TRINITY: local gyrokinetics + global transport = predictive modeling of ITER-like plasma performance

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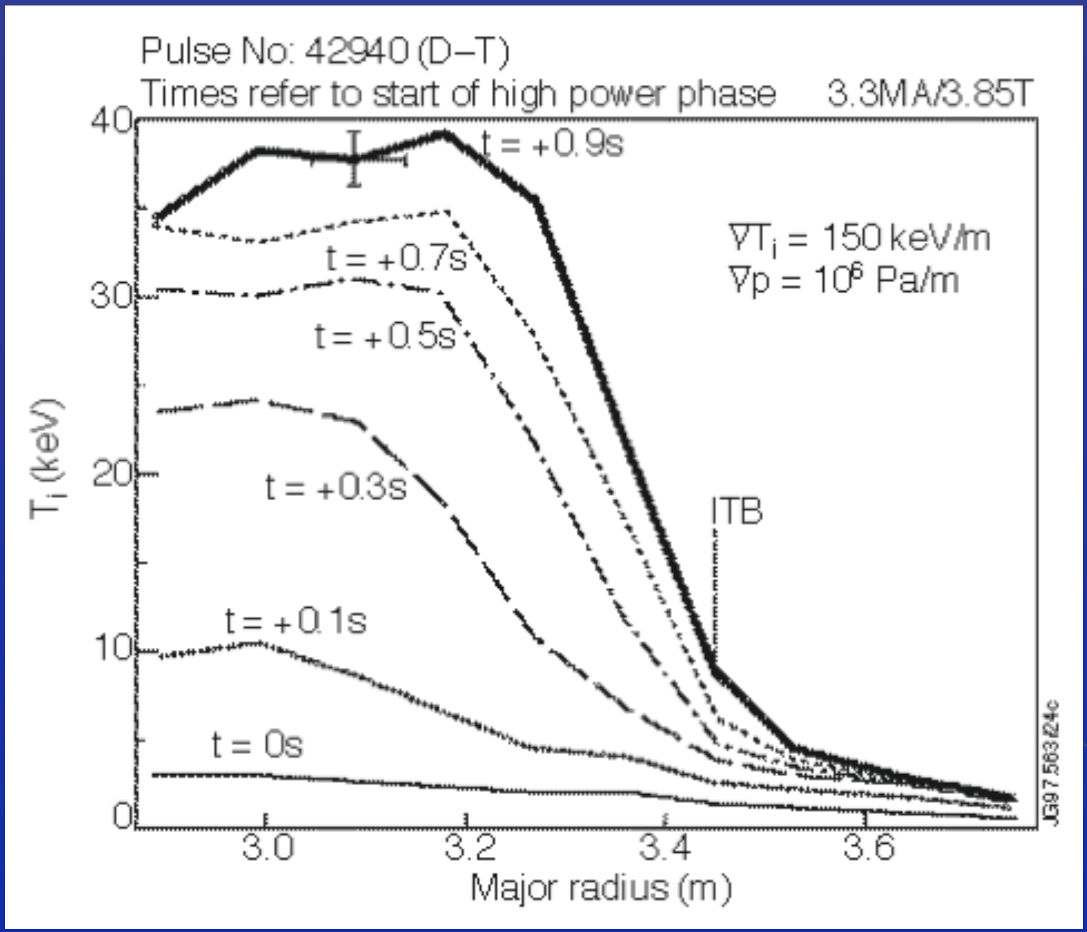
Overview

- Motivation
- Theoretical framework
- Numerical approach
- Trinity simulation results
- Future work

Objective



Connor et al. (2004)

Core:
multi-physics,
multi-scale

Edge:
multi-physics,
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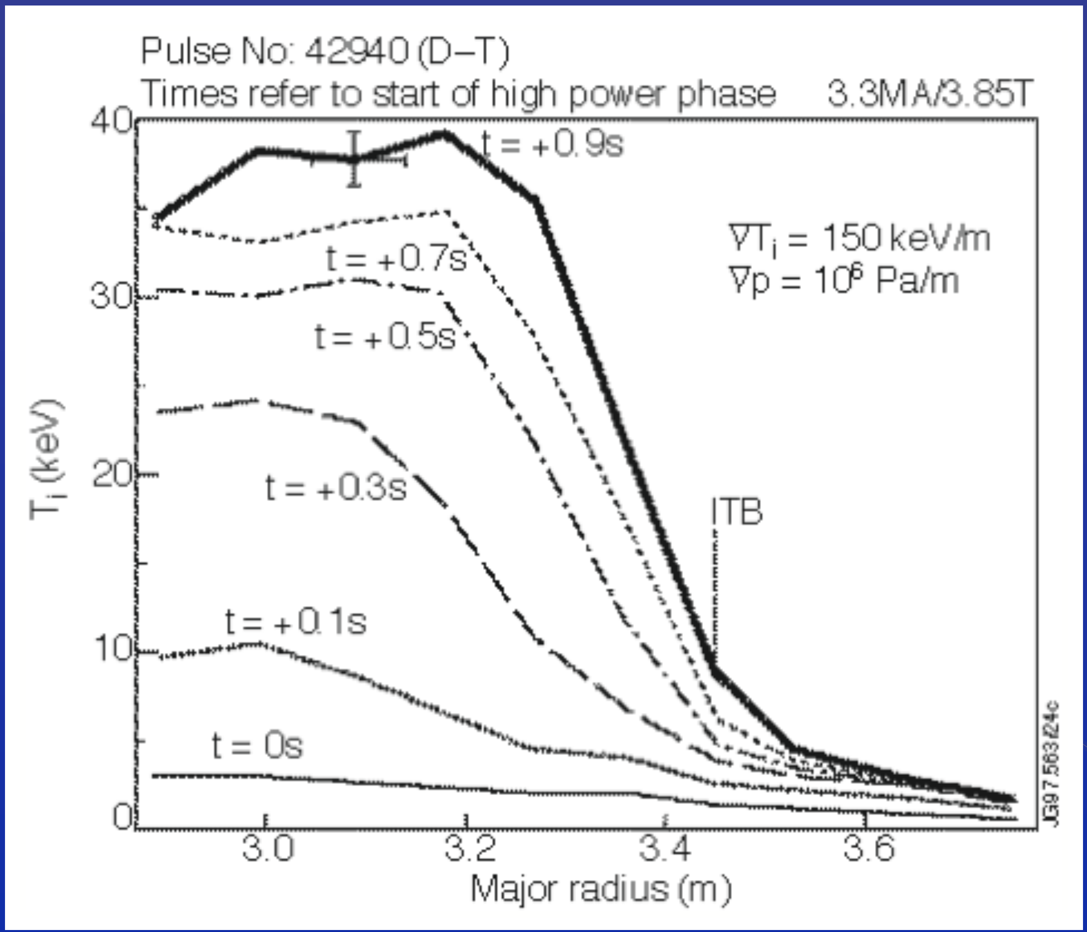
Objective



Connor et al. (2004)

Core:
multi-physics,
multi-scale

- **kinetic turbulence**
- neoclassical
- sources
- magnetic equilibrium
- MHD

Multiple scale problem

$$\frac{df}{dt} = C[f] \quad + \quad \frac{\omega}{\Omega_i} \sim \frac{\rho}{L} \ll 1$$

Physics	Perpendicular spatial scale	Temporal scale
Turbulence from ETG modes	$k_\perp^{-1}$ ~ 0.001 - 0.1 cm	ω_* ~ 0.5 - 5.0 MHz
Turbulence from ITG modes	$k_\perp^{-1}$ ~ 0.1 - 8.0 cm	ω_* ~ 10 - 100 kHz
Transport barriers	Measurements suggest width ~ 1 - 10 cm	100 ms or more in core?
Discharge evolution	Profile scales ~ 100 cm	Energy confinement time ~ 2 - 4 s

Full-f simulation cost

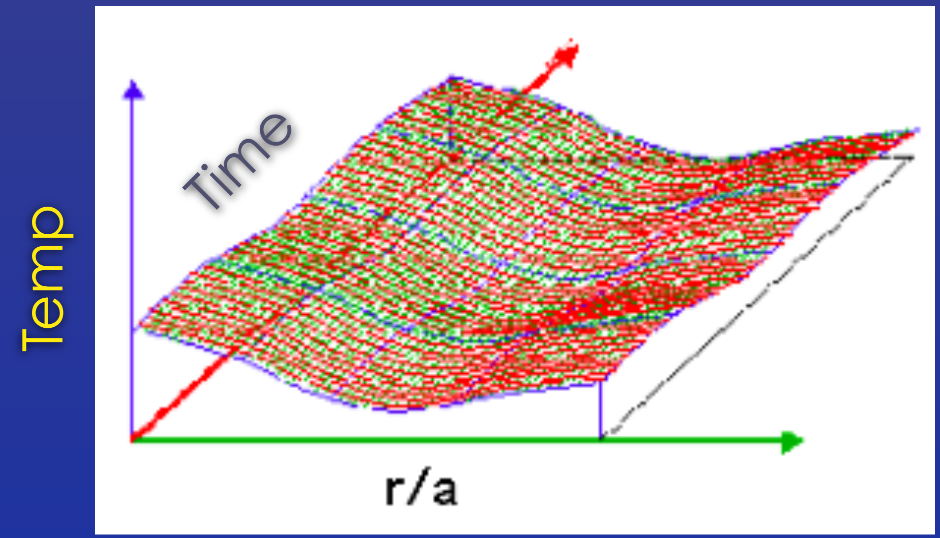
- Grid spacings in space (3D), velocity (2D) and time:

$$\Delta_\perp \sim 0.001\ cm, \quad L_\perp \sim 100\ cm$$

$$\Delta_{||} \sim 10\ cm, \quad L_{||} \sim 10\ m$$

$$\Delta v \sim 0.1\ v_{th}, \quad L_v \sim v_{th}$$

$$\Delta t \sim 10^{-7}\ s, \quad L_t \sim 1\ s$$



- Grid points required:

$$(L_{||}/\Delta_{||}) \times (L_\perp/\Delta_\perp)^2 \times (L_v/\Delta_v)^2 \times (L_t/\Delta t) \sim 10^{21}$$

- Factor of ~10^6 more than largest fluid turbulence calculations

- Direct simulation not possible; need physics guidance

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Gyrokinetic multiscale assumptions

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial f}{\partial \mathbf{R}} + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} + \frac{dE}{dt}\frac{\partial f}{\partial E} + \frac{d\vartheta}{dt}\frac{\partial f}{\partial \vartheta} = C[f]$$

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$$f = F + \delta f \qquad\qquad \delta f \sim \epsilon f$$

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- Separation of space scales:

$$\nabla F \sim F/L, \quad \nabla_{\|} \delta f \sim \delta f / L, \quad \nabla_{\perp} \delta f \sim \delta f / \rho$$

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- Separation of space scales:

$$\nabla F \sim F/L, \quad \nabla_{\|} \delta f \sim \delta f/L, \quad \nabla_{\perp} \delta f \sim \delta f/\rho$$

- "Smooth" velocity space:

$$\epsilon \lesssim \nu/\omega \lesssim 1 \Rightarrow \sqrt{\epsilon} \lesssim \delta v/v_{th} \lesssim 1$$

Key results: turbulence and transport

$$f = F_0 + h + ... \qquad F_0 = F_M(\mathbf{R}) \exp\left(-\frac{q\Phi}{T}\right)$$

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Gyrokinetic equation for turbulence:

$$\partial h/\partial t + v_\parallel \hat{\mathbf{b}} \cdot \nabla h + \langle \mathbf{v}_\chi \rangle_\mathbf{R} \cdot \nabla (F_0 + h) + \mathbf{v_B} \cdot \nabla h = \frac{qF_0}{T_0} \frac{\partial \langle \chi \rangle_\mathbf{R}}{\partial t} + \langle C[h] \rangle_\mathbf{R}$$

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Moment equations for equilibrium evolution:

$$\begin{aligned}
\frac{\partial n_s}{\partial t} &= -\frac{1}{V'} \frac{\partial}{\partial \psi} \left(V' \langle \mathbf{\Gamma}_s \cdot \nabla \psi \rangle \right) + S_n \\
\frac{3}{2} \frac{\partial n_s T_s}{\partial t} &= -\frac{1}{V'} \frac{\partial}{\partial \psi} \left(V' \langle \mathbf{Q}_s \cdot \nabla \psi \rangle \right) \\
&+ T_s \left(\frac{\partial \ln n_s}{\partial \psi} - \frac{3}{2} \frac{\partial \ln T_s}{\partial \psi} \right) \langle \mathbf{\Gamma}_s \cdot \nabla \psi \rangle + \frac{\partial \ln T_s}{\partial \psi} \langle \mathbf{Q}_s \cdot \nabla \psi \rangle \\
&- \left\langle \int d^3 v \frac{h_s T_s}{F_{0s}} \langle C[h_s] \rangle_\mathbf{R} \right\rangle + n_s \nu_\epsilon^{su} (T_u - T_s) + S_p
\end{aligned}$$

All terms in each equation same order

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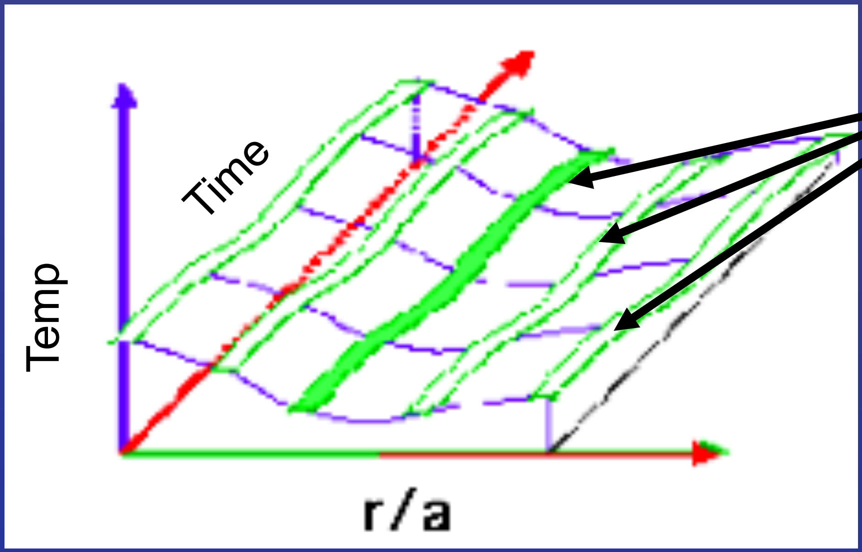
Multiscale grid

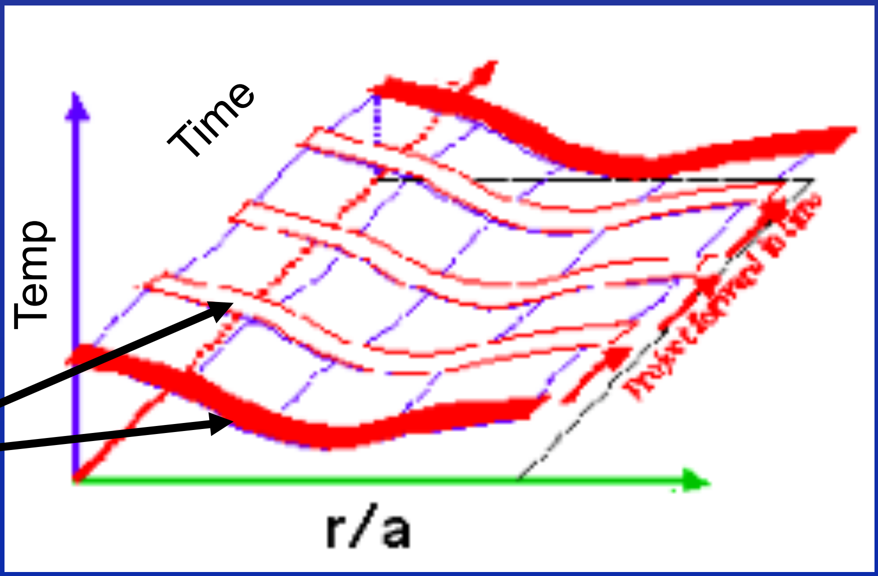


Flux tube simulation domain

- Turbulent fluxes calculated in small regions of fine grid embedded in “coarse” radial grid (for equilibrium)

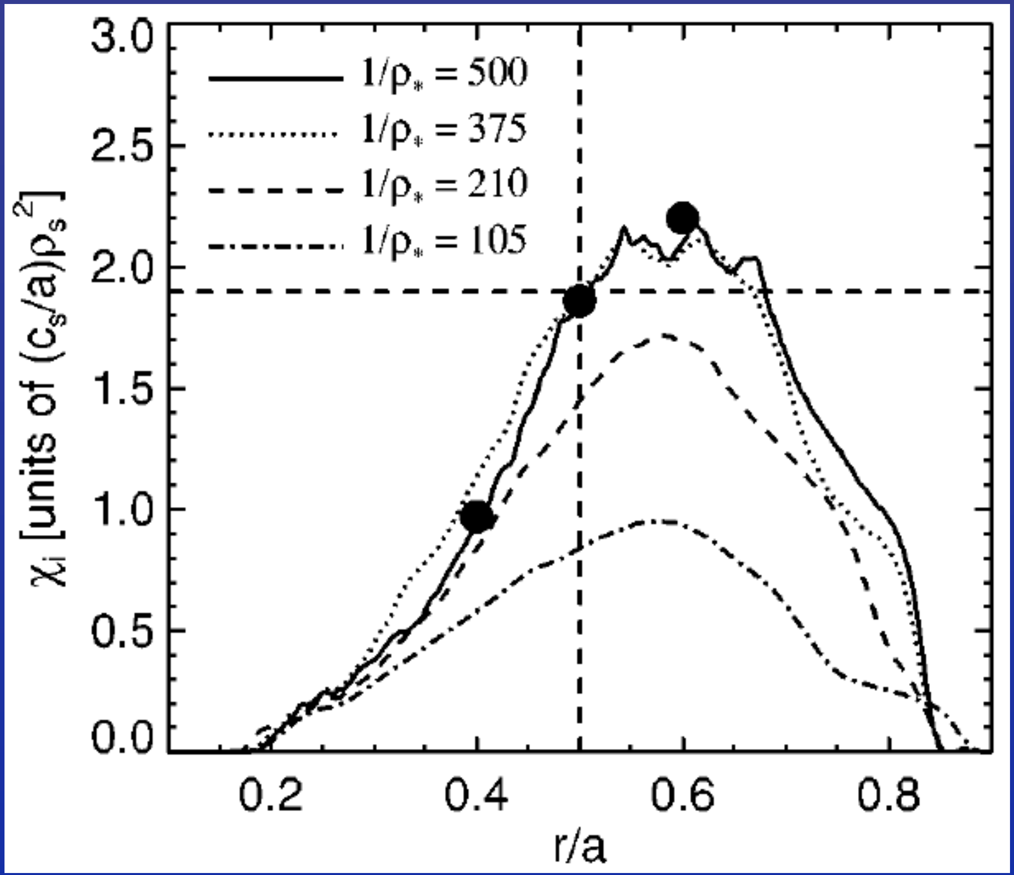
- Steady-state (time-averaged) turbulent fluxes calculated in small regions of fine grid embedded in “coarse” time grid (for equilibrium)

Flux tube simulation domain



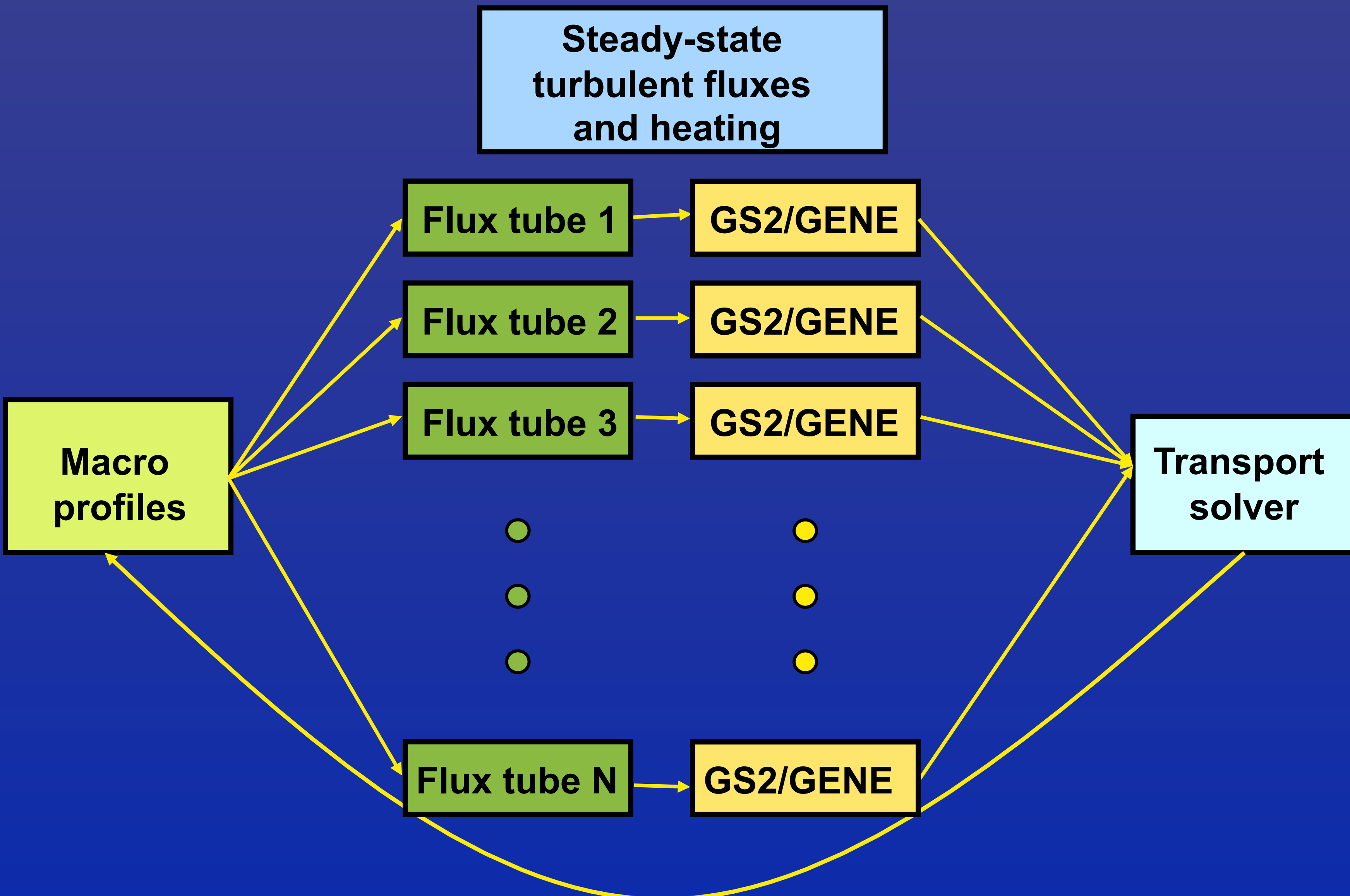
Validity of flux tube approximation

- Lines represent global simulations from GYRO
- Dots represent local (flux tube) simulations from GS2
- Excellent agreement for $\rho_* \ll 1$

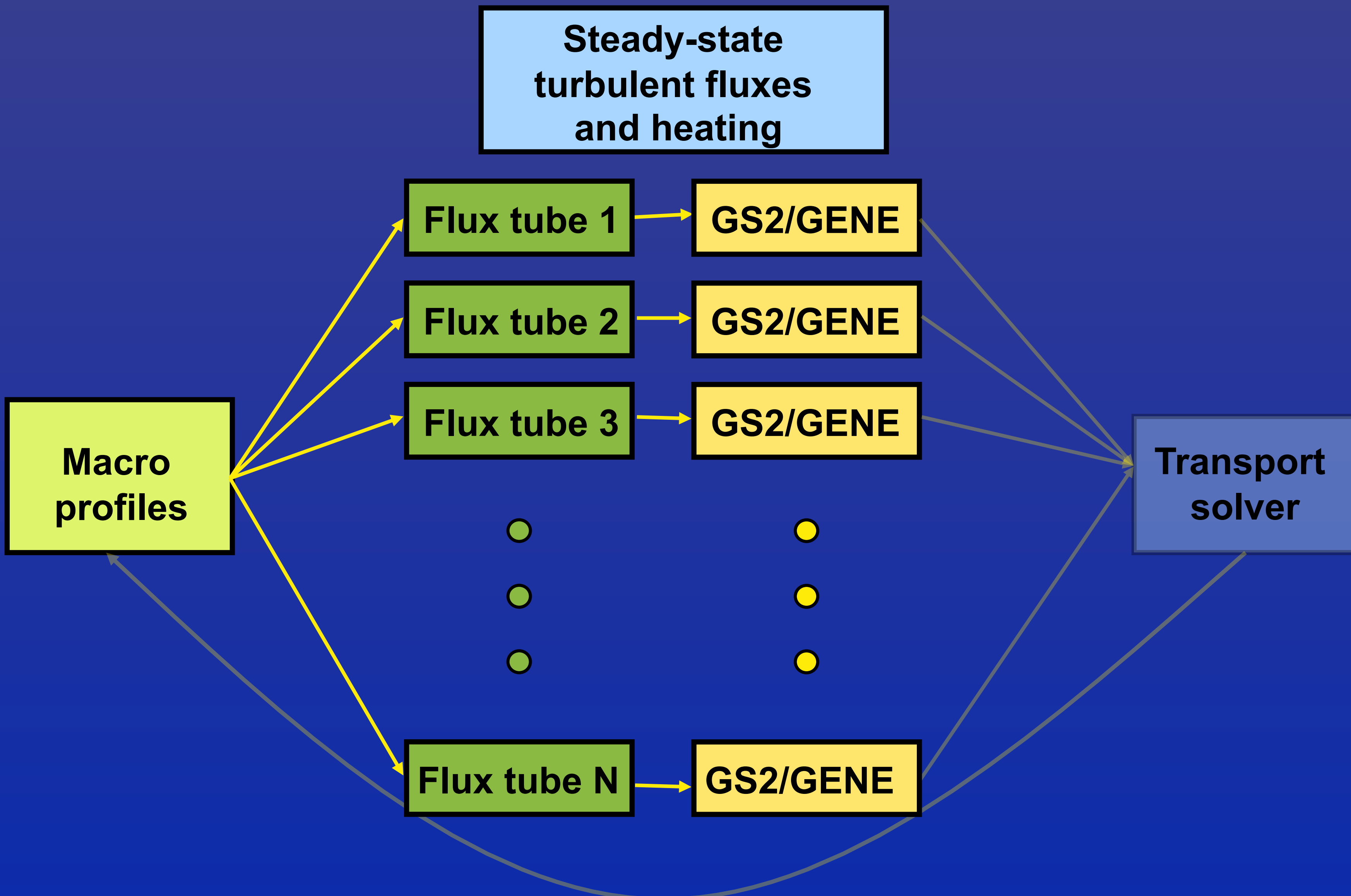


Candy et al (2004)

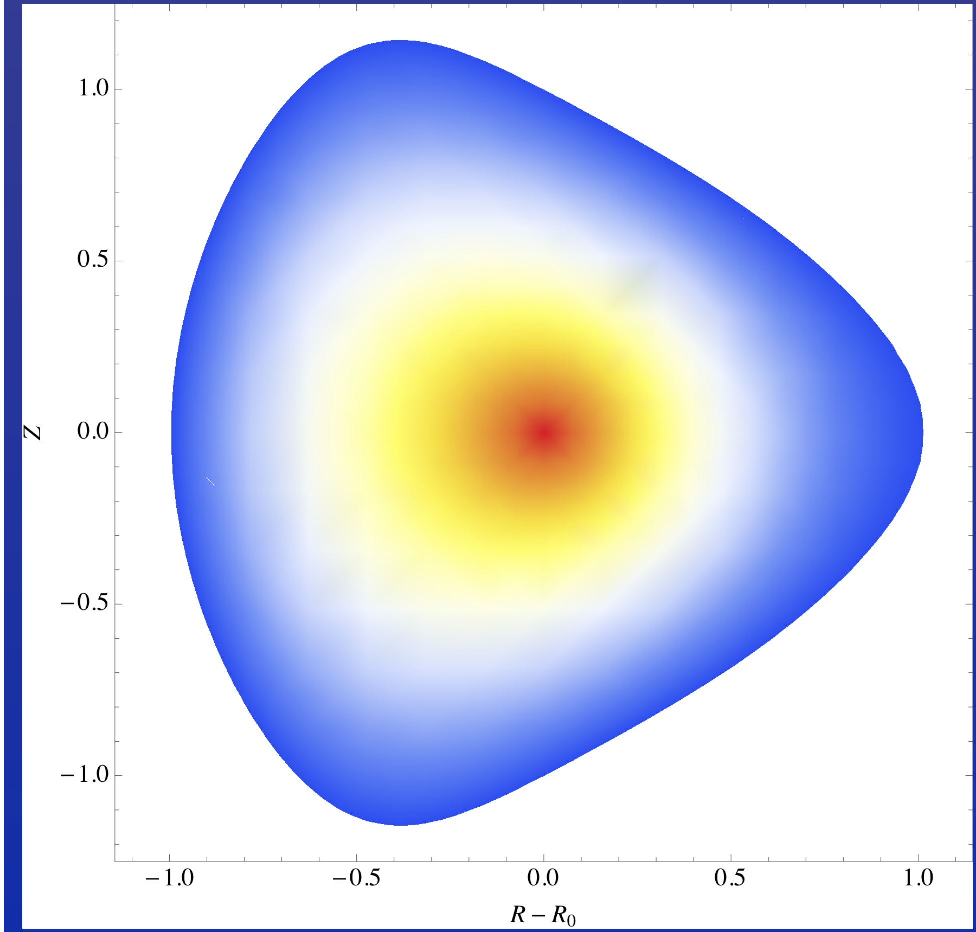
Trinity schematic

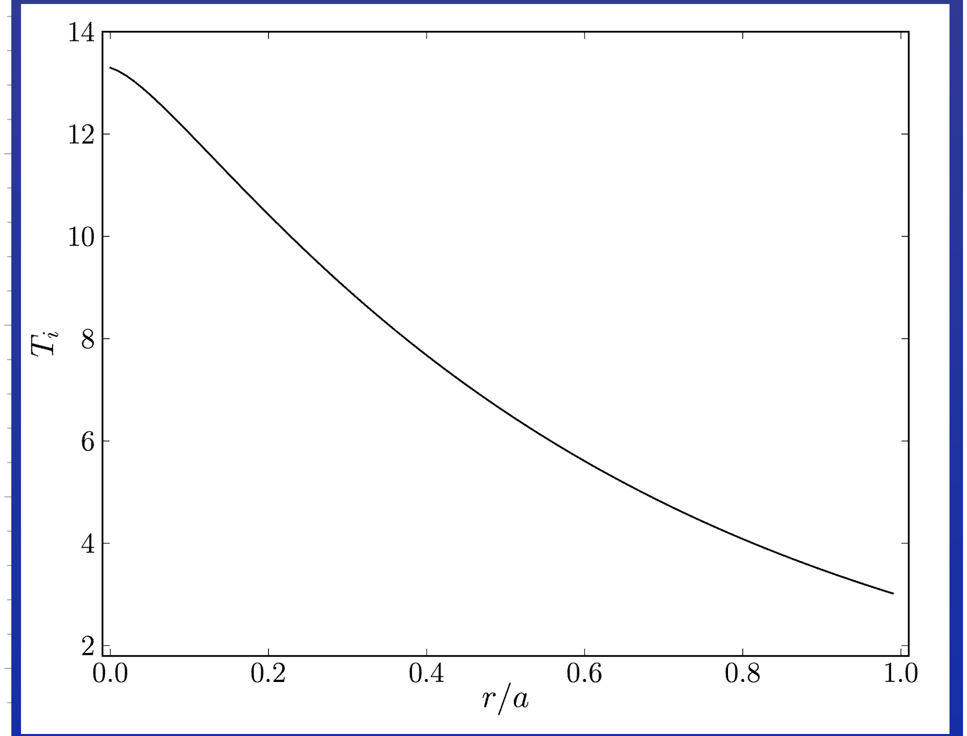


Trinity schematic



Sampling profile with flux tubes





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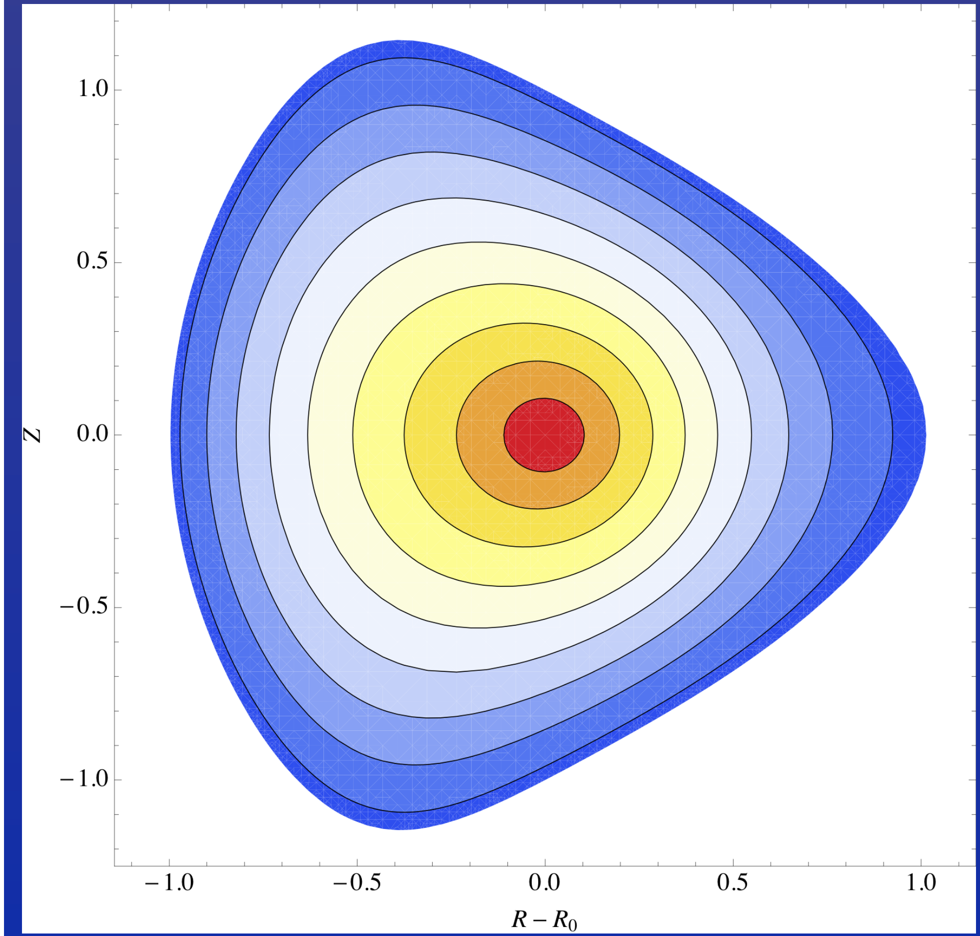

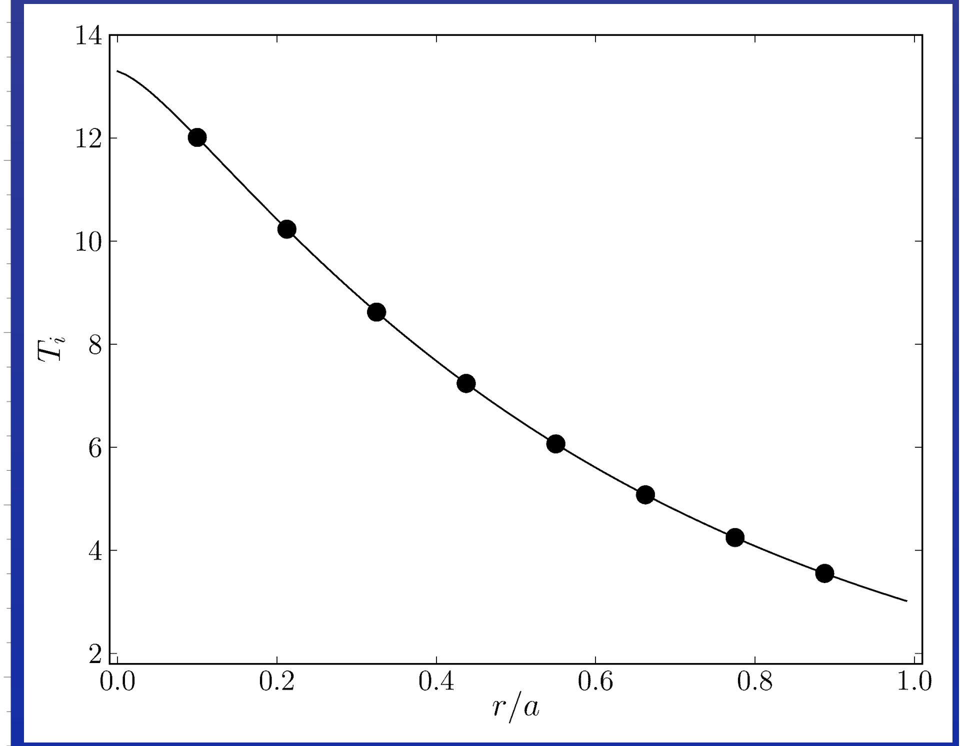

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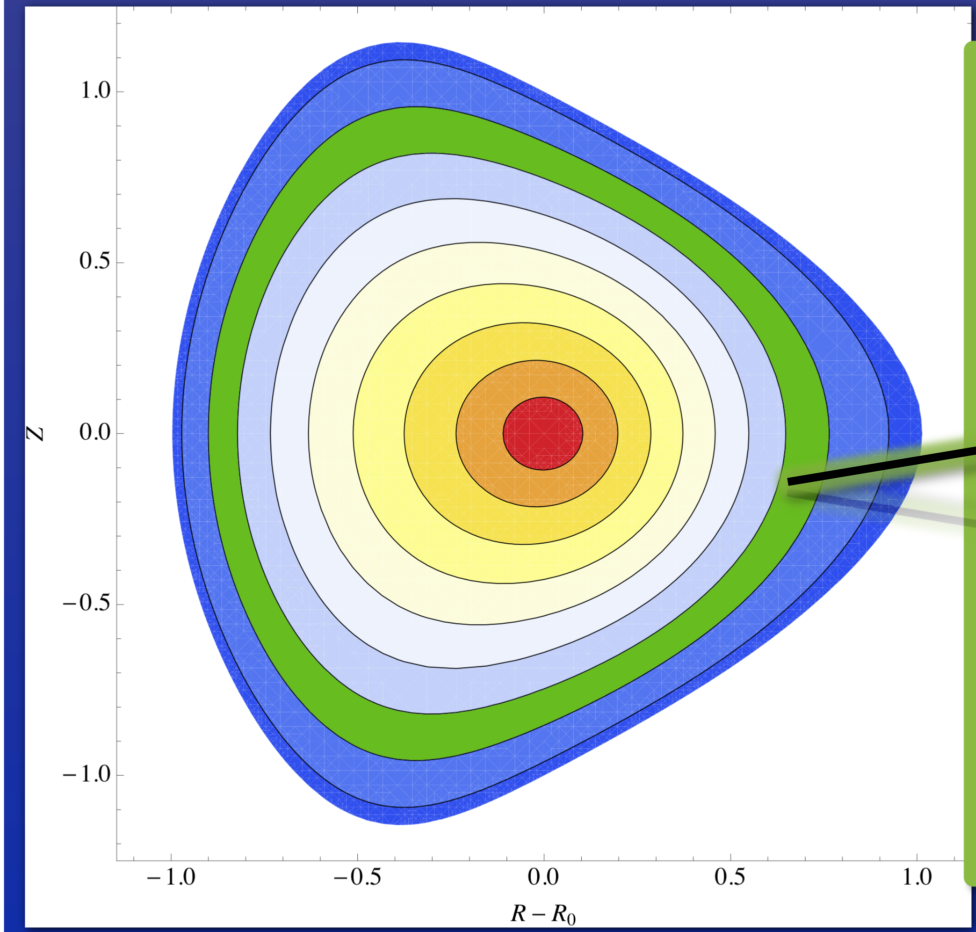
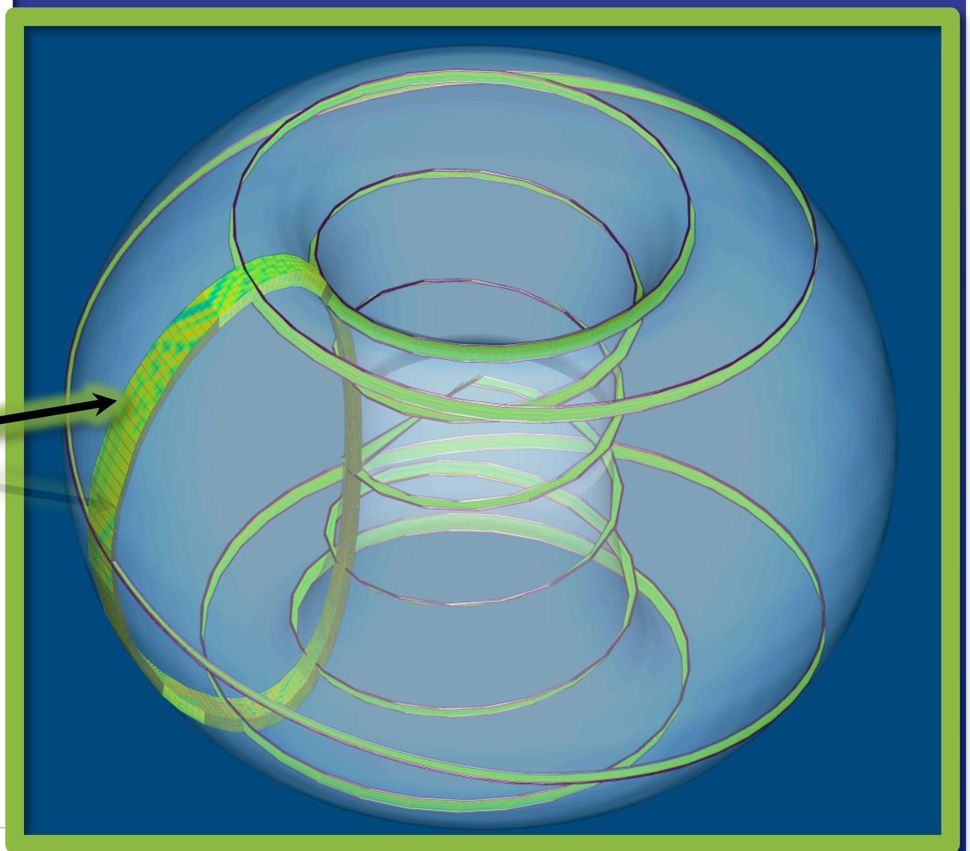
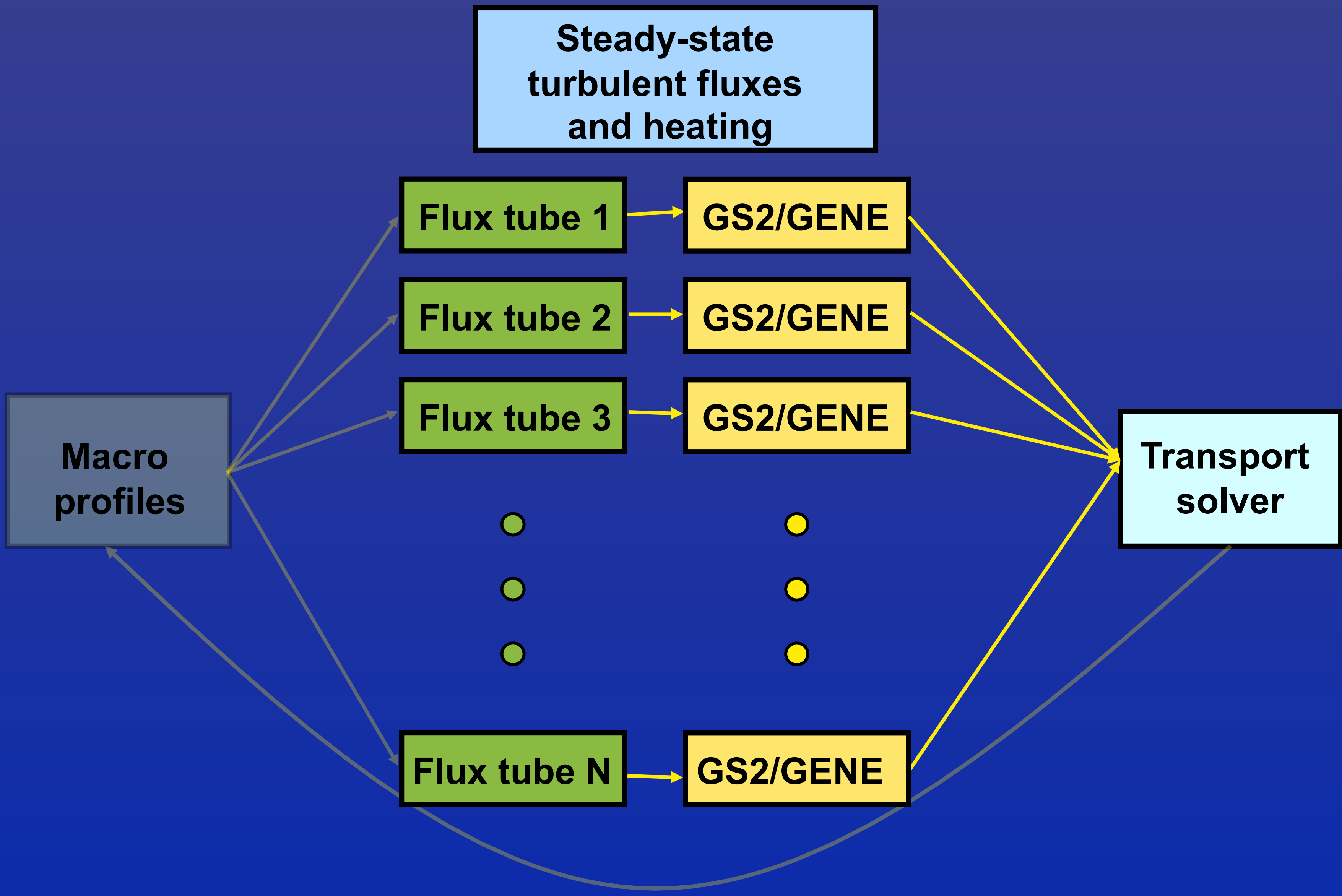




Simulation volume reduced by factor of ~100

Trinity schematic



Trinity transport solver

- Transport equations are stiff, nonlinear PDEs. Implicit treatment via Newton's Method (multi-iteration, adaptive time step) allows for time steps ~0.1 seconds (vs. turbulence sim time ~0.001 seconds)

$$\frac{\partial n}{\partial t} + \frac{\partial G(n, T, \Gamma)}{\partial r} = S_n \qquad \Gamma = \Gamma_{phys}/n v_t$$

$$G_j^{m+1} \approx G_j^m + (\mathbf{y} - \mathbf{y}^m) \left. \frac{\partial G_j}{\partial \mathbf{y}} \right|_{\mathbf{y}^m} \qquad \mathbf{y} = \left[\{n_k\}, \{p_{i_k}\}, \{p_{e_k}\}\right]^T$$

- Challenge: requires computation of quantities like

$$\frac{\partial \Gamma_j}{\partial n_k} = \frac{\partial \Gamma_j}{\partial n_j} + \frac{\partial \Gamma_j}{\partial (R/L_n)_j} \frac{\partial (R/L_n)_j}{\partial n_k}$$

- Simplifying assumption: normalized fluxes depend primarily on gradient scale lengths

Trinity transport solver

- Calculating flux derivative approximations:
  - at every radial grid point, simultaneously calculate $\Gamma_j[(R/L_n)_j^m]$ and $\Gamma_j[(R/L_n)_j^m + \delta]$ using 2 different flux tubes
  - Possible because flux tubes independent (do not communicate during calculation)
  - Perfect parallelization
  - use 2-point finite differences:

$$\frac{\partial \Gamma_j}{\partial (R/L_n)_j} \approx \frac{\Gamma_j[(R/L_n)_j^m] - \Gamma_j[(R/L_n)_j^m + \delta]}{\delta}$$

Trinity transport solver

- Example calculation with 10 radial grid points:
  - evolve density, toroidal angular momentum, and electron/ion pressures
  - simultaneously calculate fluxes for equilibrium profile and for 4 separate profiles (one for each perturbed gradient scale length)
  - total of 50 flux tube simulations running simultaneously
  - ~2000-4000 processors per flux tube => scaling to over 100,000 processors with >80% efficiency

Improved simulation cost

- Statistical periodicity in poloidal direction takes advantage of $k_{\perp}^{-1} \ll L_\theta$: savings factor of ~100
- Exploitation of scale separation between turbulence and equilibrium evolution: savings factor of ~100
- Extreme parallelizability: savings factor of ~10
- Total saving of ~10^5: simulation possible on current machines
- In addition to savings from not having to resolve $f_0, \; f_1, \; f_2$ at the same time

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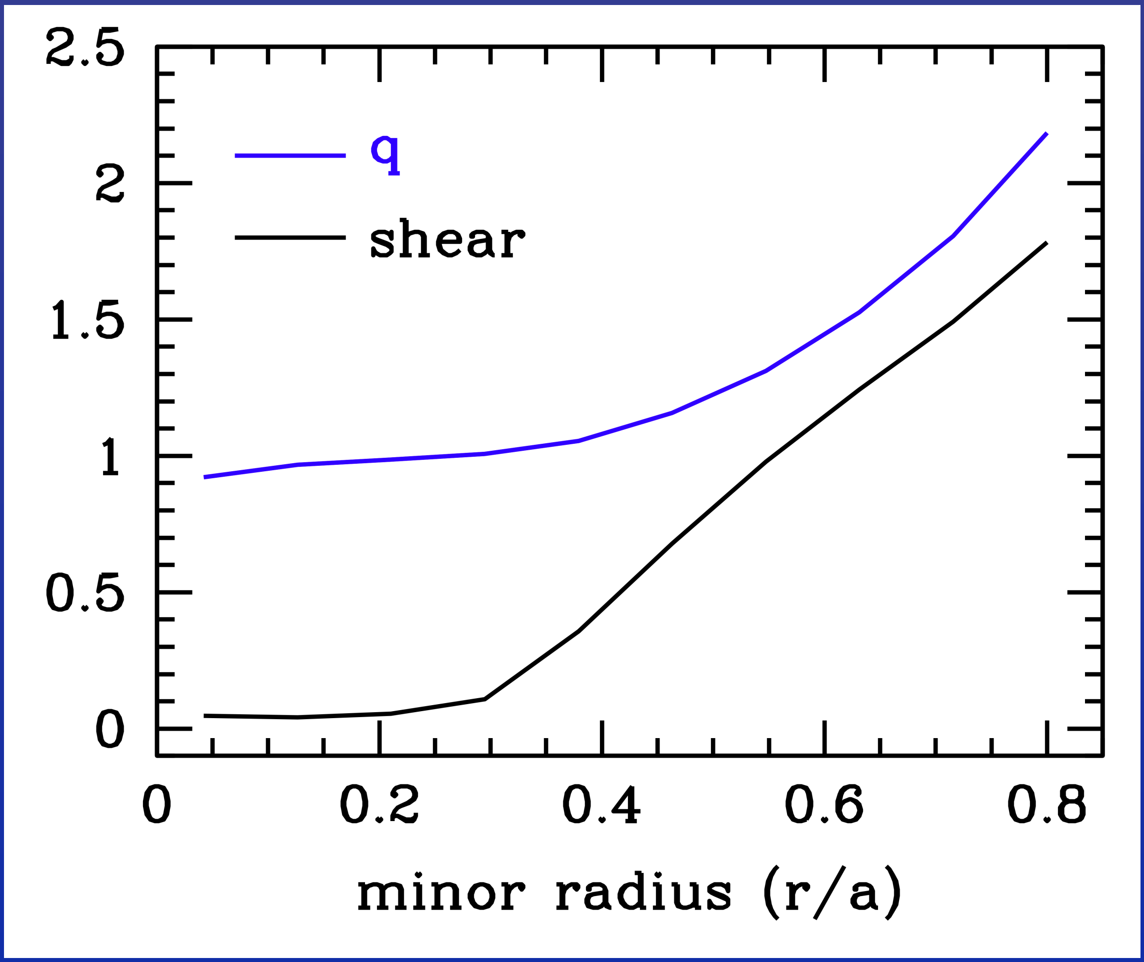
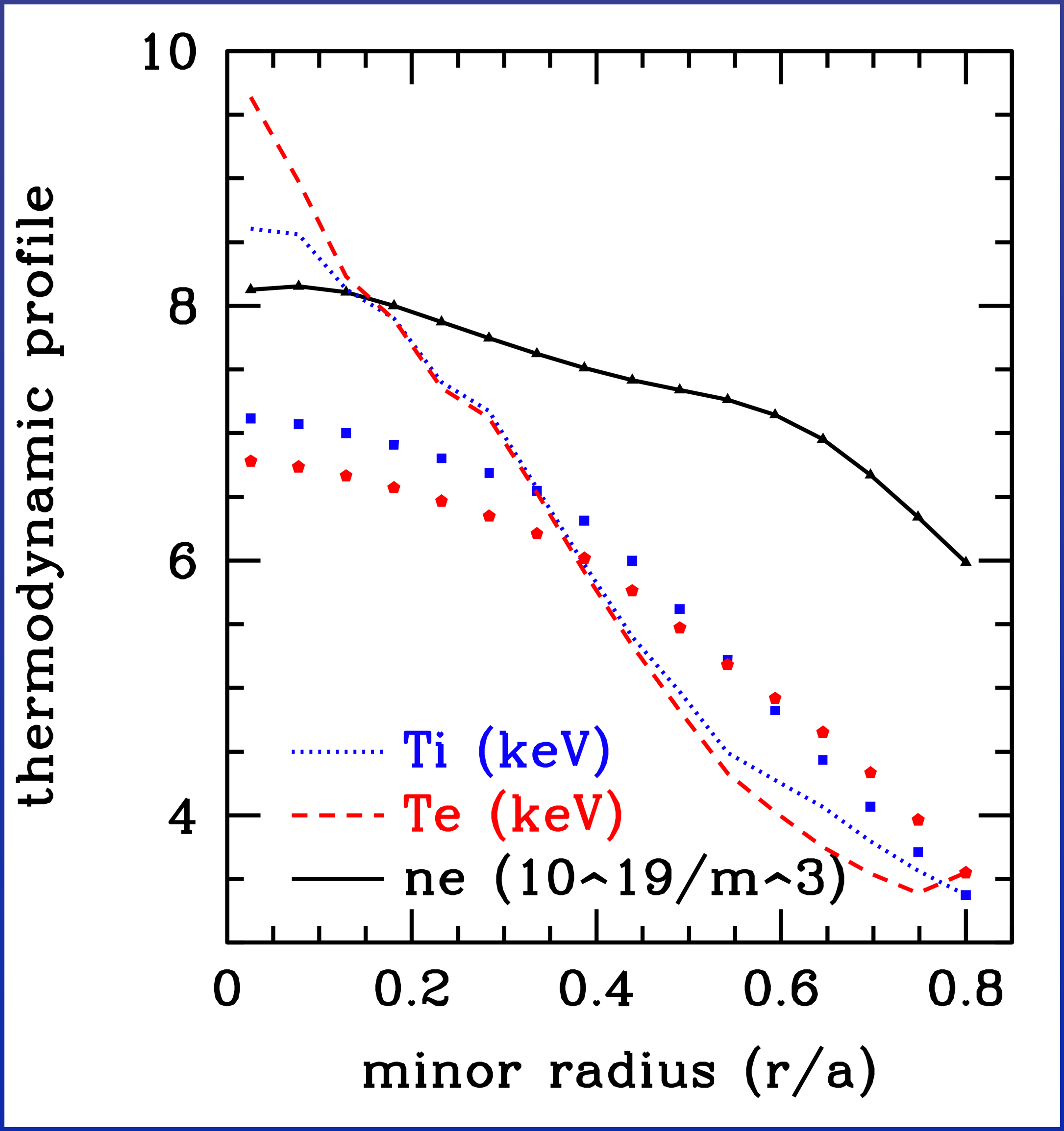
JET shot #42982



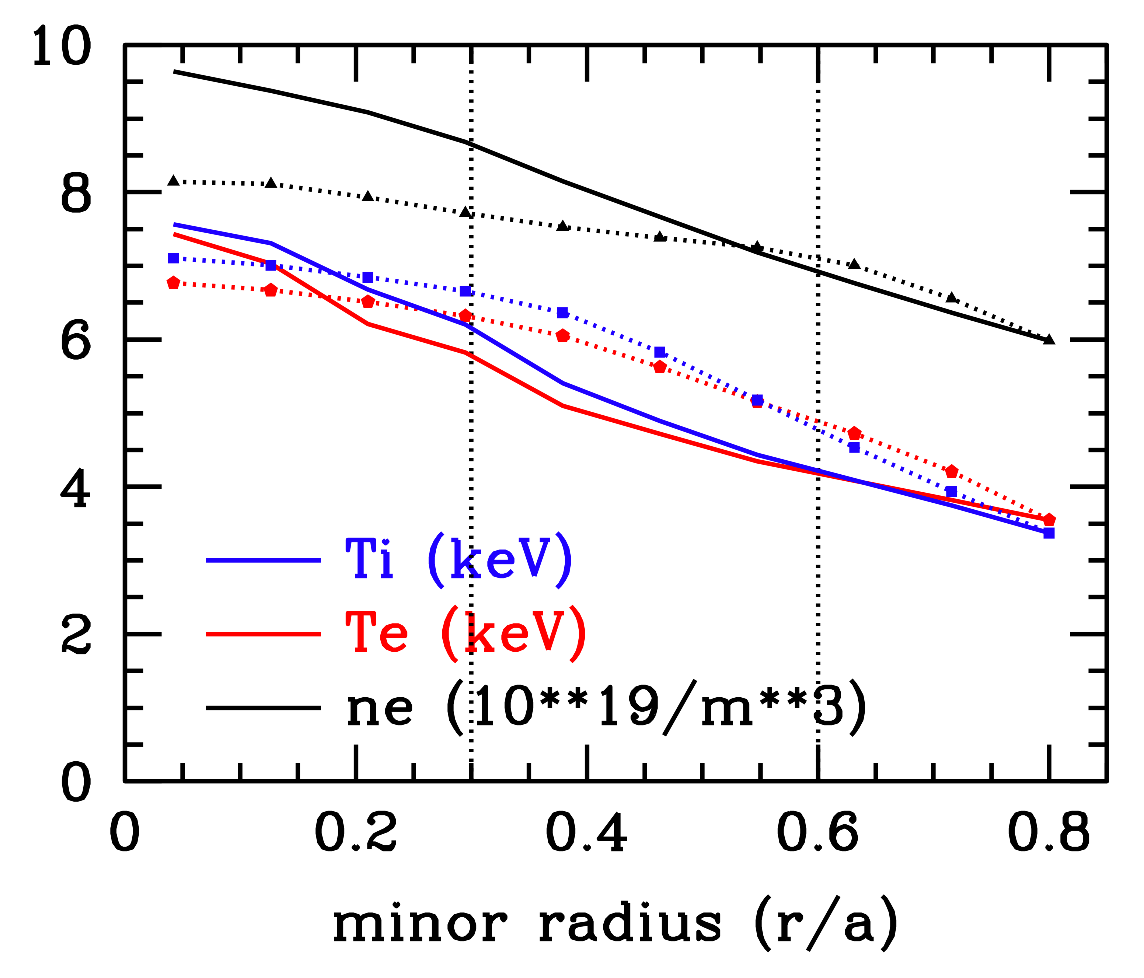
- ITER demo discharge
- H-mode D-T plasma, record fusion energy yield
- Miller local equilibrium model: q, shear, shaping, shift
- Low triangularity, elongation ~ 1.4,
- B = 3.9 T on axis
- TRANSP fits to experimental data taken from ITER profile database

Profile comparison



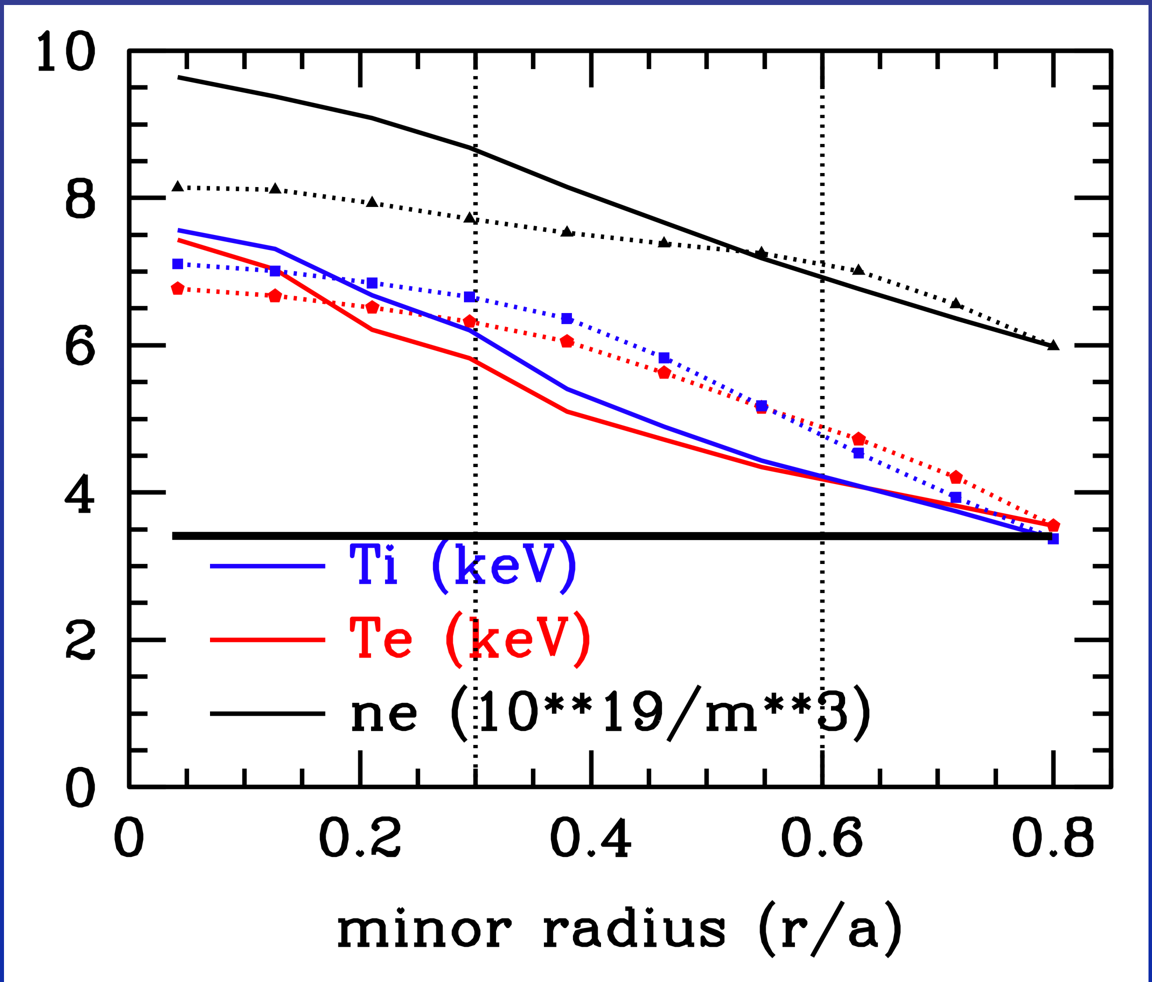
- Relatively low gs2 resolution: 9 ky's, 31 kx's, 24 field line points, 20 pitch angles, 12 energies
- ITG physics
- Electrostatic, collisionless
- 16 radial grid points
- Costs ~90k CPU hrs (<10 clock hrs)
- Fixed density profile
- Qualitative agreement, but dip in Te near edge…

Evolving density profile



- 10 radial grid points
- Costs ~120k CPU hrs (<10 clock hrs)
- Dens and temp profiles agree within ~15% across device
- Energy off by 5%
- Incremental energy off by 15%
- Sources of discrepancy:
  - Large error bars
  - Low res fluxes
  - Flow shear absent

Evolving density profile



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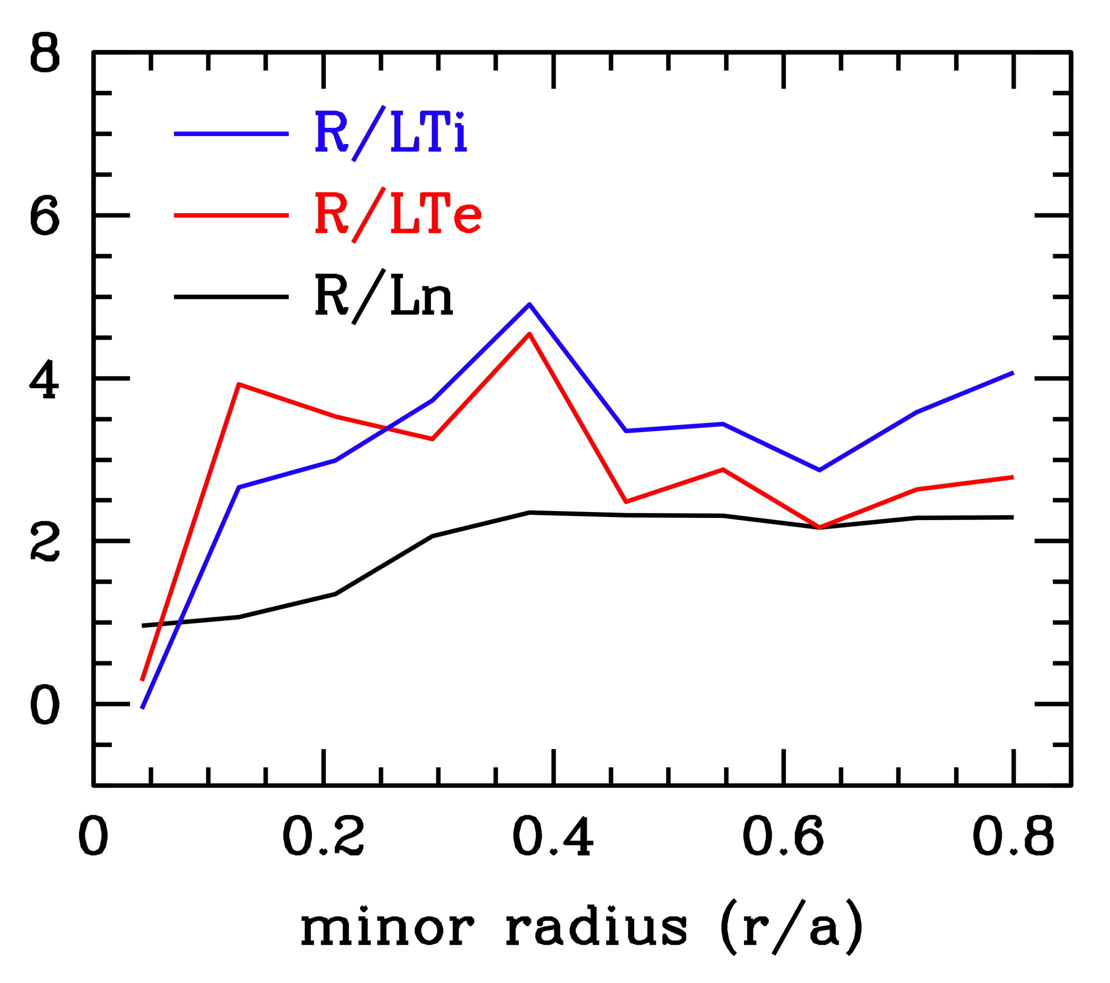
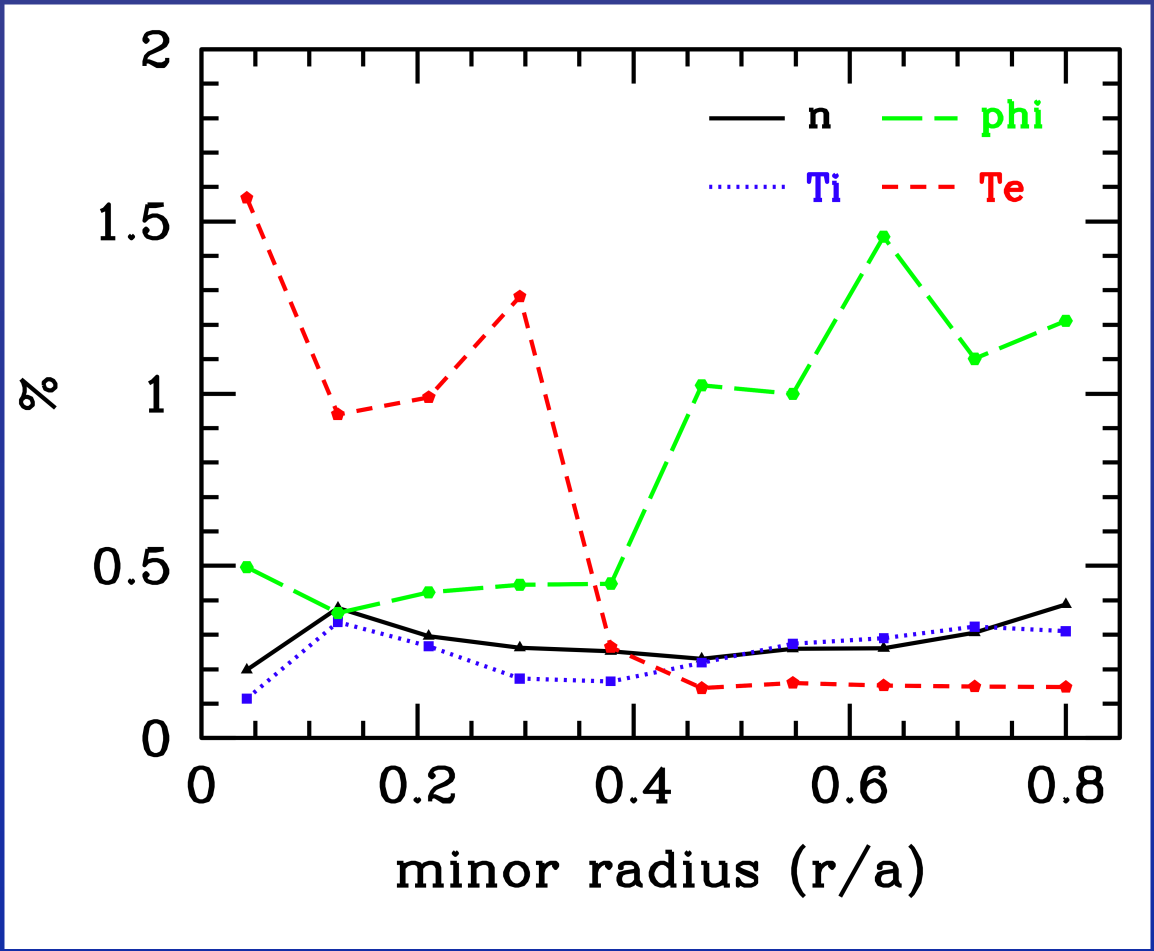
Profile stiffness



- Illustrates challenging aspect of solving for profiles (instead of gradients)
- Perhaps useful to employ smoothing?
- ~ flat grad scale lengths indicative of stiffness (near critical gradient across most of minor radius)

Fluctuations



- Steady-state fluctuation levels (time-averaged) calculated in gs2
- Fluctuations small (in agreement with experiment), so delta f/f <<1 valid

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Conclusions and future work

- Scale separation + theory provides significant savings in time and space domains
- First-principles simulations of self-consistent interaction between turbulence and equilibrium possible
- Still to do:
  - Momentum transport equations
  - Magnetic equilibrium evolution
  - MHD stability
  - Further comparisons with experimental measurements