Transport scalings for critically-balanced ITG turbulence in tokamaks

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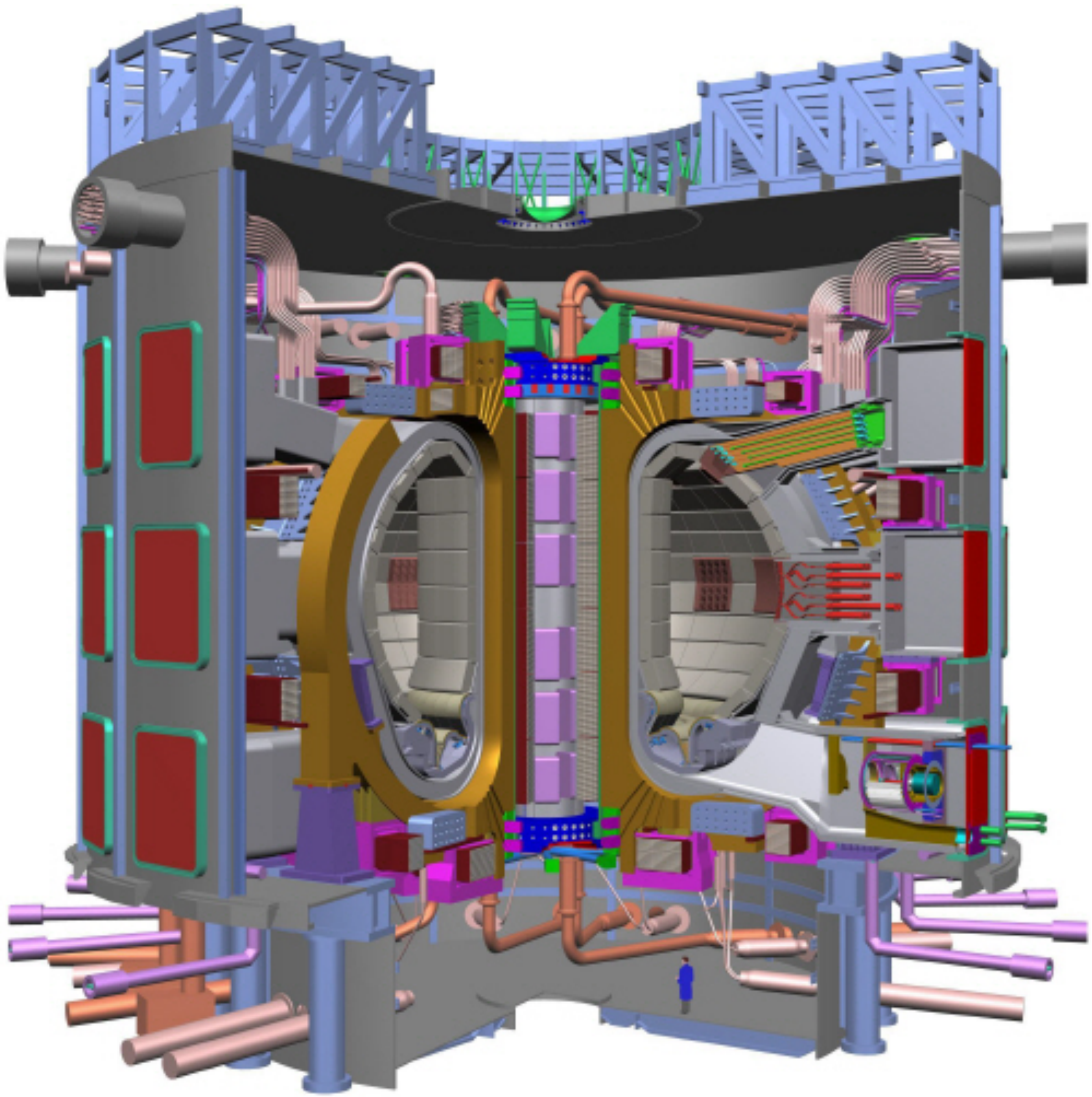
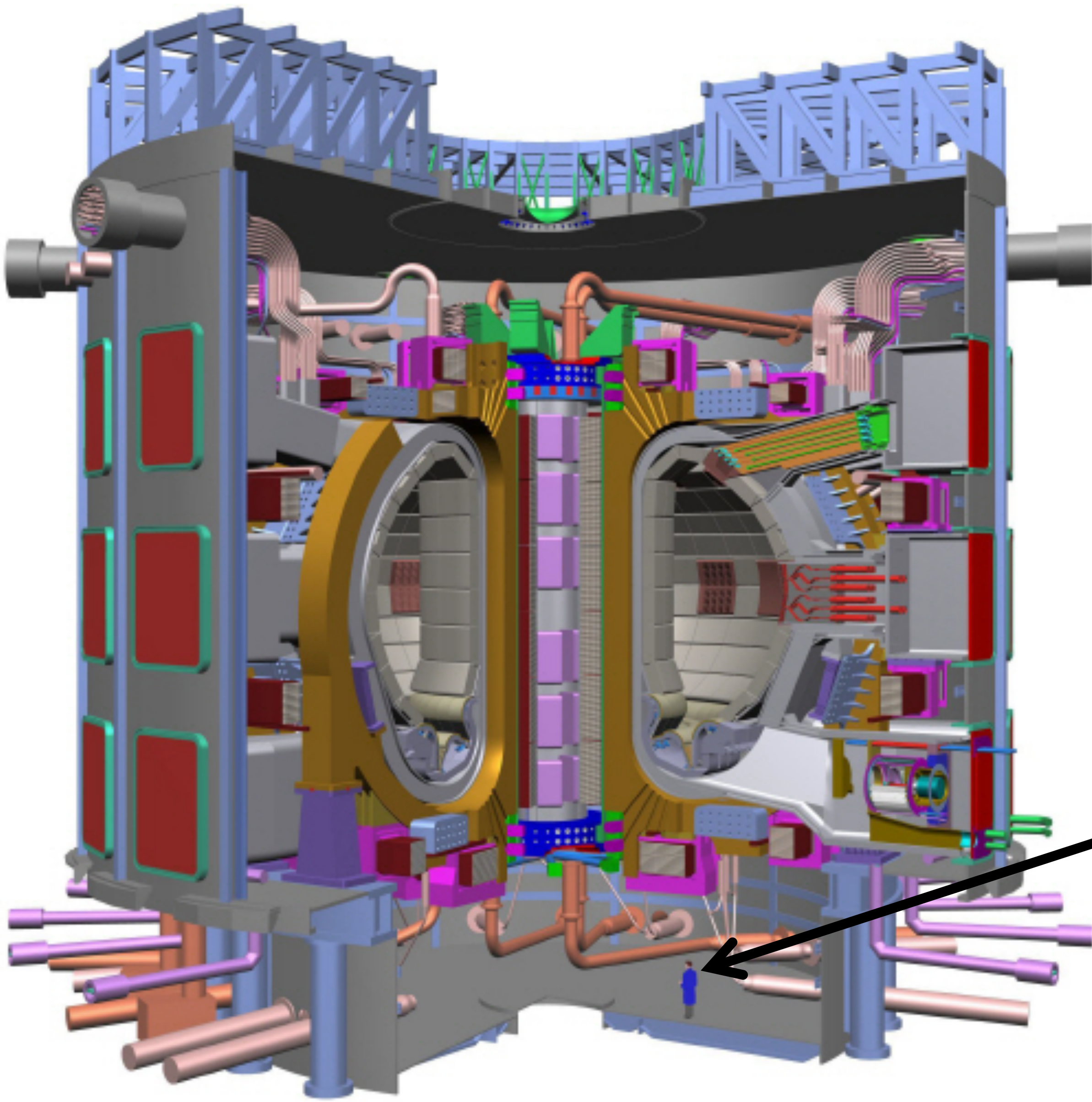
What's the point?



Core density and temperature limited by turbulent transport

ITER

What's the point?

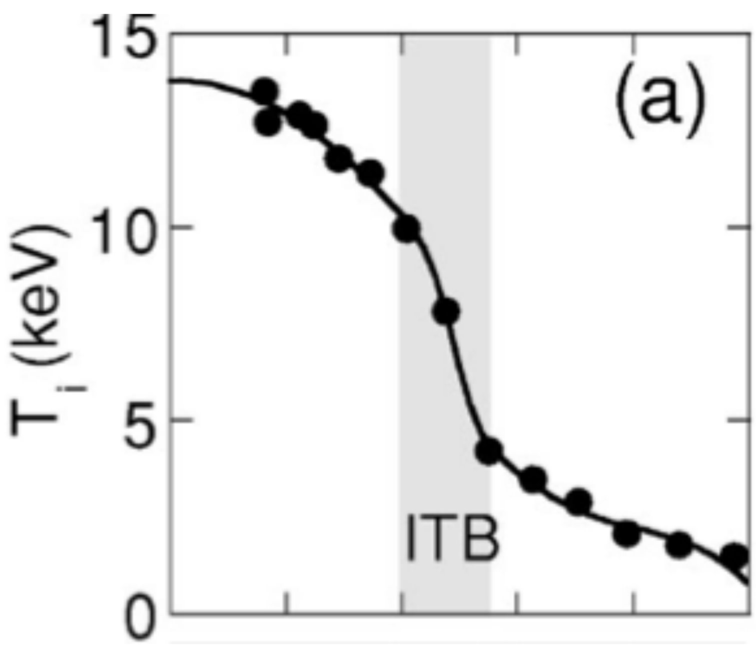


Approximate size of device without significant turbulent transport

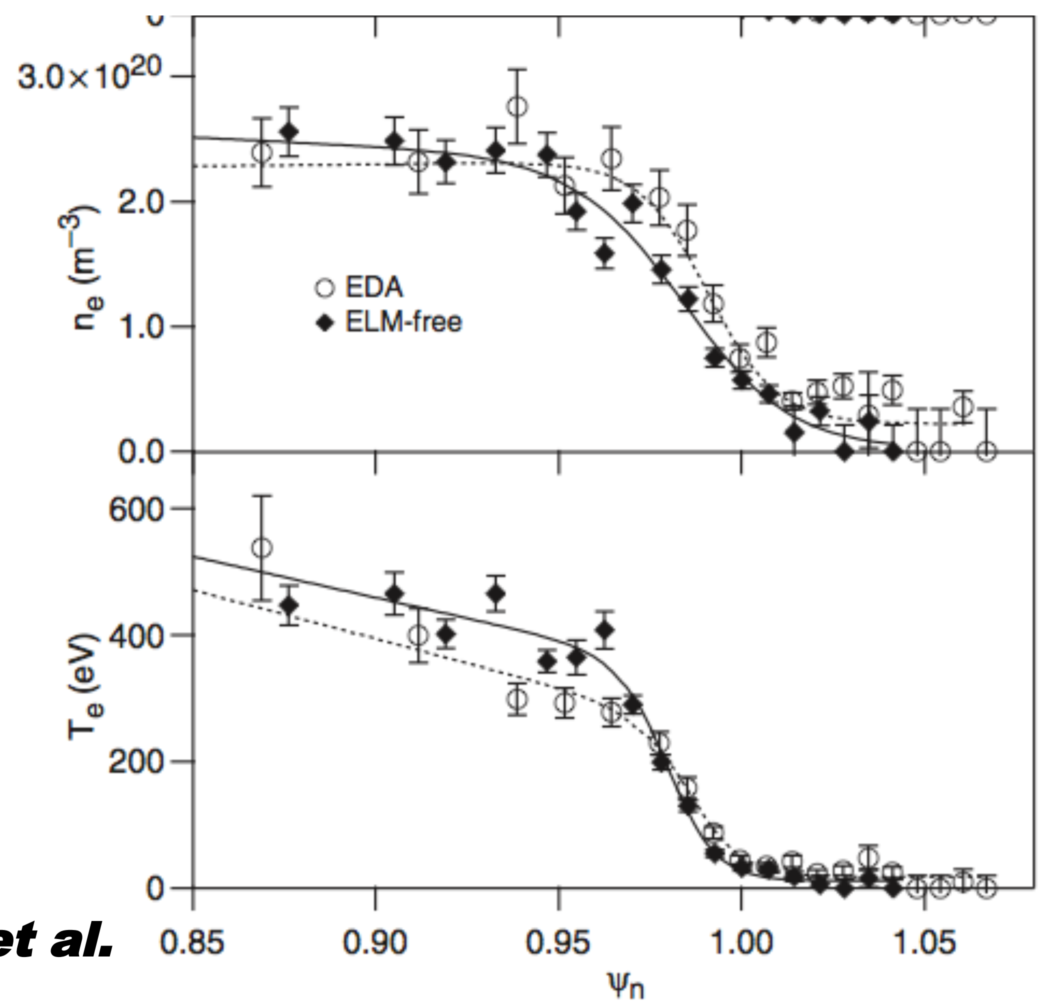
ITER

Suppression of turbulence

- We know it's possible to suppress turbulence, as reduced transport regions (transport barriers) are routinely observed in experiment



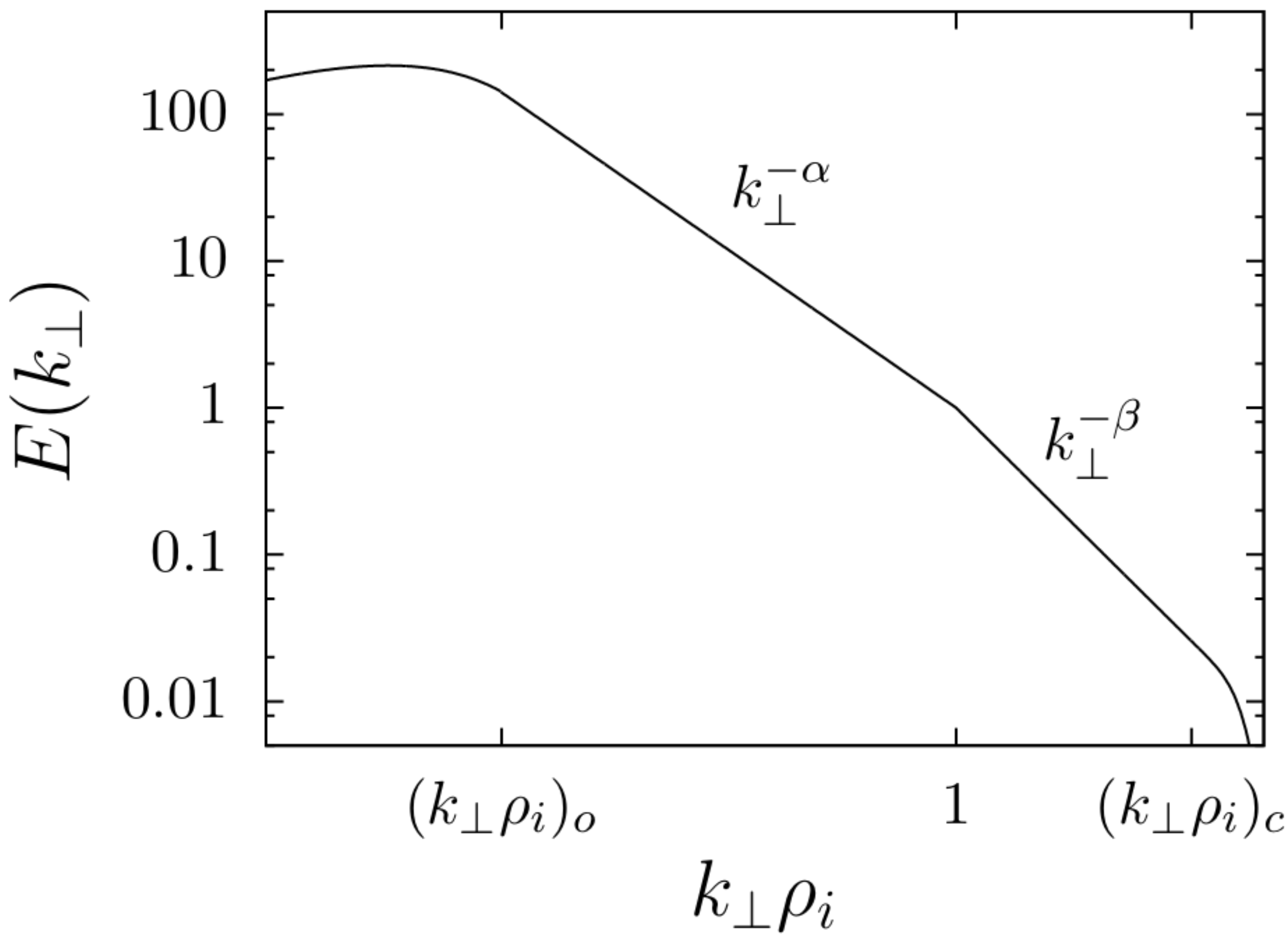
JT-60U data. Y. Miura *et al.*



C-Mod data. D. A. Mossessian *et al.*

What we have done

- Determined ITG turbulence scalings with important plasma parameters (safety factor and temp. grad.)



Outline

- Plasma turbulence in tokamaks -- description and gyrokinetic model
- Brief overview of basic turbulence concepts
- Conjectures on ITG turbulence, derivation of scaling laws, numerical tests
- Conclusions

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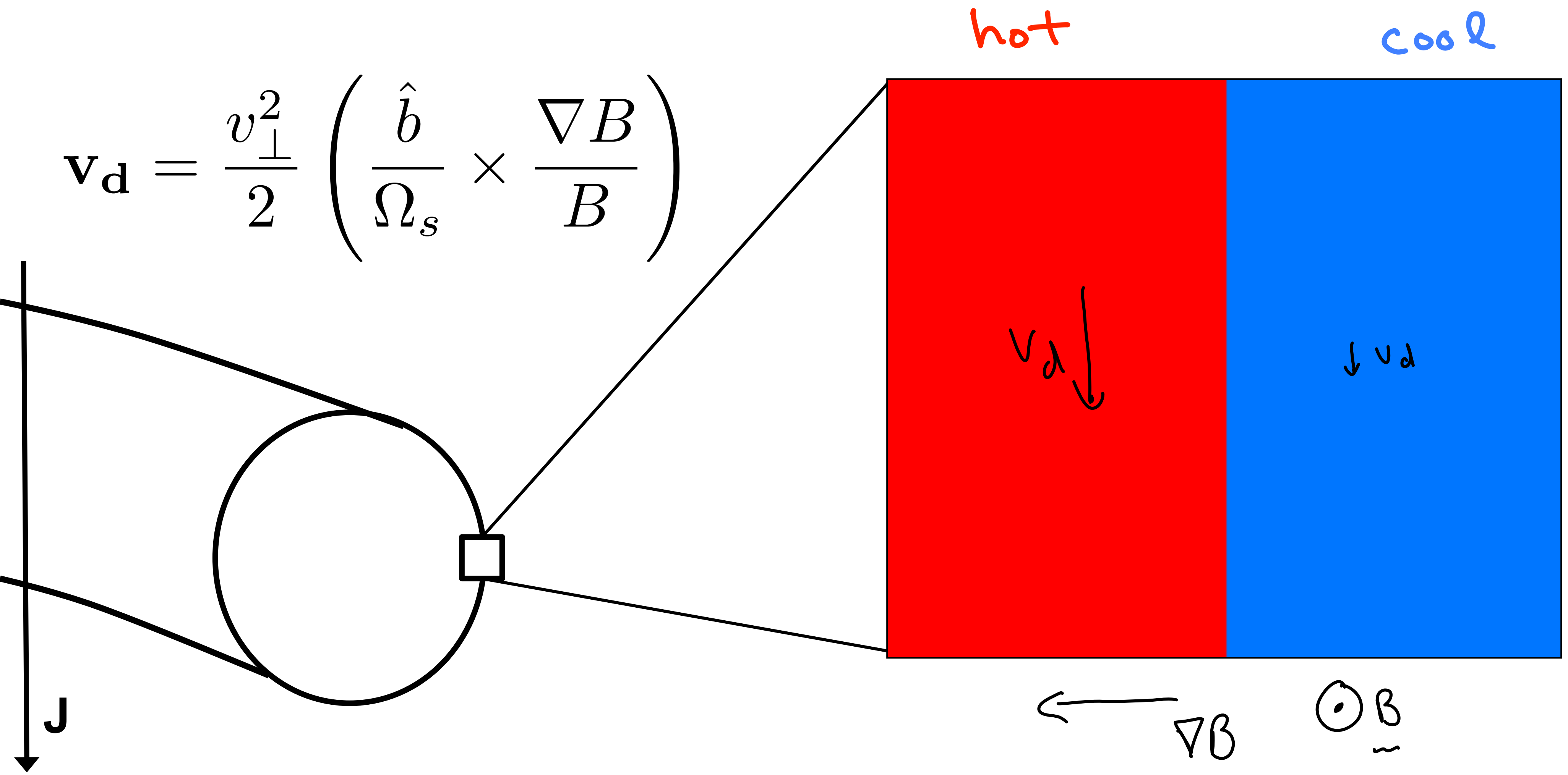
Simple picture of instability

$$\mathbf{v_d} = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega_s} \times \frac{\nabla B}{B}\right)$$



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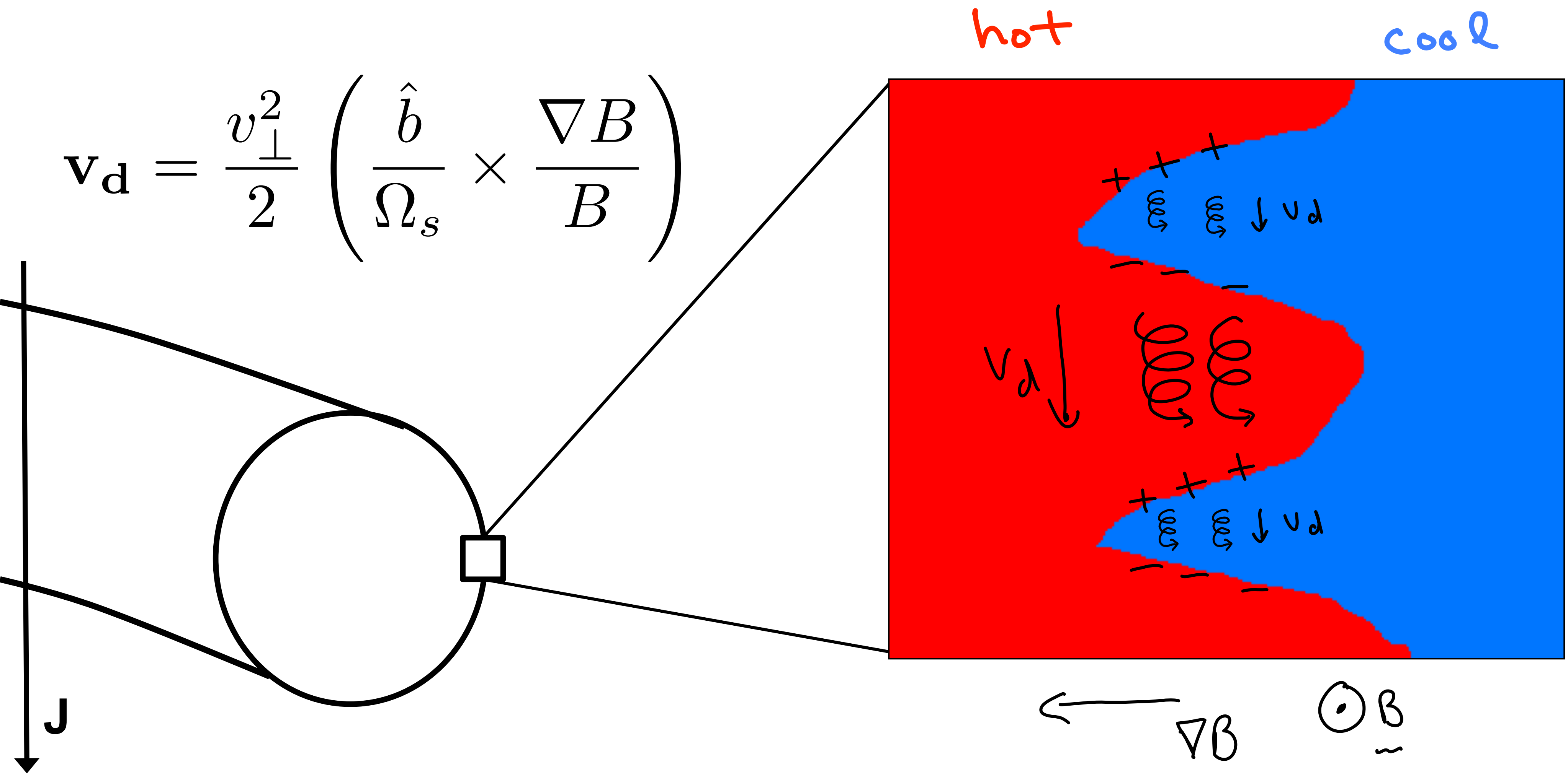
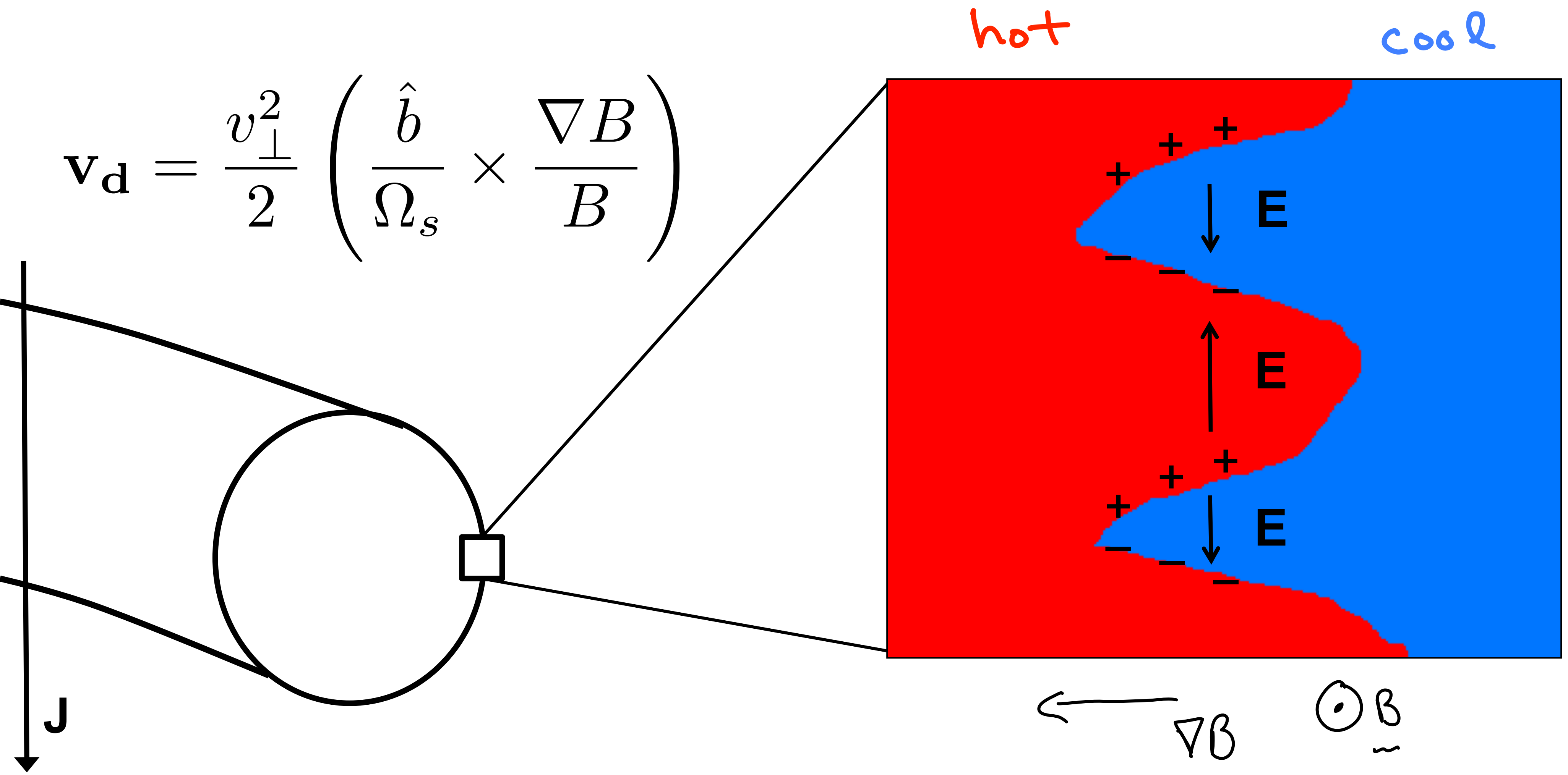


Simple picture of instability

$$\mathbf{v_E} = \frac{c}{B^2}\mathbf{E} \times \mathbf{B}$$

$$\mathbf{v_d} = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega_s} \times \frac{\nabla B}{B}\right)$$



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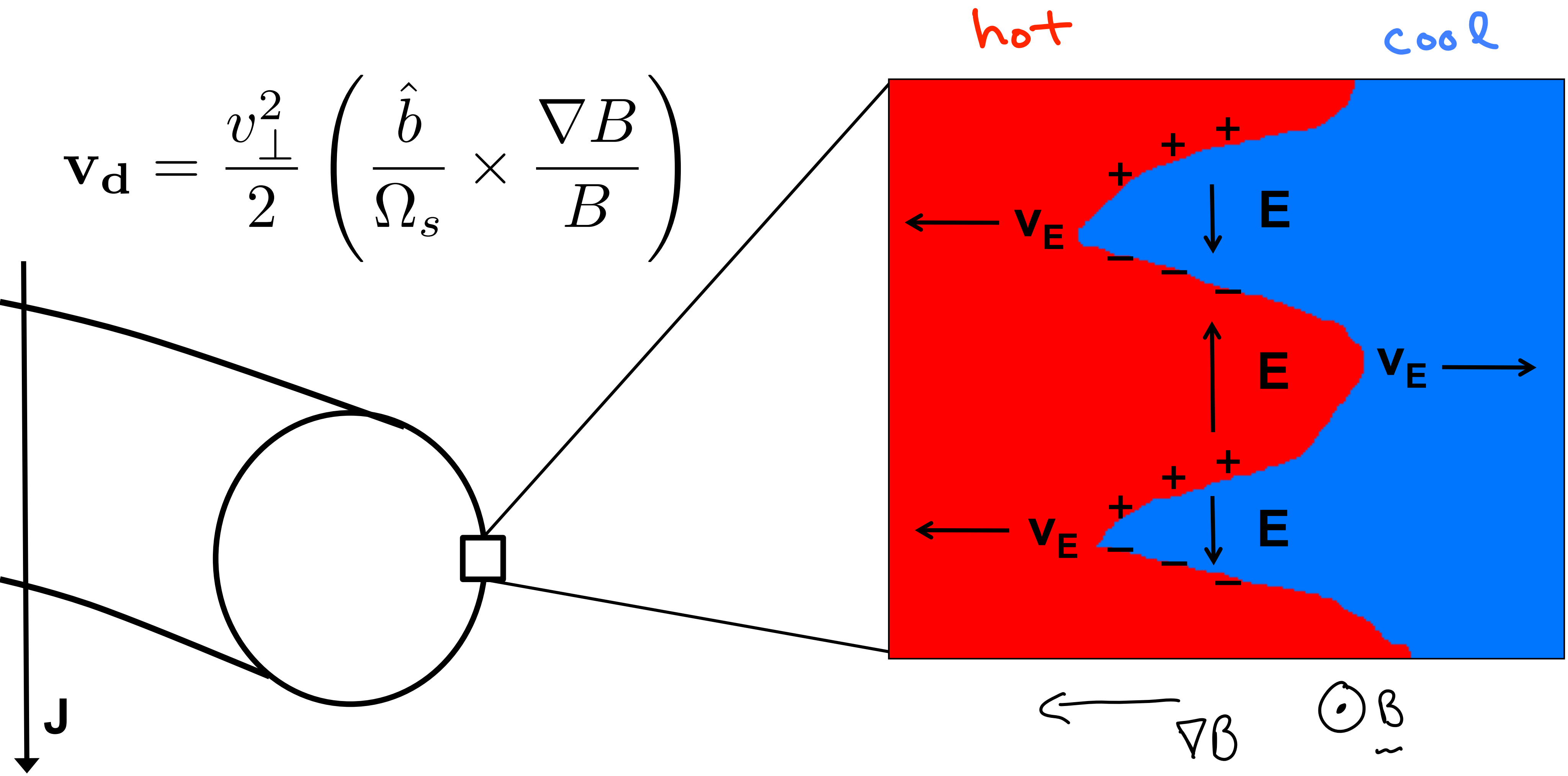
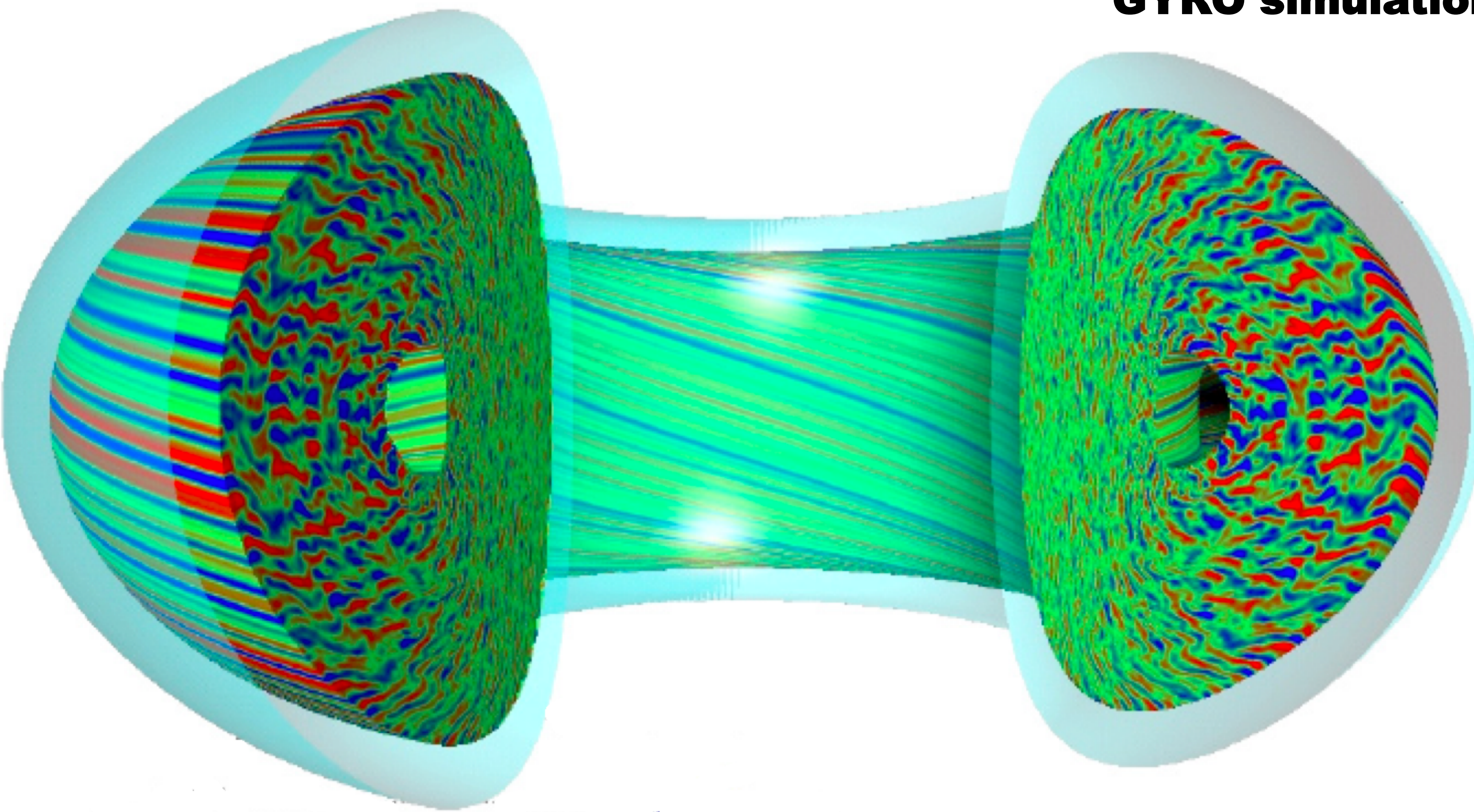


Resultant turbulence

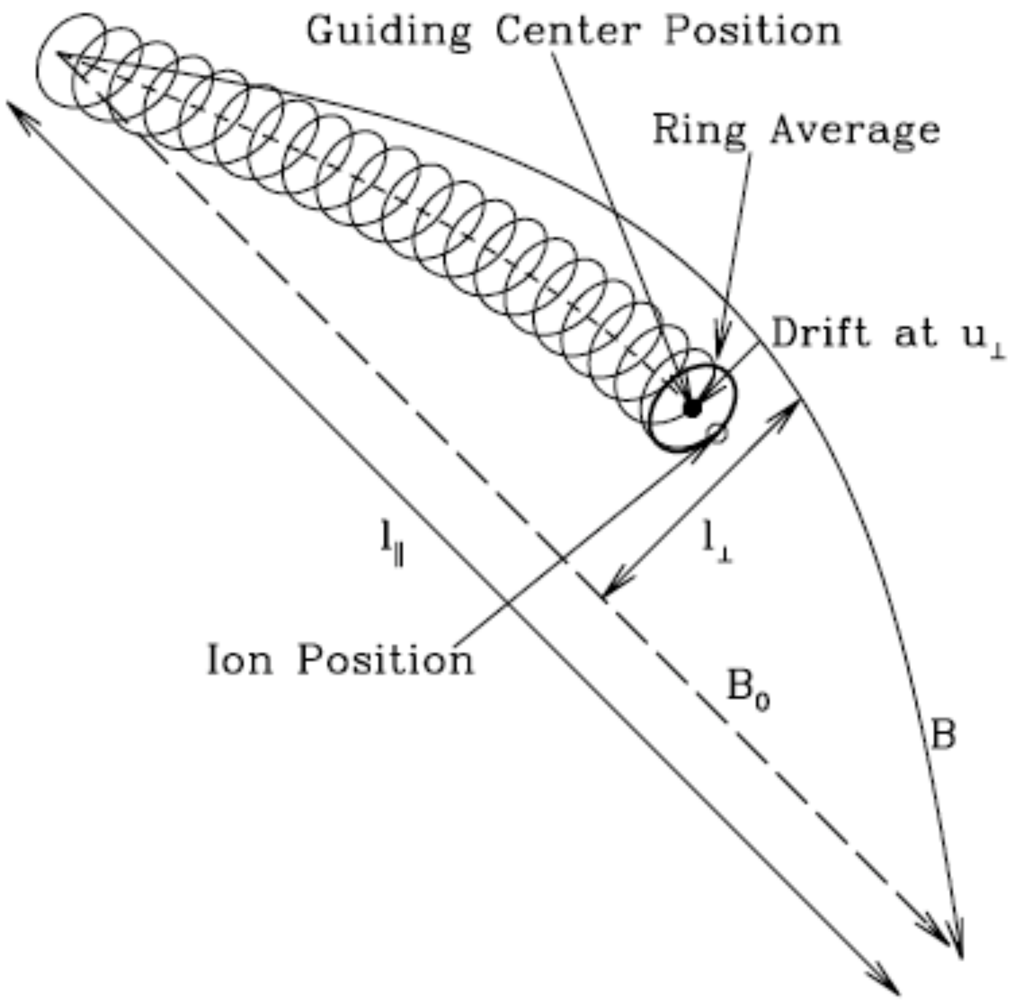
GYRO simulation



Low-amplitude, small-scale, anisotropic turbulence

Gyrokinetic model

Dynamics slow compared to Larmor frequency so gyroaverage:



Removes fast gyromotion from problem, eliminating gyroangle as a phase space variable

Now solving for trajectories of rings of charge as they stream along mean field and slowly drift across it

Delta-f gyrokinetic model

Decompose f into mean and fluctuating components:

$$f = F + \delta f$$

Turbulent fluctuations are low amplitude: $\delta f \sim \epsilon f$

Turbulence slow compared to gyrofrequency, but fast compared to mean profile evolution:

$$\frac{\partial \ln \delta f}{\partial t} \sim \omega \sim \epsilon \Omega \qquad \frac{\partial \ln F}{\partial t} \sim \epsilon^2 \omega \sim \epsilon^3 \Omega$$

Mean varies perpendicular to mean field on system size while fluctuations vary on Larmor scale:

$$\nabla_\perp \ln F \sim L^{-1} \qquad \nabla_\perp \ln \delta f \sim \rho^{-1}$$

Fluctuations are anisotropic with respect to the mean field:

$$\nabla_{||} \ln \delta f \sim L^{-1}$$

Gyrokinetic-Poisson system

$$\delta f_s = h_s - \frac{e_s \varphi}{T_s} F_{M,s}$$

Gyrokinetic equation:

$$\frac{\partial}{\partial t}\left(h_s - \frac{e\,\langle\varphi\rangle_{\mathbf{R}}}{T_s} F_{M,s}\right) + \left(\mathbf{v}_{\|} + \mathbf{v}_{M,s} + \langle\mathbf{v}_E\rangle_{\mathbf{R}}\right)\cdot\nabla h_s$$

$$= \langle C[h_s]\rangle_{\mathbf{R}} - \langle\mathbf{v}_E\rangle_{\mathbf{R}}\cdot\nabla F_{M,s}$$

Quasineutrality: $$\sum_s e_s \left(\int d^3v \; h_s - \frac{e_s \varphi}{T_s} n_s\right) = 0$$

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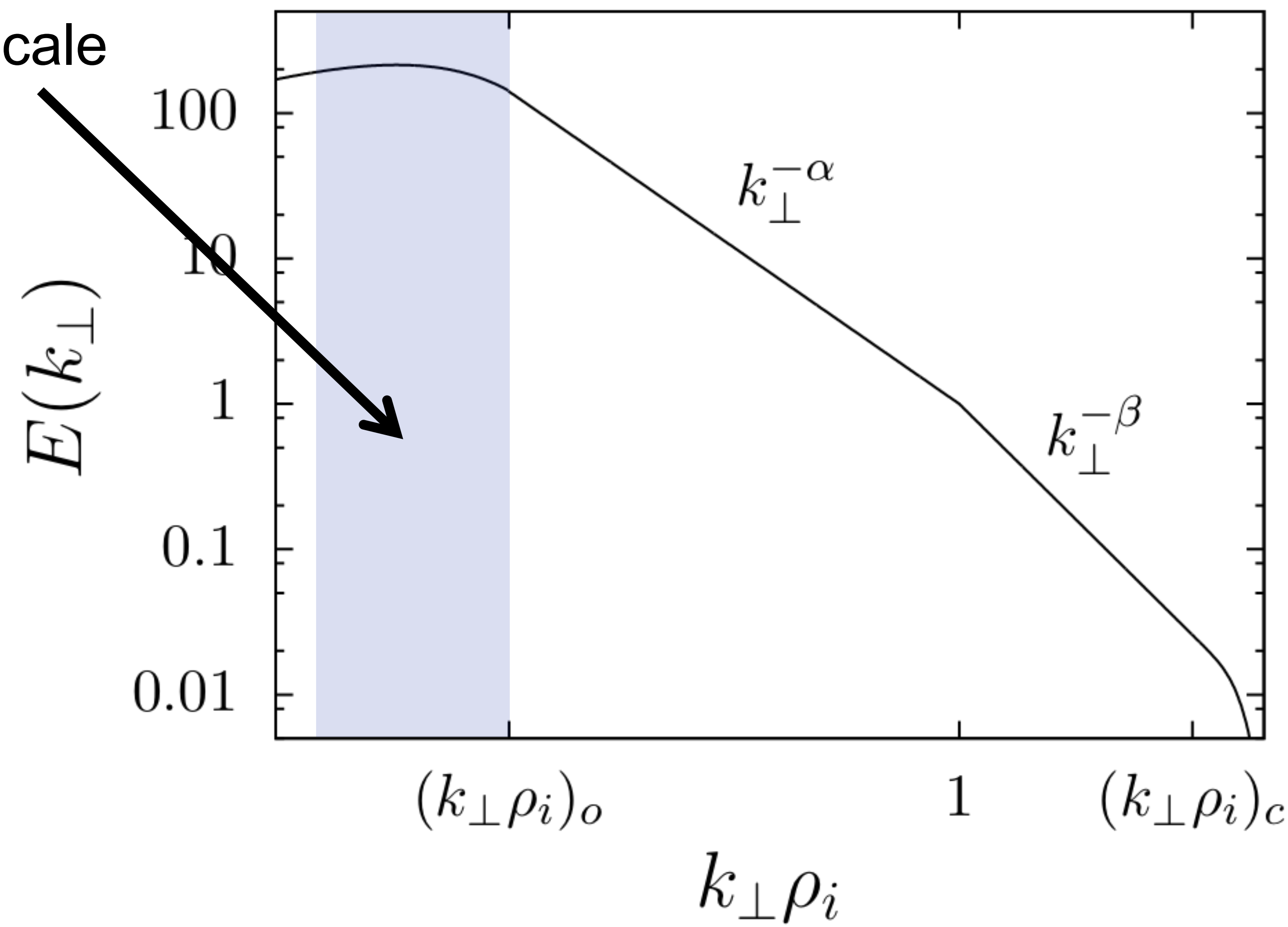
Turbulence spectrum: outer scale

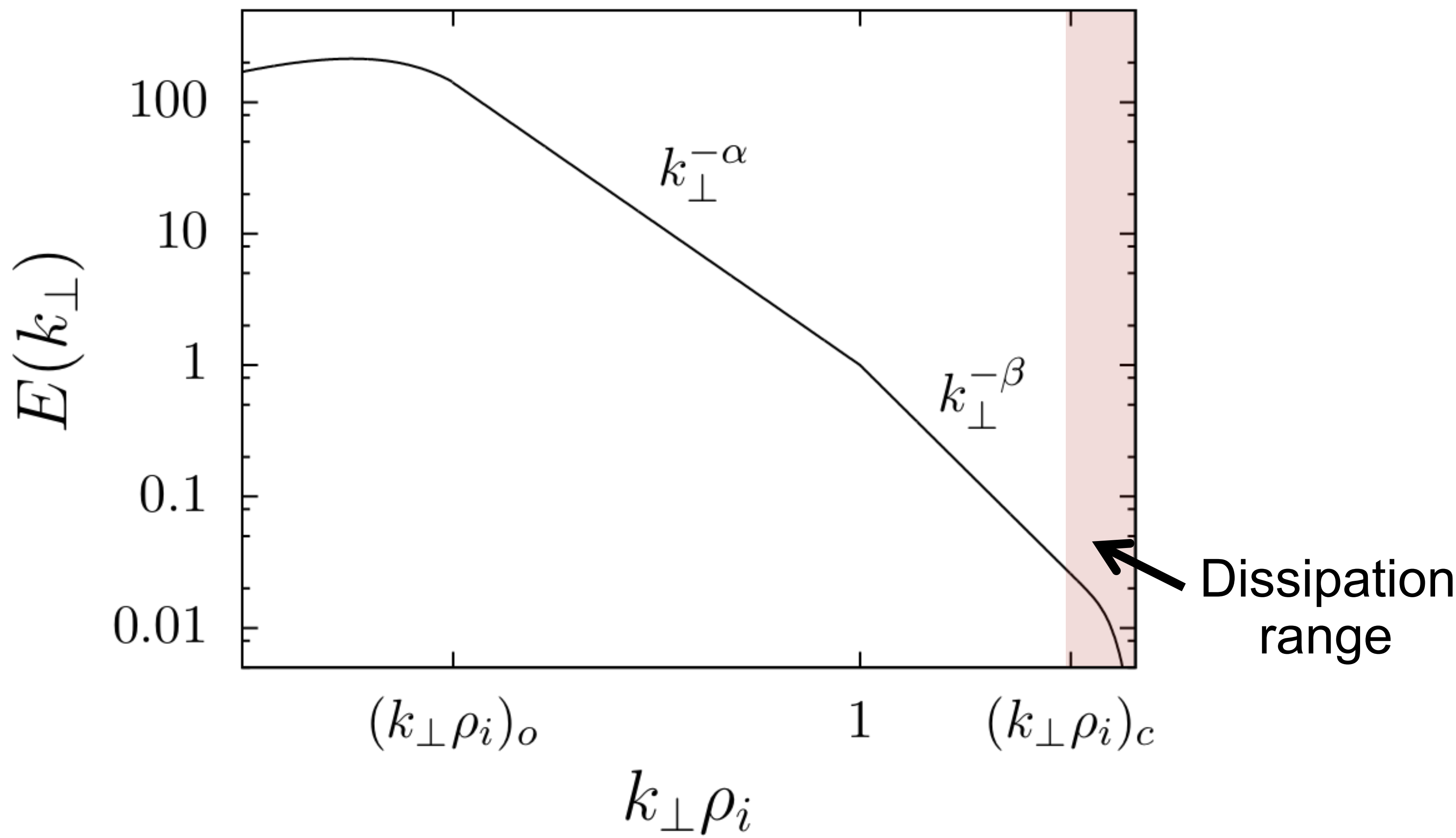
- Outer scale = energy-containing scale, roughly corresponds to where energy injected

Outer scale



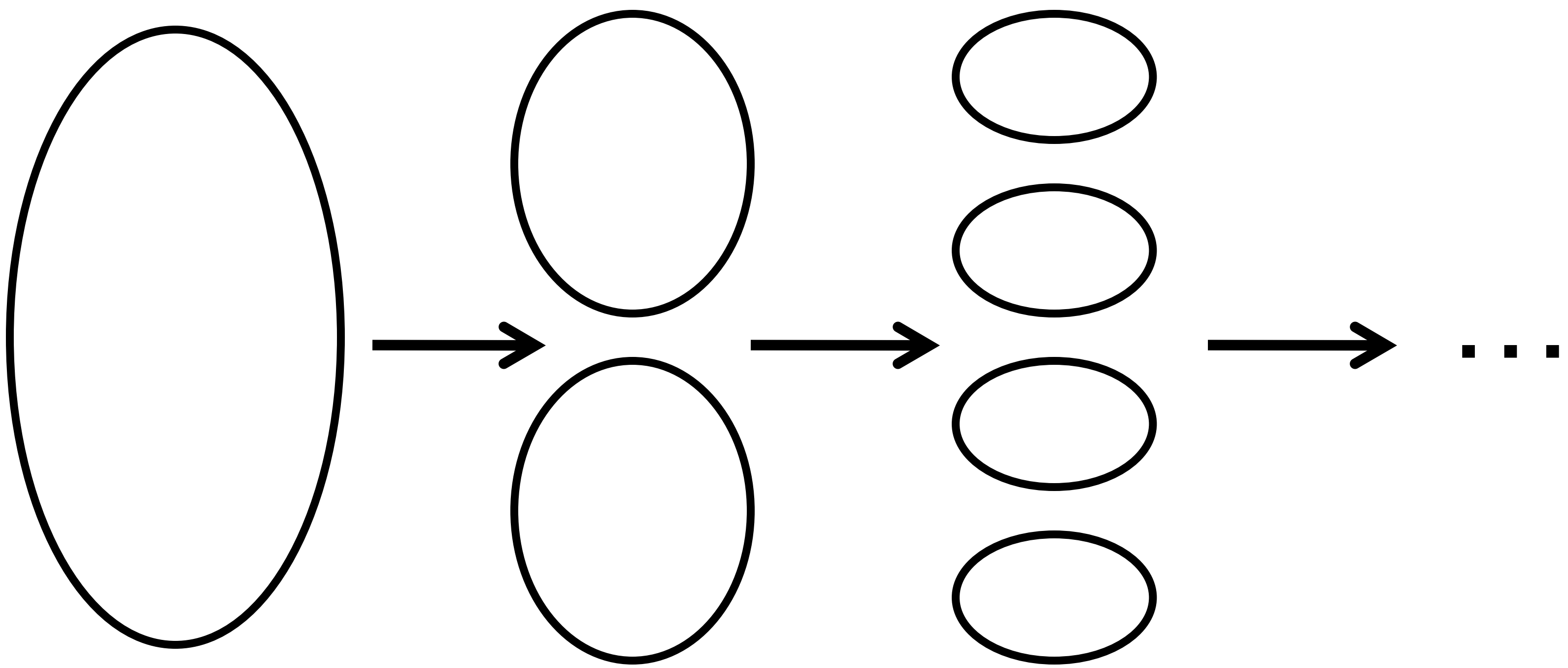
Energy balance and dissipation

$$\frac{\partial W}{\partial t} + \sum_s T_s \int_V h_s \left\langle \mathbf{v}_E \right\rangle_{\mathbf{R}} \cdot \nabla \ln F_{M,s} = \sum_s \int_V \frac{h_s T_s}{F_{M,s}} \left\langle C[h_s] \right\rangle_{\mathbf{R}}$$



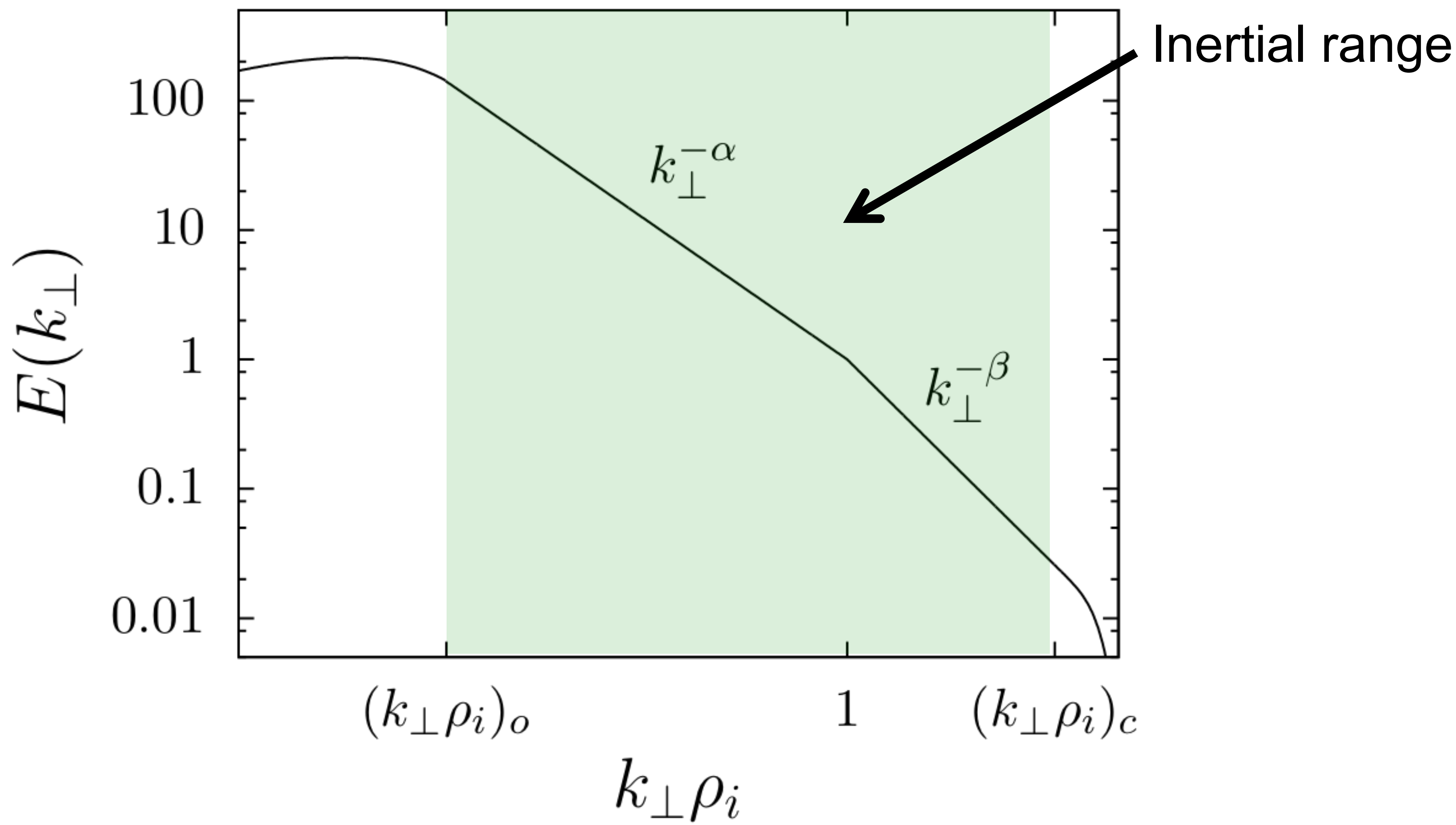
Turbulence cascade

- Large eddies break into smaller eddies repeatedly, transferring energy to smaller scales



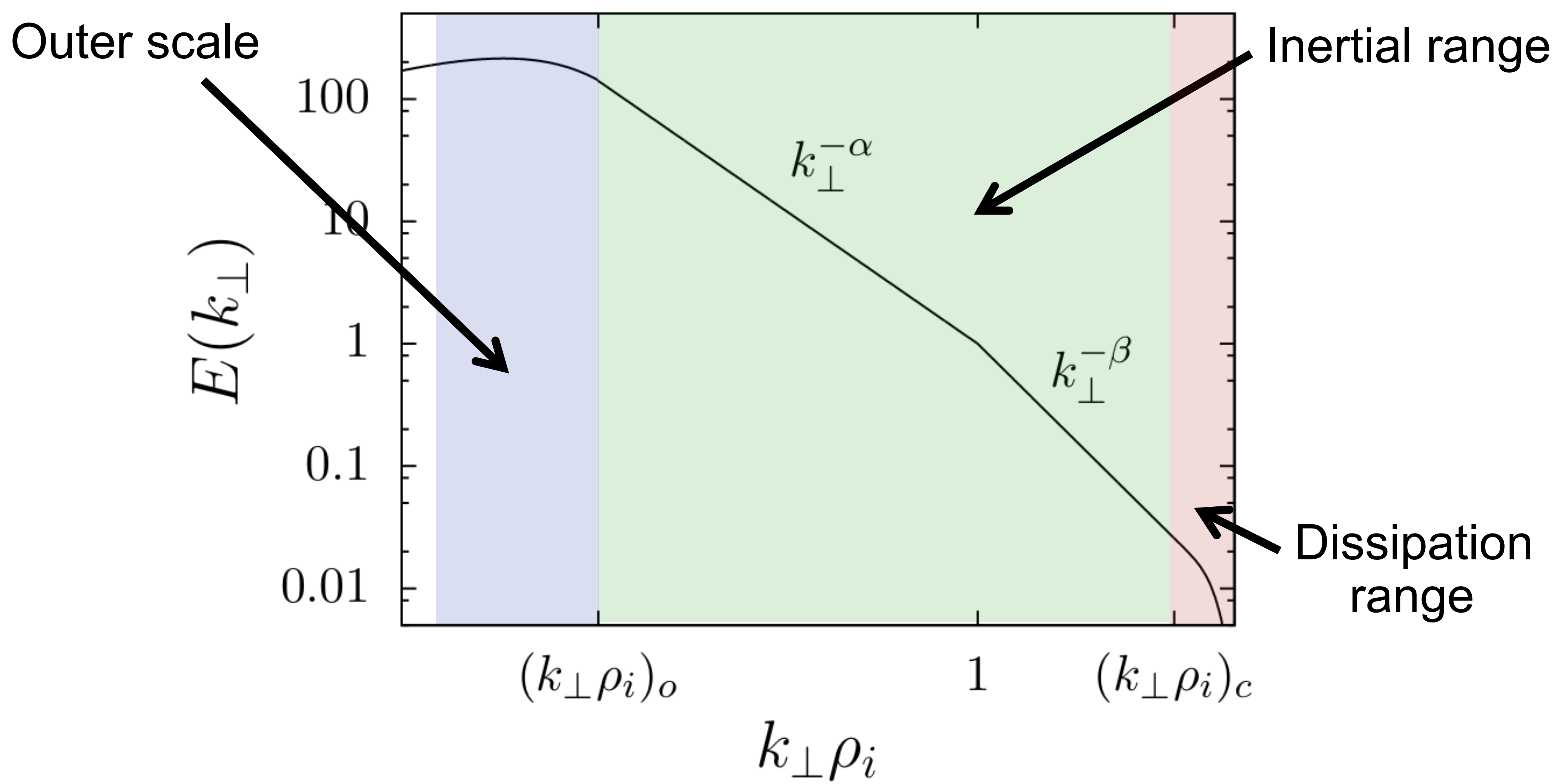
Turbulence cascade: inertial range

- Inertial range contains scales that 'feel' neither injection mechanism or dissipation mechanism



Big picture

- Want to predict how each of the quantities in this cartoon depend on safety factor and temp. gradient



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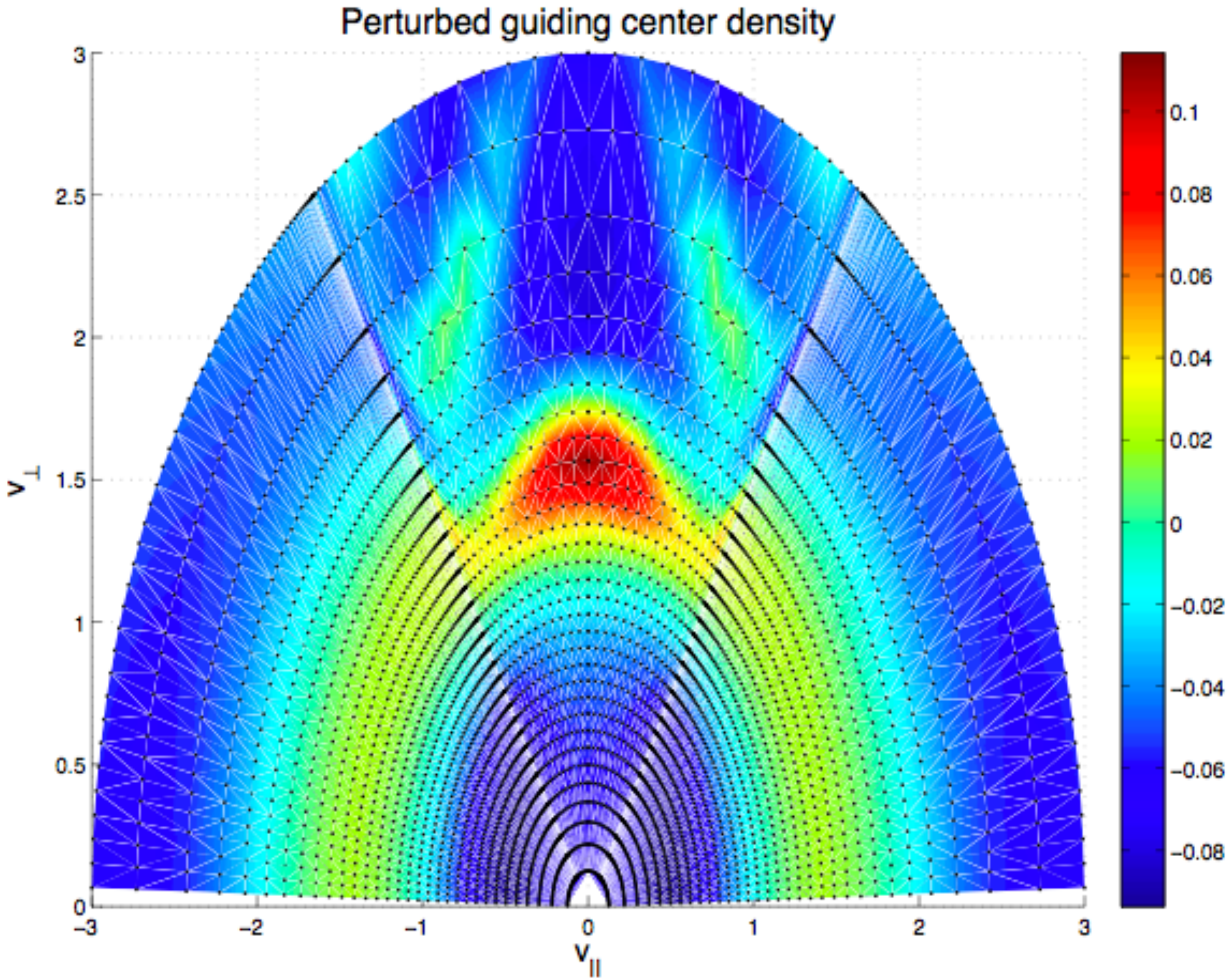
Conjectures

- Smooth, isotropic v-space
- Isotropy in plane perpendicular to B-field
- Parallel streaming time and nonlinear turnover time comparable at all scales (critical balance)
- Parallel length at outer scale set by system size (connection length)

Smooth, isotropic v-space



Smooth, isotropic v-space

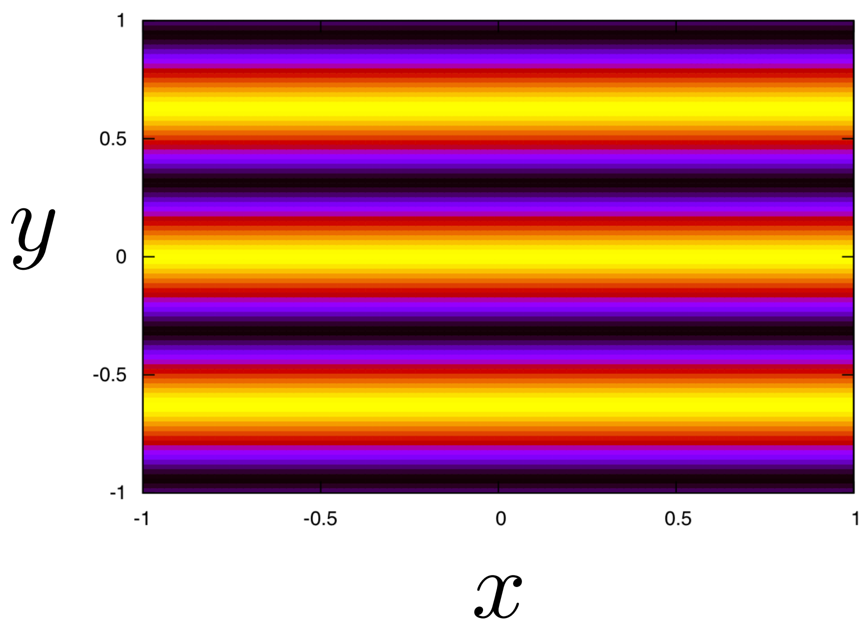
Assume $$\frac{\partial h}{\partial v_{\|}} \sim \frac{\partial h}{\partial v_{\perp}} \sim \frac{h}{v_{th}}$$

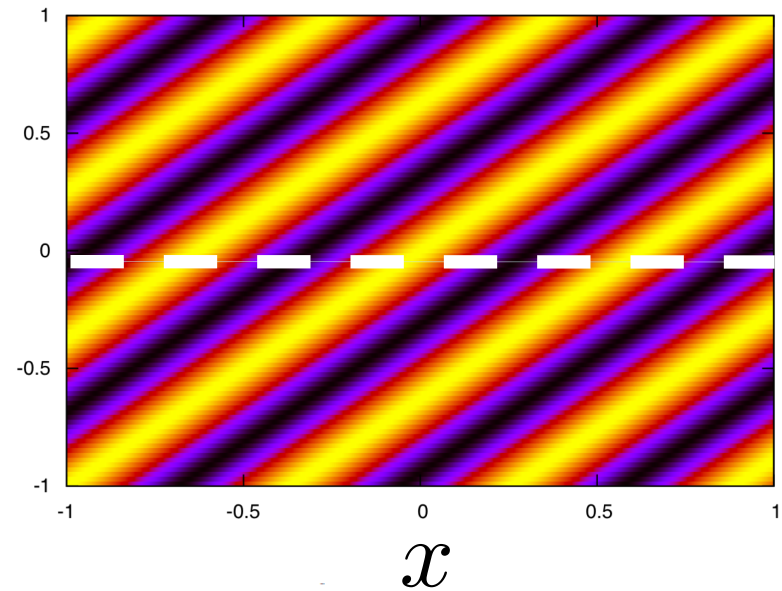
Quasineutrality: $$\sum_s e_s \left(\int d^3v \; h_s - \frac{e_s \varphi}{T_s} n_s \right) = 0$$

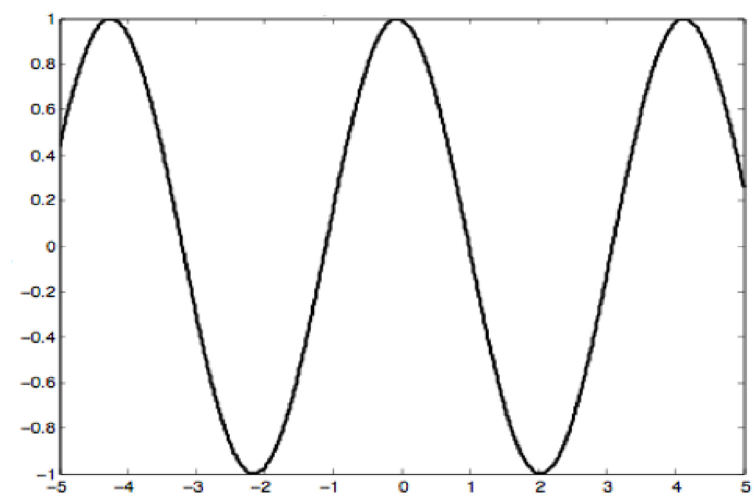
$$\int d^3v \; h \sim v_{th}^3 h \Rightarrow \frac{h}{F_M} \sim \frac{e\varphi}{T}$$

Spatial isotropy

- Physical idea: linear drive favors structures with $k_x \lesssim k_y$. Larger k_x formed through magnetic and flow shear:







Critical balance

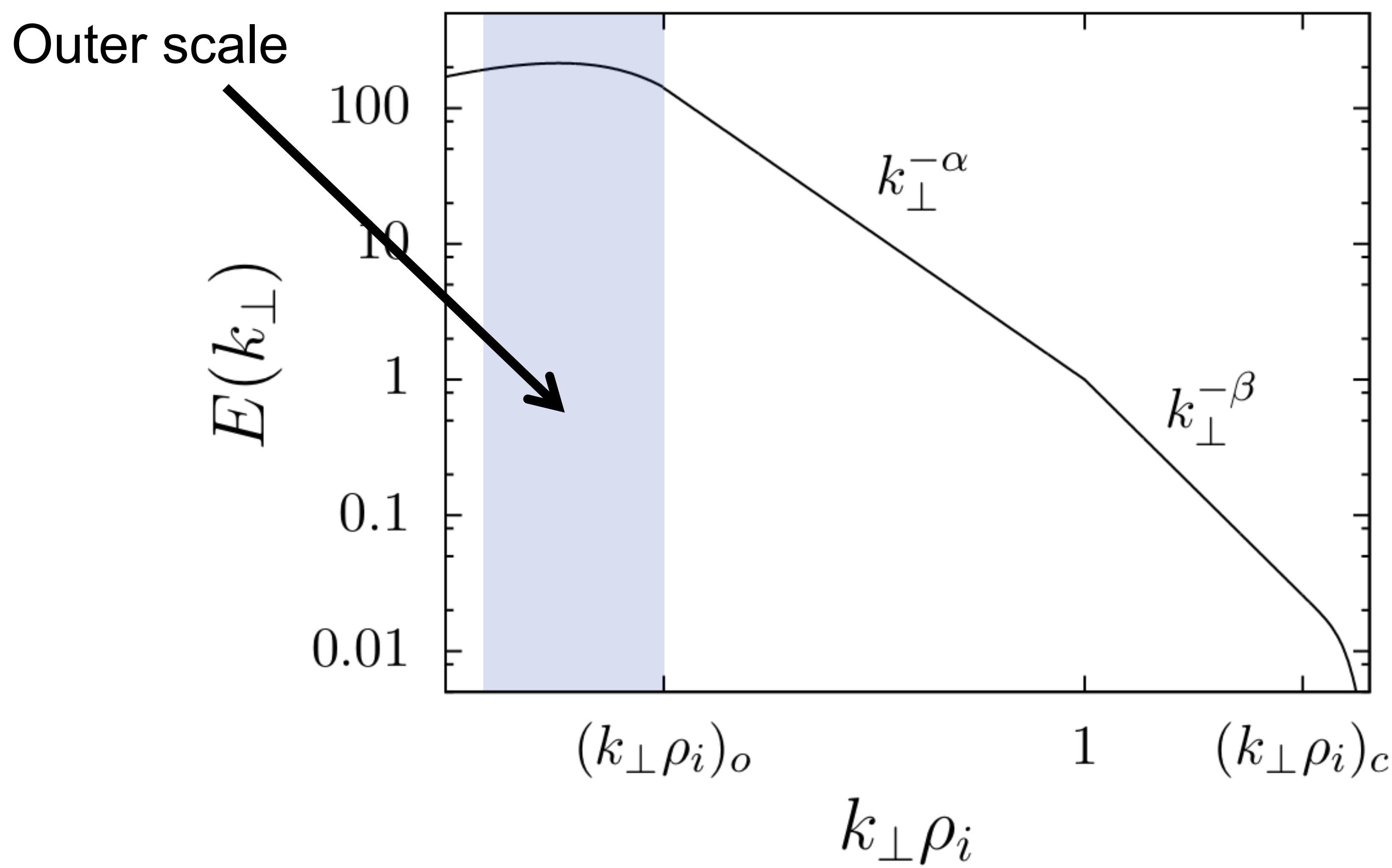
- Physical idea: two points along field correlated only if information propagates between them before turbulence decorrelated in perpendicular plane

$$\frac{h_k}{\tau_k} \sim (\mathbf{v}_E \cdot \nabla h)_k \sim \frac{v_{th}}{R} (k_\perp \rho_i)^2 \Phi_k h_k$$

Critical balance: $$k_{\|} v_{th} \sim \tau_k^{-1} \sim \frac{v_{th}}{R} (k_\perp \rho_i)^2 \Phi_k$$

$$\Phi_k \equiv \frac{e\varphi_k}{T} \frac{R}{\rho_i}$$

Big picture


Outer scale
100
10
1
0.1
0.01
$E(k_\perp)$
$k_\perp^{-\alpha}$
$k_\perp^{-\beta}$
$(k_\perp \rho_i)_o$
1
$(k_\perp \rho_i)_c$
$k_\perp \rho_i$

Outer scale

$$\omega_* h_k \sim (\mathbf{v}_E \cdot \nabla F_M)_k \sim k_\perp \rho_i \frac{v_{th}}{L_T} \frac{\rho_i}{R} \Phi_k F_M$$

- Injection rate comparable to nonlinear decorrelation time:

$$\tau_k^{-1} \sim \omega_*^{out} \sim k_\perp^{out} \rho_i \frac{v_{th}}{L_T}$$

- Combine with critical balance:

$$k_\parallel^{out} v_{th} \sim \tau_k^{-1} \sim \omega_*^{out}$$

$$\Rightarrow k_\parallel^{out} L_T \sim k_\perp^{out} \rho_i$$

Outer scale

$$k_{\|}^{out} L_T \sim k_{\perp}^{out} \rho_i$$

- Take parallel length scale to be connection length
- Physical idea: Good curvature and magnetic shear limit parallel extent of turbulence

$$k_{\|}^{out} \sim (qR)^{-1} \Rightarrow k_{\perp}^{out} \rho_i \sim \frac{L_T}{qR}$$

- With $k_{\|}^{out}$ and $k_{\perp}^{out}$, can solve for Φ_k^{out} and τ_k^{-1}

$$\Rightarrow \Phi_k^{out} \sim q \left(\frac{R}{L_T} \right)^2 \quad \text{and} \quad \tau_k^{-1} \sim \frac{v_{th}}{qR}$$

Heat flux

$$k_\perp^{out}\rho_i \sim \frac{L_T}{qR} \quad \text{and} \quad \tau_k^{-1} \sim \frac{v_{th}}{qR}$$

- Turbulence scale is
$$1/k_\perp^{out} \sim q\rho_i \sim (B/B_\theta)\rho_i \sim \rho_{\theta,i}$$
- Assume most energy contained in outer scale and use diffusive estimate for heat flux:

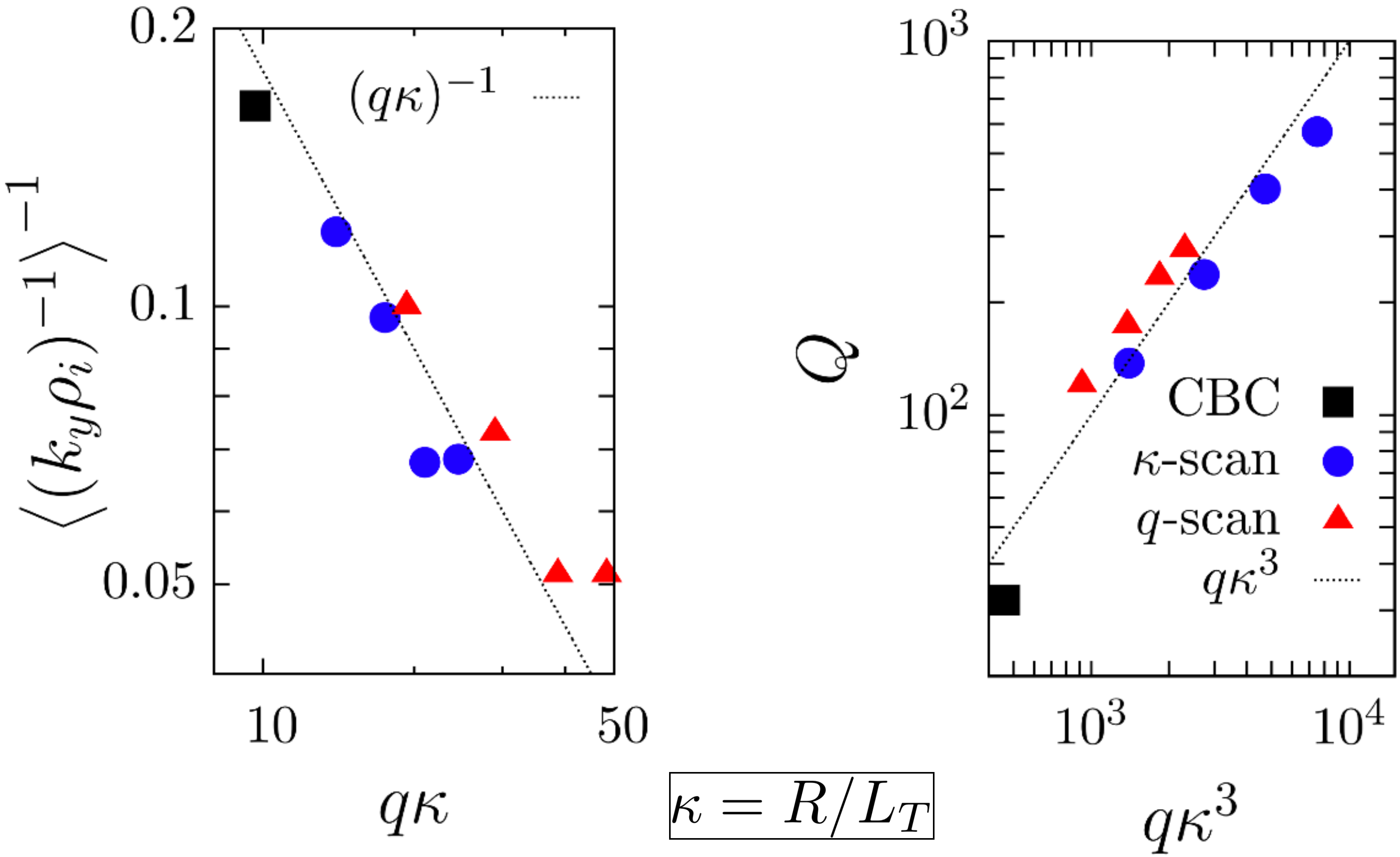
$$Q \sim \chi\frac{R}{L_T} \quad \text{with} \quad \chi \sim \frac{(\Delta x)^2}{\Delta t} \sim \frac{(k_\perp^{out})^{-2}}{\tau_k}$$

$$\Rightarrow Q \sim q\left(\frac{R}{L_T}\right)^3$$

Simulation system

- Use continuum, local, delta-f GK code GS2
- Base case is Cyclone (widely benchmarked)
  - Unshifted, circular flux surface
  - Safety factor is 1.4, magnetic shear=0.8, R/Ln=2.2, R/LT=6.9
  - Electrostatic
  - Modified Boltzmann response for electrons
- Fix R/LT and vary q from 1.4 up to 7.0
- Fix q and vary R/LT from 6.9 to 17.5

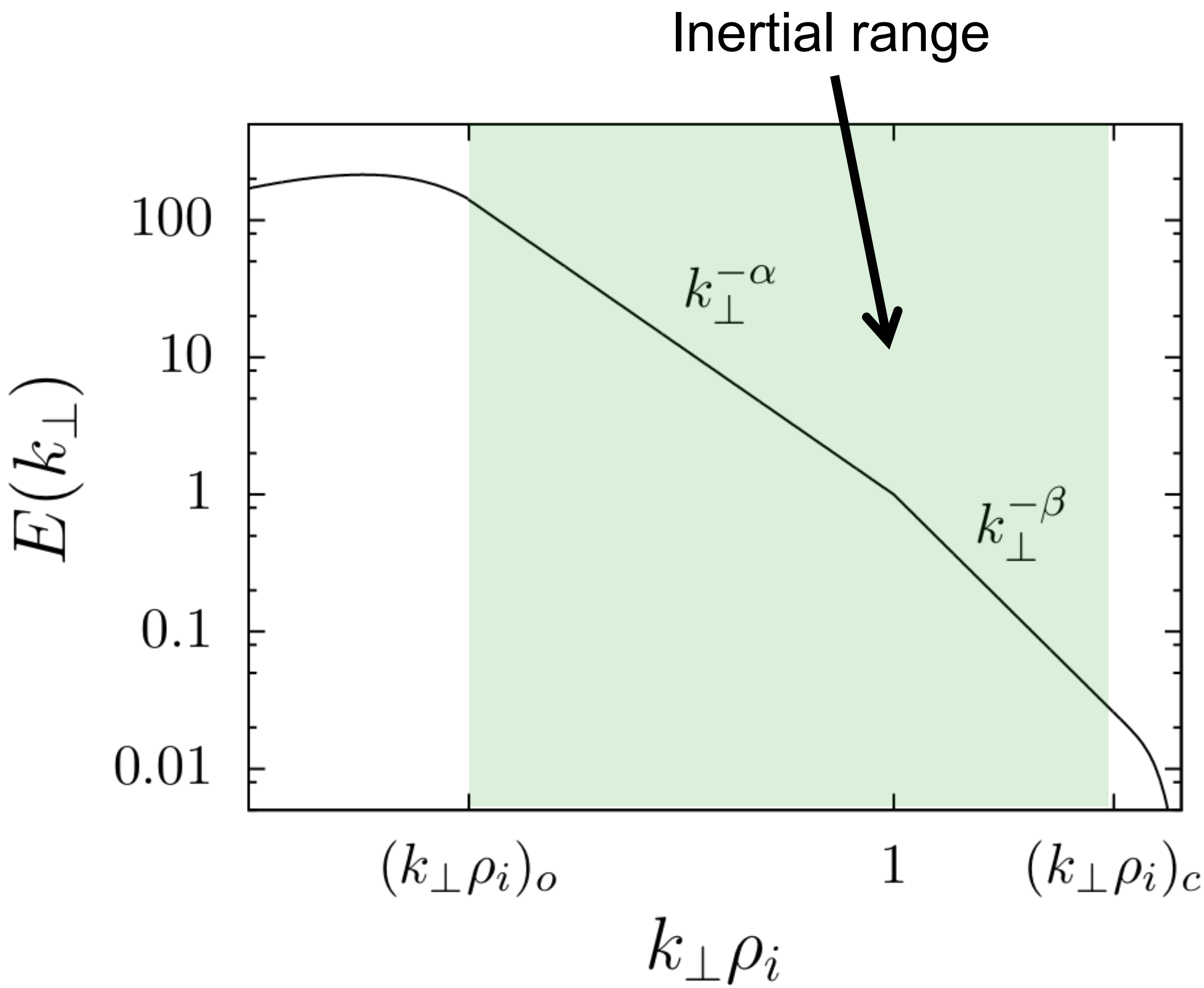
Turbulence scaling tests



$\boxed{\kappa = R/L_T}$

Note that Q at large R/L_T much larger than found in previous studies (box size used here for $R/L_T \approx 20$ was $\approx 1000\rho_i$)

Big picture



Inertial range
$k_\perp^{-\alpha}$
$k_\perp^{-\beta}$
100
10
1
0.1
0.01
$E(k_\perp)$
$(k_\perp \rho_i)_o$
1
$(k_\perp \rho_i)_c$
$k_\perp \rho_i$

Inertial range

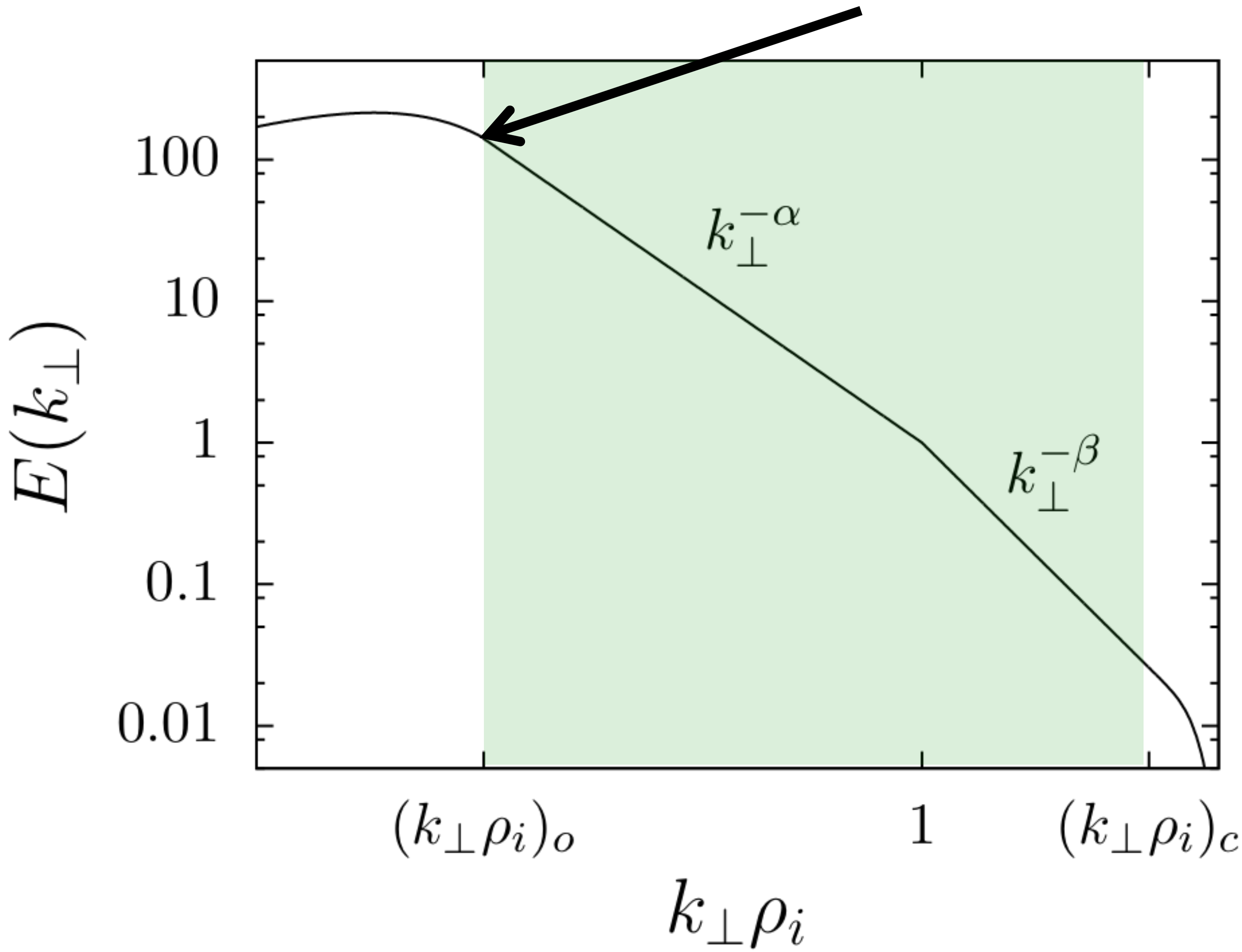
Free energy: $$W = V^{-1} \sum_s \int d^3r \int d^3v \left(\frac{T_s \delta f_s^2}{F_{M,s}} \right)$$

- Flux of free energy (nonlinear invariant) scale-independent in inertial range:

$$\frac{W_k}{\tau_k} \sim (k_\perp \rho_i)^2 \frac{v_{th}}{R} \left(\frac{\rho_i}{R} \right)^2 \Phi_k^3 \sim \text{constant}$$

Matching

Match solution from outer and inertial scales



Inertial range

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$$\Phi_k \sim q^{1/3} \left(\frac{R}{L_T} \right)^{4/3} (k_\perp \rho_i)^{-2/3}$$

Inertial range

- Use critical balance with Φ_k to relate $k_{\|}$ and $k_\perp$

$$k_{\|} qR \sim \left(k_\perp \rho_i \frac{qR}{L_T} \right)^{4/3}$$

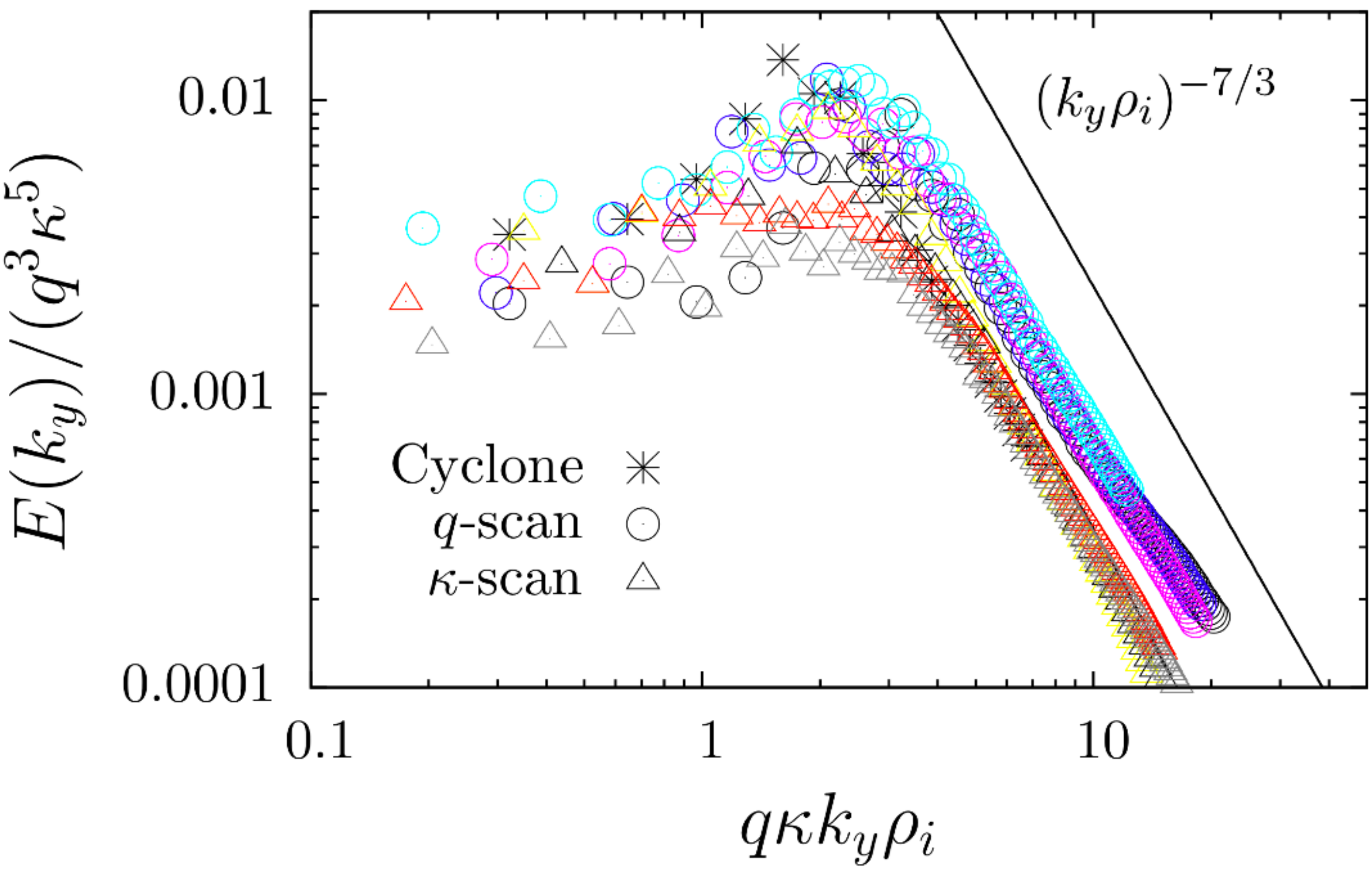
- Convert expression for Φ_k into 1D spectrum

$$\int dk_y \;\, \rho_i E(k_y) = V^{-1} \int d^3 r \;\, \Phi^2$$

$$E(k_y) \sim q^{2/3} \left(\frac{R}{L_T} \right)^{8/3} (k_y \rho_i)^{-7/3}$$

Inertial range spectra

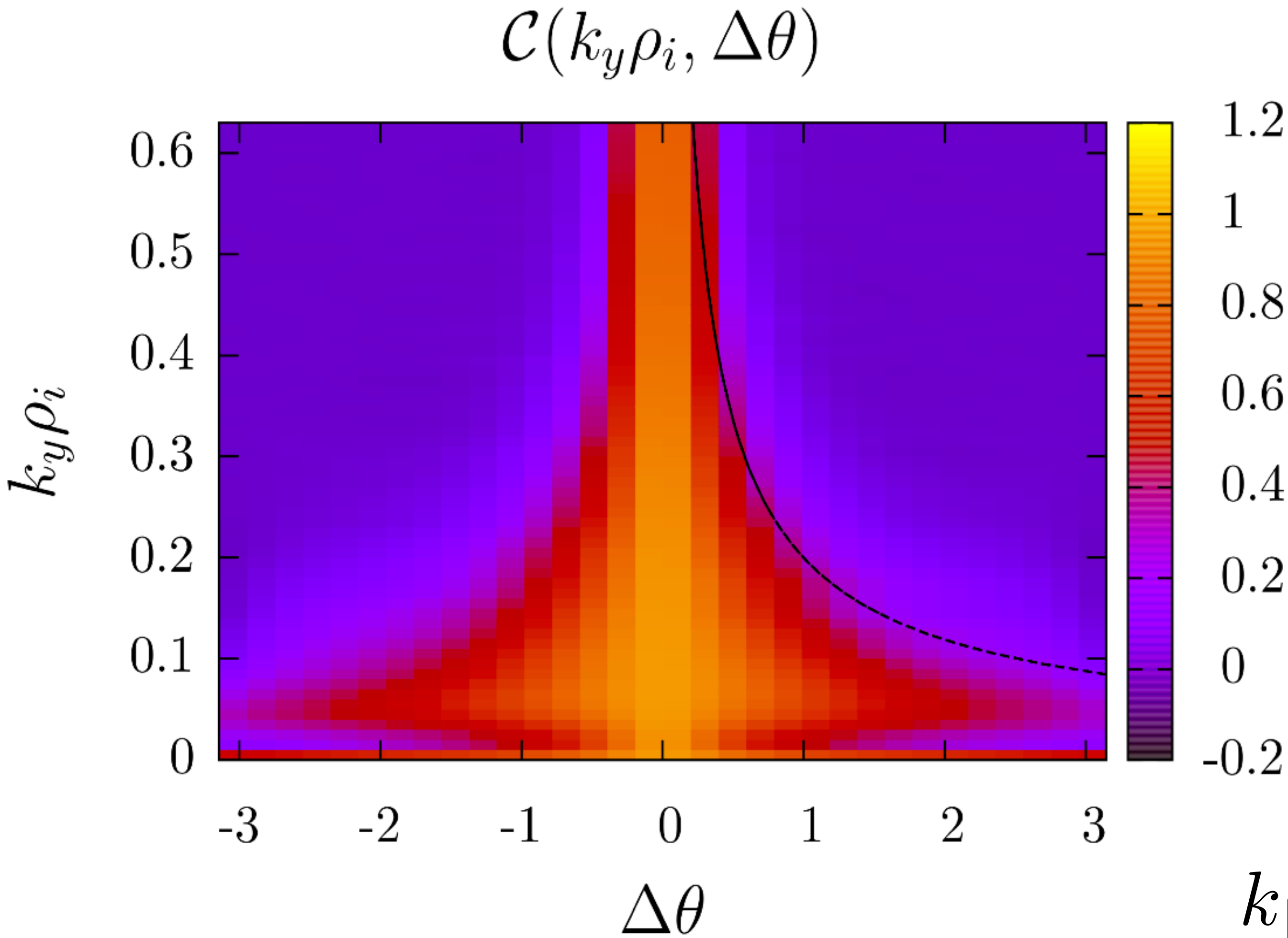
$$\int dk_y\ \rho_i E(k_y) = V^{-1} \int d^3r\ \Phi^2$$



Critical balance test

Correlation function:

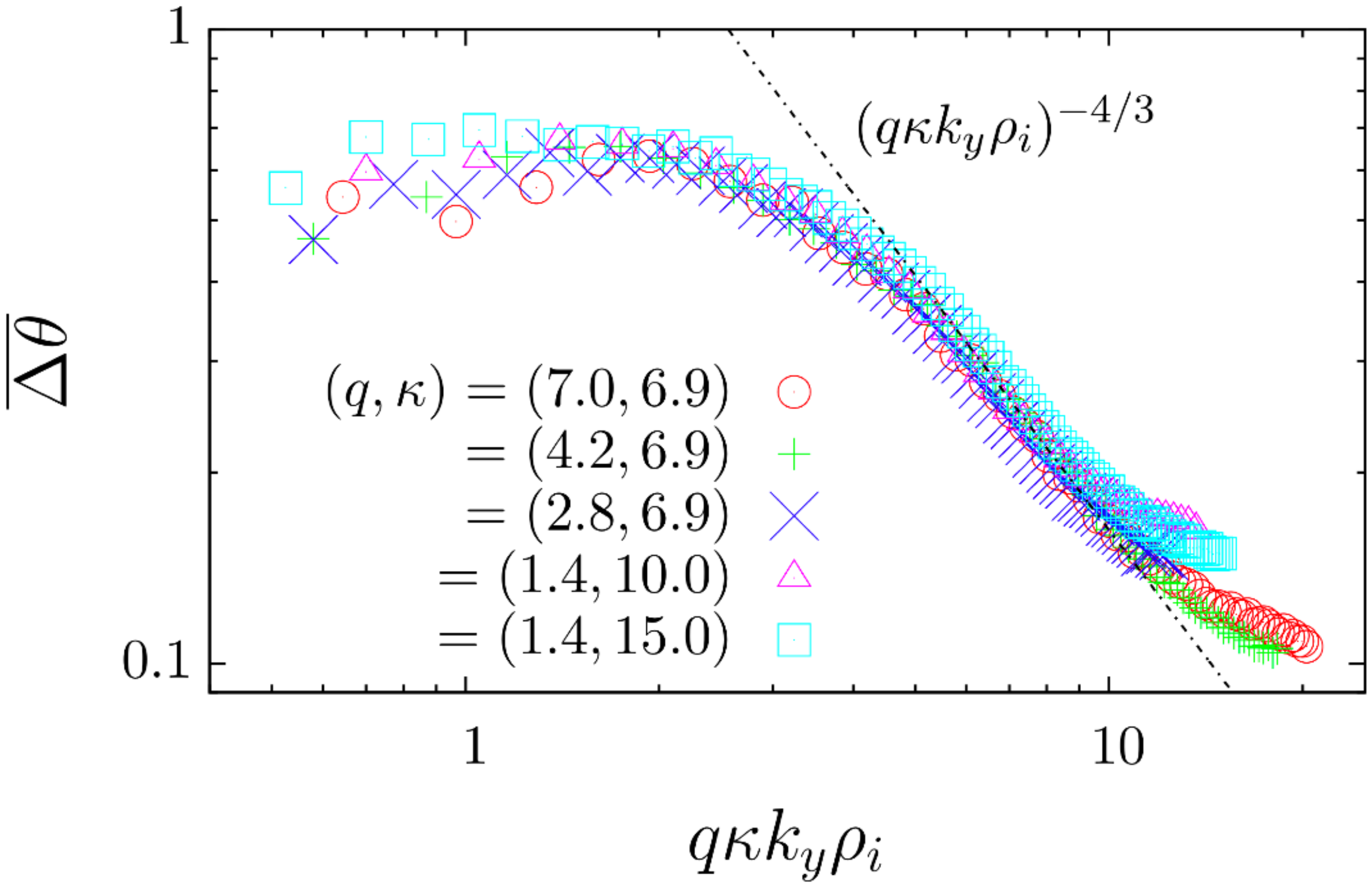
$$\mathcal{C}(k_y, \Delta\theta) = \frac{\sum_{k_x} \Phi(\theta = 0)\Phi^*(\theta = \Delta\theta)}{\sum_{k_x} |\Phi(\theta = 0)|^2}$$



$$k_\parallel qR \sim (k_\perp\rho_i)^{4/3}$$

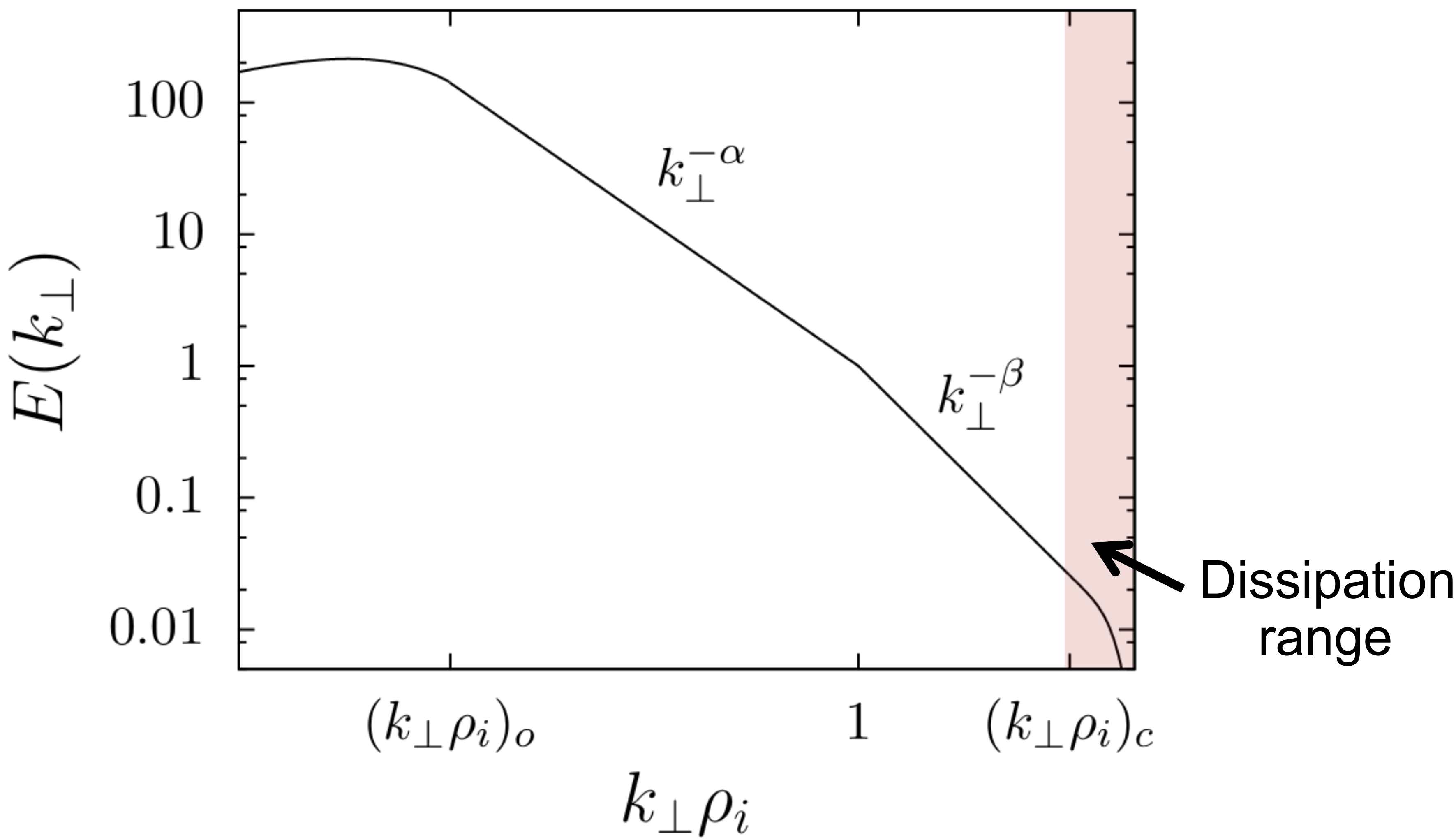
Inertial range critical balance

$$k_\parallel qR \sim \left(k_\perp \rho_i \frac{qR}{L_T}\right)^{4/3} \qquad \overline{\Delta\theta}(k_y) = \int d(\Delta\theta)\mathcal{C}(k_y, \Delta\theta)$$

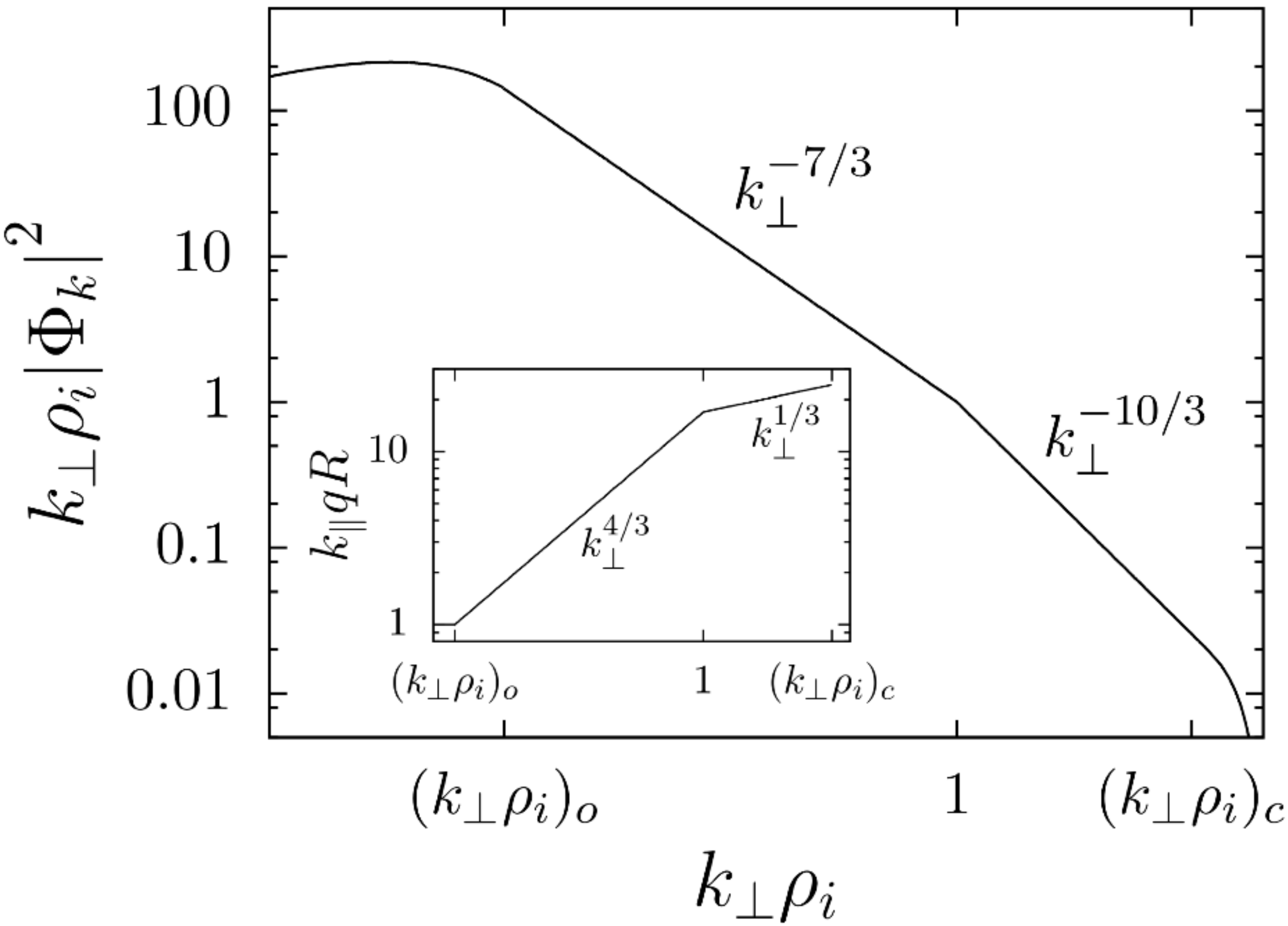


Dissipation scale

$$(k_\perp \rho_i)_c \sim q^{1/5} \left(\frac{R}{L_T}\right)^{4/5} \left(\frac{v_{th}}{\nu_i R}\right)^{3/5}$$



Back to big picture



Conclusions

- Simple scalings for turbulence spatial scales and amplitudes derived and numerically confirmed. Predictions for scalings of:
  - Turbulence amplitude, heat flux, peak space scale, spectrum, decorrelation time, cutoff scale
- Critical balance robustly satisified – plasma turbulence three dimensional
- Scalings allow for B/Bp expansion of higher order GK Eq., making it tractable to solve numerically. This allows us to address problem of intrinsic rotation.

Dissipation scale

- In analogy with Reynolds number, define

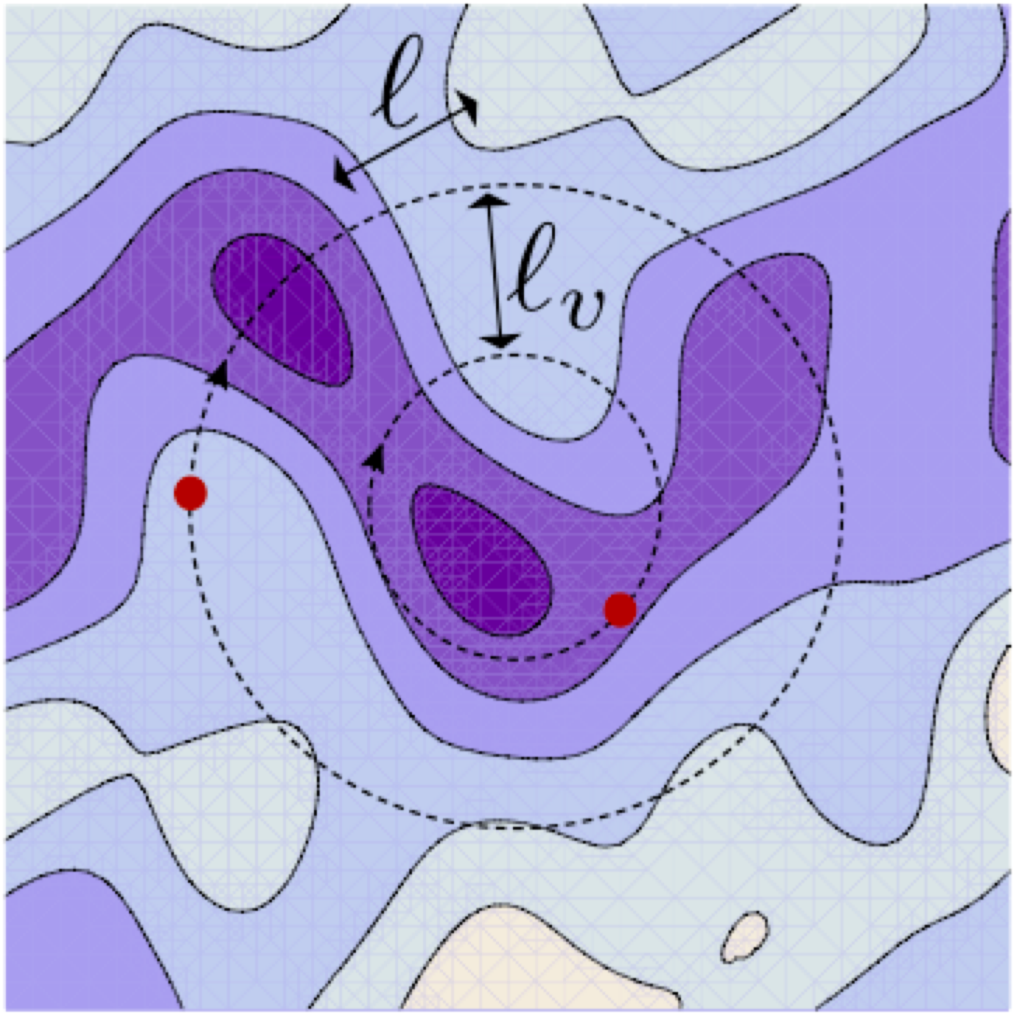
$$\mathrm{Do} \equiv (\nu_i \tau_{\rho_i})^{-1} \sim q^{1/3} \left(\frac{R}{L_T} \right)^{4/3} \left(\frac{v_{th}}{\nu_i R} \right)$$

- At dissipation scale, dissipation rate comparable to nonlinear decorrelation rate

$$\nu_i \left(\frac{v_{th}}{\delta v} \right)^2 \sim \tau_{nl}^{-1} \Rightarrow \mathrm{Do} \sim \left(\frac{v_{th}}{\delta v} \right)^2 \frac{\tau_{nl}}{\tau_{\rho_i}} \sim \left(\frac{v_{th}}{\delta v} \right)^2 \left(\frac{\ell_\perp}{\rho_i} \right)^2 \frac{\Phi_{\rho_i}}{\Phi_\ell}$$

- Dissipation scale assumed below ion Larmor scale

Perpendicular phase mixing



Schekochihin *et al.*, PPCF 2008

- Drift velocity = $F[\langle\Phi\rangle]$
- Particles with Larmor orbits separated by turbulence wavelength 'see' different averaged potential
- Drift velocities decorrelated, thus phase mixing

$$\frac{k_\perp \delta v_\perp}{\Omega} \sim 1$$

$$\Rightarrow \frac{\delta v_\perp}{v_{th}} \sim (k_\perp \rho_i)^{-1}$$

Dissipation scale

$$\mathrm{Do} \sim \left(\frac{v_{th}}{\delta v}\frac{\ell_\perp}{\rho_i}\right)^2 \frac{\Phi_{\rho_i}}{\Phi_\ell} \qquad\qquad \frac{\delta v}{v_{th}} \sim \frac{\ell_\perp}{\rho_i}$$

- Carrying out inertial range analysis (as before, but with $J_0(k_\perp \rho_i) \sim (k_\perp \rho_i)^{-1/2}$) gives*

$$\Phi_\ell \sim \left(\frac{\ell_\perp}{\rho_i}\right)^{7/6} \Rightarrow (k_\perp \rho_i)_c \sim \mathrm{Do}^{3/5}$$

$$(k_\perp \rho_i)_c \sim q^{1/5} \left(\frac{R}{L_T}\right)^{4/5} \left(\frac{v_{th}}{\nu_i R}\right)^{3/5}$$

*Schekochihin *et al*., PPCF 2008