Transport scalings for critically-balanced ITG turbulence in tokamaks

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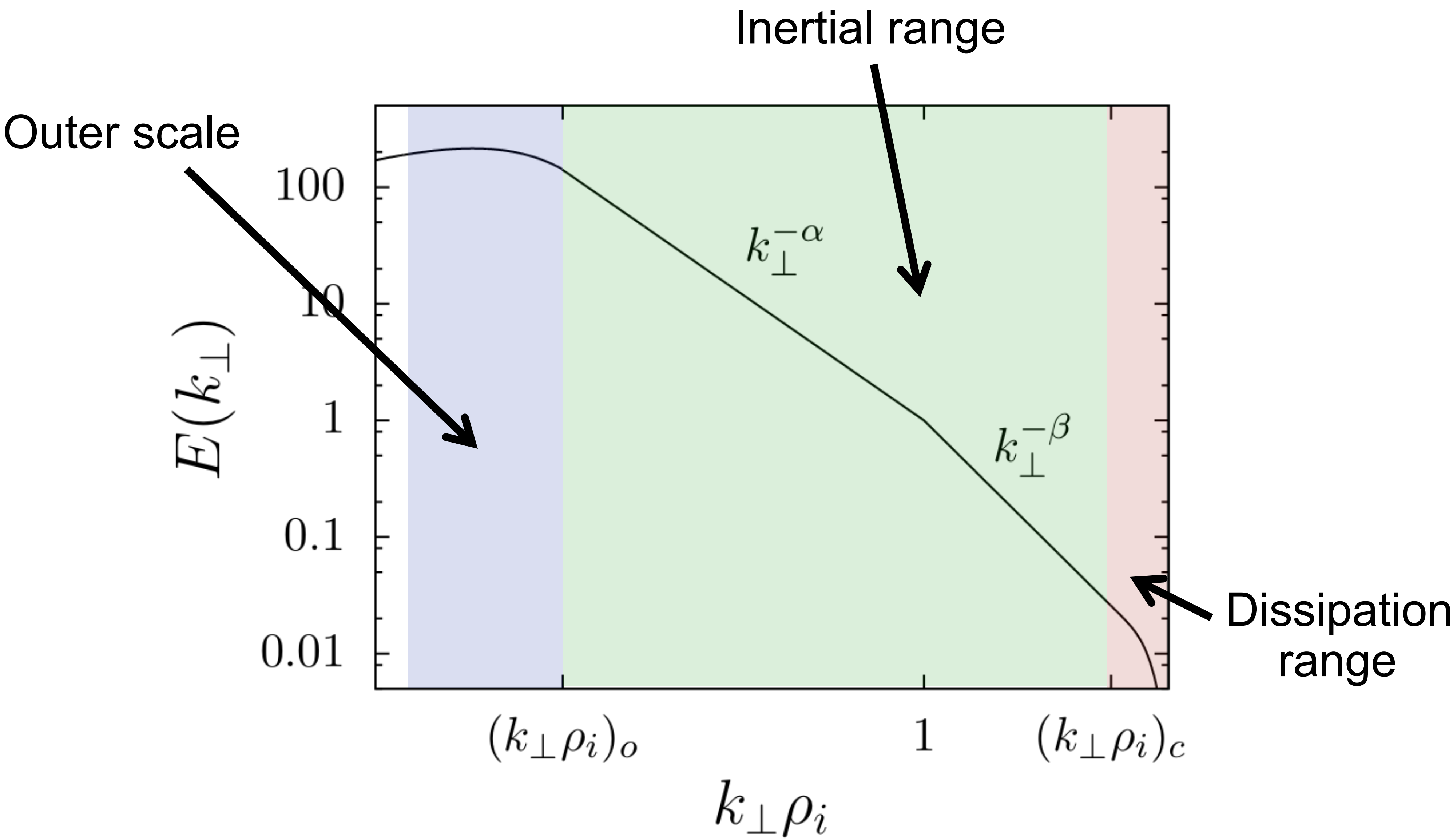
Why do we care?

- Plasma confinement properties depend strongly on quantities such as mean plasma current and ion temperature gradient
- Analytical results for turbulence are rare, and direct numerical simulations are costly
- Scaling laws useful indicators of gross plasma performance and provide guidance for numerical simulations
- Provides guidance for reduced gyrokinetic models (example: intrinsic rotation)

Big picture



Inertial range
Outer scale
Dissipation range
100
10
1
0.1
0.01
$E(k_\perp)$
$k_\perp^{-\alpha}$
$k_\perp^{-\beta}$
$(k_\perp \rho_i)_o$
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$k_\perp \rho_i$

Gyrokinetic model

Gyrokinetic variables: $\mathbf{R},\ E = \dfrac{mv^2}{2},\ \mu = \dfrac{mv_\perp^2}{2B}$

$$\frac{\partial}{\partial t}\left(h_s - \frac{Z_s e \langle\varphi\rangle_{\mathbf{R}}}{T_s} F_{M,s}\right) + \left(\mathbf{v}_\| + \mathbf{v}_{M,s}\right)\cdot\nabla h_s + \frac{c}{B}\left\{\langle\varphi\rangle_{\mathbf{R}}, h_s\right\}$$
$$= \langle C[h_s]\rangle_{\mathbf{R}} - \langle \mathbf{v}_E\rangle_{\mathbf{R}}\cdot\nabla F_{M,s}$$

Quasineutrality: $\displaystyle\sum_s Z_s\left(\int d^3v\ J_0 h_s - \frac{Z_s e\varphi}{T_s} n_s\right) = 0$

Assume $\displaystyle\int d^3v\ J_0 h \sim v_{th}^3 J_0 h \Rightarrow J_0 \frac{h}{F_M} \sim \frac{Ze\varphi}{T}$

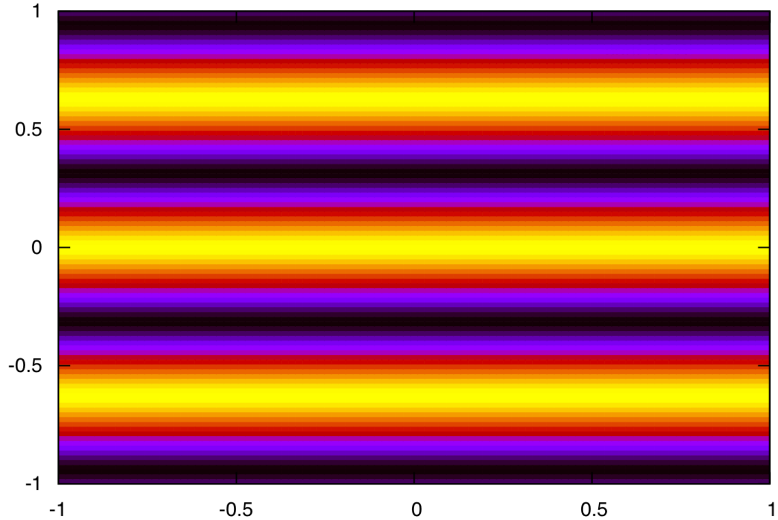
Conjectures

- Five unknowns (3 space scales, potential, distribution function) determined by two equations (GK + QN) and three conjectures:
- Fluctuation scale lengths in two dimensions of plane perpendicular to B-field are comparable
- Parallel streaming time and nonlinear turnover time comparable at all scales (critical balance)
- Parallel length at outer scale set by system size (connection length)

Isotropy

- Conjecture: fluctuation scale lengths in two dimensions of plane perpendicular to B-field are comparable
- Physical idea: linear drive favors structures with $\ell_x \gtrsim \ell_y$. Smaller ℓ_x formed through magnetic and zonal flow shear:

$$\ell_x^{-1} \sim (S_{\rm zf}\tau_{nl} + \hat{s}\theta)\ell_y^{-1} \sim \ell_y^{-1} \sim \ell_\perp$$



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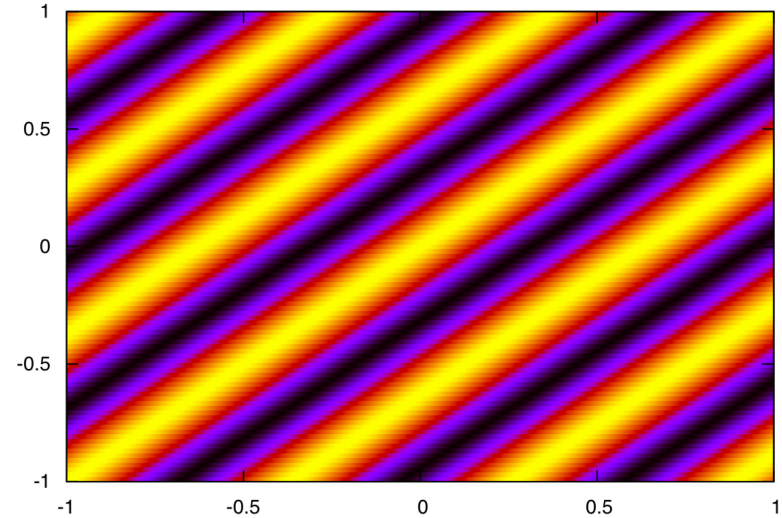


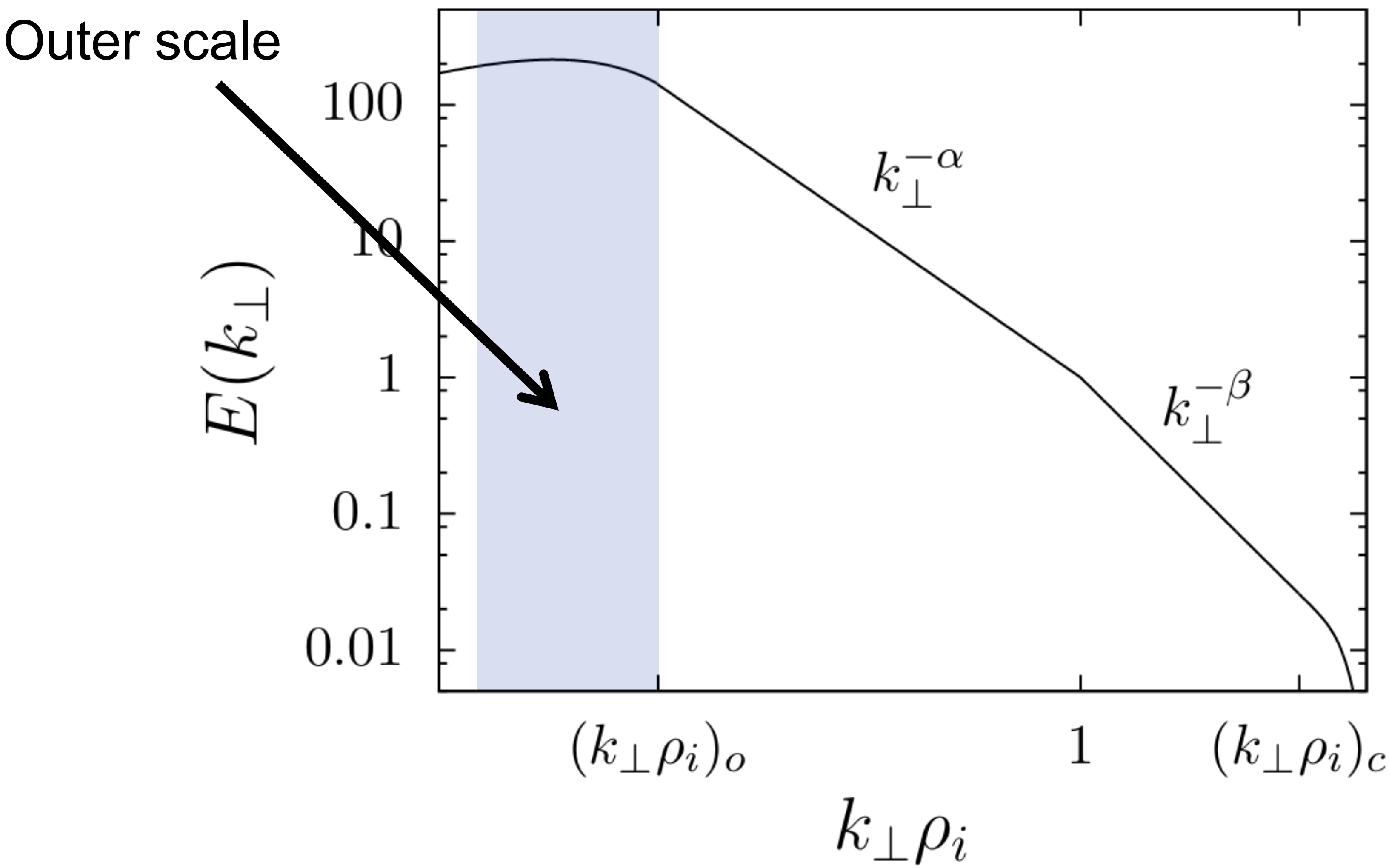
Critical balance

- Conjecture: characteristic time associated with particle streaming and wave propagation along mean field is comparable to nonlinear decorrelation time at each scale
- Physical idea: two points along field correlated only if information propagates between them before turbulence decorrelated in perpendicular plane

$$\frac{v_{th}}{\ell_{||}} \sim \tau_{nl}^{-1} \sim \frac{v_{th}}{R} \frac{\rho_i^2}{\ell_\perp^2} \langle \Phi_\ell \rangle$$

$$\Phi_\ell \equiv \frac{e\varphi_\ell}{T} \frac{R}{\rho_i} \qquad \varphi_\ell \equiv \varphi(\mathbf{r} + \ell) - \varphi(\mathbf{r})$$

Big picture



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Outer scale

- Define outer scale as range where injection rate comparable to nonlinear decorrelation time:

$$\tau_{nl}^{-1} \sim \omega_*^o \sim \frac{\rho_i v_{th}}{\ell_\perp^o L_T} J_{0\ell} \text{ ==> } \frac{\ell_\perp^o}{\rho_i} \sim \frac{\ell_\parallel^o}{L_T} J_{0\ell}$$

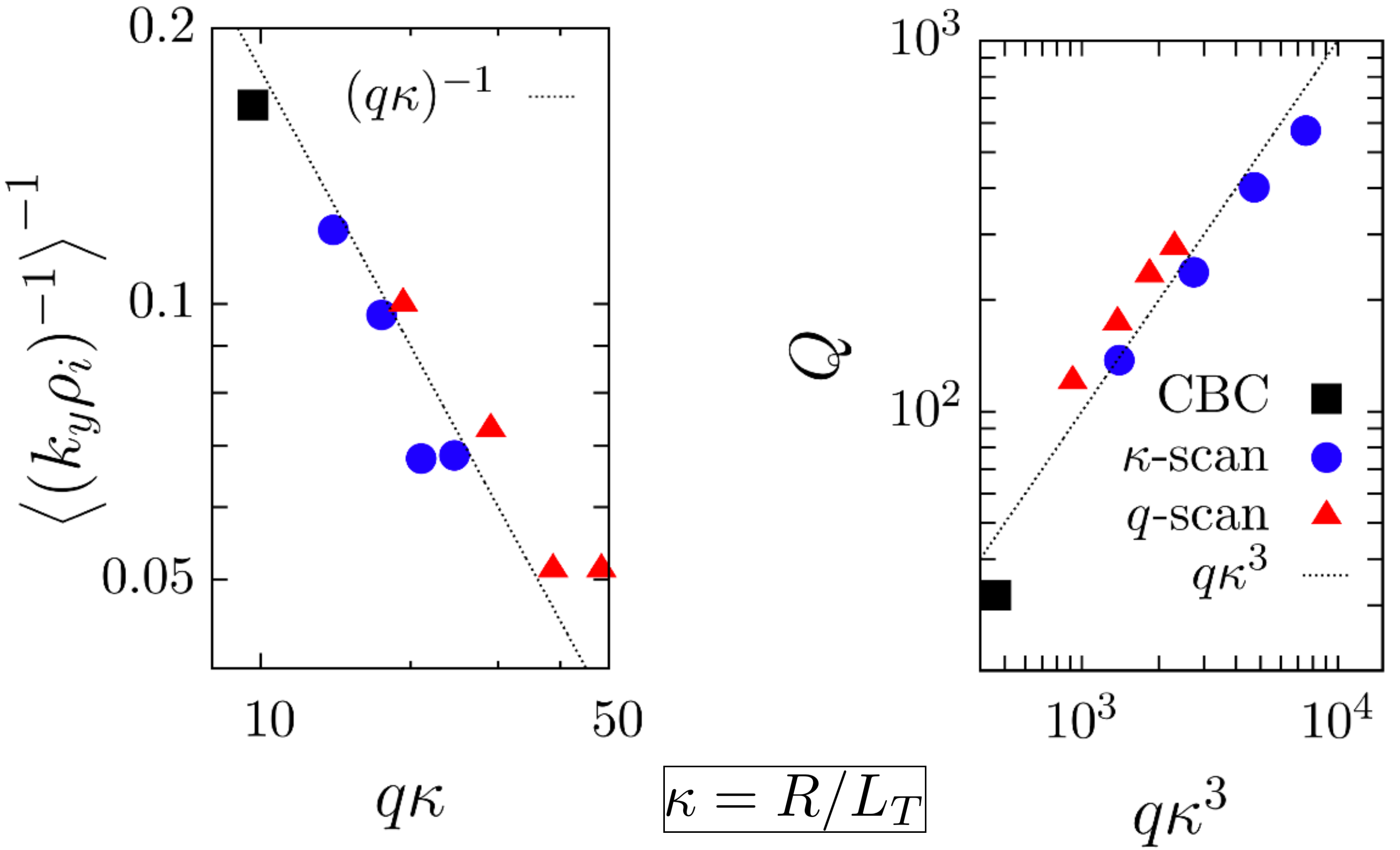
- Conjecture: characteristic parallel length scale of turbulence at outer scale is the connection length
- Physical idea: modes cannot extend much beyond connection length due to stabilizing effect of good curvature

$$\text{==> } \frac{\ell_\perp^o}{\rho_i} \sim \frac{qR}{L_T}\,, \;\; \Phi_o \sim q\left(\frac{R}{L_T}\right)^2 \text{ ==> } Q \sim \frac{\rho_i}{\ell_\perp^o}\Phi_o^2 \sim q\left(\frac{R}{L_T}\right)^3$$

Simulation system

- Use continuum, local, delta-f GK code GS2
- Base case is Cyclone (widely benchmarked)
  - Unshifted, circular flux surface
  - Safety factor is 1.4, magnetic shear=0.8, R/Ln=2.2, R/LT=6.9
  - Electrostatic
  - Modified Boltzmann response for electrons
- Fix R/LT and vary q from 1.4 up to 7.0
- Fix q and vary R/LT from 6.9 to 17.5

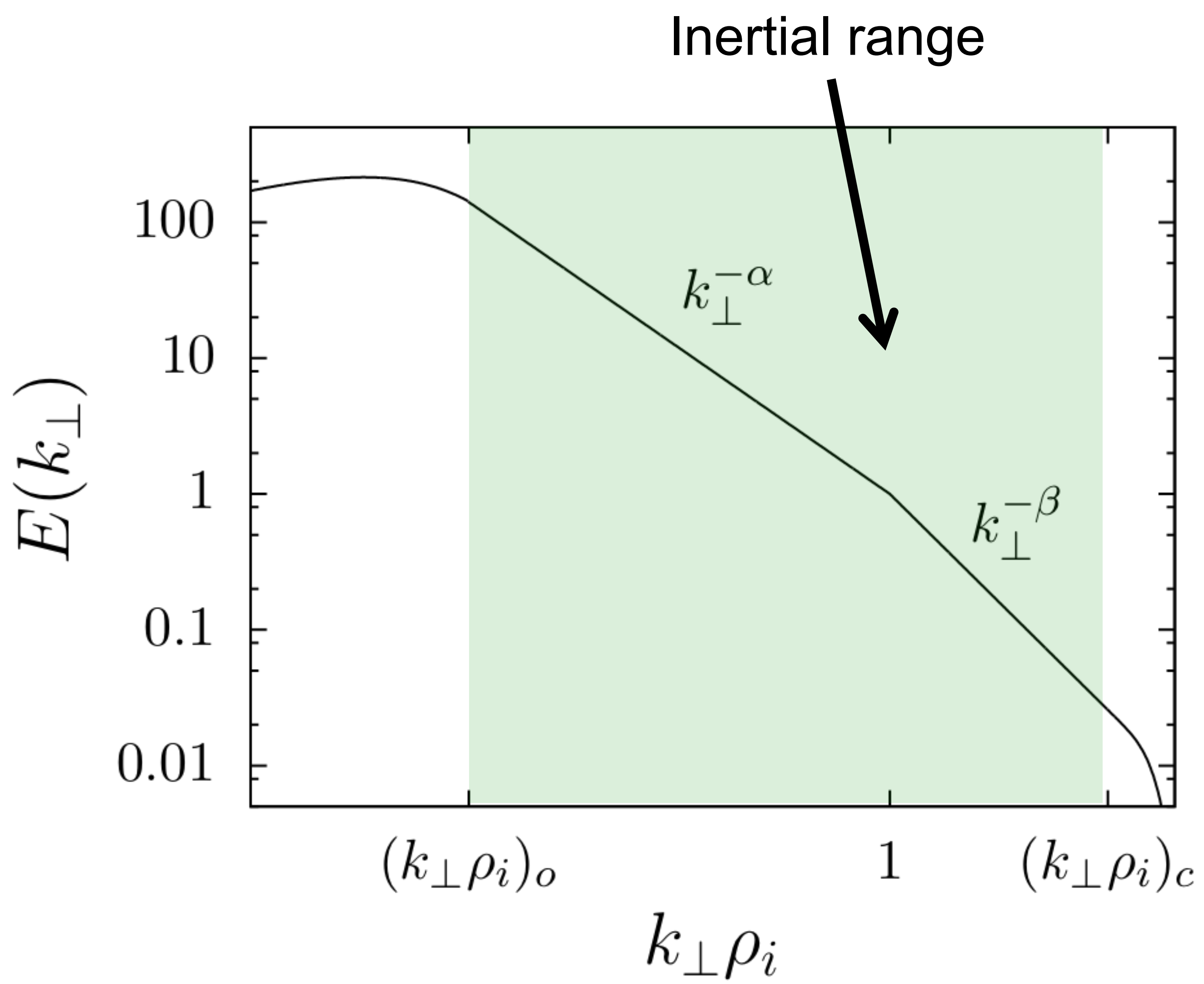
Turbulence scaling tests



$\kappa = R/L_T$

Note that Q at large R/L_T much larger than found in previous studies (box size used here for $R/L_T \approx 20$ was $\approx 1000\rho_i$)

Big picture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Inertial range

- No significant drive or dissipation between outer and dissipation scales
- Flux of free energy (nonlinear invariant) scale-independent in inertial range:

$$W = V^{-1} \sum_s \int d^3r \int d^3v \left(\frac{T_s \delta f_s^2}{F_{M,s}} \right)$$

$$\frac{W_\ell}{\tau_{nl}} \sim \left(\frac{\rho_i}{R} \right)^2 \frac{v_{th}}{R} \frac{\rho_i^2}{\ell_\perp^2} \Phi_\ell^3 \sim \text{constant}$$

==> $$\Phi_\ell \sim \Phi_o \left(\frac{\ell_\perp}{\ell_\perp^o} \right)^{2/3} \sim q^{1/3} \left(\frac{R}{L_T} \right)^{4/3} \left(\frac{\ell_\perp}{\rho_i} \right)^{2/3}$$

Inertial range

- Use critical balance and expression for Φ_ℓ to get relationship between parallel and perpendicular length scales

$$\frac{\ell_{\|}}{qR} \sim \left(\frac{\ell_\perp}{\rho_i} \frac{L_T}{qR} \right)^{4/3}$$

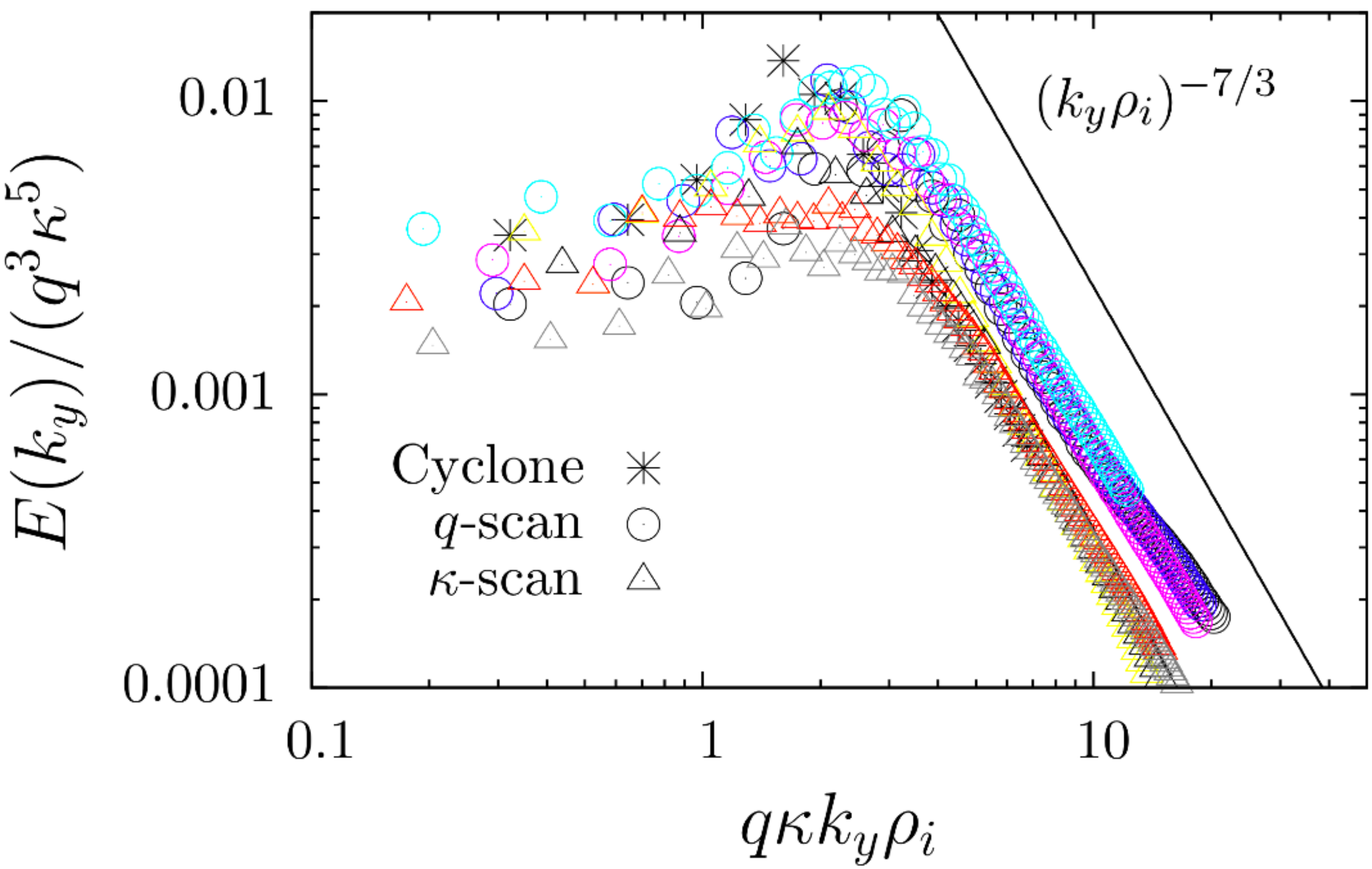
- Convert expression for Φ_ℓ into 1D spectrum using Parseval's theorem

$$\int dk_y \; \rho_i E(k_y) = V^{-1} \int d^3 r \; \Phi^2$$

$$E(k_y) \sim k_y \rho_i \left| \Phi_k \right|^2 \sim q^{2/3} \left(\frac{R}{L_T} \right)^{8/3} (k_y \rho_i)^{-7/3}$$

Inertial range spectra

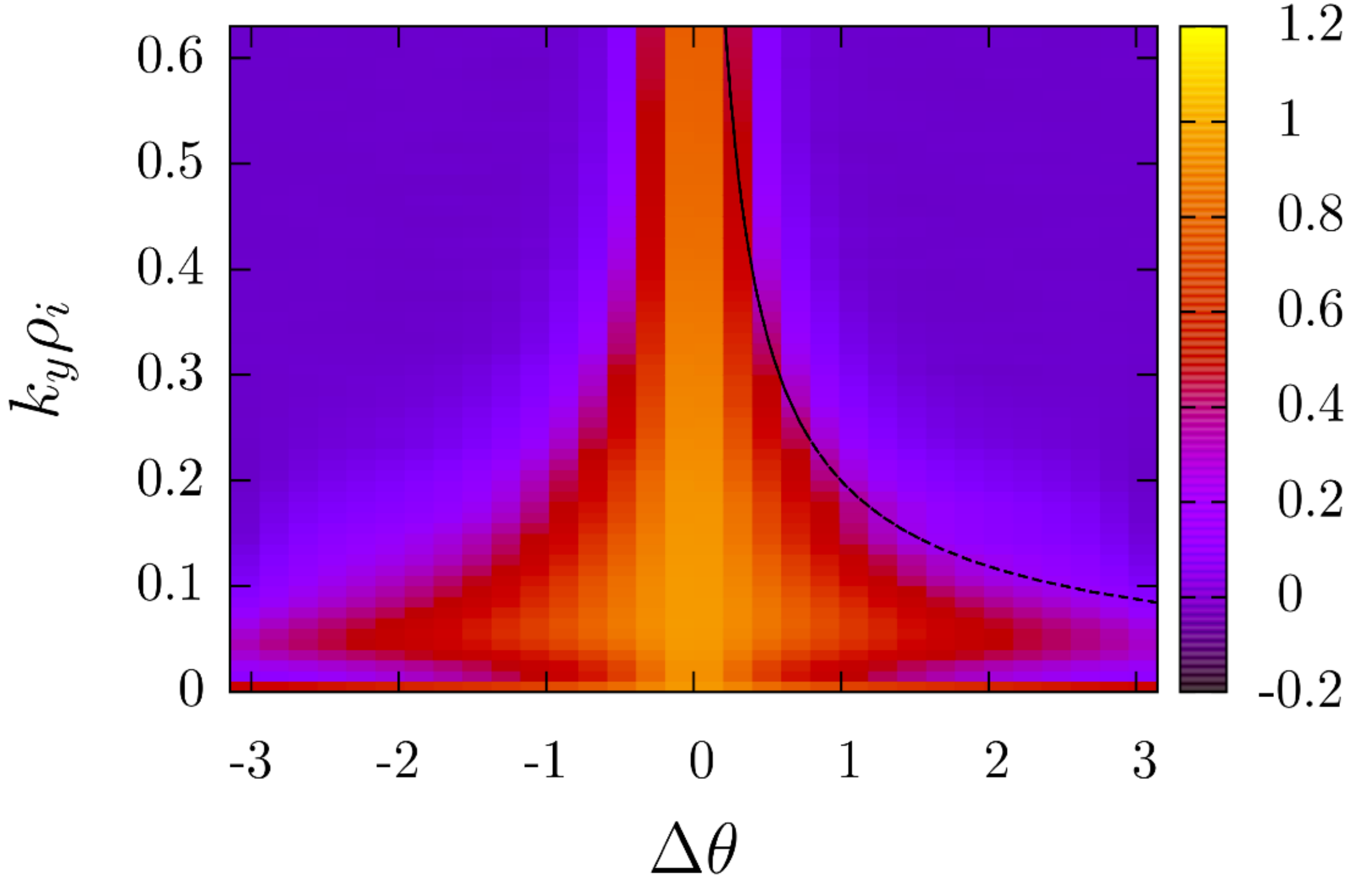
$$\int dk_y \; \rho_i E(k_y) = V^{-1} \int d^3 r \; \Phi^2$$



Critical balance test

Correlation function: $$\mathcal{C}(k_y, \Delta\theta) = \frac{\sum_{k_x} \Phi(\theta = 0)\Phi^*(\theta = \Delta\theta)}{\sum_{k_x} |\Phi(\theta = 0)|^2}$$

$\mathcal{C}(k_y\rho_i, \Delta\theta)$

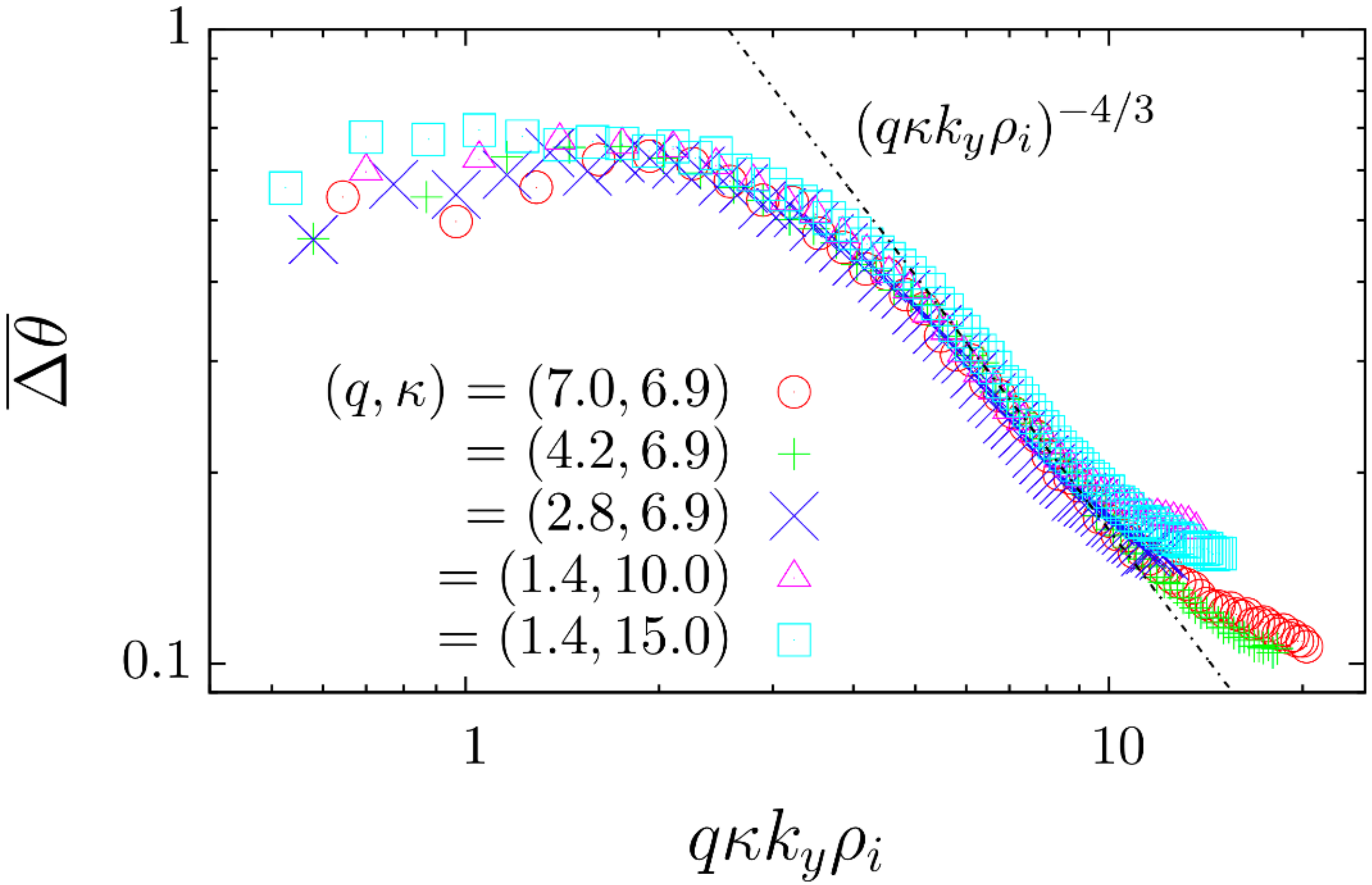


$$\frac{\ell_\parallel}{qR} \sim \left(\frac{\ell_\perp}{\rho_i}\right)^{4/3}$$

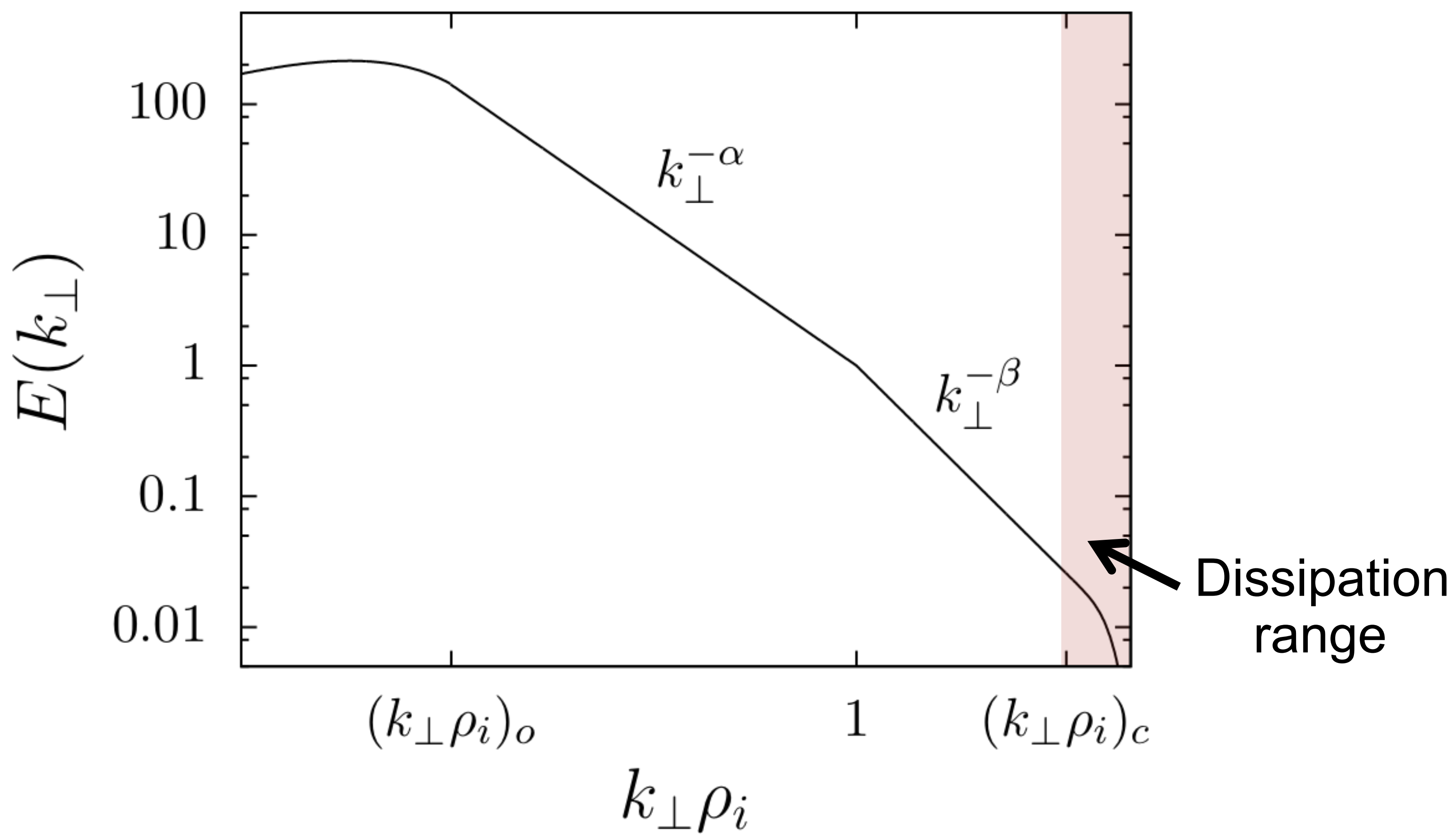
Inertial range critical balance

$$\frac{\ell_\parallel}{qR} \sim \left(\frac{\ell_\perp}{\rho_i}\frac{L_T}{qR}\right)^{4/3}$$

$$\overline{\Delta\theta}(k_y) = \int d(\Delta\theta)\mathcal{C}(k_y, \Delta\theta)$$



Big picture



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Dissipation range

Dissipation scale

- In analogy with Reynolds number, define

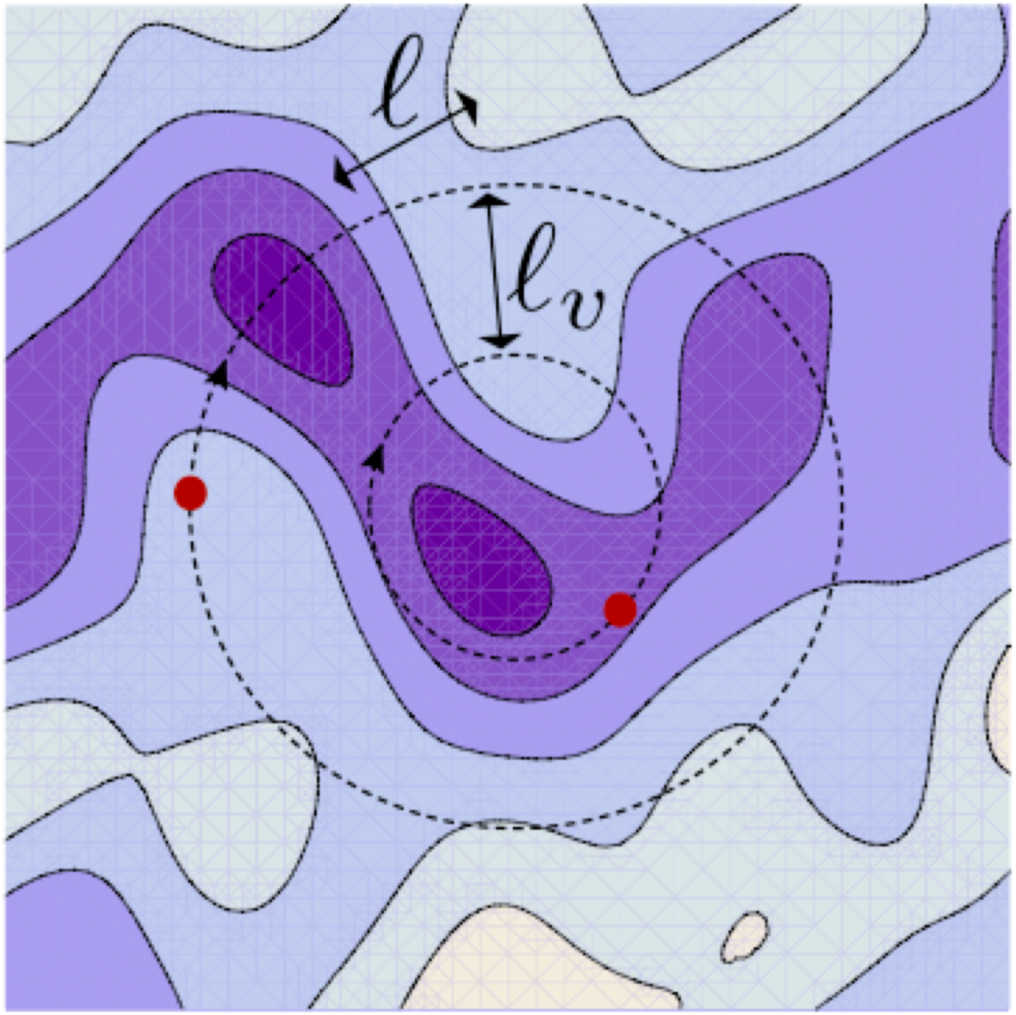
$$\mathrm{Do} \equiv (\nu_i \tau_{\rho_i})^{-1} \sim q^{1/3} \left(\frac{R}{L_T}\right)^{4/3} \left(\frac{v_{th}}{\nu_i R}\right)$$

- At dissipation scale, dissipation rate comparable to nonlinear decorrelation rate

$$\nu_i \left(\frac{v_{th}}{\delta v}\right)^2 \sim \tau_{nl}^{-1} \Rightarrow \mathrm{Do} \sim \left(\frac{v_{th}}{\delta v}\right)^2 \frac{\tau_{nl}}{\tau_{\rho_i}} \sim \left(\frac{v_{th}}{\delta v}\right)^2 \left(\frac{\ell_\perp}{\rho_i}\right)^2 \frac{\Phi_{\rho_i}}{\Phi_\ell}$$

- Dissipation scale assumed below ion Larmor scale
- Perpendicular space and velocity scales related via phase mixing: $\dfrac{\delta v}{v_{th}} \sim \dfrac{\ell_\perp}{\rho_i}$

Perpendicular phase mixing



Schekochihin *et al*., PPCF 2008

- Drift velocity = $F[\langle\Phi\rangle]$
- Particles with Larmor orbits separated by turbulence wavelength 'see' different averaged potential
- Drift velocities decorrelated, thus phase mixing

$$\frac{k_\perp \delta v_\perp}{\Omega} \sim 1$$

$$\Rightarrow \frac{\delta v_\perp}{v_{th}} \sim (k_\perp \rho_i)^{-1}$$

Dissipation scale

$$\mathrm{Do} \sim \left(\frac{v_{th}}{\delta v}\frac{\ell_\perp}{\rho_i}\right)^2 \frac{\Phi_{\rho_i}}{\Phi_\ell} \qquad\qquad \frac{\delta v}{v_{th}} \sim \frac{\ell_\perp}{\rho_i}$$

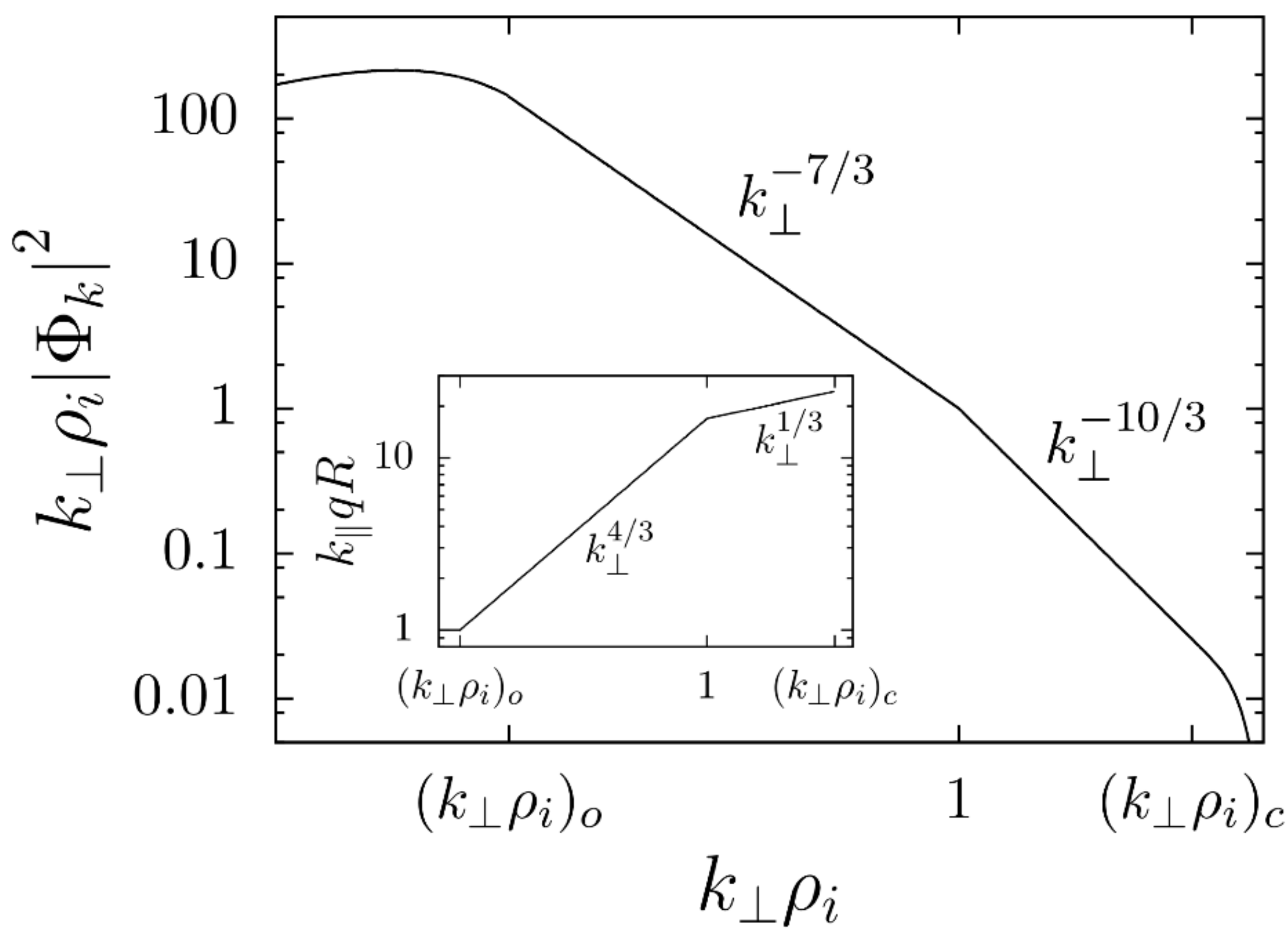
- Carrying out inertial range analysis (as before, but with $J_0(k_\perp\rho_i) \sim (k_\perp\rho_i)^{-1/2}$) gives*

$$\Phi_\ell \sim \left(\frac{\ell_\perp}{\rho_i}\right)^{7/6} \Rightarrow (k_\perp\rho_i)_c \sim \mathrm{Do}^{3/5}$$

$$(k_\perp\rho_i)_c \sim q^{1/5}\left(\frac{R}{L_T}\right)^{4/5}\left(\frac{v_{th}}{\nu_i R}\right)^{3/5}$$

*Schekochihin *et al.*, PPCF 2008

Back to big picture



Application: intrinsic rotation

- Significant ‘intrinsic’ rotation observed in experiments with no obvious momentum injection
- Rotation profiles depend on heating mechanism and can reverse sign – can’t be explained solely by ‘pinch’ from edge of plasma
- Lowest-order GK equation gives no momentum flux for up-down symmetric plasma without flow
- Must include higher-order terms to calculate flux generating intrinsic rotation
- Horrible mess – best to simplify…

Higher-order GK equation

- Using our scalings + $B_p/B \ll 1$ and $\nu\tau_{nl} \ll 1$:

$$\frac{dg_s}{dt} + \mathbf{v}_{\parallel} \cdot \nabla \left(g_s - Z_s e \langle \varphi \rangle \frac{\partial F_{0s}}{\partial E} \right) + \langle \mathbf{v}_E^{\perp} \rangle \cdot \nabla F_{0s}$$

$$+ \left(\mathbf{v}_{Cs} + \mathbf{v}_{Ms} + \langle \mathbf{v}_E^{\perp} \rangle \right) \cdot \nabla_{\perp} \left(g_s - Z_s e \langle \varphi \rangle \frac{\partial F_{0s}}{\partial E} \right)$$

$$= - \mathbf{v}_{Ms} \cdot \nabla\theta \frac{\partial}{\partial\theta} \left(g_s - Z_s e \langle \Phi \rangle \frac{\partial F_{0s}}{\partial E} \right) - \left\langle \mathbf{v}_E^{\parallel} \right\rangle \cdot \nabla_{\perp} g_s$$

$$- \langle \mathbf{v}_E^{\perp} \rangle \cdot \nabla\theta \frac{\partial g_s}{\partial\theta} - \left\langle \mathbf{v}_E^{\parallel} \right\rangle \cdot \nabla F_{0s} - \langle \mathbf{v}_E^{\perp} \rangle \cdot \nabla F_{1s}$$

$$+ Z_s e \left(\mathbf{v}_{\parallel} \cdot \nabla \langle \Phi \rangle + \mathbf{v}_{Ms} \cdot \nabla_{\perp} \langle \Phi \rangle \right) \left(\frac{\partial g_s}{\partial E} + \frac{\partial F_{1s}}{\partial E} \right)$$

$$+ \psi\text{-profile variation}$$

(R,E,μ) variables

Momentum flux

- Neglecting neoclassical contributions, momentum flux given by

$$\Pi = \Pi^{tb}_{-1} + \Pi^{tb}_{0} + \frac{m_s c}{2 Z_s e} \left\langle R^2 \right\rangle_\psi \frac{\partial p_s}{\partial t}$$

$$\Pi^{tb}_{-1} = \frac{1}{\langle |\nabla \psi| \rangle_\psi} \left\langle \int d^3 v \; m_s R^2 \mathbf{v} \cdot \nabla \phi \left(\mathbf{v}_E^\perp \cdot \nabla \psi \right) \delta f(\mathbf{R}) \right\rangle_\psi$$

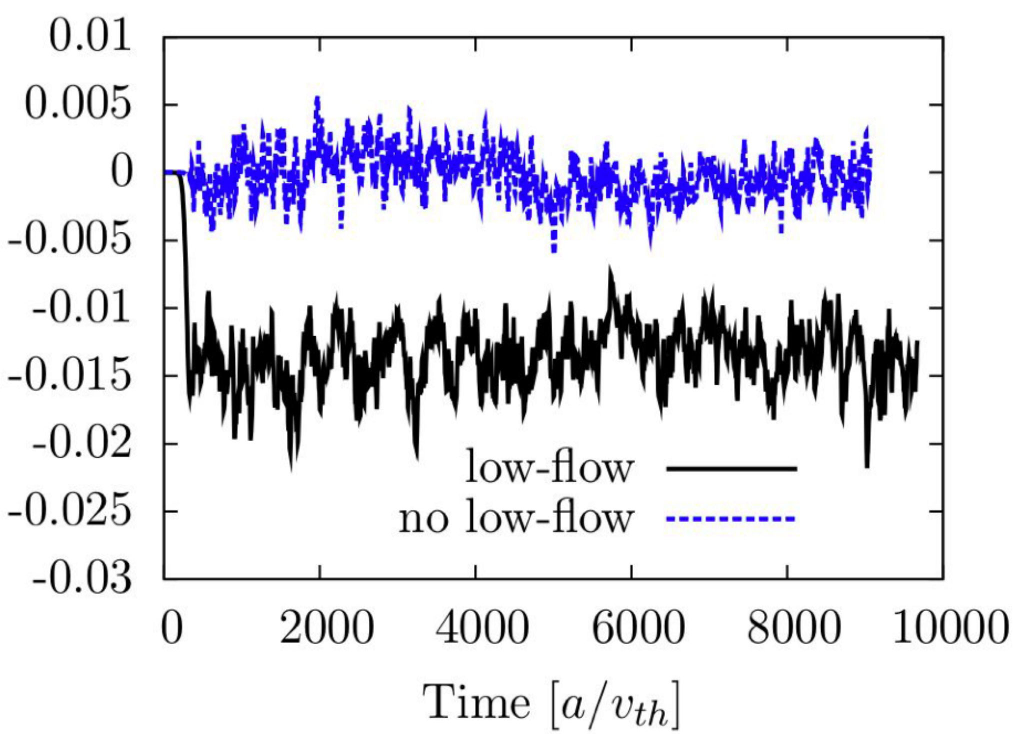
$$\Pi^{tb}_{0} = \frac{m_s^2 c^2}{2 Z_s e} \frac{1}{V' \langle |\nabla \psi| \rangle_\psi} \frac{\partial}{\partial \psi} \left(V' \left\langle \hat{\mathbf{b}} \times \nabla_\perp \varphi \cdot \nabla \psi \frac{I^2}{B^3} \int d^3 v \; v_\parallel^2 \delta f \right\rangle_\psi \right)$$
$$- \frac{1}{\langle |\nabla \psi| \rangle_\psi} \frac{m_s^2 c}{2 Z_s e} \left\langle \frac{I^2}{B^2} \int d^3 v \; C^l_{ii}[v_\parallel^2] F^{tb}_2 \right\rangle_\psi$$

Preliminary results

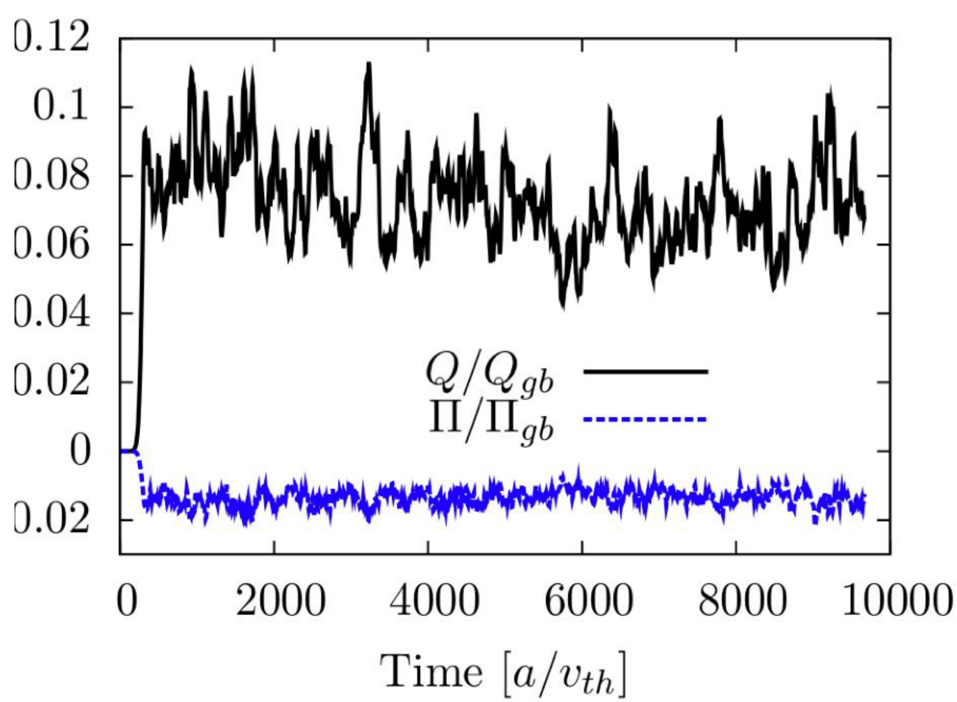
JET shot 19649 (L-mode) at ρ=0.16, no equilibrium flow

Small momentum flux generated by including neoclassical correction to F_0 in GK equation:

Momentum flux (Π^{tb}_{-1})



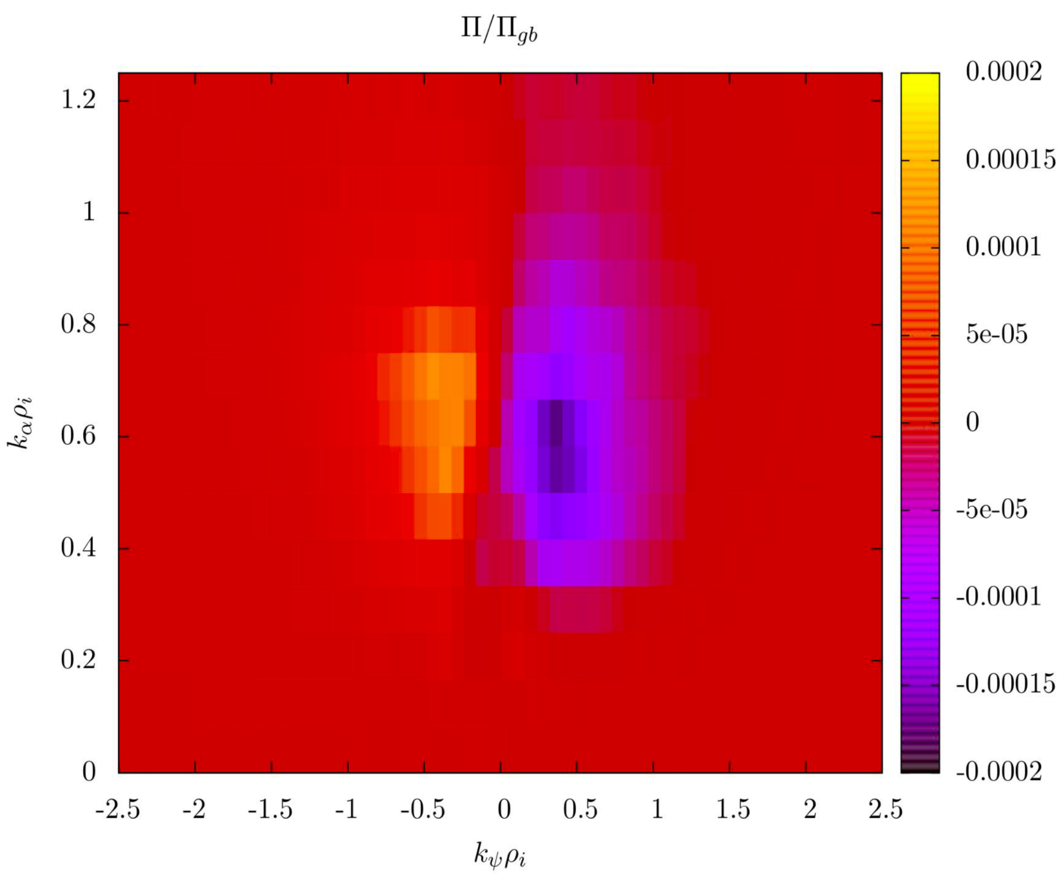
Fluxes



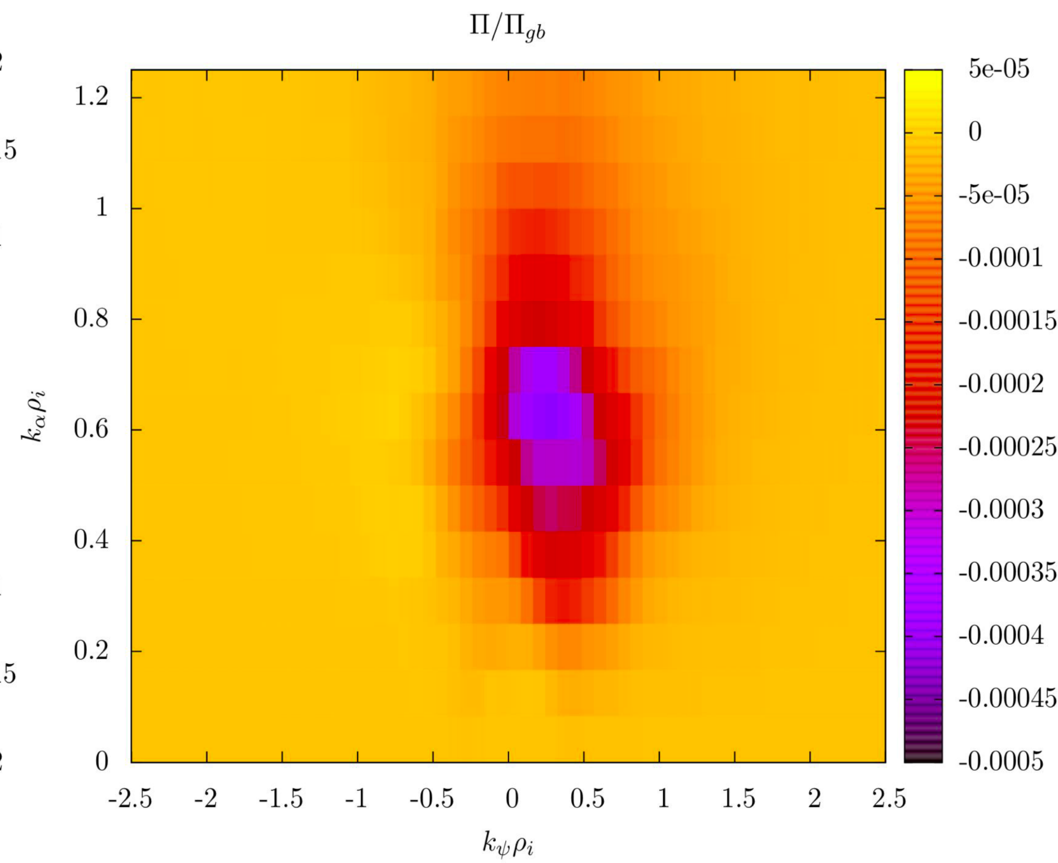
Symmetry breaking

JET shot 19649 (L-mode) at ρ=0.16, no equilibrium flow

k_ψ symmetry broken

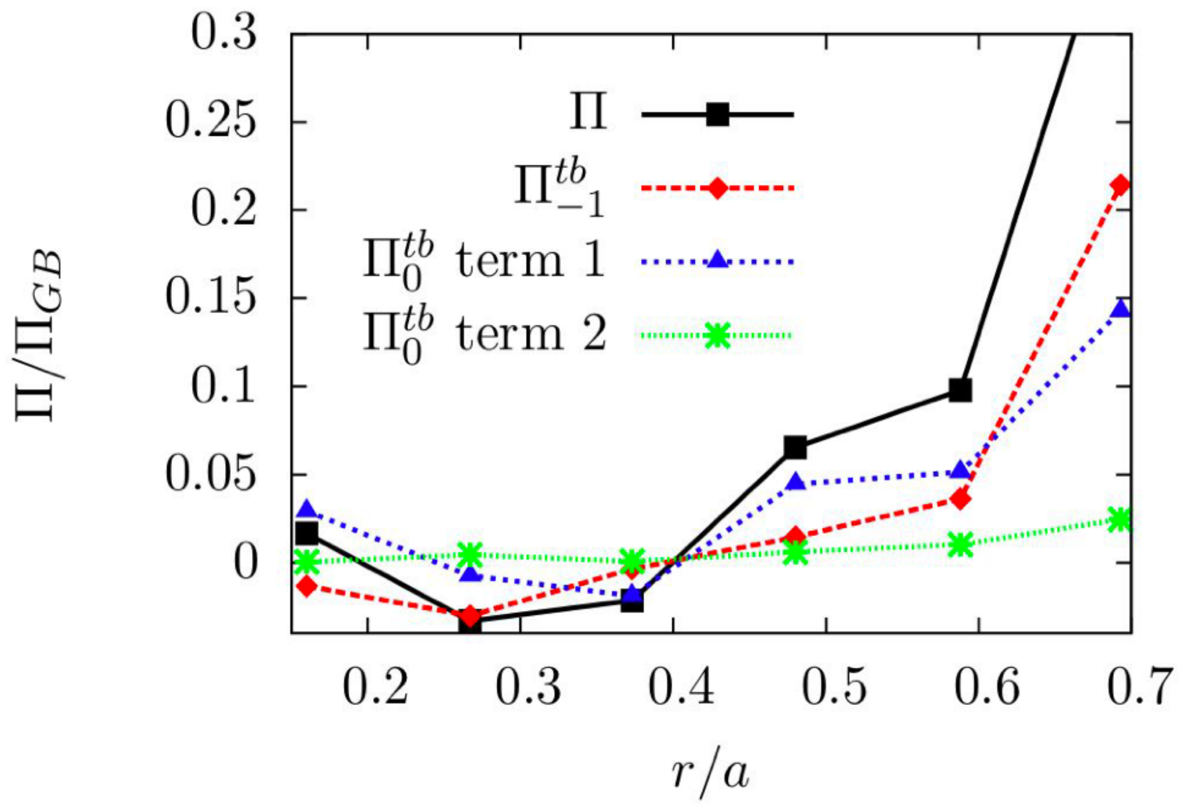


No low-flow



low-flow

Radial momentum flux profiles



JET shot 19649

- Momentum flux sign reversal allows for both co- and counter-rotating regions of plasma
- Different flux contributions of same order and possibly different sign

Conclusions

- Simple scalings for turbulence spatial scales and amplitudes derived and numerically confirmed. Predictions for scalings of:
  - Turbulence amplitude, heat flux, peak space scale, spectrum, decorrelation time, cutoff scale
- Critical balance robustly satisified – plasma turbulence three dimensional
- Scalings allow for B/Bp expansion of higher order GK Eq., making it tractable to solve numerically. This allows us to address problem of intrinsic rotation.