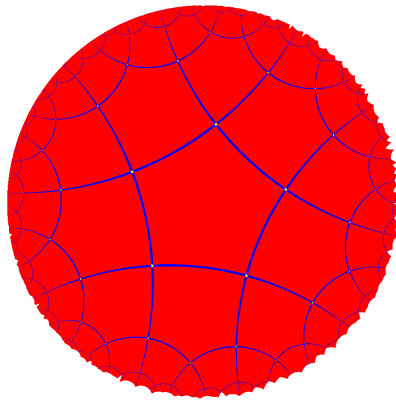


String theory, Black Holes and Quark-Gluon Plasma



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RESEARCH INSTITUTE FOR FUNDAMENTAL SCIENCE – TÜBİTAK TBAE, TURKEY

Outline

String theory and black holes: explaining black hole's entropy

Perturbative and non-perturbative results: a simple example

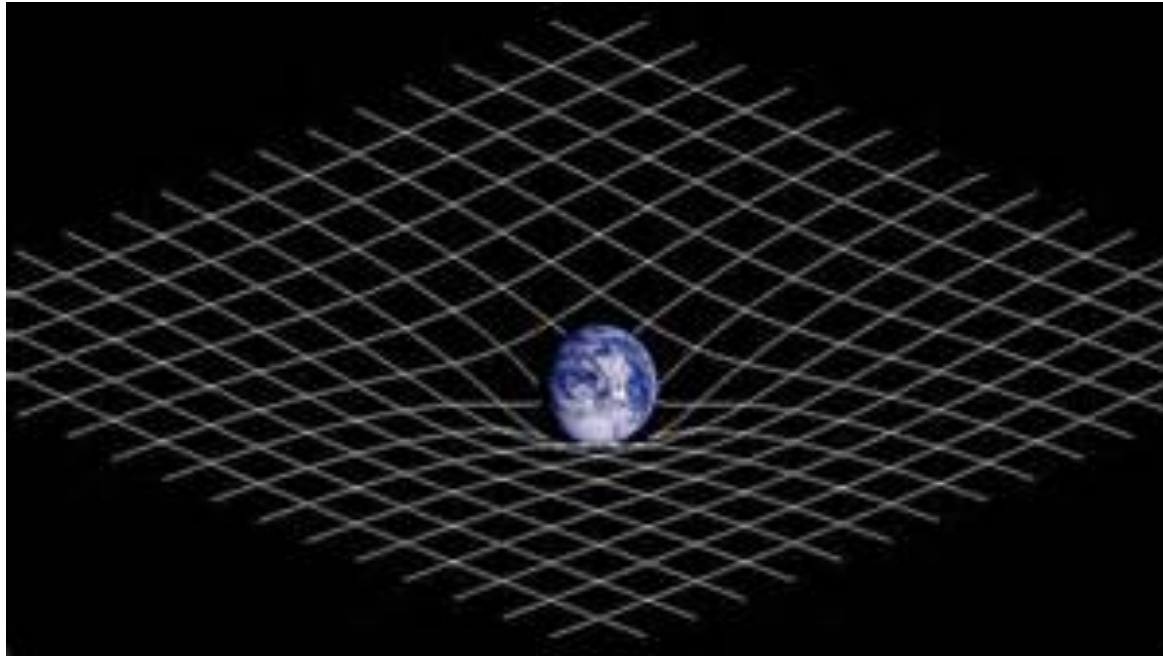
Simple example (continued): Duality as change of variables

AdS/CFT (holographic) duality

Physics of heavy ion collisions: quark-gluon plasma

How string theory helps to understand behavior
of hot and dense nuclear matter

Gravity and the metric



$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Geometry (metric) = Energy (and/or mass)

Black holes

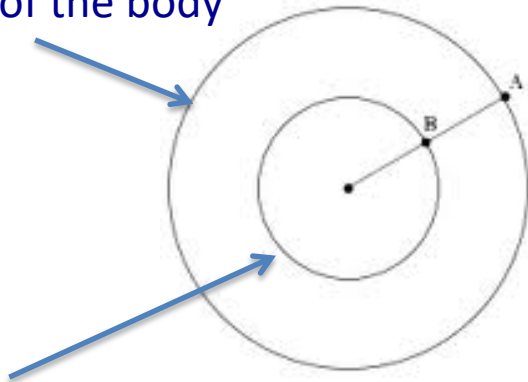
$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r} \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r} \right)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

Describes the metric of space-time OUTSIDE of a body of mass M

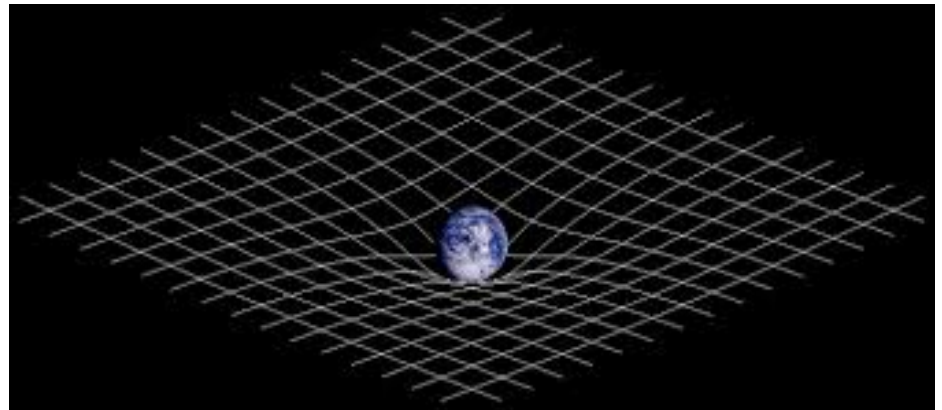
$$r = r_S = \frac{2GM}{c^2}$$

For most objects, the Schwarzschild radius is located deep inside the object and thus is not relevant since the solution is valid only outside (inside the metric is different). For example, for Earth the Schwarzschild radius is about 1 cm, for Sun – 3 km.

Surface of the body



Schwarzschild radius



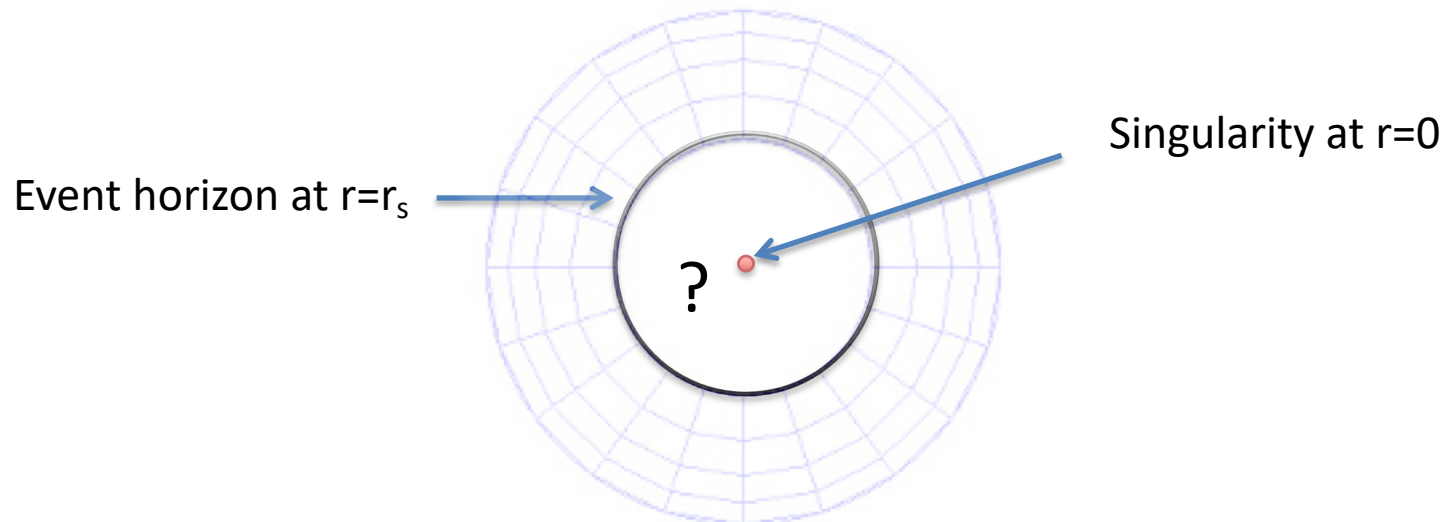
Black holes (continued)

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r} \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r} \right)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

Describes the metric of space-time OUTSIDE of a body of mass M

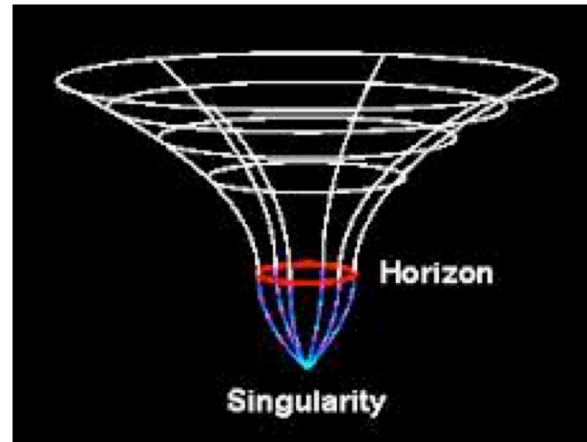
$$r = r_S = \frac{2GM}{c^2}$$

However, if the matter is squeezed inside its Schwarzschild radius (e.g. in the process of a gravitational collapse of a star), we get a black hole



Black hole singularities and the limits of modern physics

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r} \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r} \right)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$



$$S = \int d^4x \sqrt{-g} (R - 2\Lambda + c_1 R^2 + c_2 R_{ij} R^{ij} + \dots)$$

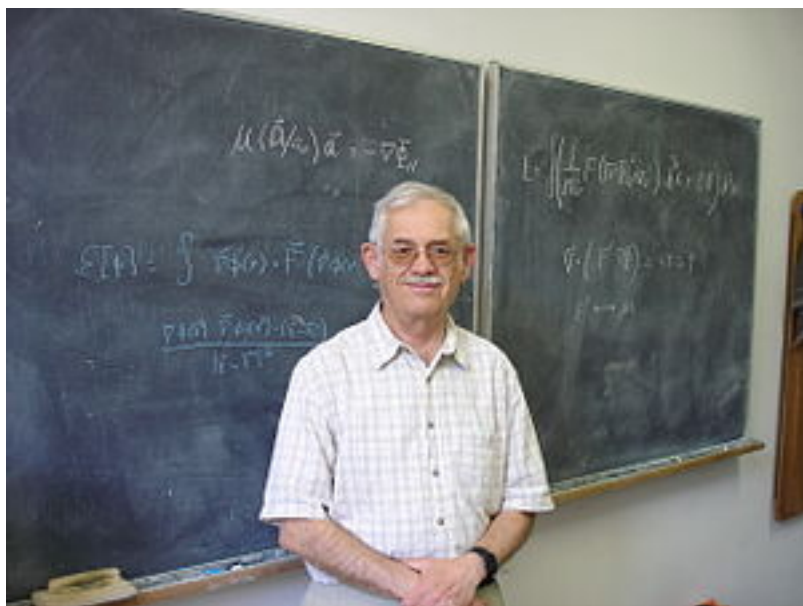
Corrections to the metric are expected near the singularity (quantum gravity, strings?)

Gravity as an effective theory at large distances & times

(difficult to test experimentally; no complete theory currently exists)

Black hole thermodynamics

In 1972, Jacob Bekenstein suggested that a black hole should have a well defined entropy proportional to the horizon area. He was laughed at by some (many...) since black holes are BLACK (do not radiate) and thus cannot have a temperature associated with them



Jacob Bekenstein



Stephen Hawking

However, in 1974 Hawking demonstrated that black holes do emit radiation at a quantum level and so one can in fact associate a temperature with them

Hawking temperature and Bekenstein-Hawking entropy

Hawking showed that black holes emit radiation with a black-body spectrum at a temperature

$$T = \frac{\hbar c^3}{8\pi k_B G M} \approx \frac{1.2 \times 10^{23} \text{ kg}}{M} K$$

(a black hole of one solar mass has a Hawking temperature of about 50 nanoKelvin)

This fixes the coefficient of proportionality in Bekenstein's conjecture:

$$S_{BH} = \frac{c^3 A}{4G\hbar}$$


Immediate consequences and problems:

- Black holes “evaporate” with time
- Information loss paradox
- *What are the microscopic degrees of freedom underlying the BH thermodynamics?*

Entropy and microstates

In statistical mechanics, entropy is related to the number of microstates:

N=1 particle with spin:  2 microstates

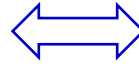
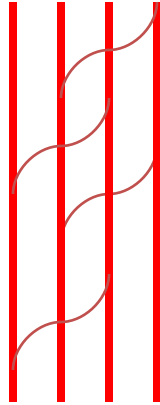
N=2 particles with spin:  4 microstates

N particles with spin: 2^N microstates

Boltzmann entropy:
$$S = k_B \ln W = k_B \ln 2^N = k_B N \ln 2$$

Can we count the microstates of a black hole and recover the Bekenstein-Hawking result?

Counting microstates of a black hole



A. Strominger and C. Vafa (1996) were able to count the microstates of a very special (supersymmetric) black hole in 5 dimensions. **The result coincides EXACTLY with the Bekenstein-Hawking thermodynamic entropy.**

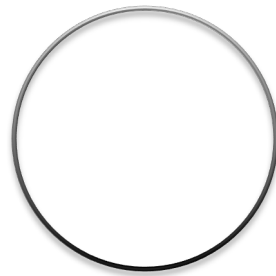
Strominger-Vafa result has been generalized in many ways since 1996.

However, we still do not know how to count the microstates of “normal” BHs, e.g. Schwarzschild BH in four dimensions...

Holographic principle

In thermodynamic systems without gravity, the entropy is extensive (proportional to volume)

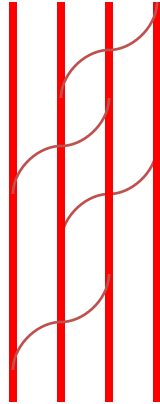
In gravitational systems, it is proportional to the AREA



$$S_{BH} = \frac{c^3 A}{4G\hbar}$$

It seems that gravitational degrees of freedom in **D** dimensions are effectively described by a theory in **D-1** dimensions ('tHooft, Susskind, 1992)

Gauge-string duality (AdS-CFT correspondence)



Open strings picture:
dynamics of strings & branes at low energy
is described by a
quantum field theory without gravity

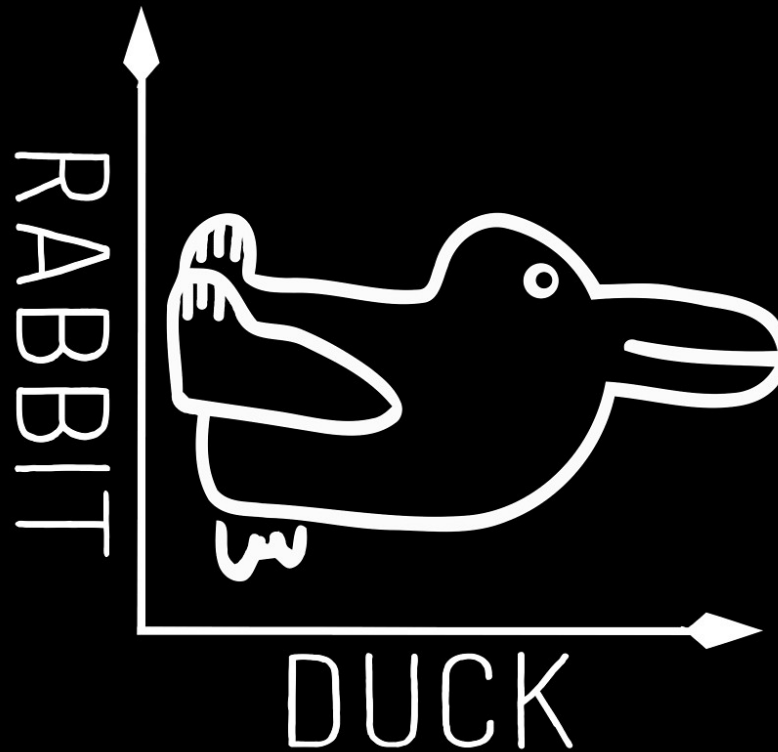
Closed strings picture: dynamics of strings
& branes at low energy is described by
gravity and other fields
in higher dimensions



conjectured
exact equivalence

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

Ludwig Wittgenstein's view of duality (1892; 1953)



The-Nerd-Shirt

(The analogy stolen from Shamit Kachru's talk at Simons Foundation, New York, Feb 27, 2019)

Quantum theory: the partition function

One way to define a quantum theory is to use Feynman path integral:

$$Z[\alpha] = \int [d\Phi] e^{-S[\Phi, \alpha]}$$

Here the partition function $Z[\alpha]$ depends on parameters of the theory α

The integration is over the fields of the theory Φ – e.g. scalar, vector, tensor or spinor fields, and $S[\Phi, \alpha]$ is the action of the theory

Note that Φ is a “dummy variable”: $Z[\alpha]$ depends only on α

The theory is “solved” completely, if we know how to calculate the path integral

Usually, this is a highly nontrivial task, and one resorts to approximations – for example, expanding for small α

Perturbative and non-perturbative approaches: a toy model

$$Z[\alpha] = \int_0^1 \frac{dx}{\sqrt{1 - \alpha^2 x^2}} = ?$$

Note that x is a “dummy variable” – the result $Z[\alpha]$ depends ONLY on α !

Suppose we only know how to compute very simple integrals such as

$$\int dx = x + C \qquad \int_0^1 dx = 1$$

Prediction for very small α :

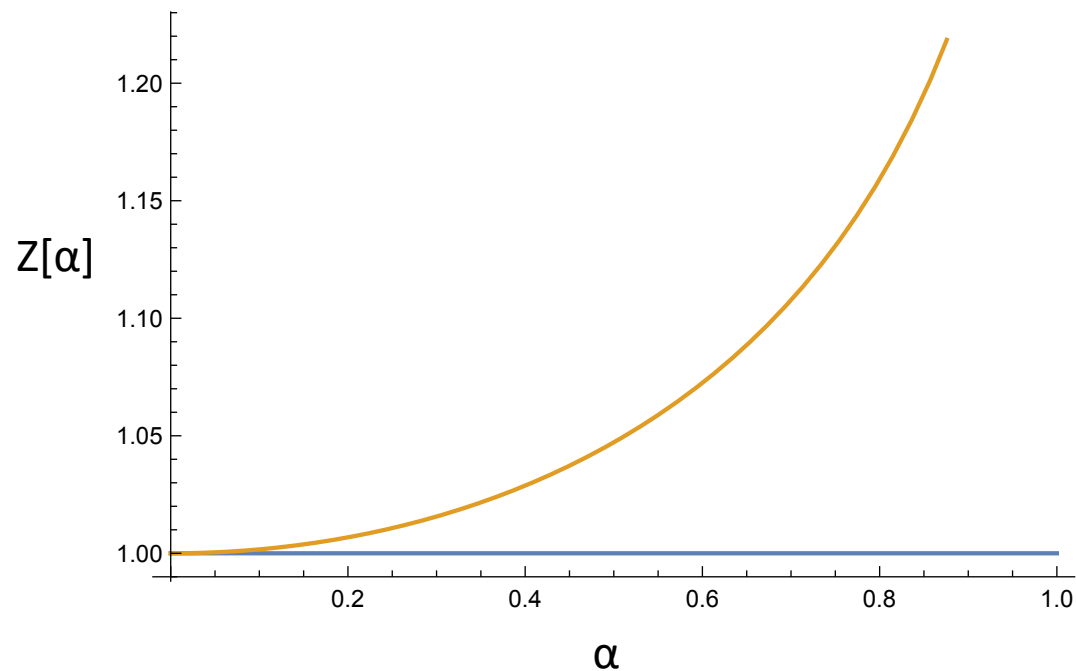
$$Z[\alpha] \approx \int_0^1 dx = 1, \quad \alpha \ll 1$$

Prediction for very small α :

$$Z[\alpha] = \int_0^1 \frac{dx}{\sqrt{1 - \alpha^2 x^2}} = ?$$

$$Z[\alpha] \approx \int_0^1 dx = 1, \quad \alpha \ll 1$$

Compare this prediction to the exact plot $Z[\alpha]$



Our prediction works well for small α but we want to know $Z[\alpha]$ for any α

$$Z[\alpha] = \int_0^1 \frac{dx}{\sqrt{1 - \alpha^2 x^2}} = ?$$

We can do a clever change of the dummy variable x . Notice that:

$$1 - \sin^2 y = \cos^2 y$$

Change variables $x \rightarrow y$ as:

$$\alpha x = \sin y$$

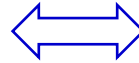
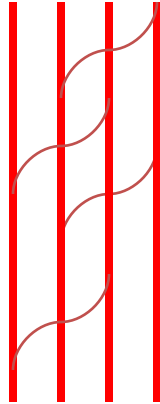
Then:

$$\alpha dx = \cos y dy$$

$$Z[\alpha] = \frac{1}{\alpha} \int_0^{\arcsin \alpha} \frac{\cos y dy}{\cos y} = \frac{1}{\alpha} \int_0^{\arcsin \alpha} dy = \frac{\arcsin \alpha}{\alpha}$$

This result is exact.

Gauge-string duality (AdS-CFT correspondence)



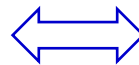
Open strings picture:
dynamics of strings & branes at low energy
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Closed strings picture: dynamics of strings
& branes at low energy is described by
gravity and other fields
in higher dimensions

$$Z_{\text{field theory}}[\alpha]$$

Partition function of
field theory in 3+1 dim

strong coupling



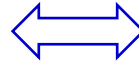
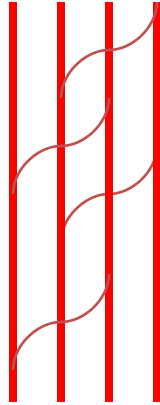
conjectured
exact equivalence

$$Z_{\text{string theory}} \left[\frac{1}{\alpha} \right]$$

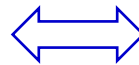
Partition function of
string theory in 10 dim

weak coupling

Gauge-String Duality, Gauge-Gravity Duality, AdS-CFT correspondence, Holography



$Z_{field\ theory}[\alpha]$



$Z_{string\ theory} \left[\begin{array}{c} 1 \\ \alpha \end{array} \right]$

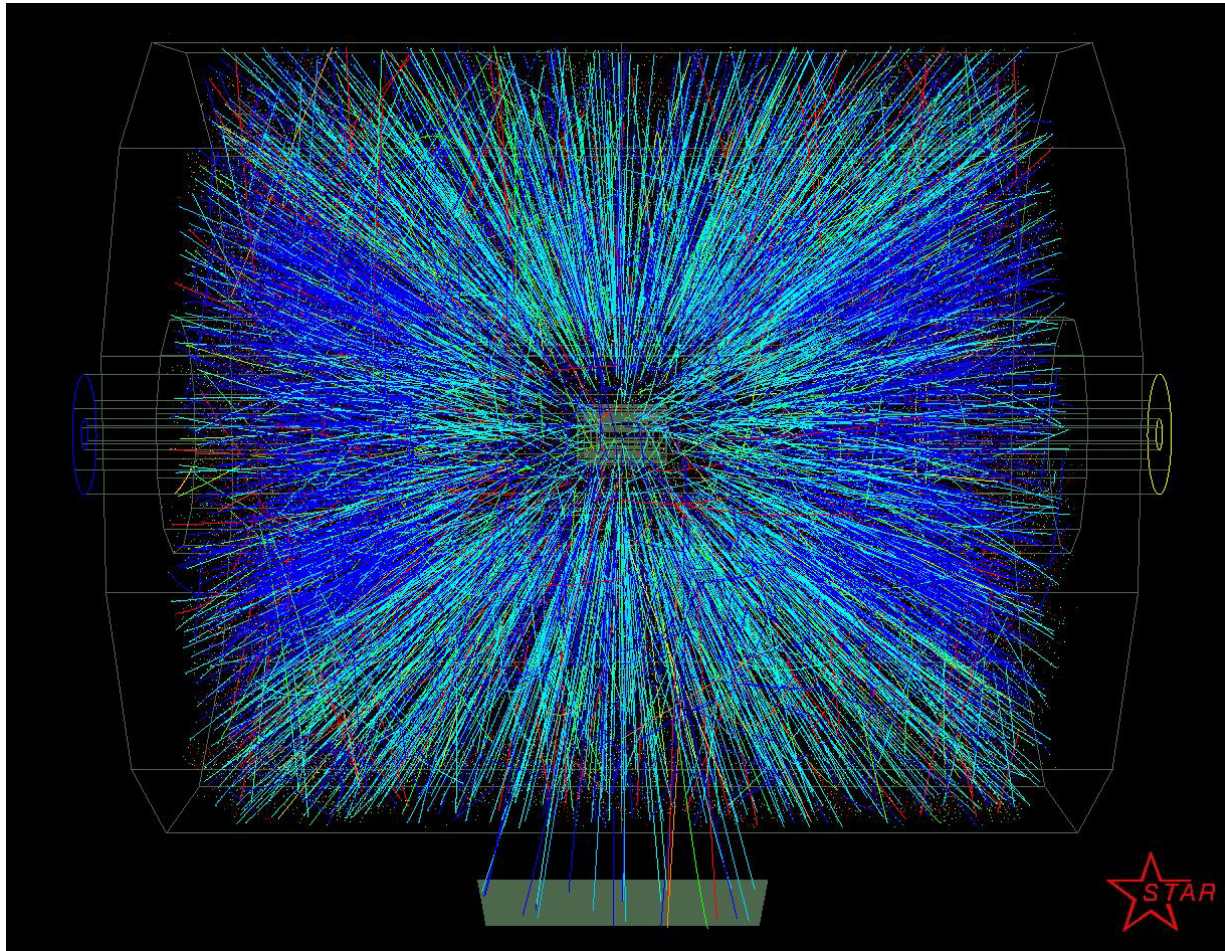
conjectured
exact equivalence

Using the duality, we can find various quantities in quantum field theory at strong coupling by doing the actual computation in the dual string theory at weak coupling

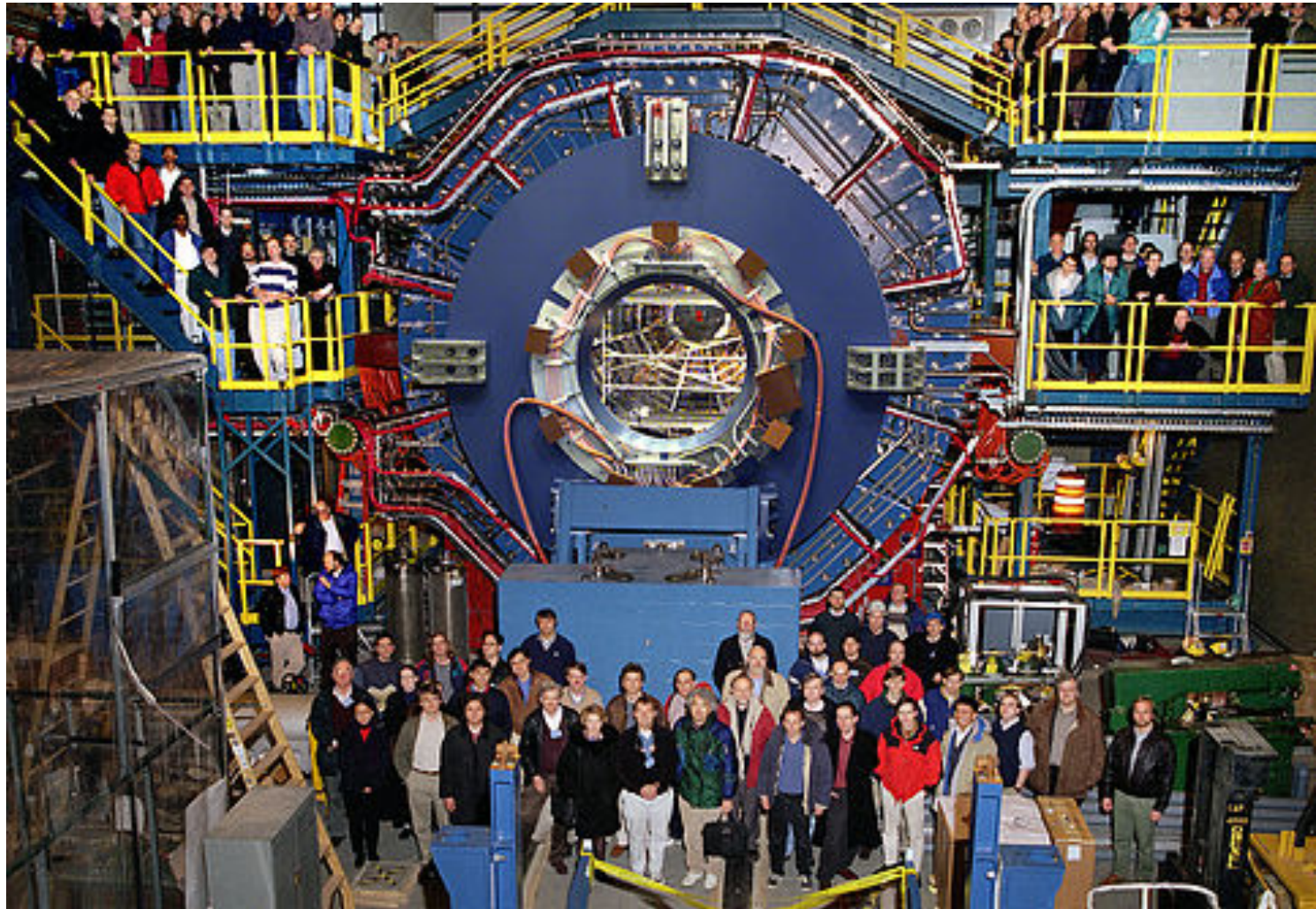
For some quantities, this is the only non-perturbative tool available

Note that the duality is independent of the status of String theory as THE Theory of Nature

Heavy ion collisions: RHIC/LHC

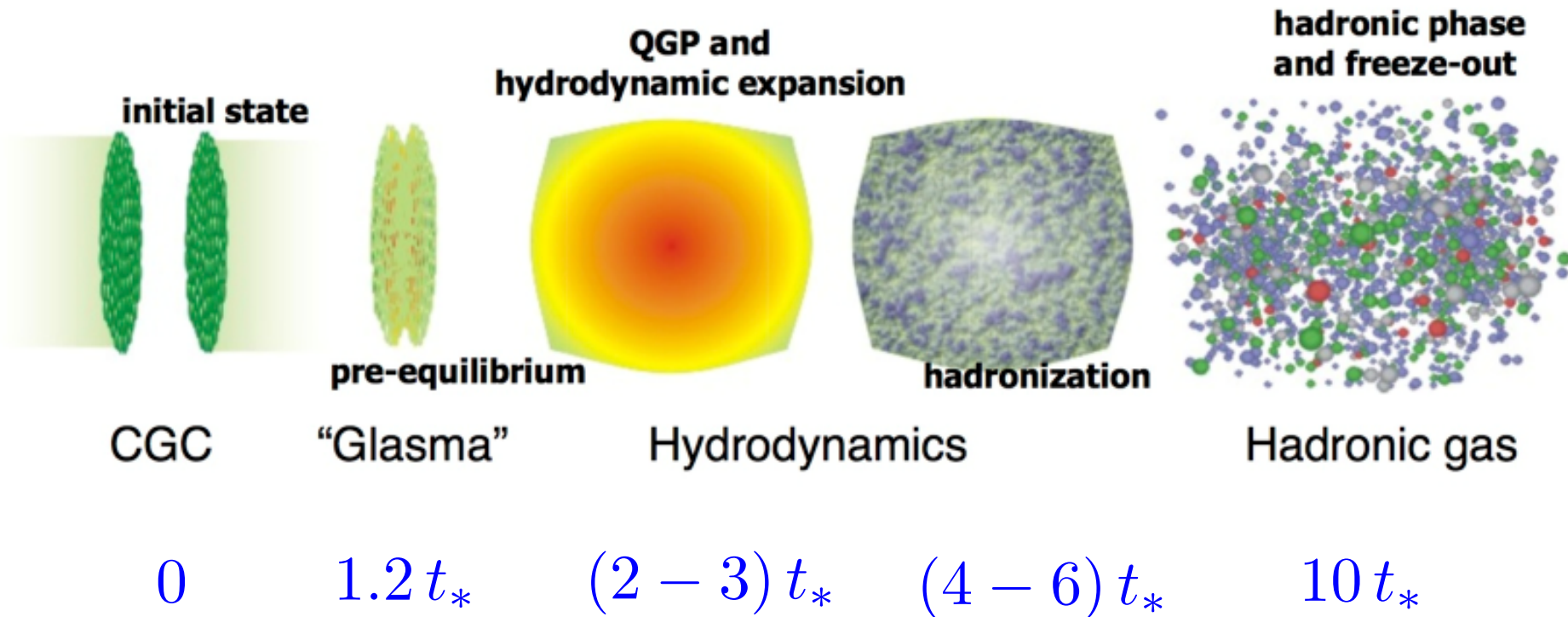


STAR: The Solenoidal Tracker at RHIC



Heavy ion collisions: evolution of the quark-gluon plasma

$$t_* \sim 10^{-24} \text{ sec}$$



Heavy ion collision experiments at **RHIC** (2000-current) and **LHC** (2010-current) create hot and dense nuclear matter known as the “quark-gluon plasma”

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma “fireball” is described by relativistic fluid dynamics
(relativistic Navier-Stokes equations)

Need to know

thermodynamics (equation of state)

kinetics (first- and second-order transport coefficients)

in the regime of intermediate coupling strength:

$$\alpha_s(T_{\text{RHIC}}) \sim 1$$

initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Energy density vs temperature for various gauge theories

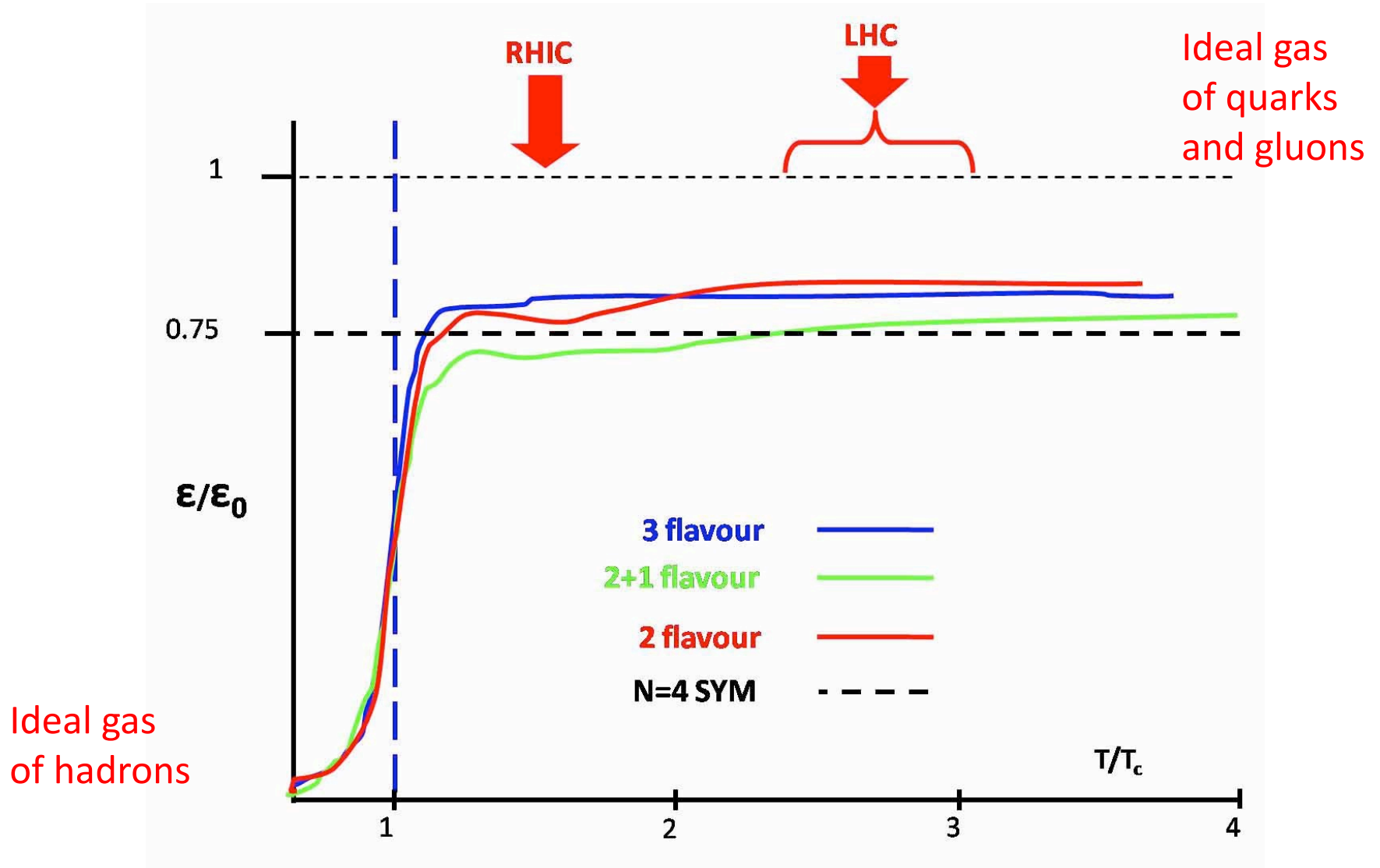
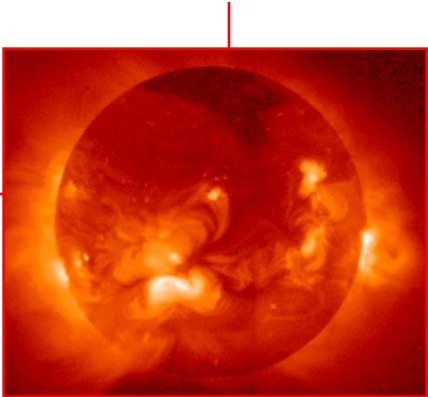


Figure: an artist's impression from Myers and Vazquez, 0804.2423 [hep-th]

Quantum field theories at finite temperature/density

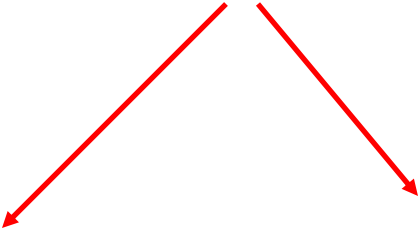
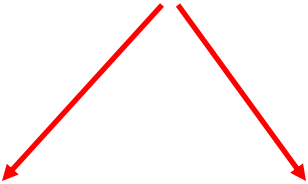
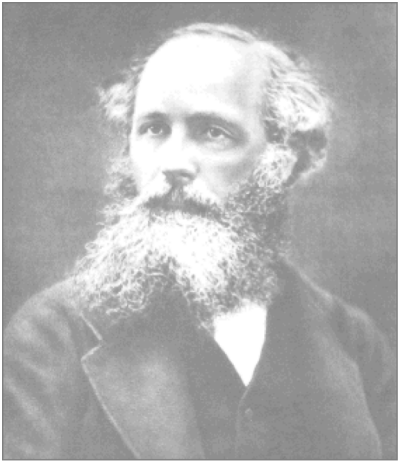


Equilibrium

Near-equilibrium

entropy
equation of state
.....

transport coefficients
emission rates
.....



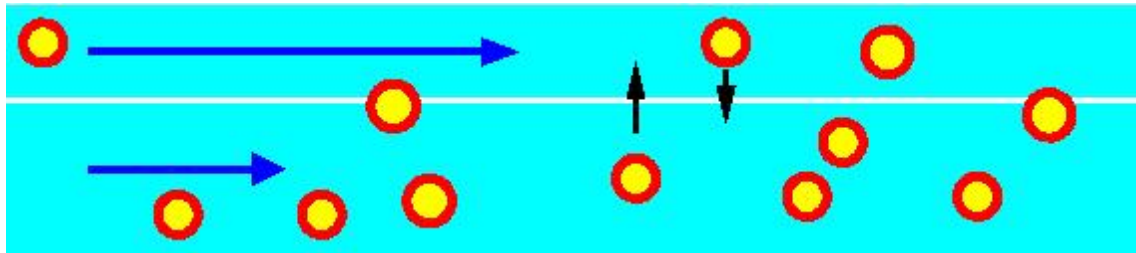
perturbative non-perturbative
pQCD **Lattice**

perturbative non-perturbative
kinetic theory **????**

Shear viscosity

One of the most important characteristics of quark-gluon plasma is shear viscosity

Shear viscosity can be understood as the measure of internal friction in liquid or gas



Momentum transfer between two layers of liquid or gas moving with different velocities leads to gradual equilibration of the velocities

Viscosity of gases and liquids

Gases (Maxwell, 1867): $\eta \sim \rho \bar{v} l_{mfp} \sim \frac{m_o \bar{v}}{\sigma} \sim \frac{m_o^{1/2}}{\sigma} \sqrt{T}$

Viscosity of a gas is

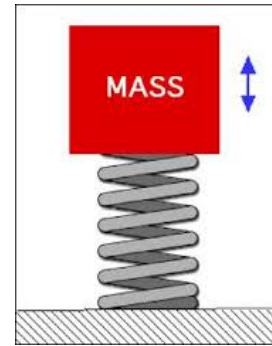
- independent of pressure
- scales as square of temperature
- inversely proportional to cross-section

Liquids (Frenkel, 1926): $\eta \sim A(P, T) \exp \frac{W}{T}$

- In practice, A and W are chosen to fit data
- No good theory to determine viscosity of liquids from first principles
- Can try AdS/CFT correspondence to compute it at strong coupling

Black holes beyond equilibrium

Undisturbed black holes are characterized by global charges: M , Q , J ...



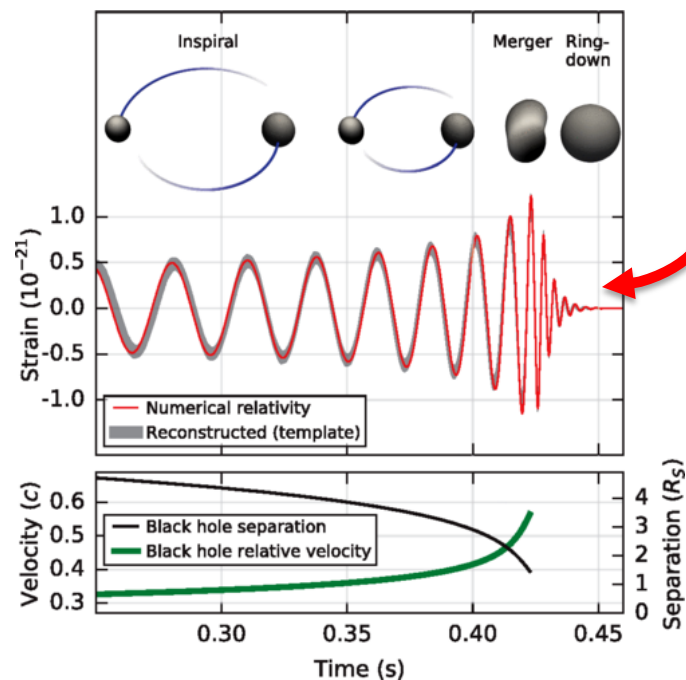
A thermodynamic system **in equilibrium** is characterized by conserved charges: E , Q , J ...

If one perturbs a non-gravitational system (e.g. a spring pendulum), it will oscillate with eigenfrequencies (normal modes) characterizing the system

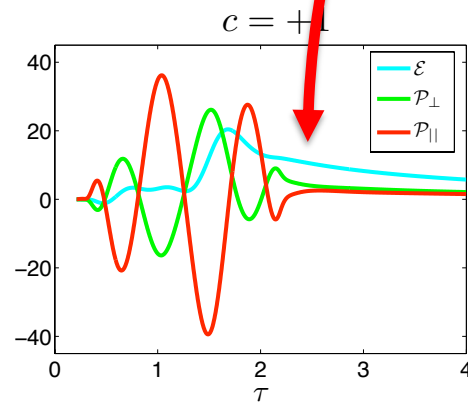
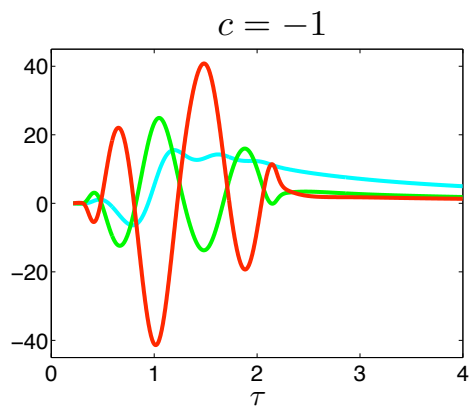
$$\omega = \sqrt{\frac{k}{M}}$$

What happens if one perturbs a black hole?

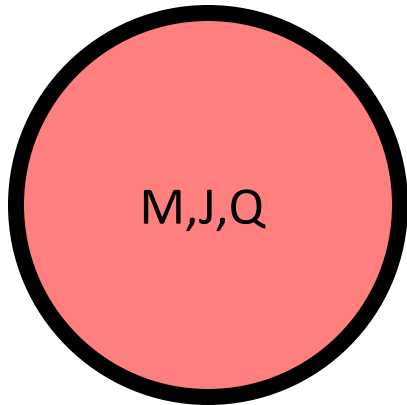
Quasinormal modes in “real life” and beyond



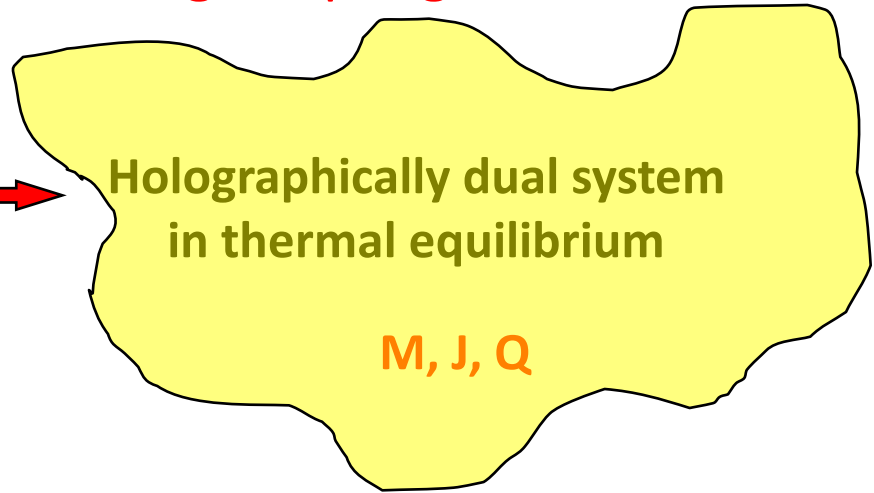
Quasinormal modes region



10-dim gravity



4-dim gauge theory – large N, strong coupling



T_{Hawking}

$S_{\text{Bekenstein-Hawking}}$



T S

Gravitational fluctuations



Deviations from equilibrium

$$g_{\mu\nu}^{(0)} + h_{\mu\nu}$$



????

"□" $h_{\mu\nu} = 0$ and B.C.



$$j_i = -D\partial_i j^0 + \dots$$

$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum



$$\omega = -iDq^2 + \dots$$

Black hole's quasinormal spectrum encodes properties of a dual microscopic system

Comparing eigenfrequencies of a black hole

$$\omega = \pm \frac{c}{\sqrt{3}} k - \frac{i}{6\pi T} k^2 + \frac{3 - 2 \ln 2}{24\pi^2 \sqrt{3} T^2} k^3 + \dots$$

with eigenfrequencies of a dual microscopic system (described by fluid mechanics)

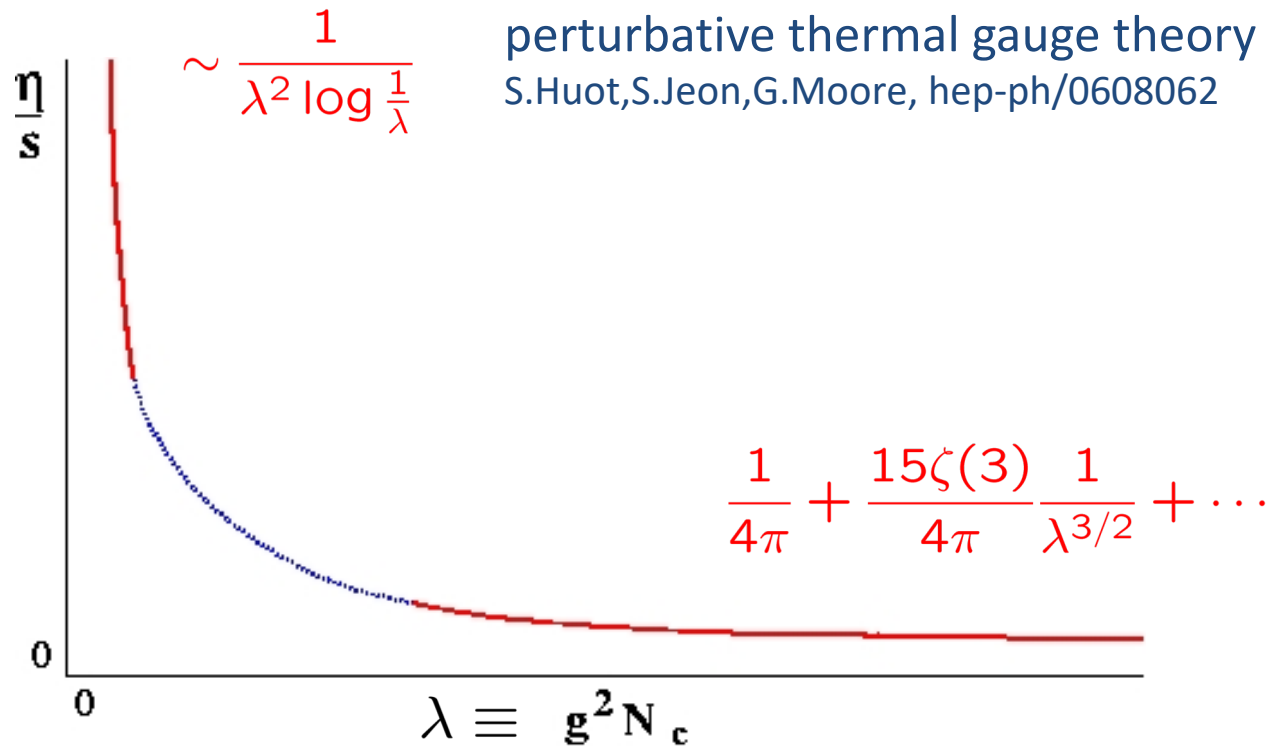
$$\omega = \pm v_s k - i \frac{2\eta}{3sT} k^2 + \dots$$

one can compute viscosity-entropy ratio and other quantities of a microscopic system

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} + \dots$$

Moreover, one can relate Navier-Stokes equations and Einstein's equations...

Shear viscosity in $\mathcal{N} = 4$ SYM

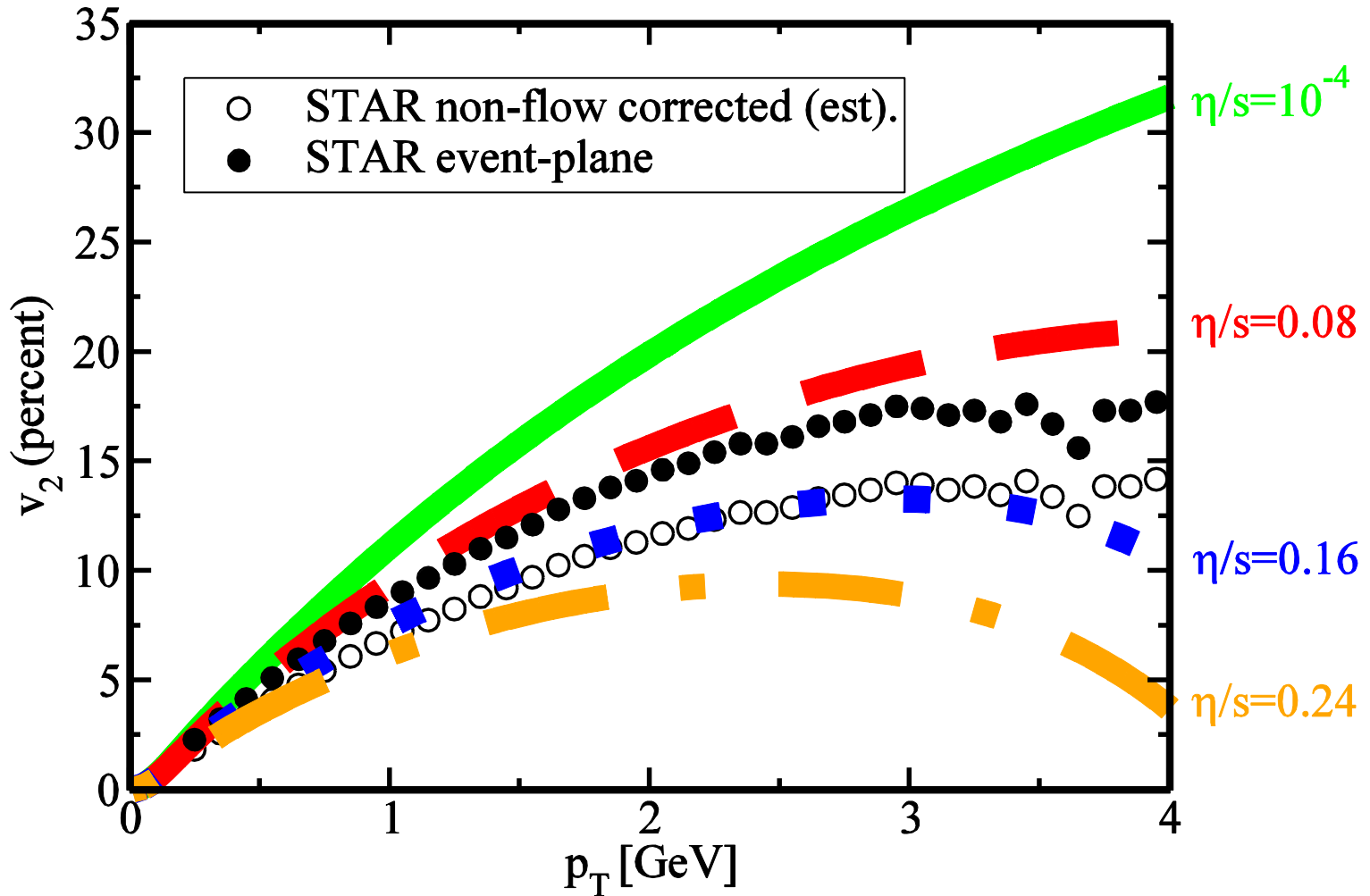


Correction to $1/4\pi$ Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

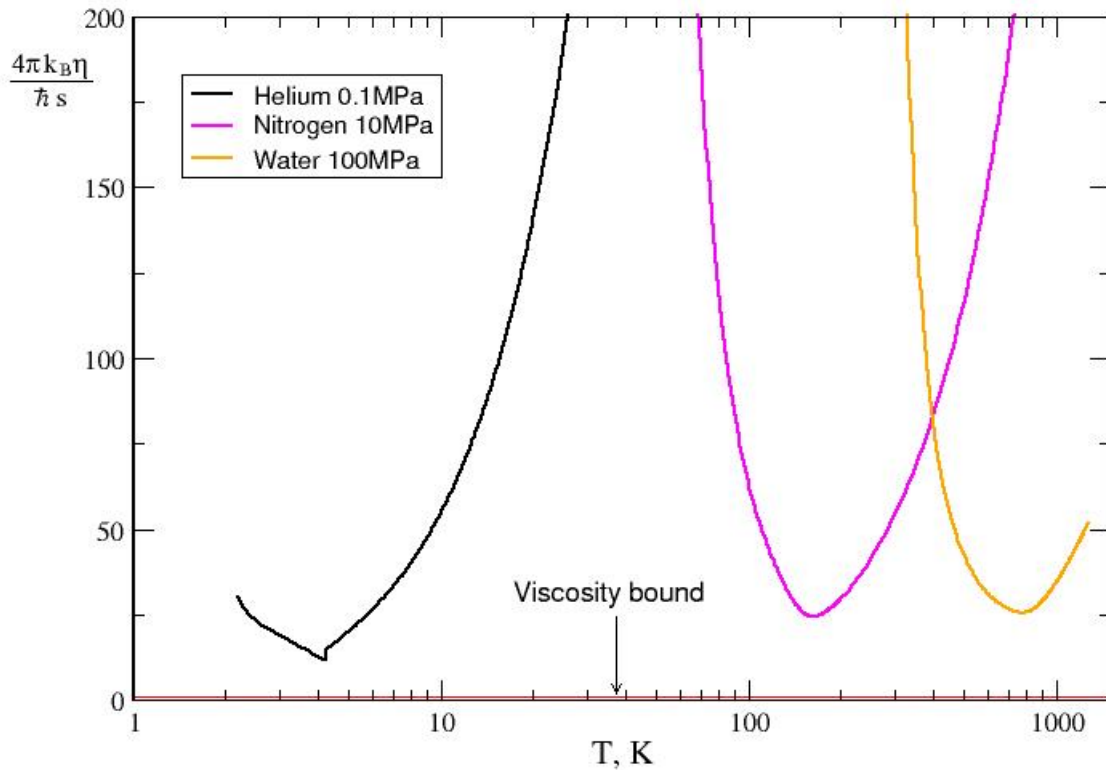
Shear viscosity “measurements” at RHIC and LHC

CGC



A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe 84

Kr 57

CO₂ 32

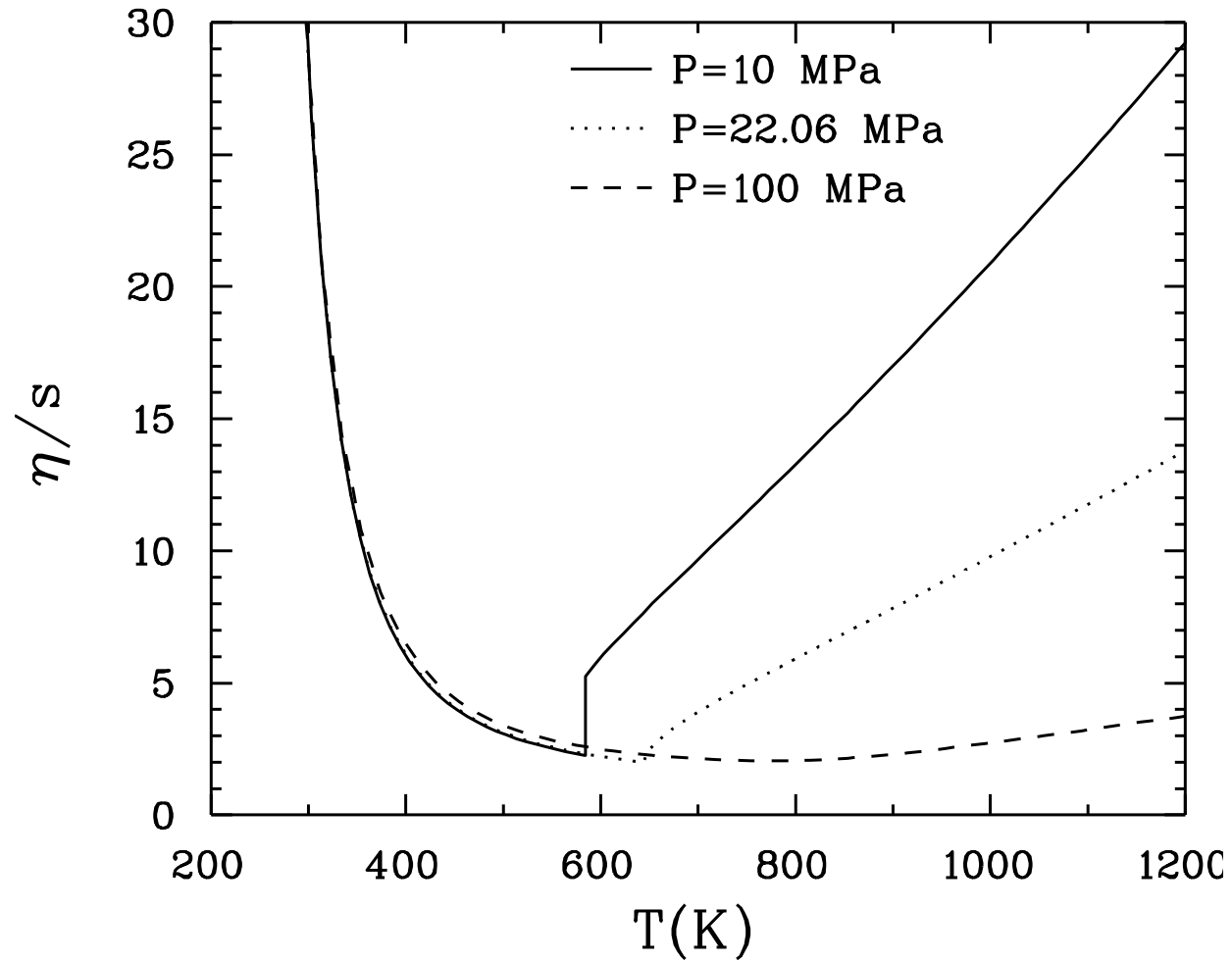
H₂O 25

C₂H₅OH 22

Ne 17

He 8.8

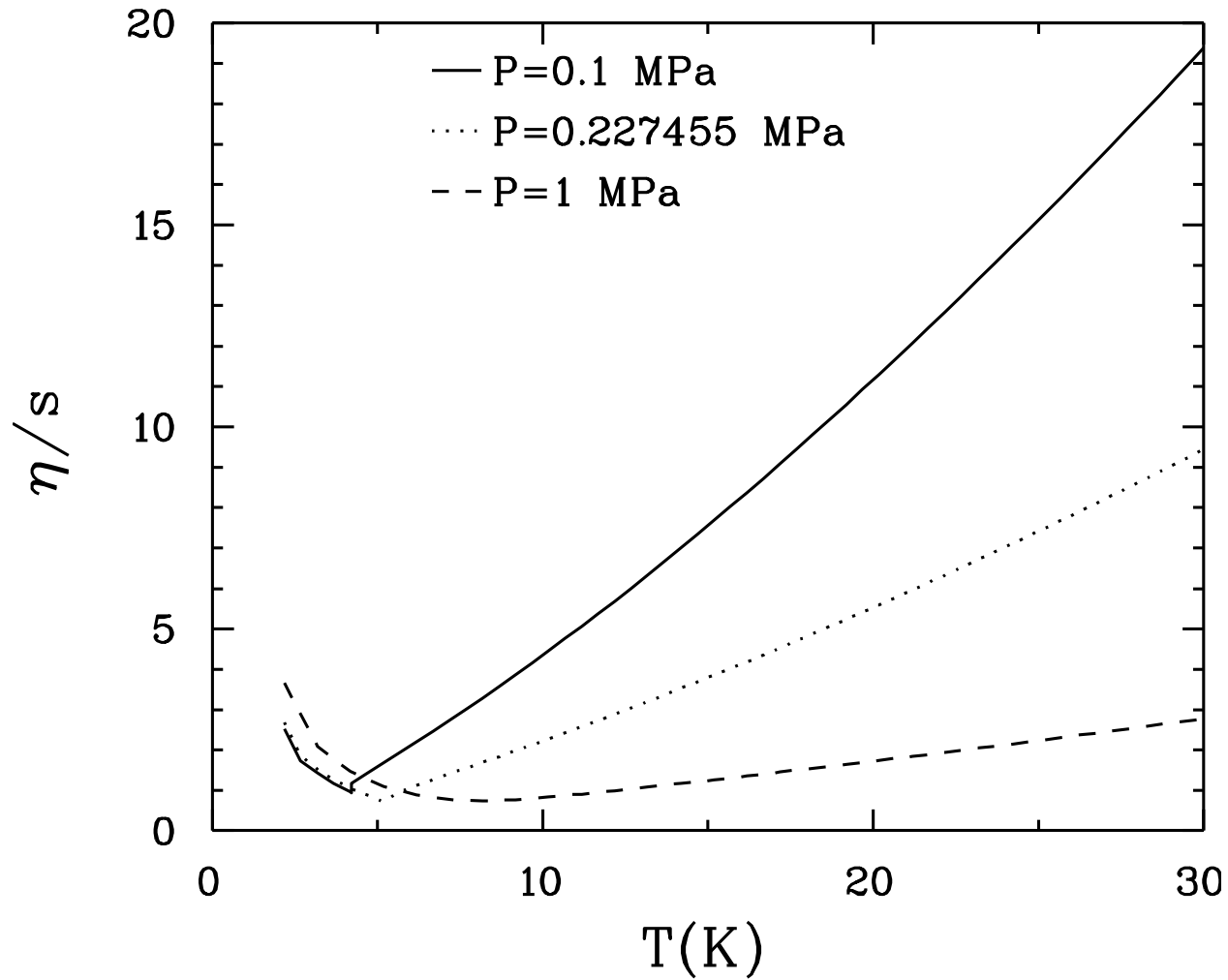
H₂O



$(\eta/s)_{\min} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$

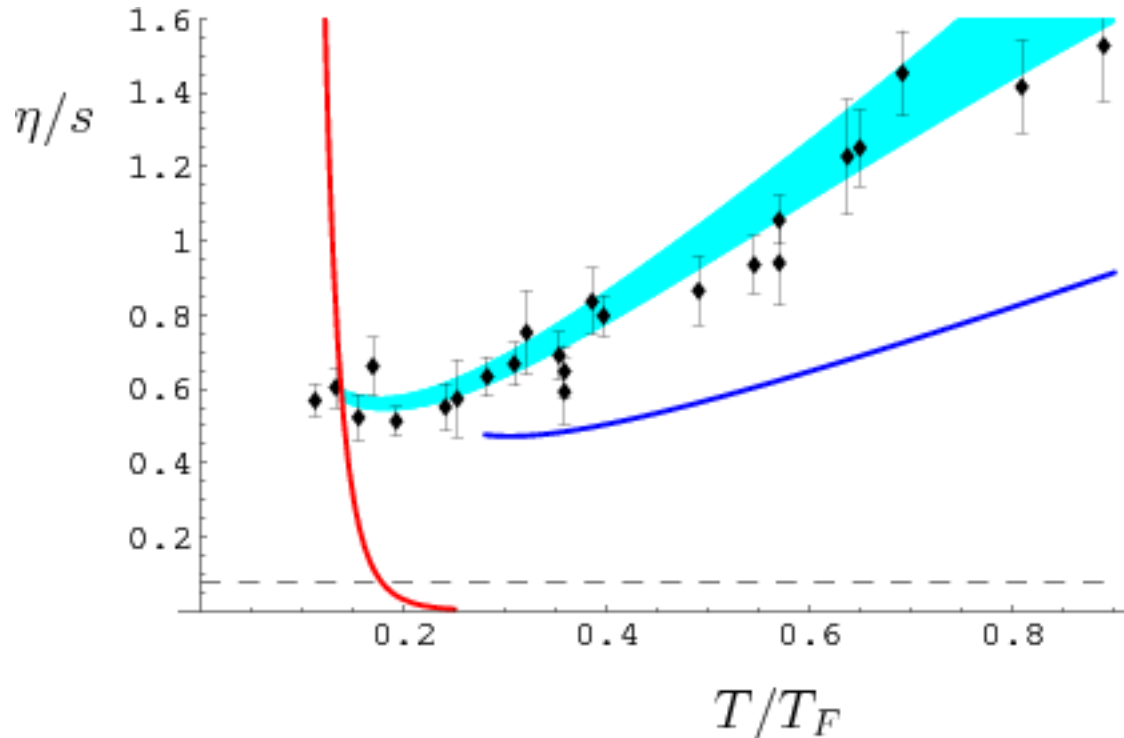
Chernai, Kapusta, McLerran, nucl-th/0604032

Helium



$(\eta/s)_{\min} \sim 8.8$ in units of $\frac{\hbar}{4\pi k_B}$

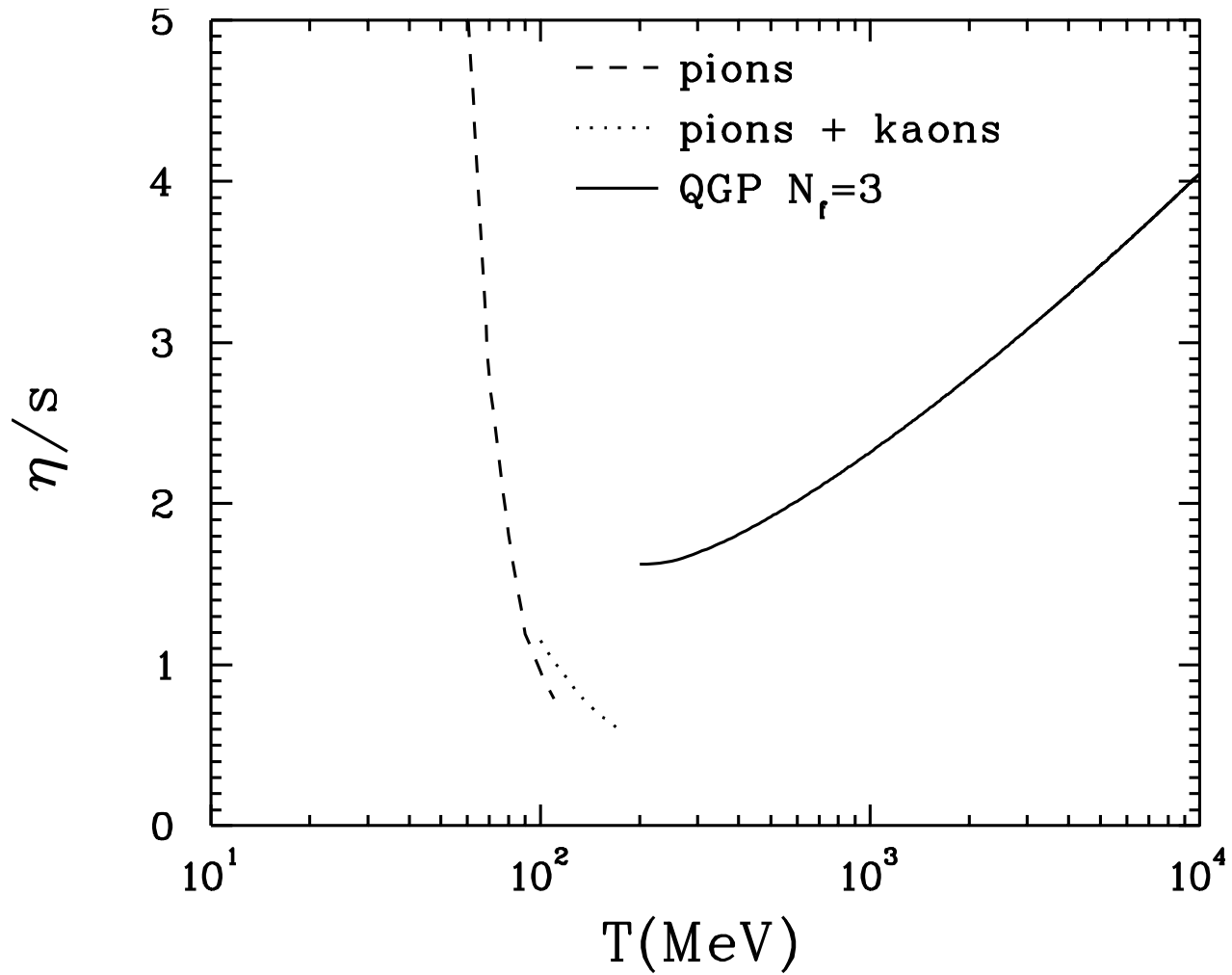
Viscosity-entropy ratio of a cold Fermi gas



$$\eta/s \sim 5.5 \text{ in units of } \frac{\hbar}{4\pi k_B} \quad \text{T.Schafer, cond-mat/0701251}$$

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

QCD



Conclusions

String theory was (partially) successful in explaining entropy of black holes

Black holes have entropy and temperature and behave like thermodynamic systems,
and we think we know why (holographic principle)

String theorists' work on black hole physics has led to the (accidental) discovery of
AdS/CFT (or gauge-string) duality

The duality can be viewed as a non-trivial change of variables in path integral
We know it works (it has been tested in numerous examples) but rigorous proof is still lacking

Gauge-string duality can compute field theory quantities at strong coupling

These computations helped to explain properties of quark-gluon plasma
(e.g. low shear viscosity-entropy density ratio)
and are of great interest for other strongly interacting systems

The duality is a two-way street – can it be used to understand strongly coupled gravity?

THANK YOU!