Holographic methods in near-equilibrium physics

Goal: to understand strongly interacting quantum systems in and out of equilibrium

Experiments:

- RHIC/LHC heavy ion collisions

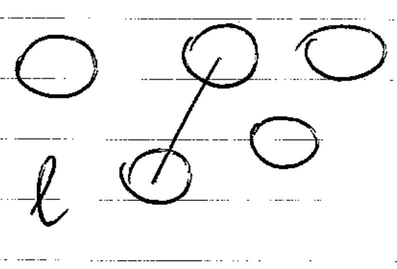
  $T \sim 2 \cdot 10^{12}\ K$

- ultra-cold quantum gases (^6Li)

  $T \sim 10^{-8}\ K$

Features: systems are many-body ($N \sim 10^4$),

quantum ($\lambda_{\text{de Broglie}} \gtrsim l$)



$l \sim \left(V/N\right)^{1/3}$

or $l \sim l_{mfp}$

strongly interacting

e.g. $\alpha_s(T_{RHIC}) \sim 1$

or $\lambda_{\mathcal{N}=4\ SYM} \gg 1$

in local thermal equilibrium

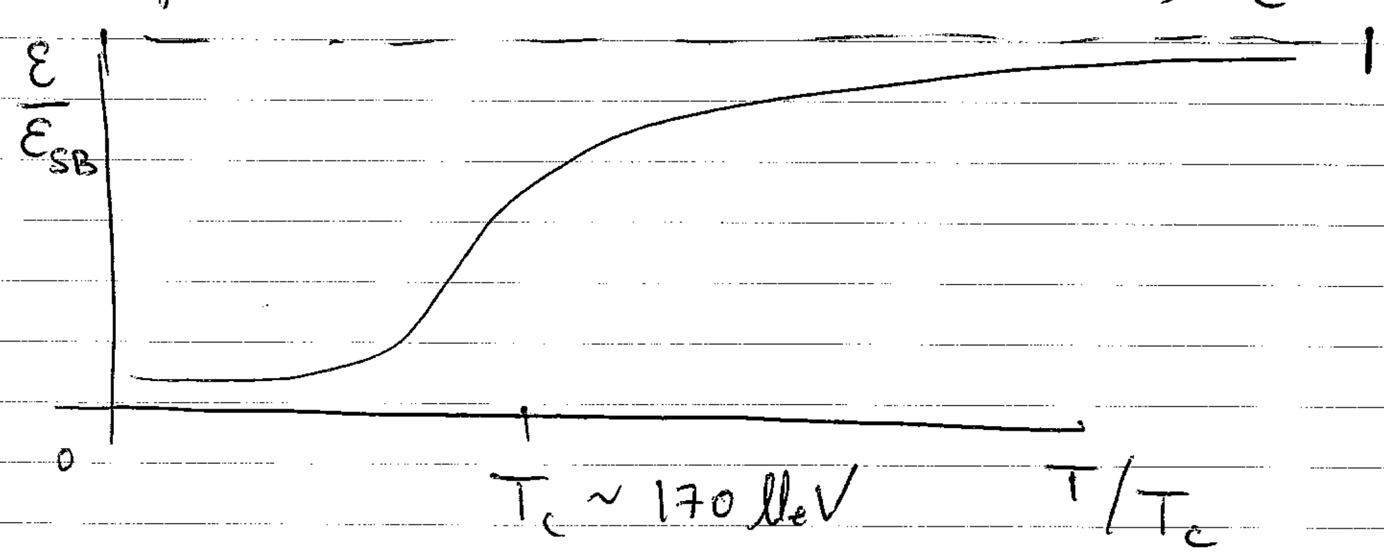
$L_{\text{var in } T} \gg \ell_{mfp}$

Evolution of the RHIC "fireball":

Need TD and transport

Thermodynamics

for QCD at $T \sim (2-4)T_c$



Transport - ?

Lecture 2

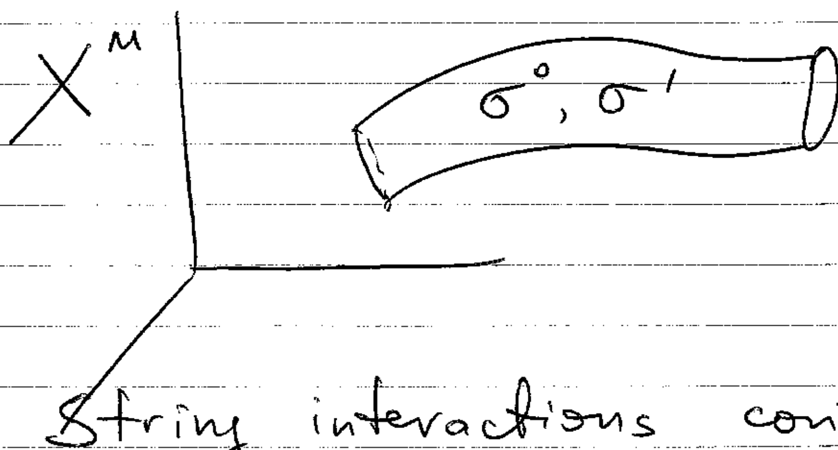
Elements of gauge-string duality

String theory = interacting strings (open and closed) and branes

Fundamental parameter $T = \dfrac{1}{2\pi\alpha'}$

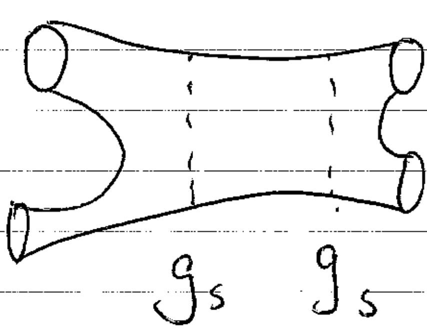
$\alpha' = l_s^2$

$$S = -T\int d^2\sigma \sqrt{-\det g}$$

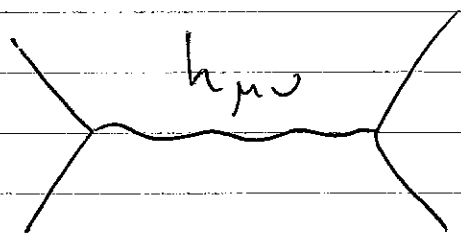


$X^M(\sigma^0, \sigma^1)$

String interactions controlled by g_s



$\longrightarrow$



$E \ll 1/l_s$

$16\pi G_{10} = (2\pi)^7 g_s^2 l_s^8$

g_s is related to the expect. value of the dilaton

Effectively, 2 parameters : l_s, g_s

Approximating gauge-string duality by gauge-grav. duality :

$$L \gg l_s \qquad , \qquad g_s \ll 1$$

where L is the typical size of geometry, e.g. L_{AdS}.

Branes enter as non-perturb. (in g_s) with $p+1$-dim world-volume and

$$T_{Dp} \sim \frac{1}{g_s l_s^{p+1}} .$$

Generically, all these d.o.f. are excited.

But at $E \ll 1/l_s$ only the lowest states are of interest

E.g. massless fields of type IIB str. th.

$g_{\mu\nu}$ graviton

ϕ, C dilaton, axion

$B_{\mu\nu}$, $A_{\mu\nu}$ rank 2 antisymm

$A^{+}_{\mu\nu\lambda\sigma}$ rank 4 antisym, self-dual

$\psi^{I=1,2}_{\mu,\alpha}$ gravitini

$\lambda^{I=1,2}_{\alpha}$ dilatini

($\mathcal{N}=2$ supra $d=10$)

Preserving symmetries of string action at quantum level leads to conditions on β - functionals:

$$\beta_{\mu\nu}(X) = R_{\mu\nu} + \frac{1}{4} H_{\mu}^{\ \lambda\sigma} H_{\nu\lambda\sigma} - 2 D_\mu D_\nu \phi$$
$$+ O(\alpha') = 0$$

Here $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}$

More precisely, type IIB low energy eom

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}e^{-\phi}\left(H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} - \frac{1}{2}g_{\mu\nu}H^2\right) + e^{2\phi}\frac{1}{2}\partial_\mu C\partial_\nu C +$$

$$+ e^{\phi}\frac{1}{4}\left(\tilde{F}_{\mu\lambda\sigma}\tilde{F}_\nu{}^{\lambda\sigma} - \frac{1}{2}g_{\mu\nu}\tilde{F}^2_{(3)}\right) +$$

$$+ \frac{1}{96}\tilde{F}_{\mu\lambda\rho\sigma\varkappa}\tilde{F}_\nu{}^{\lambda\rho\sigma\varkappa}$$

$$\nabla^2\phi = e^{2\phi}\partial_\mu C\partial^\mu C - \frac{1}{12}e^{-\phi}H_3^2 + \frac{1}{12}e^{\phi}\tilde{F}_3^2$$

+ Maxwell-like eqs for $\tilde{F}_3$, $\tilde{F}_5$, H_3,

where $F_3 = dA_2$, $H_3 = dB_2$,

$\tilde{F}_3 = F_3 - C H_3$ etc +

condition $\tilde{F}_5 = *\tilde{F}_5$.

<u>Note</u>: there are corrections to e.o.m. at $O(\alpha')$ or higher.

$$\mathcal{L}/\sqrt{-g} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4\cdot 5!}\tilde{F}_5^{\,2} + \dots$$

$$+ \frac{\zeta(3)}{8} e^{-\frac{3}{2}\phi} \alpha'^3 W + \dots$$

$$W = \underbrace{C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk}}_{\sim (R_{ijkl})^4} + \text{similar terms}$$

Recall $R_{ijkl} \sim 1/L^2$, $R \sim 1/L^2$

$\alpha'^3 = l_s^6$.

In AdS/CFT: $\lambda = g_{YM}^2 N_c$

$$\alpha' = l_s^2 = \frac{L_{AdS}^2}{\lambda^{1/2}} \Rightarrow \text{corrections}$$

$\sim \alpha'^3 \sim \lambda^{-3/2}$.

Indeed, for $\mathcal{N}=4$ SYM

$$\eta/s = \frac{\hbar}{4\pi k_B}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \dots\right).$$

Solutions to these eom characterized by scale L (e.g. L_{AdS}) generically get stringy and quantum grav. corrections (modulo some exceptions)

<u>unless</u>

$$L \gg l_s \quad , \quad L \gg l_p .$$

<u>In AdS/CFT</u> : $\dfrac{L^4}{l_s^4} = \lambda \Rightarrow$

$\boxed{\lambda \gg 1}$ classical (non-stringy) geometry valid

Also, $g_{YM}^2 = 4\pi g_s$ and

$L^4 \gg l_p^4 \sim g_s l_s^4 \Rightarrow$ OK when $\boxed{N_c \gg 1}$

(no quantum grav. corrections)

Consider a solution of type IIB eom with all fields but $g_{\mu\nu}$, F_5 and $\phi = \text{const}$ set to zero.

This is a self-consistent choice.

$$\begin{cases} R_{\mu\nu} = \frac{1}{96} F_{\mu\rho\sigma\lambda\varkappa} F_{\nu}{}^{\rho\sigma\lambda\varkappa} \\ F_{(5)} = * F_{(5)} \end{cases}$$

Solution:

$$ds^2_{10} = H^{-1/2}(r)\left[-f(r)\,dt^2 + dx^2 + dy^2 + dz^2\right] + H^{1/2}(r)\left(\frac{dr^2}{f} + r^2 d\Omega_5^2\right)$$

$$H(r) = 1 + L^4/r^4$$

$$f(r) = 1 - r_0^4/r^4$$

$$F_5 = -\frac{4L^2}{H^2 r^5}\sqrt{r_0^4 + L^4}\,(1+*)\,dt\wedge dx\wedge dy\wedge dz\wedge dr$$

Near-hor. limit $r \ll L$:

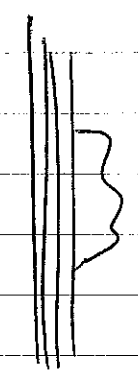
$$ds_{10}^2 = \frac{(\pi T L)^2}{u}\left(-f dt^2 + dx^2 + dy^2 + dz^2\right) +$$

$$+ \frac{L^2}{4u^2 f} du^2 + L^2 d\Omega_5^2$$

$$u \equiv r_0^2/r^2, \quad f = 1-u^2, \quad T = \frac{r_0}{\pi L^2}.$$

Gravitational dressing of D3 branes

Open string picture of N_c D3 branes



N_c

Recall: correct. to flat metric in $d=4$

$\sim GM/r$

relevant for $\frac{GM}{r} \sim 1$.

For $p+1$-dim object in d dim:

$$\sim \frac{GM}{r^{d-p-3}} \sim \frac{G_{10} N_c T_{D3}}{r^4}$$

But: $G_{10} \sim g_s^2 \, l_s^8$

$$T_{D3} \sim \frac{1}{g_s \, l_s^4}$$

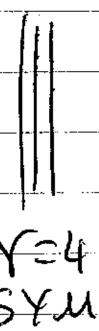
$$\Rightarrow \quad \text{corr} \sim \frac{g_s N_c \, l_s^4}{r^4}$$

- for $g_s N_c \ll 1$, corr ~ 1 for $r \ll l_s$
- for $g_s N_c \gg 1$, corr ~ 1 for $r \gg l_s$

$\Rightarrow$ cannot use open string picture

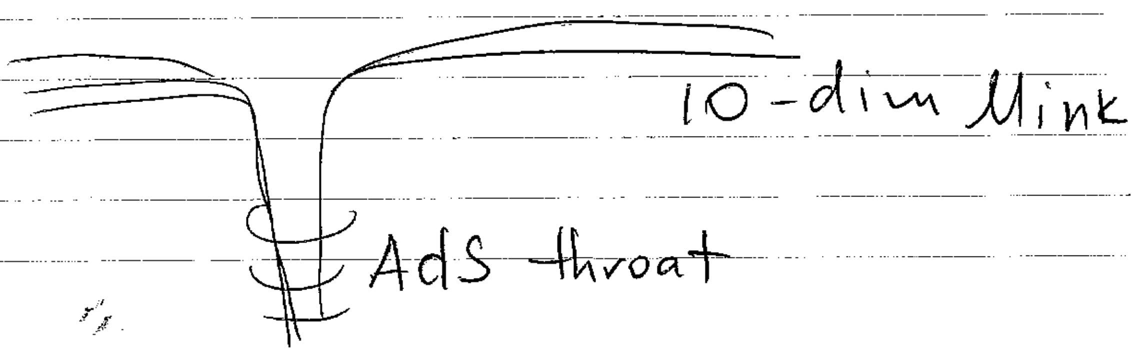
At low energy, $\omega \ll 1/l_s$!

$g_s N_c \ll 1$



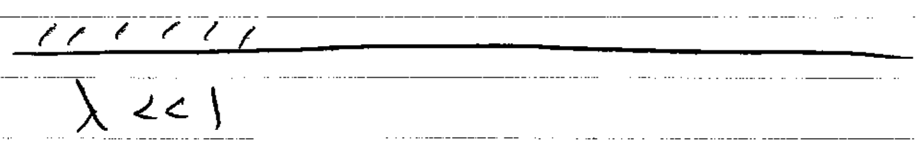
+ 10-dim Mink

$g_s N_c \gg 1$
$N_c \gg 1$

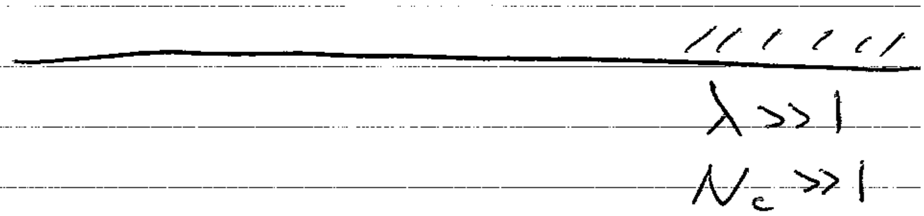


Remark : in the closed string picture, far away from the throat only the decoupled excitations of $d=10$ massless fields are relevant at low energy $(\omega \ll 1/l_s)$ but arbitrarily large excitations of type IIB strings are relevant deep down the throat since they are redshifted from the point of view of an asym. observer.

A : Open str. picture



B : closed str picture



$$\lambda = g_{YM}^2 N_c \,, \qquad g_s = 4\pi g_{YM}^2$$

Conjecture (Maldacena, 1997):

$$A = B$$

$\mathcal{N}=4$ $SO(N_c)$ SYM in $d=3+1$ = type IIB superstr theory on $AdS_5 \times S^5$.

$$\boxed{Z_{\substack{\mathcal{N}=4 \\ SYM}}[J] = Z_{\substack{IIB \\ AdS_5\times S^5}}[J]}$$

$$\downarrow$$

$$Z_{\substack{\mathcal{N}=4\ SYM \\ N_c \to \infty \\ \lambda \to \infty}}[J] = e^{-S_{\substack{grav \\ AdS_5 \times S^5}}[J]}$$

+ corr $O(1/N_c^2)$ $O(1/\lambda^{3/2})$

+ corr $O(1/N_c^2)$ $O(1/\lambda^{3/2})$

$$\boxed{\left\langle e^{\int J \mathcal{O}\, d^4x} \right\rangle_{SYM} = e^{-S_{grav}[J]}}$$

The role of J:

Recall that $g_s = 4\pi g_{YM}^2$ is the expect. value of dilaton at asympt. infinity.

$\Rightarrow$ changing coupling g_{YM}
(deforming $\mathcal{L}_{YM} \to \mathcal{L}_{YM} + J\mathcal{O}$,
where $\mathcal{O} \approx \mathcal{L}_{YM}$)

$\Rightarrow$ changing the boundary value of a bulk field (ϕ).

In general,

$$S \to S + \int d^4x\, \phi(x)\, \mathcal{O}(x),$$

$$J \equiv \phi(x) \approx \lim_{r\to\infty} \Phi_{bulk}(r, x)$$

(perhaps renormalized).

Minkowski correlation functions

Mink $(-+++)$ $\qquad k = (\omega, \vec{q})$

$$G^R(\omega, \vec{q}) = -i \int d^4x \, e^{-ikx} \, \theta(t) \langle [\hat{\mathcal{O}}(x), \hat{\mathcal{O}}(0)] \rangle$$

G^A, G^F, G (Wightman)

$$G(k) = \frac{1}{2} \int d^4x \, e^{-ikx} \langle \hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0) + \hat{\mathcal{O}}(0)\hat{\mathcal{O}}(x) \rangle$$

$$G^A = G^R(-k) = (G^R)^*$$

$$G^F = \mathrm{Re}\, G^R + i \coth\frac{\omega}{2T} \, \mathrm{Im}\, G^R$$

$$G(k) = -\coth\frac{\omega}{2T} \, \mathrm{Im}\, G^R(k)$$

$\Rightarrow$ enough to know $G^R(k)$.

„Normalizable vs non-normalizable modes"

example: $\qquad ds^2_{d+1} = \frac{L^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$

e.o.m. min coupled scalar:

$$z^{d+1}\partial_z\left(z^{1-d}\partial_z\phi\right)-k^2z^2\phi-m^2L^2\phi=0$$

$$k^2=-\omega^2+\vec{k}^2$$

~~OD~~E 2nd order:

$$\phi = A(k)\,\varphi_1 + B(k)\,\varphi_2$$

Near $z\to 0$ (boundary):

$$\phi = A(k)\,z^{d-\Delta}+\dots + B(k)\,z^{\Delta}+\dots$$

(Frobenius expansion)

$$\Delta=\frac{d}{2}+\nu \qquad \nu=\sqrt{\frac{d^2}{4}+m^2L^2}.$$

ν real for $m^2L^2\geq -d^2/4$ (BF bound)

AdS inner product:

$$(\phi_1,\phi_2)=-i\int_{\Sigma_t}dz\,d^dx\,\sqrt{-g}\,g^{tt}\left(\phi_1^*\partial_t\phi_2-\phi_2\partial_t\phi_1^*\right)$$

For $m^2 L^2 \geq -\frac{d^2}{4} + 1$:

$\varphi_1 \sim z^{d-\Delta}$ non-normalizable modes w.r.t. inner product

$\varphi_2 \sim z^{\Delta}$ normalizable ones

The non-normalizable term is a source for $\mathcal{O}(x)$ in the boundary action:

$$S_{\partial M} \to S_{\partial M} + \int d^d x \, J(x) \, \mathcal{O}(x)$$

with $J(x) \equiv A(x)$

$$J(x) = \phi \Big|_{\partial M} = \lim_{z \to 0} z^{\Delta - d} \phi(z, x)$$

Normalizable modes: quantization in curved space-time (see Birrell-Davies)

$\Rightarrow$ Hilbert space (bulk) $= \mathcal{H}$-space of the boundary theory

$\Rightarrow$ <u>normalizable modes</u> $\rightarrow$
<u>states in the boundary theory</u>

One can show: $\langle \mathcal{O}(x) \rangle_J = 2\nu B(x)$.