University of Oxford
Department of Physics

Oxford Master Course in Mathematical and Theoretical Physics

Introduction to Gauge-String Duality

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Problem Set III

Black Branes

- Check explicitly that the non-extremal three-brane background

$$\begin{aligned} ds_{10}^2 &= H^{-1/2}(r)\left[-f dt^2 + dx^2 + dy^2 + dz^2\right] \\ &\quad + H^{1/2}(r)\left(f^{-1}dr^2 + r^2 d\Omega_5^2\right) , \end{aligned} \tag{1}$$

where $H(r) = 1 + R^4/r^4$, $f(r) = 1 - r_0^4/r^4$, and the Ramond-Ramond five-form is given by

$$F_5 = -\frac{4R^2}{H^2 r^5}\sqrt{R^4 + r_0^4}\;(1 + *)\, dt \wedge dx \wedge dy \wedge dz \wedge dr \,, \tag{2}$$

is a solution of type IIB low energy equations of motion,

$$\begin{aligned} R_{MN} &= \frac{1}{96} F_{MPQRS} F_N^{PQRS} \,, \\ F_{(5)} &= *F_{(5)} \,, \end{aligned}$$

where all other supergravity fields are consistently set to zero.

- Show that the Hawking temperature of the black three-brane is given by $T = \frac{1}{\pi r_0} H^{-1/2}(r_0)$.

- Compute the Bekenstein-Hawking entropy of the black three-brane solution.

- Show that in the near-horizon limit $r \ll L$ the black three-brane metric factorizes into the five-dimensional AdS-Schwarzschild part,

$$ds_{10}^2 = \frac{(\pi T L)^2}{u}\left[-f dt^2 + dx^2 + dy^2 + dz^2\right] + \frac{L^2}{4u^2 f}\, du^2 \,,$$

and the metric on a five-sphere of radius L. Show that the AdS-Schwarzschild metrics is a solution of five-dimensional equations

$$R_{\mu\nu} = -\frac{4}{L^2}\, g_{\mu\nu} \,.$$

Show that the Hawking temperature of this metric is $T = r_0/\pi L^2$ and the Bekenstein-Hawking entropy is $S = \pi^6 T^3 L^8 V_3/4G_{10}$, where V_3 is the three-volume along x, y, z directions. With $G_{10} = (2\pi)^7 g_s^2 l_s^8/16\pi$ and the AdS/CFT identification $L^4 = 4\pi g_s N_c l_s^4$, find the entropy density $s = S/V_3$ of the dual gauge theory in the limit of infinite N_c and infinite 't Hooft coupling.

- The black p-brane metric ($p < 7$) in the Einstein frame reads

$$ds_E^2 = H^{-\frac{7-p}{8}} \left(-f dt^2 + dx_1^2 + dx_2^2 + \cdots + dx_p^2\right) + H^{\frac{p+1}{8}} \left(f^{-1} dr^2 + r^2 d\Omega_{8-p}^2\right) ,$$

where $H = 1 + \frac{L^{7-p}}{r^{7-p}}$ and $f = 1 - \frac{r_0^{7-p}}{r^{7-p}}$. The Ramond-Ramond field strength is given by

$$F_{r01\ldots p} = (p-7) \frac{L^{7-p}\sqrt{r_0^{7-p} + L^{7-p}}}{H^2(r) r^{8-p}}$$

and the dilaton is

$$e^{\Phi} = H^{\frac{3-p}{4}}(r)$$

Show that this background is a solution of type IIB low-energy equations of motion. Compute the Bekenstein-Hawking entropy and show that the Hawking temperature is given by

$$T = \frac{7-p}{4\pi r_0} H^{-1/2}(r_0) .$$