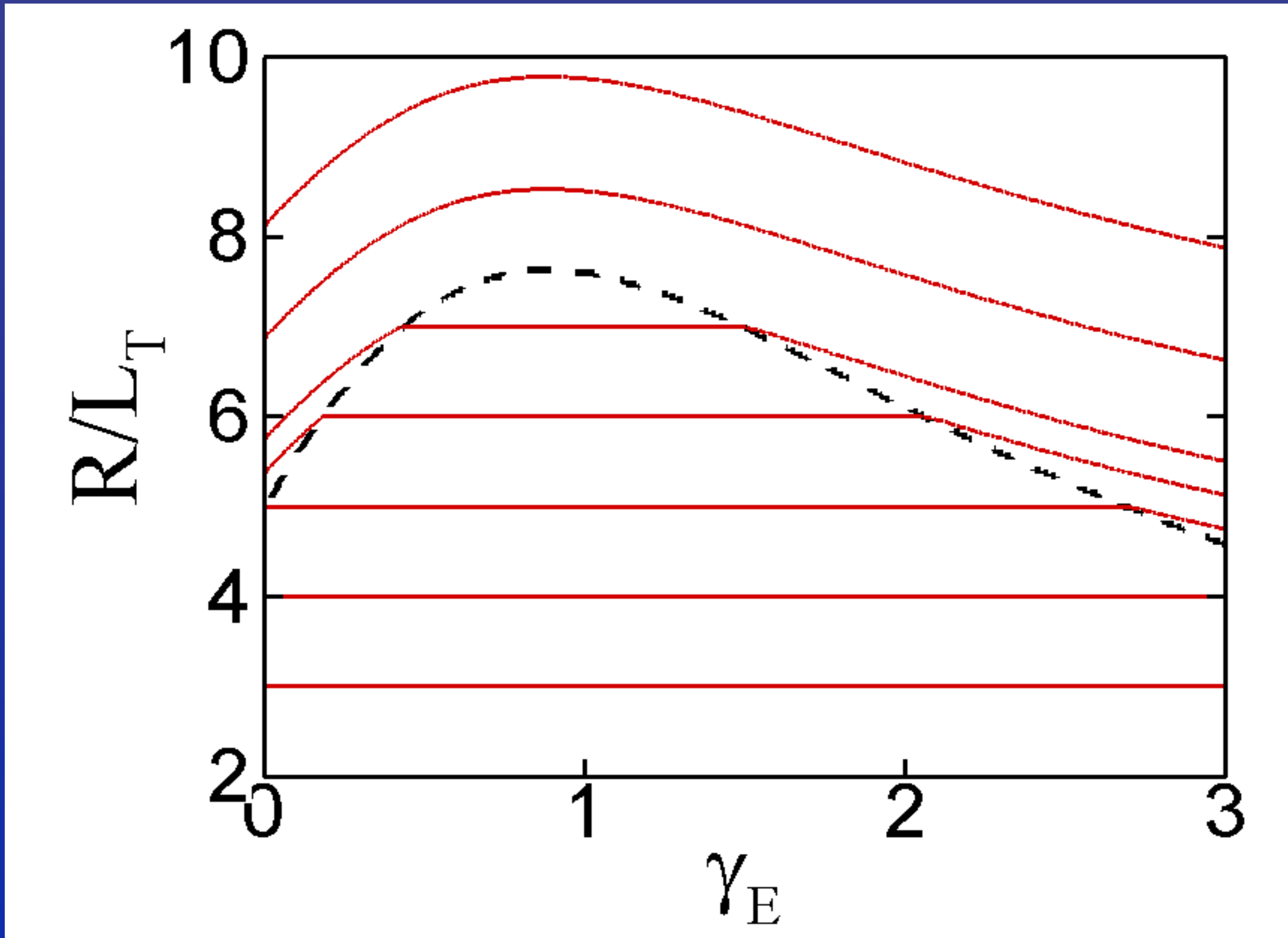
- Critical gradient = dashed line
- For given $\overline{\Pi}/\overline{Q}$ and $\overline{Q}$, only one solution

  No bifurcation!

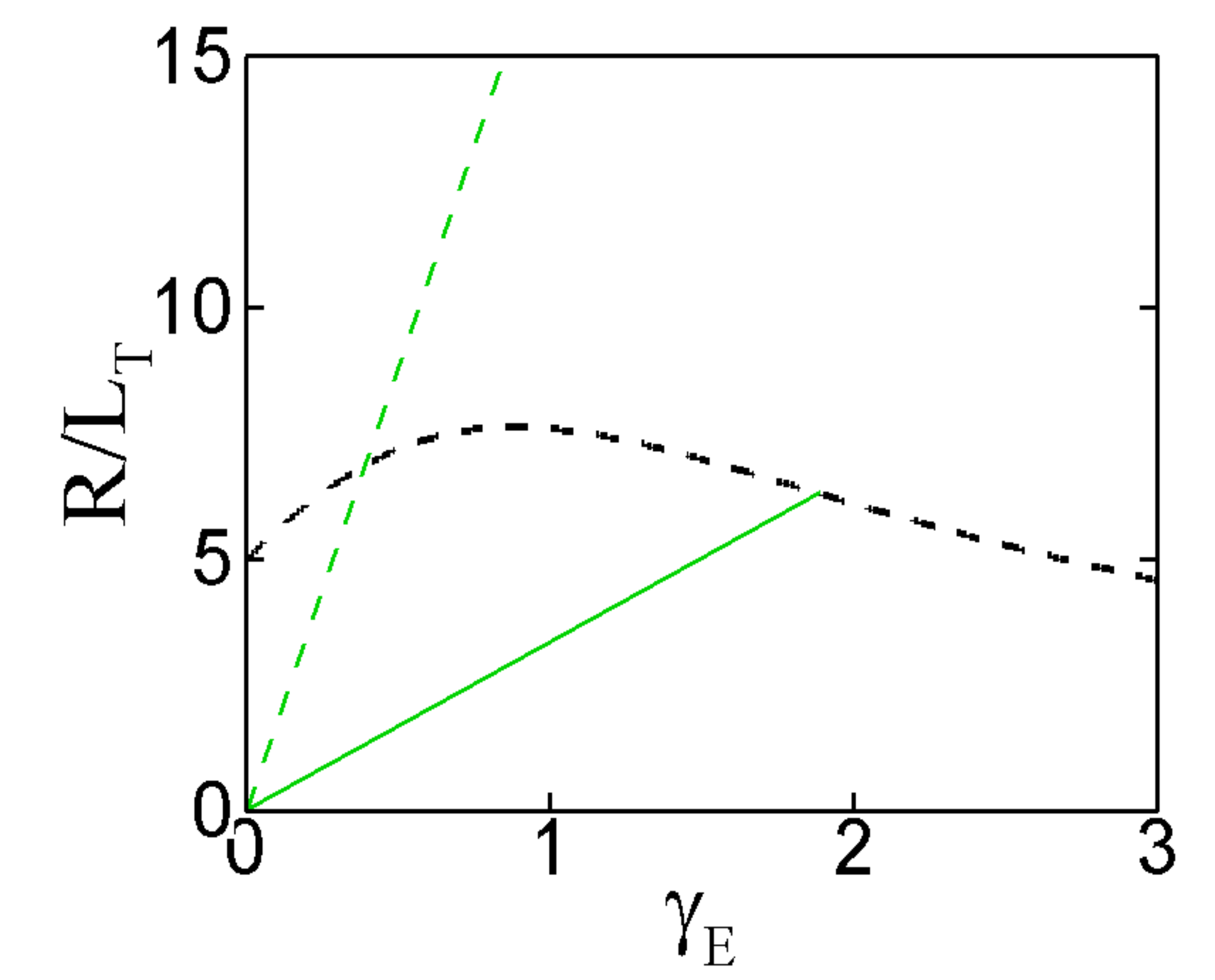
Neoclassical energy flux



Balance with neoclassical



Curves of constant $\overline{\Pi}/\overline{Q}$



- Neoclassical

$$\frac{R}{L_t} = \frac{\mathrm{Pr}_n}{\overline{\Pi}/\overline{Q}}\gamma_E$$

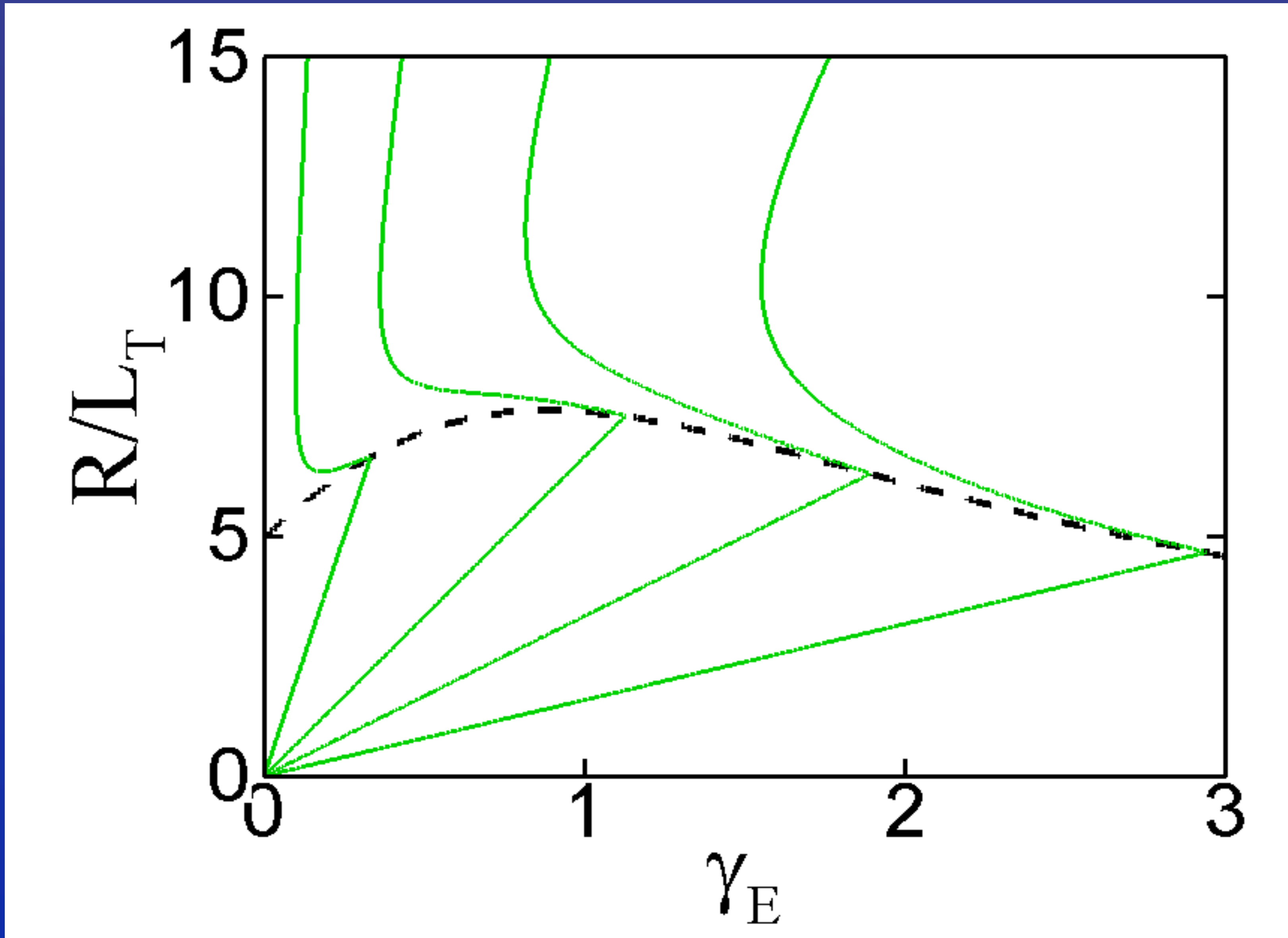
- Turbulent

$$\frac{R}{L_t} = \frac{\mathrm{Pr}_t}{\overline{\Pi}/\overline{Q}}\gamma_E$$

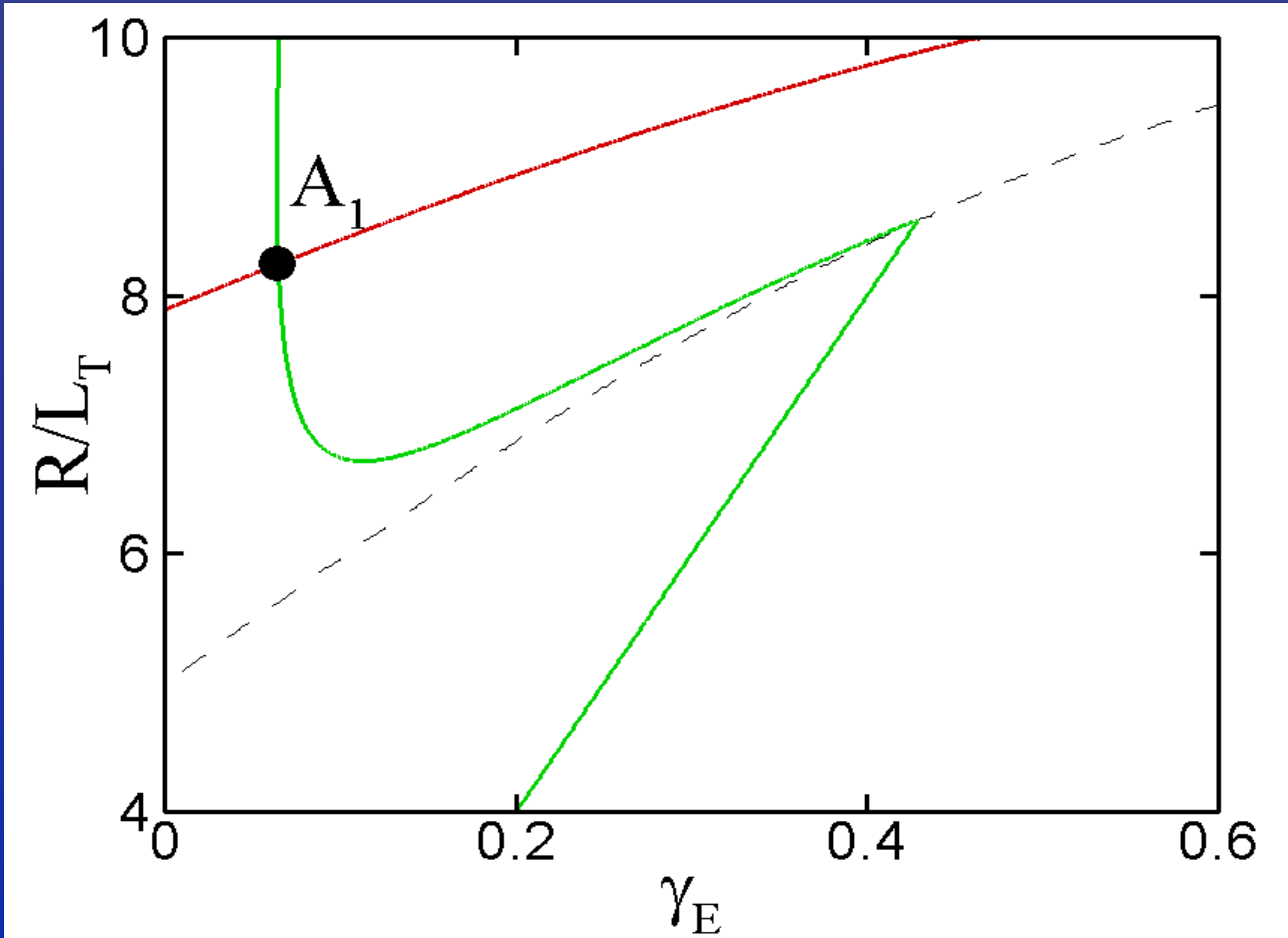
- Prandtl numbers

$$\mathrm{Pr}_n \ll \mathrm{Pr}_t$$

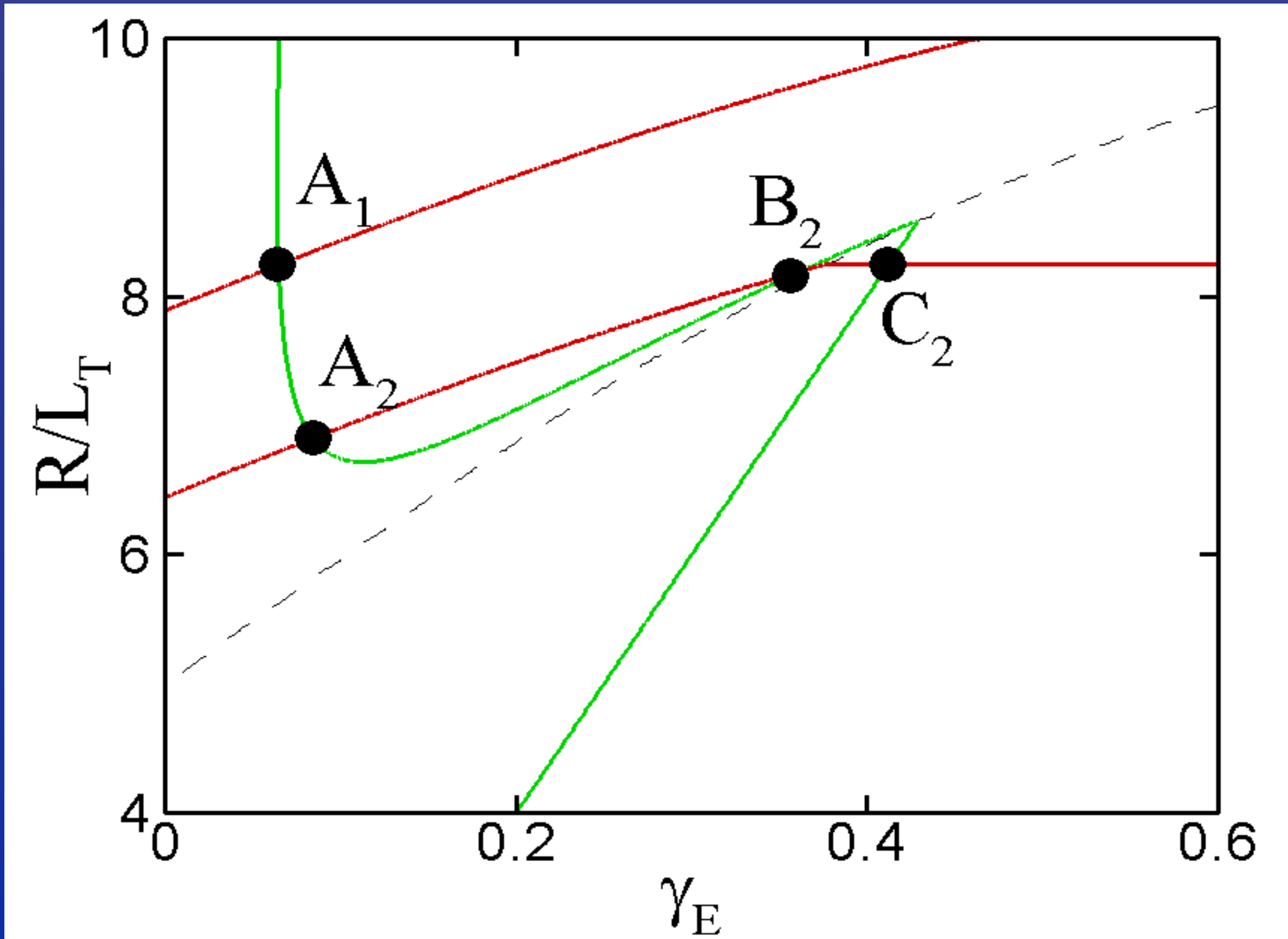
Curves of constant $\overline{\Pi}/\overline{Q}$



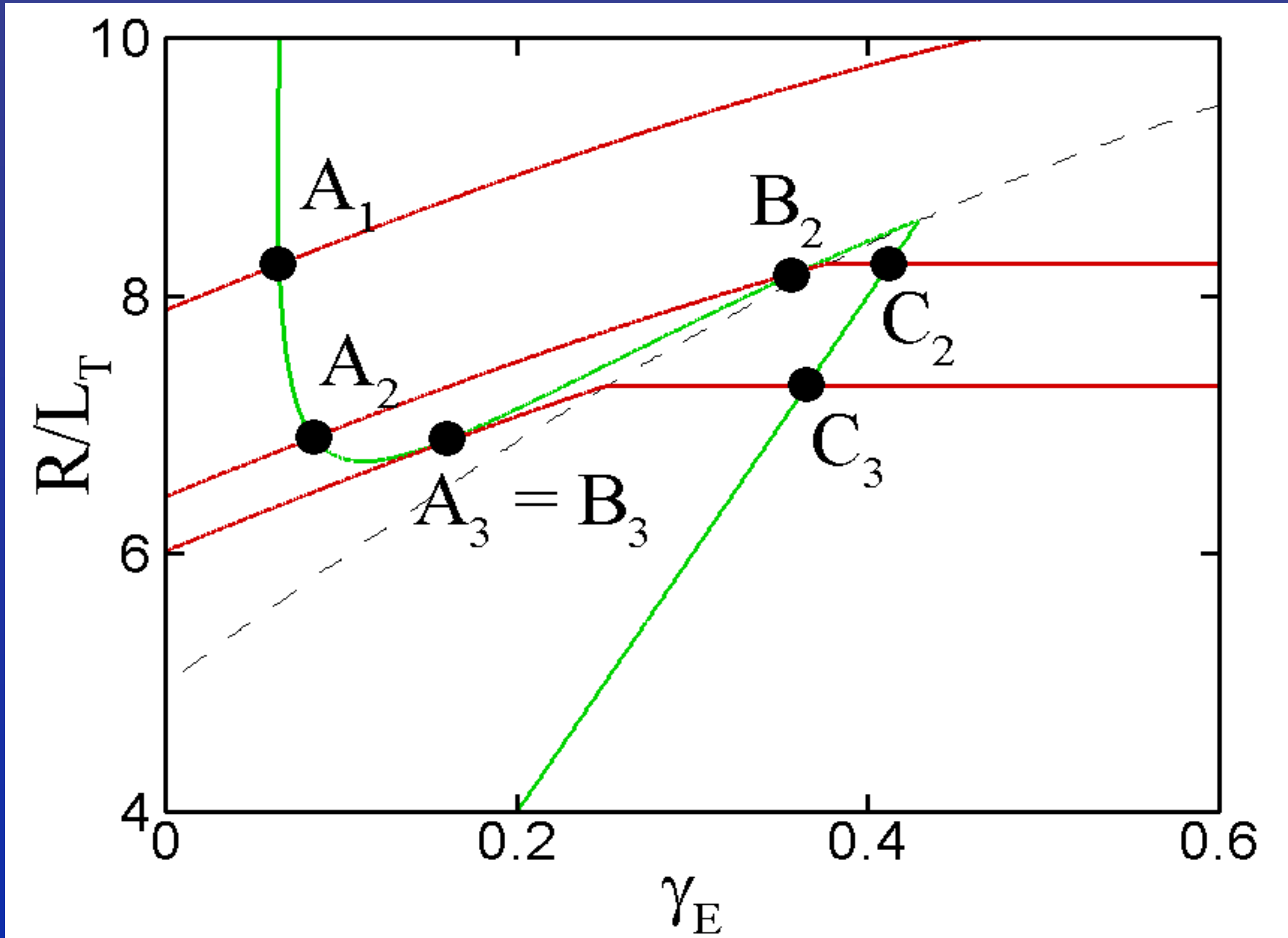
Possible solutions



Possible solutions



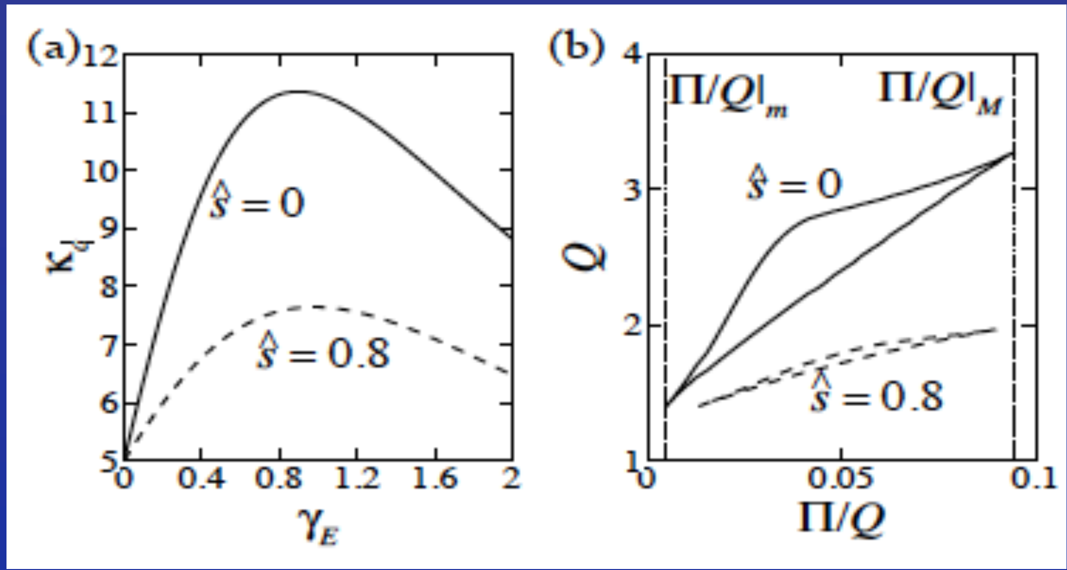
Possible solutions



Bifurcation condition

Bifurcations only occur when $Q_t \sim Q_n$ so take $R/L_T \approx R/L_{Tc}$

$$\left.\frac{d(R/L_T)}{d\gamma_E}\right|_Q \leq \left.\frac{d(R/L_T)}{d\gamma_E}\right|_{\Pi/Q}$$

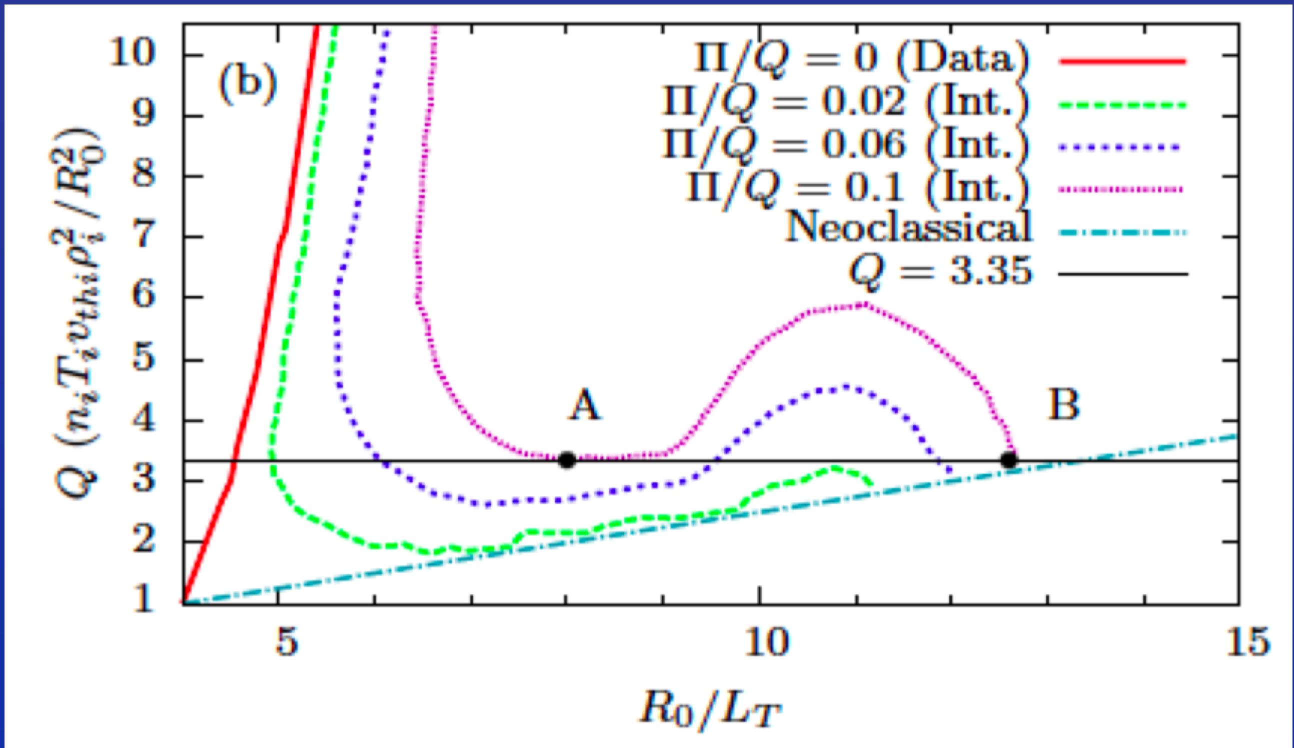


$$\Rightarrow \frac{\mathrm{Pr}_n^2}{\mathrm{Pr}_t}\frac{qR}{r}\left(\left.\frac{d(R/L_{Tc})}{d\gamma_E}\right|_{\gamma_E=0}\right)^{-1} < \frac{\Pi}{Q} \leq \mathrm{Pr}_t \frac{qR}{r}\left(\frac{\gamma_{E,max}}{R/L_{Tc,max}}\right)^2 \left.\frac{d(R/L_{Tc})}{d\gamma_E}\right|_{\gamma_E=0}$$

Parra *et al.*, PRL submitted (2010), arXiv:1009.0733

Bifurcations in GS2

- Use many nonlinear GS2 simulations to generate constant Pi/Q contours



- With inclusion of neoclassical fluxes, we see potential bifurcations to much larger flow shear and R/LT
- Very similar to simplified model predictions

Highcock PRL (2010)

Overview

- Theoretical and numerical model
- Effect of rotational shear on turbulent transport
- Implications for local gradients (0D)
- Extension to radial profiles (1D)

Solving for radial profiles

- Expressions for fluxes:

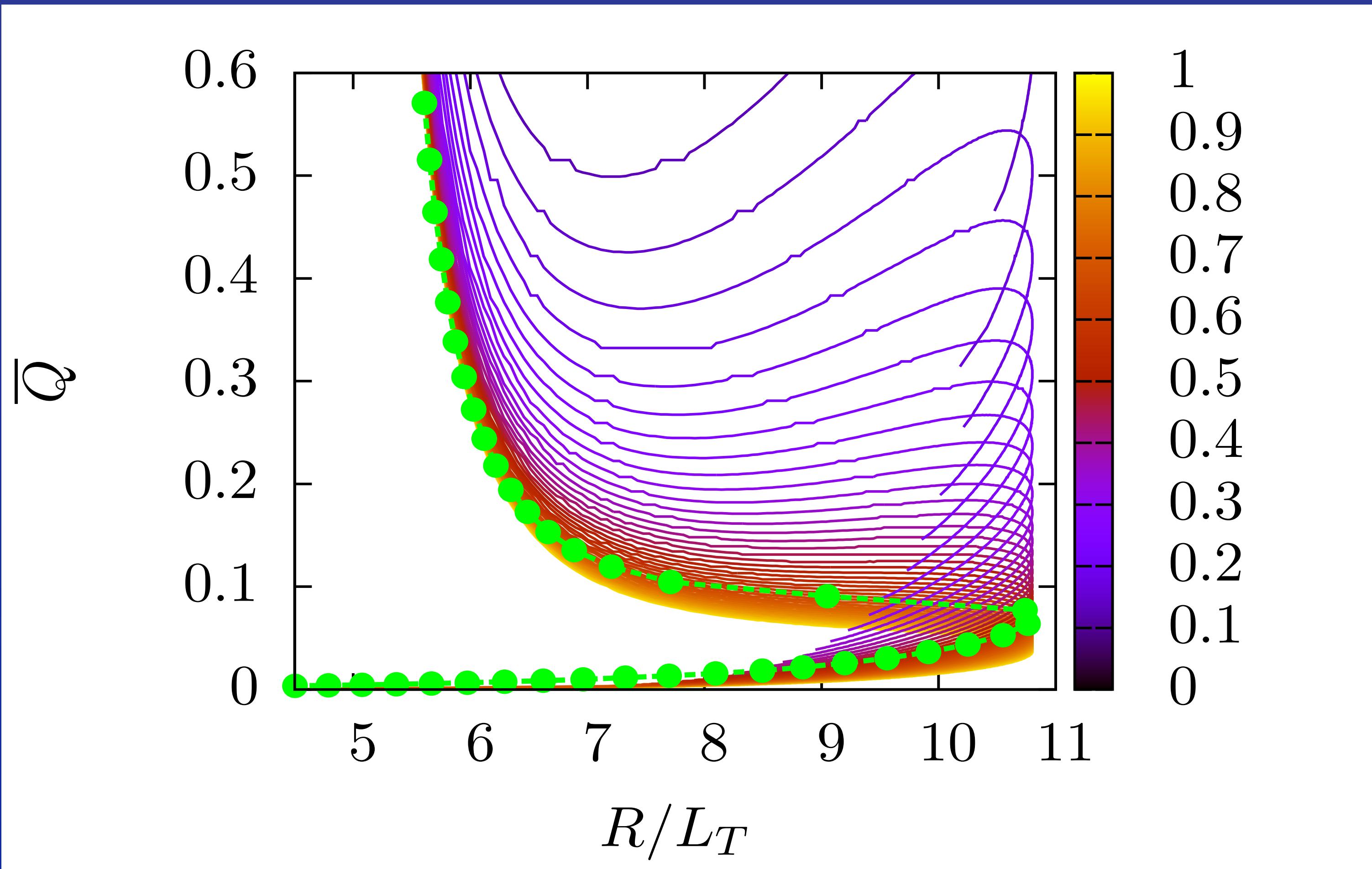
$$\hat{Q}(\kappa, \gamma_E, T) = \hat{T}^{5/2}\left(\hat{\chi}_t\left(\kappa - \kappa_c\right) + \frac{\hat{\chi}_n}{\hat{T}^2}\kappa\right)$$

$$\hat{\Pi}(\kappa, \gamma_E, T) = \gamma_E\left(\hat{\chi}_t\left(1 - \frac{\kappa_c}{\kappa}\right)\mathrm{Pr}_t\hat{T}^2 + \hat{\chi}_n\mathrm{Pr}_n\right)$$

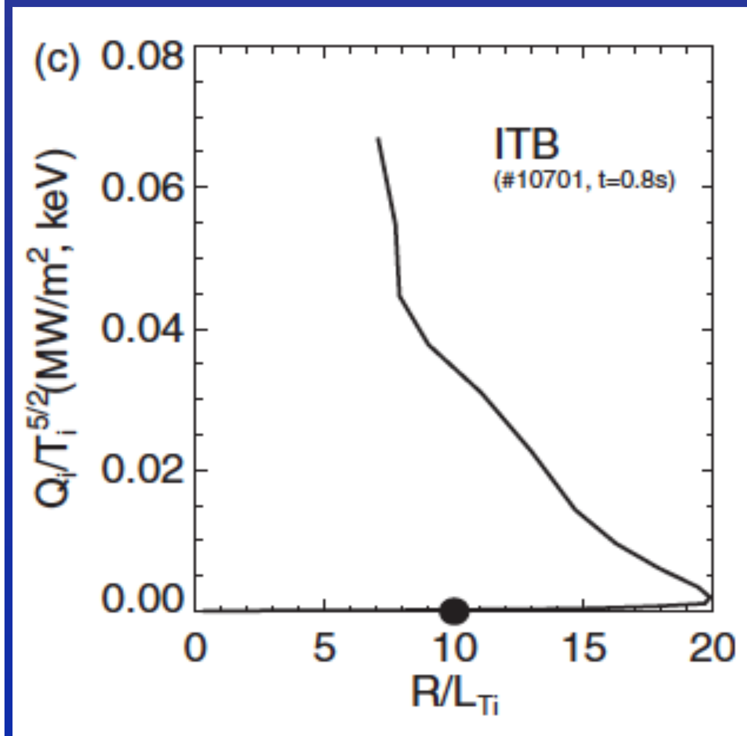
- Radial profiles of $\hat{Q}$ and $\hat{\Pi}$ are inputs. Given $\hat{T}$ at one radius, we can solve for γ_E and κ at that radius. With $\hat{T}$ and κ, we can obtain $\hat{T}$ at nearby radii. Repeat process to construct radial profiles.

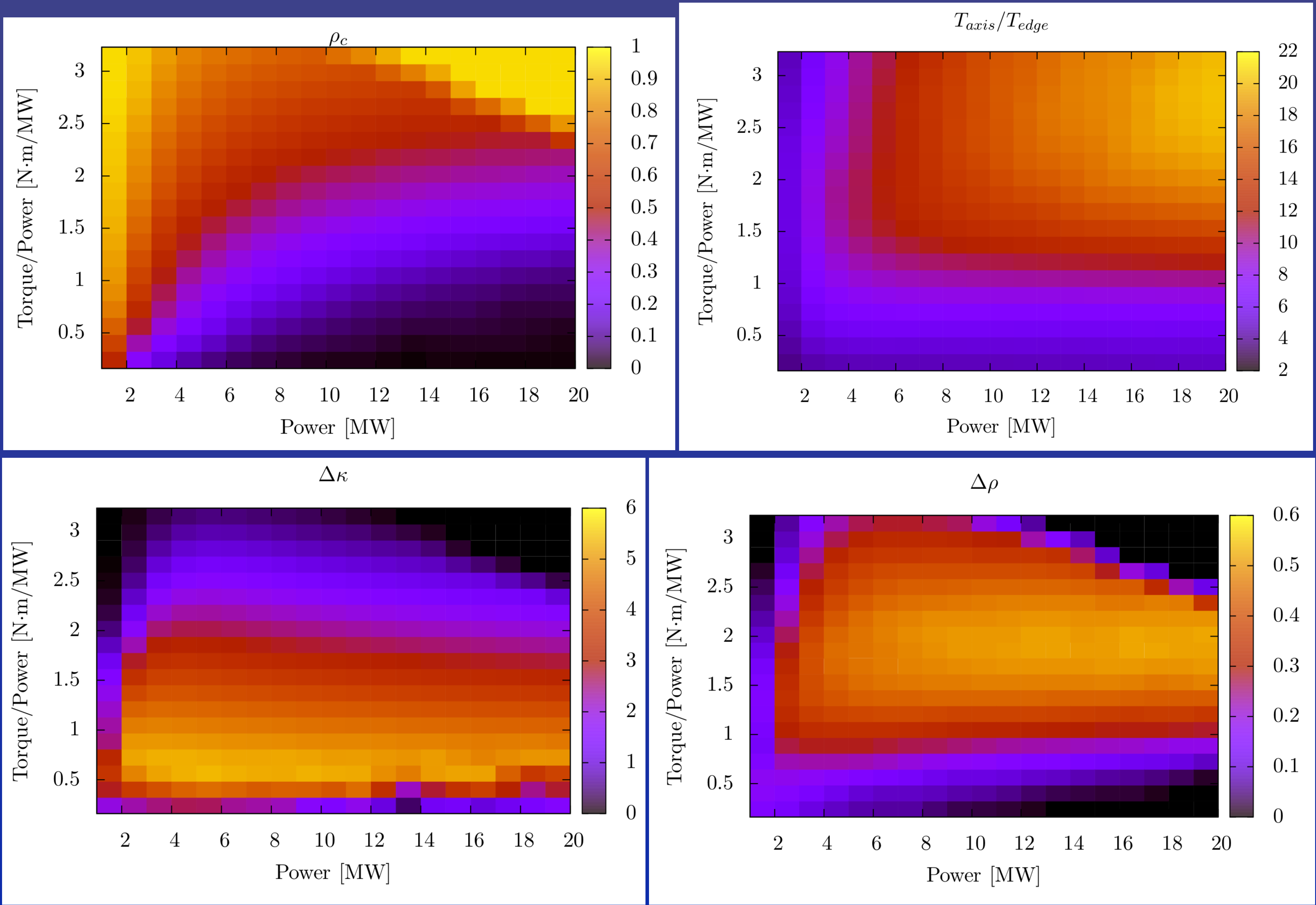
Numerical results

- Here, Q~sqrt(r/a), Pi/Q=0.1, Edge T=2 keV



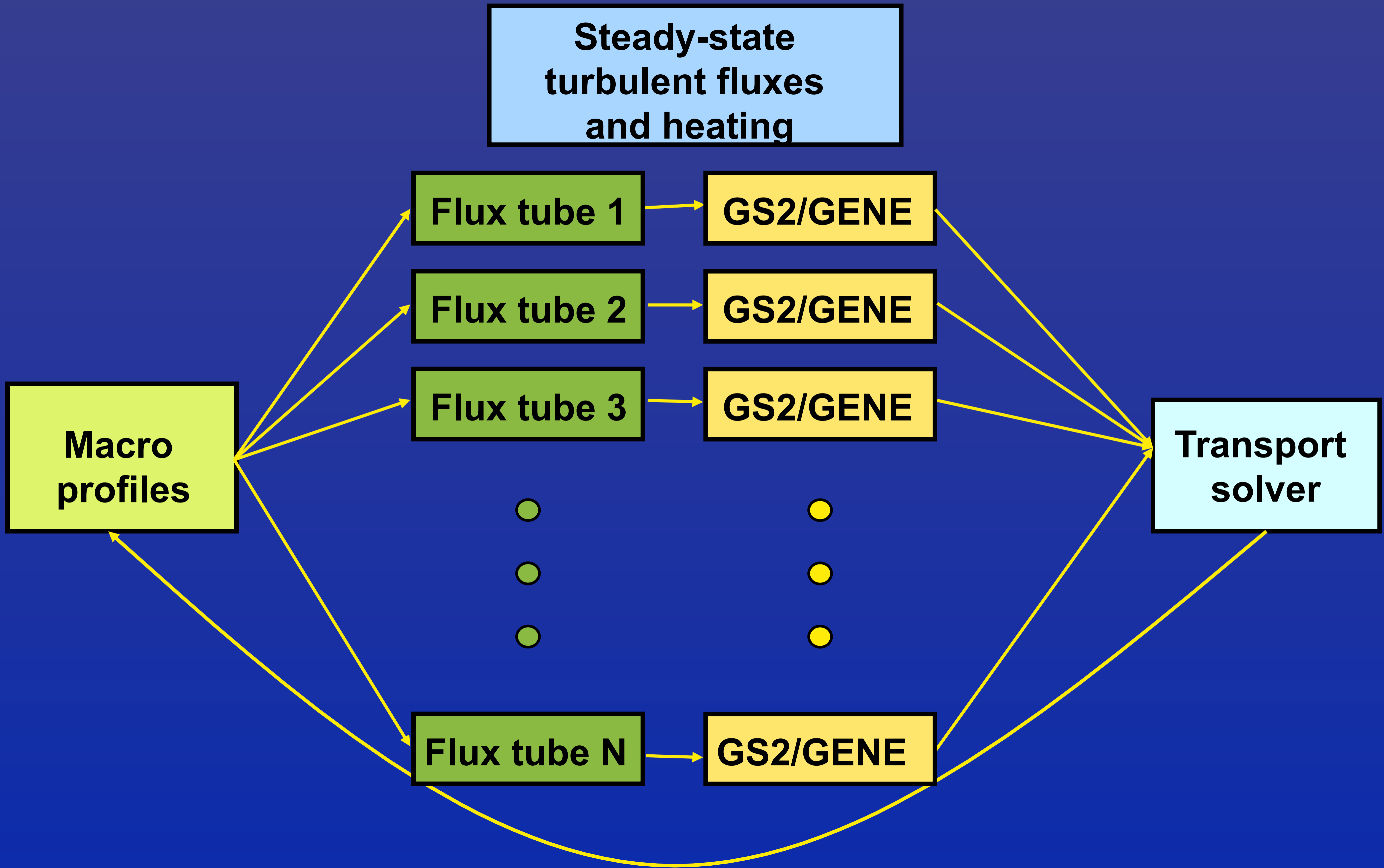
Wolf, PPCF **45** (2003)



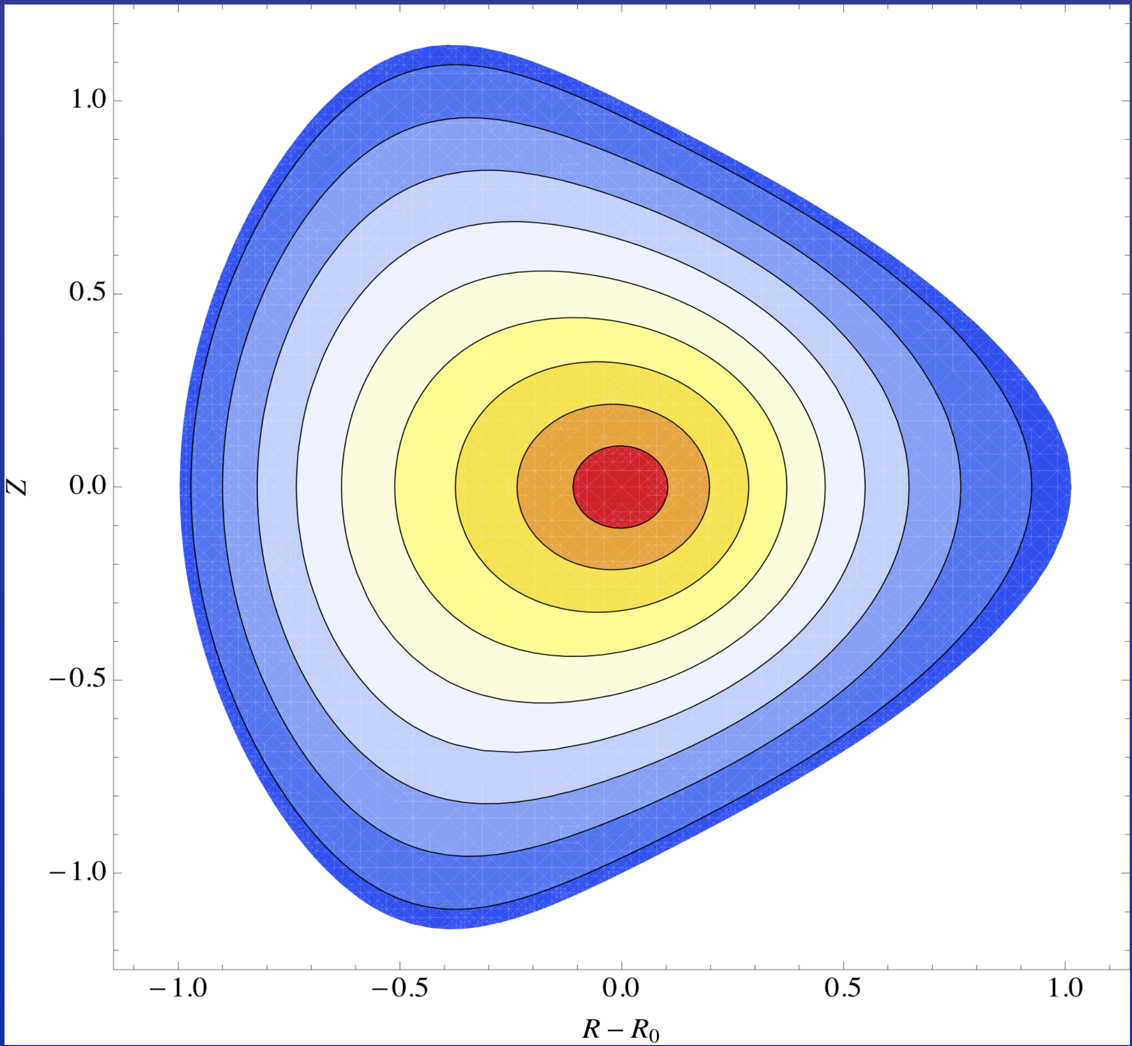


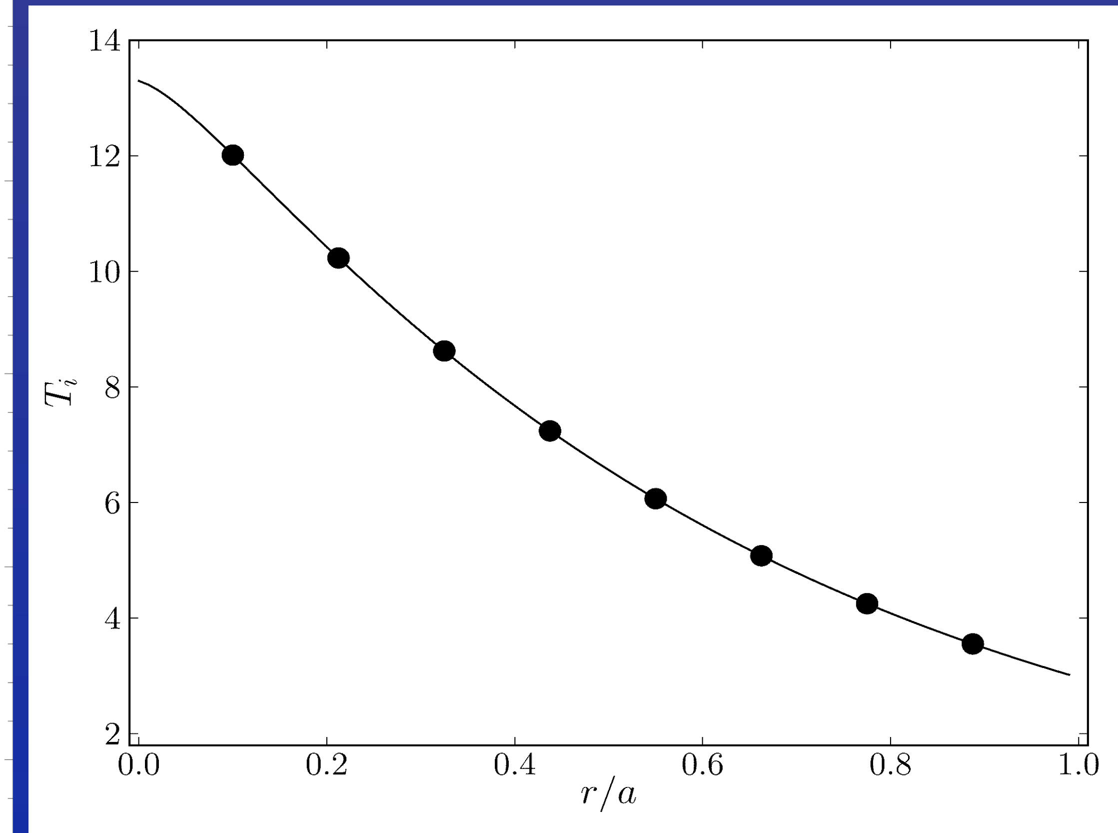
ρ_c
T_{axis}/T_{edge}
$\Delta\kappa$
$\Delta\rho$
Torque/Power [N·m/MW]
Power [MW]

TRINITY schematic



Sampling profile with flux tubes





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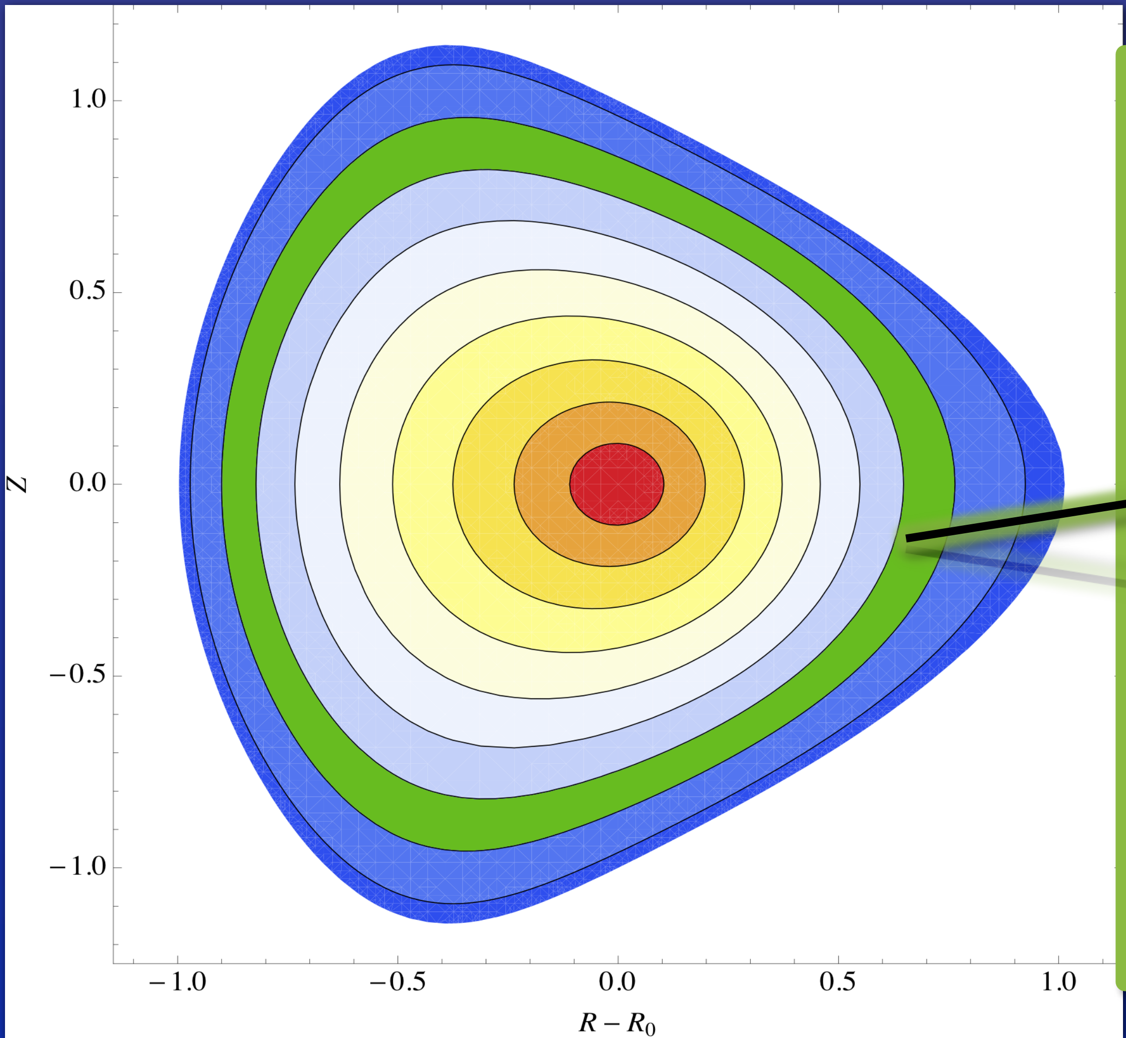
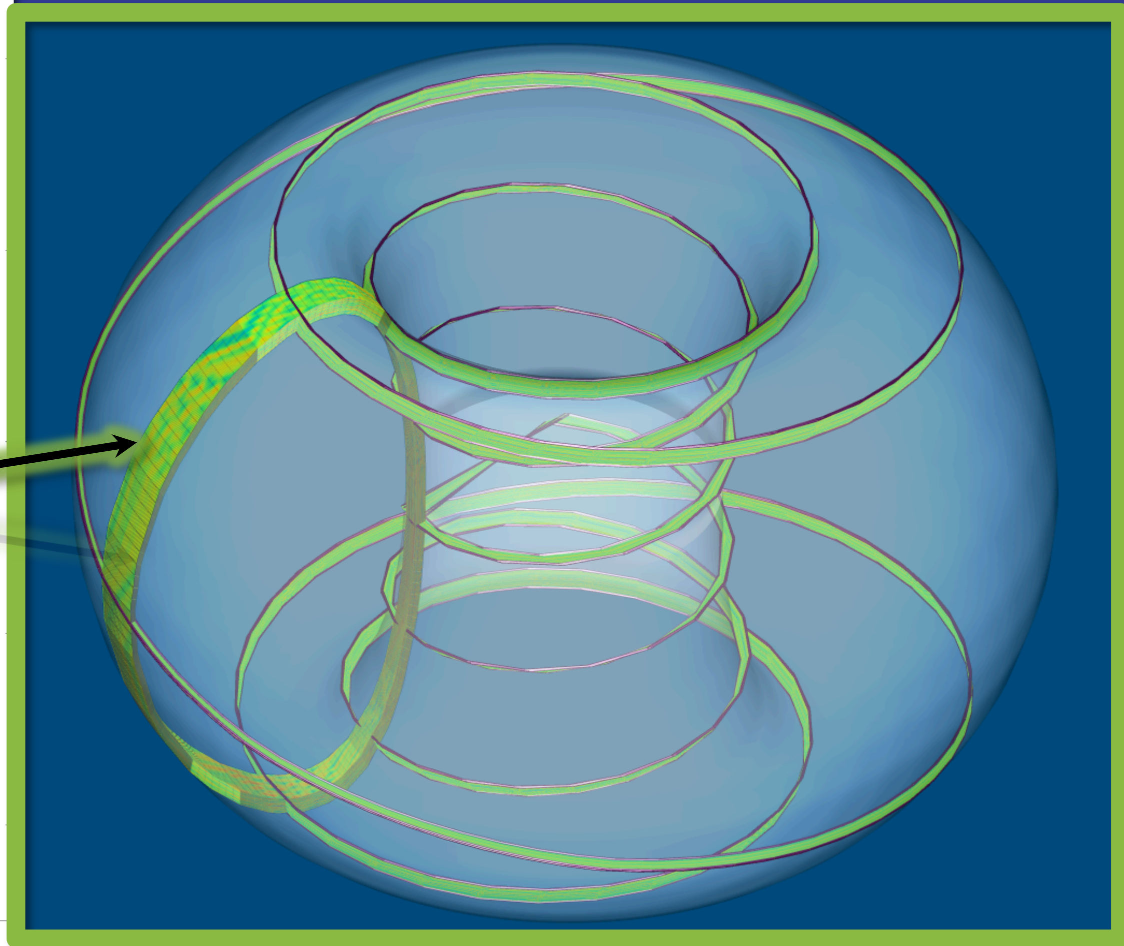
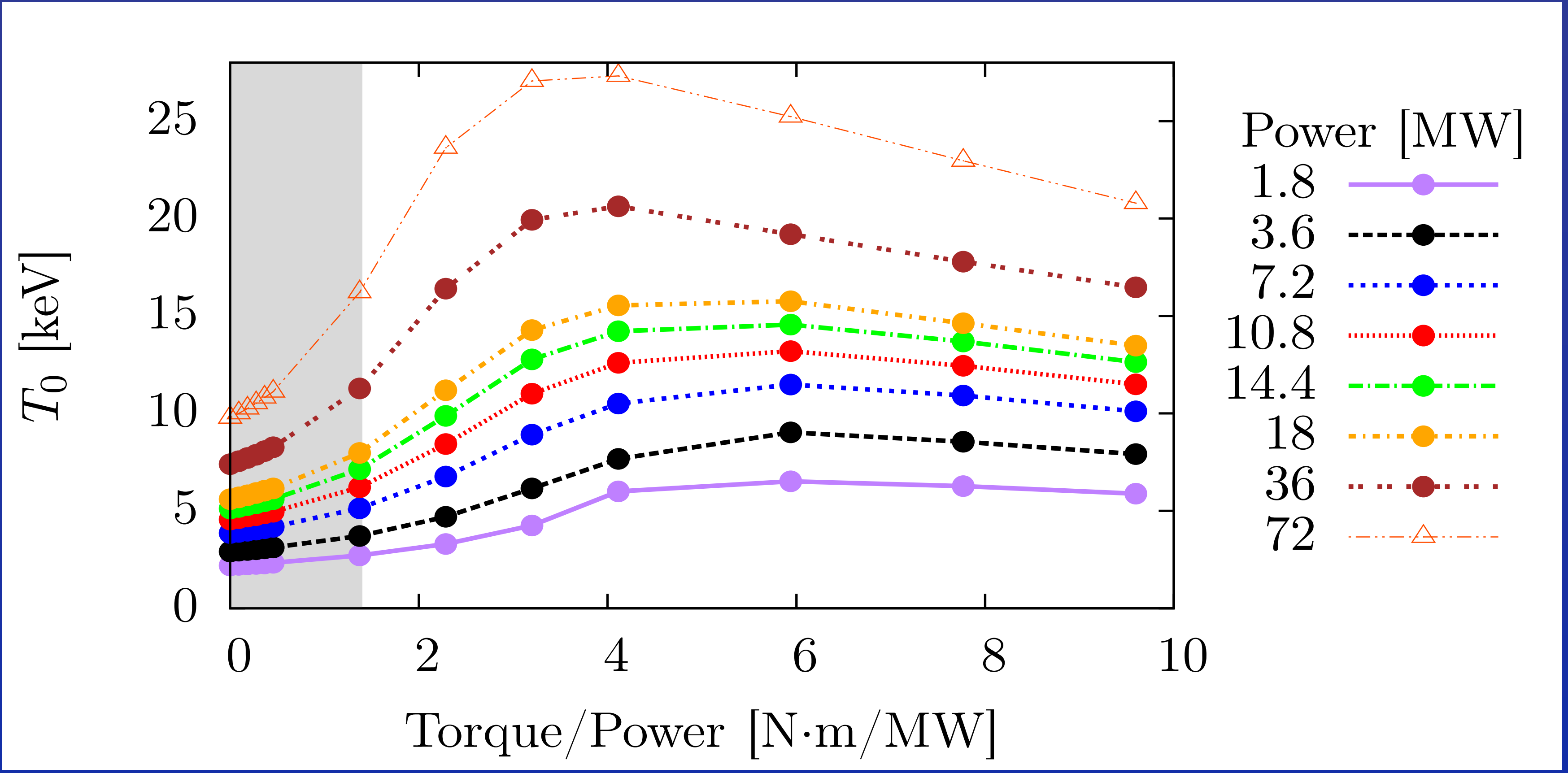




Simulation volume reduced by factor of ~10s

Results with model fluxes



Conclusions and future directions

- Mean flow shear can fully suppress turbulence in tokamak plasmas (in certain parameter regimes)
- Turbulence suppression can give rise to bifurcation in flow shear and temperature gradient
- Such bifurcations are candidates for thermal transport barriers in core of tokamak experiments
- Still a lot of work to be done in understanding underlying theory and determining parametric dependencies
- Need self-consistent treatment including back-reaction of turbulence on mean flow (evolution of mean profiles)