Elements of string theory (continued-2)

Perturbatively, string theory is a theory of open and closed strings. Non-perturbatively (i.e. for arbitrary g_s), a variety of higher-dimensional solitonic objects appear.

Dp-branes are (dynamical) hypersurfaces with p+1-dim worldvolume on which open strings can end

type IIA : stable Dp branes exist for

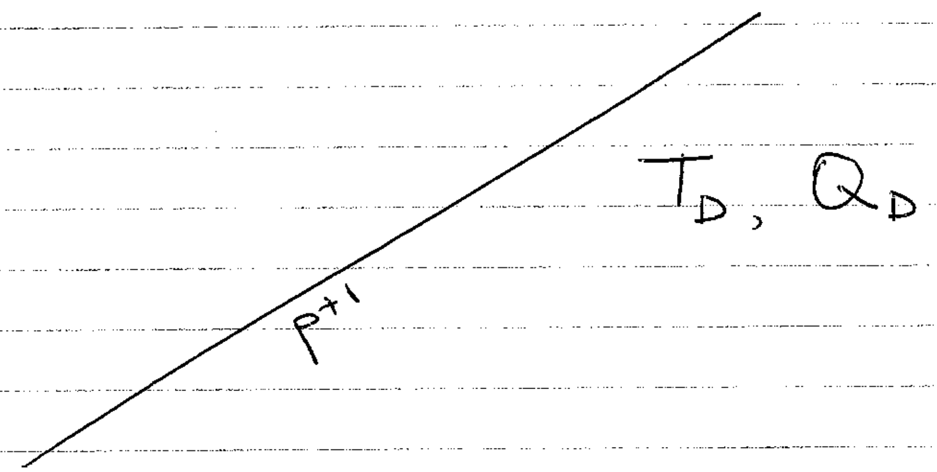
$$p = 0, 2, 4, 6, 8$$

type IIB : $p = 1, 3, 5, 7, 9$ *

* some conditions apply

Remark: Dp branes are charged objects (e.g. D3 is charged w.r.t. $A^+_{(4)}$) and have finite mass / unit volume $T_D \sim 1/g_s$.

<u>Remark</u>: not all branes are D-branes, there are also NS-branes and M-branes (in 11-dim SUGRA). There are also various „phenomenological branes" (as in braneworlds) not necessarily related to str. theory.



Fluctuations of a Dp brane are determined by the quantum spectrum of open strings attached to it.

<u>Massless modes</u>: $A_\mu(x)$ $\mu = 0, \dots p$
(Abelian field) x : $p+1$-dim. coordinates on world-volume

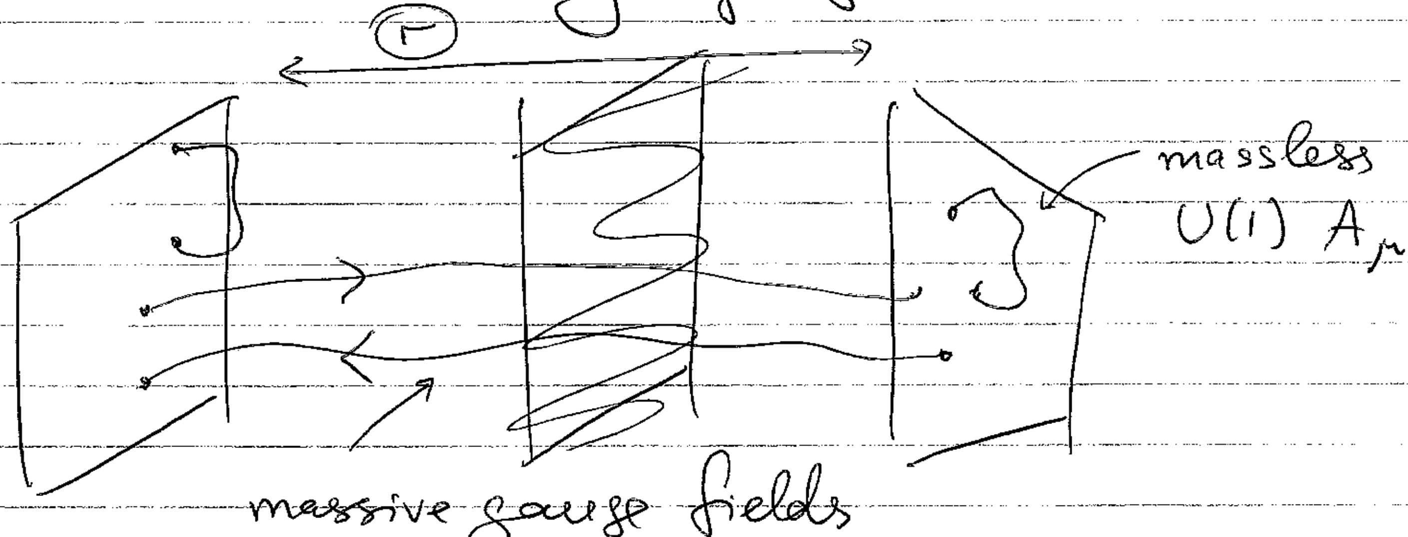
$\Phi^i(x)$: $i = 1, \dots 9-p$ scalars

+ superpartners

Massive modes: $m_s \sim 1/l_s$

Remark: massless modes are in fact familiar collective coordinates of solitons, as in QFT (see Harvey; Rubakov "Classical theory of gauge fields", Rajaraman). They are Goldstone bosons associated with the spontaneous breaking of symmetries by the brane.

Multiple branes: appearance of non-Abelian gauge fields



$m \sim r/2\pi\alpha'$

$r \to 0$: 4 massless fields

$(A_\mu)^a{}_b \in U(2)$

$a, b = 1, 2$

Scalars: $(\phi^i)^a{}_b$ in the adjoint of $U(2)$

Witten, hep-th/9510135

„Bound states of strings and p-branes"

Dynamics of massless modes = non-Abelian gauge theory

Consider N_c D3 branes in type IIB theory

Massless spectrum: A_μ, ϕ^i, $i = 1, \ldots 6$,

4 Weyl fermions in the adjoint of $U(N_c)$

Low energy effective action =

$\mathcal{N}=4$ $U(N_c)$ SYM in $d=4$

(theory well known before)

$$\mathcal{L} = -\frac{1}{g_{YM}^2} \operatorname{tr}\left(\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} D_\mu \phi^i D^\mu \phi^i + [\phi^i, \phi^j]^2 \right) + \text{fermions},$$

and $g_{YM}^2 = 4\pi g_s$.

This theory has interesting properties...

E.g. $\beta = 0$

<u>Remark</u>: three-loop result computed by Avdeev, Tarasov, Vladimirov in 1980 with the help of computer algebra:

Phys Lett B 96 (1980) p. 94-96

See References for papers showing $\beta = 0$ to all orders in pert. theory and non-perturbatively

<u>Remark</u>: the relationship $g_{YM}^2 = 4\pi g_s$ implies a particular choice of normaliza-tion of YM generators: in general,

for $\mathrm{tr}\, T^a T^b = c\, \delta^{ab}$ we have $g_{YM}^2 = 2\pi g_s / c$. The standard choice in gauge theory is $c = 1/2$.

The moral: with $\beta = 0$, g_{YM} is scale-independent and does not run with energy: the coupling constant is truly a constant in this theory.

Remark: $U(N_c) = SU(N_c) \times U(1)$, the $U(1)$ part can be decoupled and describes motion of the center of mass of the brane system.

Remark: parameters of $\mathcal{N}=4$ SYM:

N_c and g_{YM}

In the large N_c limit, the perturbative expansion of any amplitude can be written as

$$A(g_{YM}, N_c) = \sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda), \quad \lambda = g_{YM}^2 N_c$$

is the 't Hooft coupling.

(See S. Coleman and Y. Makeenko ~~books~~ for large N gauge theories discussion.)

Ingredients of $\mathcal{N}=4$ SYM

$A_\mu(x)$ 2 d.o.f.

$\phi^i(x)$ $i=1,\dots 6$ 6 d.o.f.

$\lambda^a_\alpha(x)$ $4 \times 2 = 8$ d.o.f.

All fields in the ad rep. of $SU(N_c)$.

The theory is superconformal:

supergroup $PSU(2,2|4)$

- conformal symmetry $SO(2,4)$

generators: P_μ, $L_{\mu\nu}$, D, K_μ

- R-symmetry $SO(6)_R \sim SU(4)_R$

T^A, $A=1,\dots 15$

- Poincare SUSY: Q^a_α, $\bar{Q}_{a\dot{\alpha}}$ $a=1,\dots 4$
- Conformal SUSY: $S_{\alpha a}$, $\bar{S}^a_{\dot{\alpha}}$, $a=1,\dots 4$

- the theory is UV finite, $\beta = 0$, $g_{YM} = \text{const}$
- the theory has exact Montonen-Olive duality (self-dual under $g_{YM} \to \frac{2\pi}{g_{YM}}$, $SL(2,\mathbb{Z})$). Evidence: spectra of elem. excit. = spectra of monopoles, see J. Harvey's lectures.

The action receives higher-derivative corrections suppressed by $\alpha' \omega^2$ (see below).

There are also closed strings in the bulk interacting with $G_{10} \sim g_s^2 l_s^8$, i.e. interaction terms are $\sim \omega^8 l_s^8$ (small for $\omega \ll 1/l_s$). Open-closed string interactions are controlled by the same parameter, $\omega^8 G_{10}$.

At low energies, closed strings do not interact and decouple from the open string sector which in this limit is described by $\mathcal{N}=4$ SYM in $d=4$.

Remark: for a single Dp-brane with constant (or slowly varying) world-volume fields, higher-order α' corrections can be resummed

$$S_{DBI} = -T_{D_p} \int d^{p+1}x \, e^{-\phi} \sqrt{-\det\left(g^{ind}_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)}$$

$$T_{D_p} = \frac{1}{(2\pi)^p g_s \, l_s^{p+1}}$$

induced metric on the brane

Remark: expanding this to quadratic order in F, one recovers (Abelian) Lagrangian and the relation $g^2_{YM} = 4\pi g_s$

Remark: non-abelian version of the DBI action is not known, see e.g.

A. Tseytlin, hep-th/9908105.

Remark: one can consider corrections to DBI beyond slowly-varying field approx.
(Tseytlin; Green; Bachas).